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ELECTRICAL ENGINEERING

AND

MANUAL FOR ELECTRICAL TESTING

FOR ENGINEERS AND FOR STUDENTS IN
ENGINEERING LABORATORIES

BY
V. KARAPETOFF

VOL. I.

THIRD EDITION — COMPLETELY REVISED AND RESET
TOTAL ISSUE TWENTY-SIX THOUSAND

NEW YORK
JOHN WILEY & SONS, INC.
LONDON: CHAPMAN & HALL, LIMITED

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BY
V. KARAPETOFF

First Edition — Entered at Stationers' Hall

Printed in U. S. A.

Stanhope Press
TECHNICAL COMPOSITION COMPANY
F. H. GILSON COMPANY
BOSTON, U. S. A.

12-29

Engineering
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PREFACE TO THE THIRD EDITION

In 1904 the author was called to Cornell University to equip a new laboratory in electrical engineering and to organize instruction in the testing of electrical machinery. As a result, he issued, in 1905, a preliminary set of blueprinted laboratory notes for his students. In 1906 these notes were enlarged and privately printed in Ithaca, N. Y., in two volumes. The First Edition of the present work, based on these notes, appeared in 1909, in one large volume; in 1910–11, the Second Edition was issued, in two volumes.

It was found more convenient for students to handle two smaller books than one large one; moreover, the non-electrical students did not usually need the second volume, because all the elementary experiments were put into Vol. I. The same general arrangement between the two volumes has been preserved in the present edition; this will explain why certain subjects, such as alternating-current machinery, are divided, the elementary portions being treated in Vol. I, and the advanced topics in Vol. II.

The present Third Edition has been thoroughly revised and completely reset. Before beginning the revision, the author sent the list of experiments, contained in the second edition, to over one hundred teachers of experimental electrical engineering, asking them (*a*) to indicate the experiments which they actually used, and the relative importance of the tests in each group; (*b*) to send their mimeographed notes and instructions; (*c*) to give their criticisms and suggestions for the next edition. The response was most generous and hearty, and the tabulated replies gave a clear picture of what was wanted. Therefore, the present edition is a composite picture of what is actually taught in the electrical laboratories in this country, with such additions as were deemed desirable in order to make each chapter complete for reference purposes.

After this, individual chapters of the Second Edition were sent to various specialists for corrections and suggestions. For example, the chapter on storage batteries went to some four or five prominent makers; the chapters on direct-current machinery were sent to large manufacturers who use approved methods of testing and issue their own instruction booklets. Here again, the response was most generous, some engineers having written voluminous reports on what they considered to be the best up-to-date practice.

While arranging the materials, thus gathered, into a coherent whole, the author had to adopt certain principles, and some of these are submitted below in the hope that their understanding may facilitate the use of the book.

1. Actual experience in many colleges has shown that, in addition to a textbook, mimeographed notes are desirable. These notes usually contain information on the local equipment, detailed instructions, questions for the reports, etc. The book does not aim to give such varying details, but emphasizes the general principles and relationships from the point of view of a practicing engineer who desires to build on a sound theoretical foundation.

2. The preceding editions of the book having been used to a considerable extent by consulting, operating, and test engineers, industrial physicists, etc., it has been deemed desirable to make the new edition still more serviceable as a reference work, by including certain topics which would hardly be covered in a regular undergraduate course.

3. With a few exceptions, the individual chapters of the book have been made as independent of each other as possible. It is not necessary to study the book consecutively, page by page, like a book on mathematical theory. The beginner can perform one or two elementary experiments in each of several chapters of Vol. I, and thus get a general idea of the most important measurements. Then he may go over some experiments of medium difficulty. More advanced students will find the same method applicable to Vol. II, while graduate students and those interested in research will find suitable subjects among the last experiments in several chapters.

4. After mature consideration and several trials, literature references have been practically omitted, except in a few special and advanced topics. With the tremendous increase in the number of books and magazine articles on electrical subjects, a change has become necessary in the method of obtaining a desired bit of information. Let us suppose that a user of this book, five years after it has been written, desires some additional information on a piece of apparatus or a method of testing. Whatever references to books and articles this work may contain, will at that time be neither complete nor up-to-date, and in some cases will be positively misleading. Fortunately, there are well-known methods of seeking information, methods with which every librarian is familiar, and whereby a thoughtful reader can orient himself without any difficulty as to the latest information on a technical subject. Library catalogs, *Science Abstracts,* indexes of the leading periodicals, special

published bibliographies, electrical handbooks, catalogs of book publishers, trade literature, etc., are sources at least some of which are readily available to most readers. Attention is called in particular to the *Standards* of the American Institute of Electrical Engineers, and to similar publications of other electrical societies in this country and abroad. These rules are frequently revised and should be consulted in important tests and specifications.

To numerous fellow-teachers and practicing engineers, who have helped with their material and suggestions, the author wishes to express his sincere appreciation. He is also glad to acknowledge his gratitude to Mr. O. K. Marti, who read the proofs in a painstaking manner and made many valuable suggestions.

THE AUTHOR.

Cornell University, Ithaca, N. Y.
October, 1922.

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EXPERIMENTAL ELECTRICAL ENGINEERING

CHAPTER I

MEASUREMENT OF RESISTANCES

1. EXPERIENCE shows that electrical conductors offer a certain opposition to the passage of an electric current. It is necessary, therefore, to apply a difference of potential, or an electromotive force, at the ends of the conductor to produce a flow of current in it. This is analogous to a difference of pressure necessary at the ends of a pipe in order that water may flow through it. It is also found that the electrical pressure E to be applied at the ends of a conductor is proportional to the current I desired to be produced in the conductor. In other words,

$$E = RI \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where R is a coefficient of proportionality, called the *resistance* of the conductor. This experimentally determined relation between electromotive force and current is called Ohm's law. The greater the resistance R, the higher must be the electromotive force E to produce a certain current I. In this respect the electrical resistance R is analogous to friction between the walls of a pipe and the water flowing through it; the greater the friction, the higher the pressure required to produce the same flow of water in the pipe.

The resistance of a conductor is its most important electrical feature, and various methods have been devised for accurately measuring and comparing resistances. The practical unit of resistance is called *the ohm.* According to the definition of this unit established by an international agreement, it is represented by the resistance of a column of mercury of a definite length and cross-section and at a definite temperature. The reasons for selecting this unit, and the relation of the ohm to the absolute or c.g.s. (centimeter-gram-second) system, possess little interest for practical engineers. To them it is an arbitrary unit of which the prototype is carefully preserved by the respective governments, and with which prototype secondary standards in practical use are compared.

Two methods for measuring resistances are most often used in practice:

(*a*) Drop-of-potential method;

(*b*) Wheatstone bridge.

The first method is based directly on Ohm's law given above; the second is a zero, or balance, method, the unknown resistance being compared to a standard. These, and some other methods, are described below and illustrated by applications to a few important practical problems.

DROP-OF-POTENTIAL METHOD

2. The connections for determining the resistance of a conductor by the drop-of-potential method are shown in Fig. 1. In this figure *Ba* is a storage battery or another steady source of direct current, *X* is the unknown resistance, and *A* is an ammeter for measuring the current. *V* is a voltmeter for measuring the drop of potential across the resistance *X*.

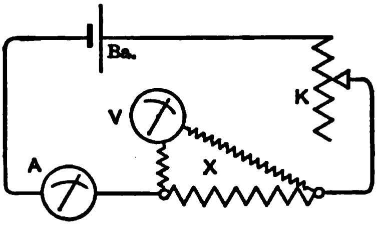


FIG. 1. Measurement of resistance by the drop-of-potential method.

If *E* is the voltmeter reading in volts, and a current of *I* amperes is read simultaneously on the ammeter, the resistance of the conductor $X = E \div I$ (in ohms). This is according to Ohm's law mentioned in § 1.

Usually several readings are taken with different values of the current, and an average calculated of the corresponding values of *X*. It is important not to use too large a current which might appreciably heat the conductor. Experience shows that the resistance of most conductors depends on their temperature; in making measurements it is therefore necessary to note to what temperature they refer.

As a further precaution, the current flowing through the voltmeter must be negligible as compared to that flowing through the resistance *X*; otherwise a correction may be necessary for the ammeter reading. Let, for example, the voltmeter read 100 volts, and the ammeter 0.5 ampere. Let the resistance of the voltmeter itself be 10,000 ohms. At a pressure of 100 volts, the current through the voltmeter is 100/10,000 = 0.01 ampere. Therefore, the true current through *X* is 0.50 − 0.01 = 0.49 ampere, and the unknown resistance $X = 100 \div 0.49 = 204$ ohms.

When the drop-of-potential method is employed for very accurate measurements, a potentiometer is used (§ 73), or an electrostatic voltmeter (§ 53), instead of an ordinary voltmeter; with either of these no current is shunted around X, and no correction is necessary.

A modification of the drop-of-potential method is shown in Fig. 2. Here a standard resistance R is used in place of the ammeter, and the voltage drop is taken in succession across R and across X. Let the readings be E_R and E_x. If the current I has not changed during the measurement, we have,

$$I = \frac{E_R}{R} = \frac{E_x}{X} \quad . \; . \; (2)$$

or

$$X = R\frac{E_x}{E_R} \quad . \; . \; (3)$$

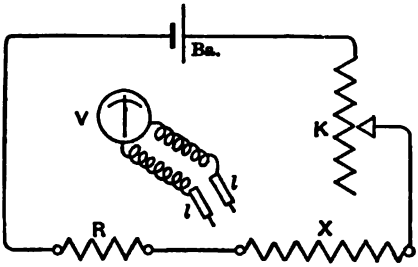


FIG. 2. Comparison of resistances, using a voltmeter.

The voltmeter is conveniently transferred from one resistance to the other by using flexible leads *ll*. The advantage of this method is that only one instrument, instead of two, needs to be calibrated, and even for this one instrument it is not necessary to know the actual values of divisions on its scale, since only the ratio of the readings enters into the result.

When the unknown resistance X is comparatively high, an appreciable fraction of the current may be shunted through the voltmeter. In using this method one should know at least the order of magnitude of the unknown resistance and the resistance of the voltmeter, so as to be able to estimate the error. An electrostatic voltmeter consumes no current and involves no error of this kind. Where extremely accurate results are required, R and X are compared by means of a potentiometer.

3. EXPERIMENT 1-A. — Resistance Measurement by the Drop-of-Potential Method. — The purpose of the experiment is to determine the value of an unknown resistance by the drop-of-potential method explained in the preceding section. (1) *Ammeter and voltmeter method.* Connect the apparatus as shown in Fig. 1. Close the circuit and adjust the regulating rheostat K to give the highest current which does not appreciably heat the resistance to be measured. Read volts and amperes. Reduce the current in steps and take similar readings at each step. In order to see the effect of the error due to the current through the voltmeter, perform the foregoing test with a high resistance, using a small current, and then with a low resistance, using a large current.

In each case measure the voltage across the resistance alone, and across the resistance including the ammeter. Read the current with the voltmeter on and off.

(2) *Voltmeter and standard resistance method.* Connect the apparatus as shown in Fig. 2 and read the voltage drops across the resistances X and R respectively. Repeat for several settings of the rheostat K. Repeat the preceding measurements for several values of the unknown resistance X, preferably the same as were used in part (1) so that a close comparison of the results of the two methods can be obtained. Select a comparatively high resistance X and connect an electrostatic voltmeter across it. Observe any changes in its indications when the electromagnetic voltmeter is connected across R, across X, and when removed. Do the same when the resistance X is quite low. Before leaving the laboratory find the resistances of the ammeter and the voltmeter and of the standard resistor R.

Report. For each unknown resistance in part 1, plot volts against amperes as abscissæ, and draw the straight line through the origin that comes the nearest to passing through all the points. Compute the unknown resistances from these straight lines. Show from the readings that the error due to the voltmeter current is greater for a higher unknown resistance, and explain this theoretically. Plot volts E_r against E_x as abscissæ, and draw the straight line through these points. Compute the unknown resistances from the ratios of these voltages. Describe the experiment with the electrostatic voltmeter, and estimate theoretically the percentage of error due to the current through the electromagnetic voltmeter with different values of the unknown resistance.

4. Influence of Length, Cross-Section, and Material of a Cylindrical or Prismatic Conductor upon its Resistance. — At a constant temperature the resistance of a cylindrical or prismatic conductor (a) increases in proportion to its length, and (b) varies in an inverse ratio to its cross-section. Hence, the resistance may be expressed by the formula

$$R = \rho \frac{l}{q} \quad \ldots\ldots\ldots\ldots \quad (4)$$

where l is the length of the conductor, q is its cross-section, and ρ a physical constant which characterizes the material of the conductor and is known as its *resistivity* or *specific resistance.* This is somewhat analogous to the flow of water through a pipe: the longer the pipe, the greater is its frictional resistance; the larger its cross-section, the easier it is to force through it a certain quantity of water per minute.

It will be seen from formula (4) that ρ is the resistance of a prismatic conductor of a given material, of unit length and of unit cross-section.

For the practical units in use, and for the numerical values of ρ for different materials, the reader is referred to almost any elementary textbook on electricity, or to one of the numerous tables of physical constants in engineering handbooks.

5. EXPERIMENT 1-B. — Determination of the Influence of Length, Cross-Section, and Material of a Conductor on its Resistance. — The purpose of the experiment is to verify the relation stated in the preceding section and to determine the specific resistance ρ for a few conducting materials used in practice, such as copper, aluminum, iron, German silver, and carbon. The connections shown either in Fig. 1 or in Fig. 2 may be used. Insert the wire to be tested, in place of X, and adjust the current by the rheostat K. Read the current on the ammeter, and the voltage drop across a certain length of the wire. Use knife-edge contacts at the end of the voltmeter leads, in order to insure a good contact and to have a definite length of wire. Reduce the current in steps, and read both the voltage and the current at each step.

Repeat the same experiment on wires (a) of different length, (b) of different cross-section, (c) of different materials. In each case vary but one factor entering into formula (4); keep the other factors constant. Take several readings of volts and amperes for each sample of wire, in order to eliminate possible errors and to obtain more accurate results. Do not use large currents which might appreciably heat the conductors under test and thus change their resistance. When accurate results are required, conductors are immersed in an oil-bath maintained at a definite temperature. Before leaving the laboratory, measure accurately the lengths and the cross-sections of the samples tested.

Report. (1) Show from the readings that the resistance of a conductor of a given cross-section is proportional to its length. To do this, plot the observed volts to amperes as abscissæ for each length used and draw the corresponding straight lines through the origin. The ratio of the ordinates for any chosen abscissa must then be equal to the ratio of the corresponding lengths of the samples. (2) Show from the readings that the resistance is inversely proportional to the cross-section. For this purpose plot again the observed volts to amperes as abscissæ for each cross-section used, and draw the corresponding straight lines through the origin. The ratio of the abscissæ for any chosen ordinate must be equal to the ratio of the corresponding cross-sections of the samples. (3) Calculate the resistivity ρ of each material tested, and compare with the values given in an available reference book.

6. Influence of Temperature upon the Resistance of a Conductor. — The electrical resistance of a metal increases with its temperature; within practical limits the increase in resistance is usually assumed to be proportional to the temperature rise. For commercial copper the resistance is taken to increase 0.42 per cent for each degree centigrade, *the resistance at* 0° C. *being assumed to be* 100 *per cent.* Thus, if the resistance of a copper conductor at 15° C. is R_{15} ohms, then the resistance R_0 at 0° C. is determined from the relationship

$$R_{15} = R_0 (1 + 0.0042 \times 15).$$

The resistance at 25° C. is

$$R_{25} = R_0 (1 + 0.0042 \times 25) = R_{15} \left(\frac{1 + 0.0042 \times 25}{1 + 0.0042 \times 15}\right) \qquad (5)$$

A constant, such as 0.0042, is called the *temperature coefficient* of the material; it has widely different values for different substances. For numerical data see an electrical handbook. Carbon and electrolytes have a negative temperature coefficient; their resistance decreases with increase in temperature.

In solving various practical problems it is of importance to know the temperature coefficient, or per cent increase of resistance with temperature, for various substances. Thus, for instance:

(1) A resistance used as a precision standard for measurements, as well as one used for purposes of accurate regulation, must be made of a material having a negligible temperature coefficient.

(2) Some protective resistors are made of a material whose resistivity increases rapidly with the temperature. This protects the apparatus in series with the resistor against an excessive current. An iron wire at the temperature of the dark red glow possesses a markedly large temperature coefficient. It is ordinarily used in an atmosphere of hydrogen to prevent oxidation.

(3) The change in the resistance of a conductor with the temperature is utilized for temperature measurements in the so-called resistance thermometers. These a e especially convenient for inaccessible places, such as the interior of an electrical winding, or where a continuous graphical record is desired.

(4) The original *carbon*-filament incandescent lamp had a negative temperature coefficient. The present-day *tungsten* lamp has a positive coefficient. For this reason the tungsten lamp is less sensitive to variations in voltage.

The temperature coefficient of a solid conductor is determined in a manner similar to that in which its resistance is measured (§ 2). The material tested is placed in an oil-bath maintained at a definite temperature. The bath is heated gradually, and readings of volts and amperes are taken as in Fig. 1, every few minutes. The corresponding temperatures are read on a thermometer placed in the oil. For more accurate measurements a Wheatstone bridge (§ 16) or a Kelvin double bridge (§ 32) may be used. For the conductivity measurement of an electrolyte, see §§ 14 and 22.

7. **EXPERIMENT 1-C. — Determination of Temperature Coefficient of Solid Conductors.** — The experiment is performed in a manner similar to Experiment 1-A; the theory is given in the preceding section. Perform this experiment with materials such as iron or copper which have an appreciable temperature coefficient, and with manganin and German silver which have a very small temperature coefficient.

Observe the rapid increase in the resistance of an iron wire as it becomes dull red. The wire may be heated by the same current with which the resistance is measured. No oil-bath need be used with this particular test as only qualitative results are expected.

Demonstrate that the resistance of carbon decreases with an increase in temperature. This can be easily shown on the filament of a carbon incandescent lamp. Gradually raise the voltage at the terminals of the lamp and read the values of the voltage and current. Refer the readings to the states of incandescence, thus: the lamp just begins to glow, dull red, red, bright red, yellow, white, brilliant white. Perform a similar experiment on a tungsten-filament lamp.

Report. (1) Calculate the values of the temperature coefficient for the materials tested, referring to 0° C. Compare the values obtained with those given in a table of physical constants. (2) Give the results of the test on the iron wire at dull incandescence. (3) Plot the currents through the carbon lamp to terminal volts as abscissæ. Compute and plot the resistance curves to the same abscissæ. On these curves mark the stages of incandescence. Plot similar curves for the tungsten-filament lamp.

8. **Determination of Temperature Rise in a Winding by the Increase in Resistance.** — This problem is the inverse of that in Exp. 1-C. The temperature coefficient is now assumed to be known, and the temperature rise of a coil of wire is calculated from the observed increase in its resistance.

Let, for example, the resistance of a coil of copper wire be 4.573 ohms at 15° C., while at some unknown temperature it was found to be 5.468 ohms. For commercial copper the resistivity increases

0.42 per cent for each degree Centigrade; hence the resistance of the coil at 0° C. is

$$\frac{4.573}{1 + 0.0042 \times 15} = 4.302 \text{ ohms.}$$

The percentage of increase in the resistance at the unknown temperature is

$$\frac{5.468 - 4.302}{4.302} = 27.1 \text{ per cent}$$

Therefore, the unknown temperature is

$$\frac{27.1}{0.42} = 64.5° \text{ C.}$$

This is one of the methods used for the determination of temperature rise in the windings of electrical machinery. A measurement by thermometers is often insufficient, because the temperature of the outside layers of the coil may be considerably below that in the center (Fig. 3). An excessive temperature rise inside the coil carbonizes the cotton insulation and may finally cause a short-circuit.

The temperature rise in a coil depends essentially upon the conditions of cooling. In some cases, as for instance in transformers, coils are cooled artificially by immersing them in oil or by subjecting them to a draft of air (oil-cooled and air-cooled transformers). The distribution of temperature within a coil (Fig. 3) may be investigated by placing exploring wires in various places, as shown by small circles. The temperature rise is determined by the increase in the resistance of the exploring wires. Iron would be the most suitable material because of its considerable temperature coefficient, if it were not for the fact that it oxidizes readily. Because of this tendency of iron to oxidize, fine copper wires are used in preference. The terminals of each exploring coil are brought out separately; the resistance is meas-

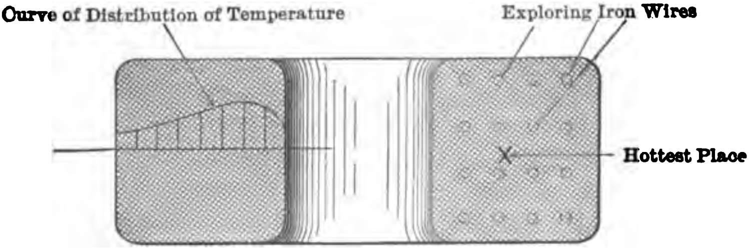


FIG. 3. Cross-section of a coil, showing exploring wires and distribution of temperature.

ured from time to time while the coil under test is being heated by a current. For very accurate measurements the temperature coefficient should be determined for a sample of the wire used.

Instead of measuring temperature rise by the increase in resistance, calibrated thermo-couples are sometimes used. For accurate temperature measurements by the resistance method the Wheatstone bridge (§ 16) or one of its modifications should be used, since a small error in the resistance determination will cause a relatively large error in the calculated value of the temperature.

9. EXPERIMENT 1-D. — Temperature Rise in a Winding by the Increase in Resistance. — The purpose of the experiment is to determine the temperature rise in a coil by means of the increase in resistance of one or more exploring coils. The theory of the method is explained in the preceding section. Provide a suitable coil, such as that shown in Fig. 3, with exploring coils placed in known parts of its cross-section. Before heating the main coil, and while it is at the room temperature, connect all the exploring coils in succession as the " unknown " arm X of a Wheatstone bridge (Fig. 10) and determine accurately their resistances. Then, after heating the main coil by means of an electric current again, measure accurately the same resistances. This test should be performed with the coil under different conditions, such as (a) in the quiet open air, (b) enclosed in a box, (c) with an electric fan blowing on it, (d) cooled in oil, etc.

Report. Calculate the rise in temperature of the various parts of the main coil by means of the increase in resistance of the exploring coils, and plot the results as a curve (Fig. 3) or mark the temperatures on a sketch of a cross-section of the coil. Indicate the effect of the different methods of cooling upon the temperature rise, and state advantages and disadvantages of air blast as compared to oil cooling.

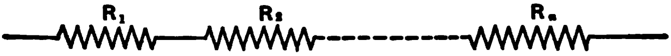


FIG. 4. Resistances in series.

10. Resistances in Series and in Parallel. — Resistances are connected in electrical circuits in various combinations; it is sometimes required to determine the value of a resistance which would produce the same electrical effect as two or more given resistances. Such a resistance is called the *equivalent* or *resultant* resistance. A few simple cases are here considered.

(1) *Resistances in series.* From the very concept of electrical resistance, it follows that two resistances, R_1 and R_2 (Fig. 4), connected in

series, are equivalent to a resistance $(R_1 + R_2)$. The same is true for any number of resistances in series, so that

$$R_{\text{equiv.}} = \sum_1^n R \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

(2) *Resistances in parallel* (Fig. 5). The equivalent resistance in this case is less than either of the component resistances, because the addition of each resistance means a new path for the current. Let the common voltage across the resistances be e, and the currents i_1, i_2, etc. We then have

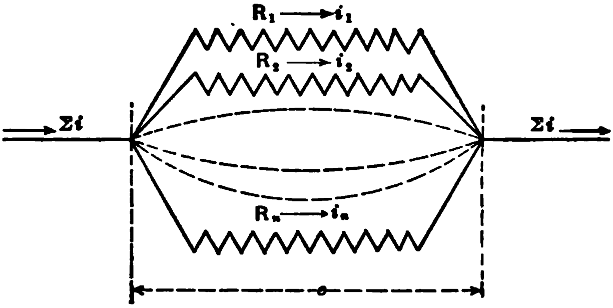


FIG. 5. Resistances in parallel.

$$i_1 = \frac{e}{R_1}; \qquad i_2 = \frac{e}{R_2}; \text{ etc.} \quad . \; . \; . \; . \; . \; . \; . \quad (7)$$

The equivalent resistance must, by definition, have such a value that the same total current Σi would flow through it with the same voltage e; hence

$$\sum_1^n i = \frac{e}{R_{\text{equiv.}}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (8)$$

Substituting the values of i_1, i_2, etc. from eq. (7) we have

$$\sum_1^n \frac{e}{R} = \frac{e}{R_{\text{equiv.}}};$$

or, dividing both sides of the equation by e:

$$\frac{1}{R_{\text{equiv.}}} = \sum_1^n \frac{1}{R} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

This formula gives the value of the equivalent resistance in terms of the component resistances. The reciprocal value of a resistance is sometimes called the *conductance*; the result (9) indicates that the equivalent conductance is equal to the sum of the conductances of the branches connected in parallel. Denoting the conductances by g, we have

$$g_{\text{equiv.}} = \sum_{1}^{n} g \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (10)$$

which is analogous to the expression (6).

(3) *Series-parallel combination of resistances.* Let it be required to find the resistance equivalent to the combination shown in Fig. 6.

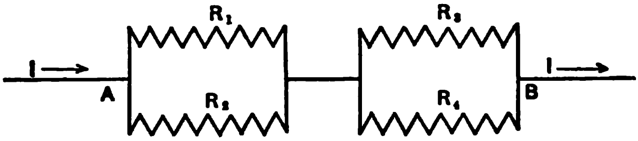


FIG. 6. Combination of resistances in series and in parallel.

First find a resistance r' equivalent to the resistance offered by R_1 and R_2, by using formula (9); in the same way find the resistance r'' equivalent to R_3 and R_4. The equivalent resistance of the whole combination is $(r' + r'')$.

A more complicated combination of resistances in series and in parallel is shown in Fig. 7. It corresponds to the practical case of a transmission line $OeO'e'$ with current-consuming devices R_1, R_2, . . . R_5 connected at different places. The problem is to find one single resistance which would allow the same total current to flow, with the same supply voltage between O and O'.

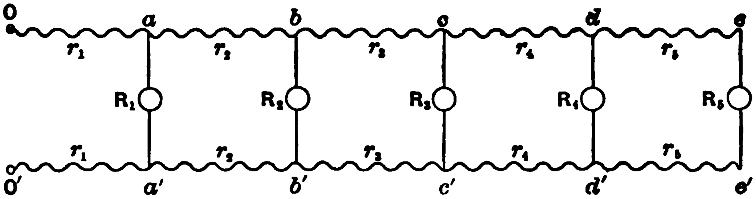


FIG. 7. Current-consuming devices R_1, R_2, etc., connected across a transmission line of appreciable resistance.

The problem is solved in steps: The resistances R_4 and $(2r_5 + R_5)$ are connected in parallel between the points d and d'. Their equivalent resistance R_4', according to the formula (9), may be calculated from the expression

$$\frac{1}{R_4'} = \frac{1}{R_4} + \frac{1}{R_5 + 2r_5};$$

In a similar manner we find

$$\frac{1}{R_3'} = \frac{1}{R_3} + \frac{1}{R_4' + 2r_4};$$

$$\frac{1}{R_2'} = \frac{1}{R_2} + \frac{1}{R_3' + 2r_3};$$

$$\frac{1}{R_1'} = \frac{1}{R_1} + \frac{1}{R_2' + 2r_2};$$

and finally

$$R_{\text{equiv.}} = R_1' + 2r_1.$$

The problem is solved by calculating R_4' from the first equation and substituting its value in the second equation; then R_3' is calculated from this equation and its value substituted in the next equation, etc.

In more complicated networks, such, for example, as an unbalanced Wheatstone bridge, the component resistances cannot be reduced to series and parallel combinations, and the unknown currents are found by applying Kirchhoff's two laws.

11. EXPERIMENT 1-E. — Exercises with Resistances in Series and in Parallel. — Take several resistances of suitable value and measure them separately by the drop-of-potential method or with a Wheatstone bridge. Connect the resistances as shown in Figs. 4 to 7, or in any other desired series-parallel combination, and measure the resultant (equivalent) resistance. Compare the results with the formulæ given in the preceding article.

7 **12. EXPERIMENT 1-F. — Measurement of Resistance of a D. C. Armature.** — A direct current is passed through the armature from an external source, and the voltage drop at the terminals is measured with a low-reading voltmeter. The ratio of this voltage to the current gives the resistance of the armature. In performing the measurement it is advisable to hold the voltmeter leads once on the terminals of the machine, and again on the commutator bars under the brushes. This will make it possible to separate the voltage drop due to the armature resistance from the contact e.m.f. of the brushes, and from that due to the resistance of the brushes themselves (§ 375).

This contact drop is not quite the same when the machine is at rest as it is when the machine is running. To determine the difference, the machine may be run at a low speed with the field circuit open, and the same measurement repeated. It must be remembered, however, that even when the field circuit is open, there is some residual magnetism in the field. This magnetism induces a voltage in the armature and may considerably affect the results. In order to eliminate its influence, the measurements must be repeated with the machine running in the opposite direction at exactly the same speed. The average of the two resistances will give the true resistance of the armature.

In stating the results it is necessary to note to which temperature they refer. The resistance of the armature, and therefore the voltage drop, will be somewhat higher when the machine is hot. In contracts the efficiency is usually guaranteed at a certain temperature of the machine, say at 90 degrees C. If the resistance was measured when the machine was cold, for example at 25 degrees C., its resistance at 90 degrees C. can be computed from the formula given in § 6. Or else, if the resistance of the armature was measured when cold and again after a continued run at some load (temperature test), the temperature rise in the armature winding may be calculated from the increase in resistance, as explained in § 8.

Report. Plot volts to amperes as abscissæ and draw the straight line that passes through these points. When the contact drop at the brushes is not included, the straight line must pass through the origin. With the contact drop it may lie above the origin, because the contact counter-e.m.f. has an appreciable value even with the smallest current. Give separately the values of the resistance of the armature proper and the voltage drop due to brush contact. Similarly, determine the contact e.m.f. at the brushes with the armature revolving. Draw a curve showing the voltage drop in the armature at higher temperatures; indicate the voltage drop due to the brush contact.

SUBSTITUTION METHOD

13. Theory of the Substitution Method. — With this method (Fig. 8) a certain current is made to flow in the circuit containing the unknown resistance *X*, and then an adjustable calibrated resistance *R* is substituted in its place and set at such a value as to give the same current as before. If the battery voltage remains the same, then the known resistance is equal to the unknown resistance. In Fig. 8, *Ba* is the battery, *Ga* is an ammeter or a galvanometer, which need not be calibrated, and *K* is a regulating resistor. The double-throw switch *S* is first thrown down and the current adjusted so as to obtain a desired deflection of the galvanometer. After this, the switch is thrown up, and *R* adjusted until the galvanometer gives the same deflection; then the resistance *R* is equal to *X*.

When using this method care should be taken that the battery e.m.f. does not change (on account of polarization) between the two readings, and also that the resistance *K* remains constant. Instead of being connected in parallel, R and X may be connected in series, as shown in Fig. 9, and short-circuited in succession.

The substitution method is sometimes used for measuring high resistances, such as insulation resistances; also for measuring the resistance of liquid conductors, as explained in § 14.

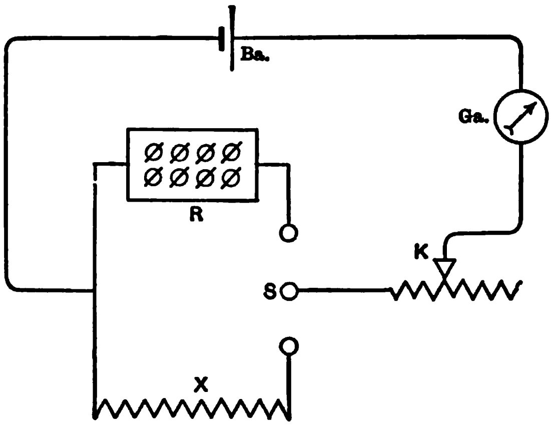


FIG. 8. Substitution method of measuring resistances.

14. Measurement of the Resistance of Electrolytes by the Substitution Method. — The difficulty in measuring the electrical resistance of most liquids is that they are decomposed by electric current. Gases are deposited on the electrodes, the resistance being thereby increased, and a counter-e.m.f. of polarization set up. One way out of this difficulty is to measure the resistance of liquids with an alternating current (see § 22). Another way is to use the substitution method shown in Fig. 9

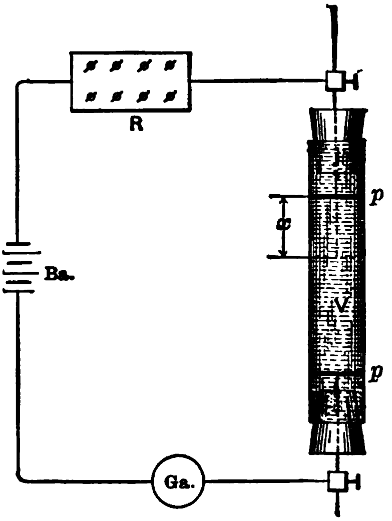


FIG. 9. Measuring the resistance of a liquid by the substitution method.

The liquid under test is placed in a glass tube V, provided with plugs at both ends, and with the electrodes pp, the upper electrode being adjustable. R is a calibrated resistance box. A certain current is sent through the circuit, until the conditions in the liquid become constant; then the deflection of the galvanometer is noted. After this, the upper electrode p is moved slowly by an amount x, as shown by the dotted lines. The current is brought back to its former value by increasing the resistance R by an amount r which is evidently equal to the resistance of the column x of the liquid. The effect of the polarization is thus eliminated, as it is present to the same extent with both positions of p.

The resistance of a liquid is usually expressed as that of a column 1 cm. high and having a cross-section of 1 sq. cm. This is called the resistivity, or the specific resistance, of the liquid. If the cross-section inside the glass tube is A sq. cm., the specific resistance of the liquid under test is rA/x. This is because the resistance is proportional to the length and inversely proportional to the cross-section of a conductor.

15. EXPERIMENT 1-G. — Measurement of Resistance by the Substitution Method. — The theory of the method and its application to the measurement of resistances of solid conductors is given in § 13. The application to liquid conductors is explained in § 14. The experiment should be so planned as to give the student some practice with the arrangements illustrated in Figs. 8 and 9.

WHEATSTONE BRIDGE

16. The combination of conductors known as the Wheatstone bridge (Fig. 10) is used for measuring resistances, and has the following features:

(1) The unknown resistance X is compared directly to a standard resistance R.

(2) No calibrated measuring instrument is required, the adjustment being made by bringing the galvanometer Ga to zero (null method).

The unknown resistance X and a standard resistance R are connected in series with each other and with the battery Ba. The combination is shunted by two other resistances M and N, the ratio of which is known. The galvanometer is bridged between the two sets of resistances, hence the name "bridge." Contact keys 1 and 2 are provided in the battery and galvanometer circuits.

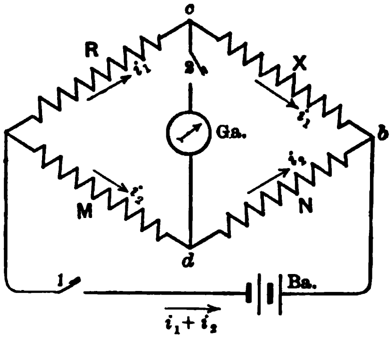


FIG. 10. The diagram of connections known as the Wheatstone Bridge.

The battery key 1 is closed first, and then the galvanometer key 2 is closed. If the galvanometer gives a deflection, the "balancing" resistance R and the "ratio" resistances M and N are varied until the galvanometer comes back to zero. Then

$$X : R = N : M. \qquad (11)$$

Proof: Suppose that the four resistances satisfy the condition that no current flows through the galvanometer; in other words, the difference of potential between the points *c* and *d* is zero. From this condition it follows that the voltage drop in the branch *ac* is equal to that in the branch *ad*; or, with the notation in the sketch,

$$Ri_1 = Mi_2.$$

Similarly

$$Xi_1 = Ni_2.$$

Dividing the second equation by the first, we obtain the expression (11).

Instead of using R for the standard resistance, we can use N for a comparison with X, while M and R may serve as ratio coils. From equation (11) we have identically

$$X : N = R : M \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (12)$$

The greatest accuracy of measurement is obtained when X is nearly equal to R, and consequently the ratio of N to M is nearly equal to unity.

17. Slide-Wire Wheatstone Bridge. — A simple Wheatstone bridge is shown in Fig. 11: the ratio resistances M and N are formed by two parts of a straight wire; the ratio is varied by moving the sliding contact *d*. The slide-wire is made of a non-corrodible metal of high specific resistance and of a small temperature coefficient. The brass blocks *a, b, c* to which R and X are connected have a negligible resistance.

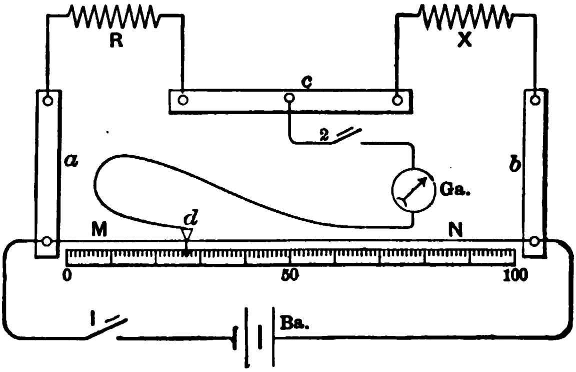


FIG. 11. Diagram of slide-wire Wheatstone bridge.

To measure the resistance X, the key 1 is closed first, then the key 2, and the contact *d* is moved along the wire until the galvanometer remains on zero when the key 2 is closed or opened. Let the standard resistance R equal 10 ohms; with the position of the contact shown in the sketch, M = 27 div., N = 73 div. According to formula (11),

$$X = 10\frac{73}{27} = 27.04 \text{ ohms.}$$

The details of construction of the bridge are shown in Fig. 12; the rider is provided with a vernier so as to read accurately the tenth parts of a division. Extra binding posts and the reversing switch in the center make it possible to use the bridge for the Carey-Foster method, described in § 24.

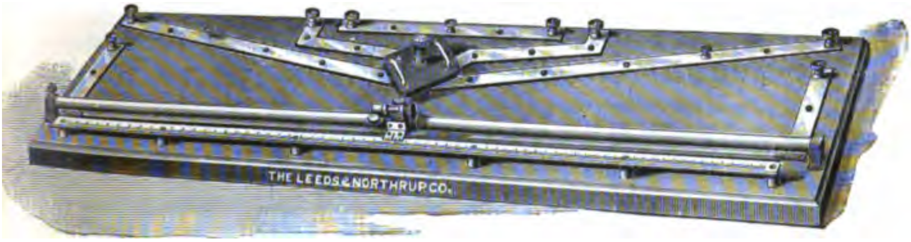


FIG. 12. Slide-wire Wheatstone bridge; also suitable to be used as a Carey-Foster bridge.

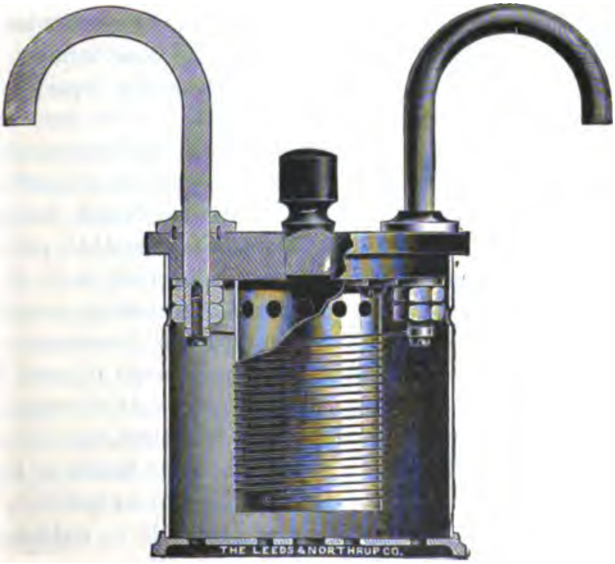


FIG. 13. A standard resistance.

The balancing resistance R may have various forms; usually it consists of a coil of wire (Fig. 13) wound on a metal spool and protected by a metal case. The heavy horn terminals are used for suspending the device from mercury cups, in order to minimize the error due to contact resistances.

18. EXPERIMENT 1-H. — Resistance Measurements with a Slide-Wire Bridge. — The theory and the use of the bridge are explained in the two preceding sections. Three kinds of problems may be solved with the bridge:

(a) Checking a standard resistance against a primary standard.
(b) Determining the value of an unknown resistance
(c) Adjusting a resistance to a desired value.

The student is expected to make himself familiar with the use of the bridge, and to practice on these three problems. No particular accuracy is expected in this exercise, it being only a preparatory one for more advanced work with the Wheatstone bridge.

19. Bridge Measurements with Alternating Currents. — A chemical decomposition takes place in some conductors when a direct current is passing through them. This results in the so-called polarization or formation of gases and a counter-e.m.f. at the electrodes; the apparent resistance of the conductor may be considerably increased thereby. Such an action is observed with many liquids known as electrolytes and in ground connections; in the latter case an electrolytic action takes place between the conductor and the earth. Where polarization is known to be present, or is suspected, it is not advisable to determine the resistance with a Wheatstone bridge using a direct current. Alternating current is then used, and the D.C. galvanometer is replaced either by an ordinary telephone receiver or by a vibration galvanometer (§ 43) responsive to small alternating currents. An alternating-current galvanometer similar to the direct-current instrument is also available. The field magnet is made of soft laminated steel and is excited with the same current as the coil; this gives a steady deflection with a sensitiveness somewhat less than the direct-current galvanometer. If a telephone is used, a noise is heard as long as the bridge is out of balance. This noise is reduced to a minimum when the bridge is exactly balanced. It is hardly possible to reduce the noise completely to zero if there are any harmonics in the supply current.

A telephone receiver may also be used with a direct-current source in place of the galvanometer. In this case a click is heard when the circuit is closed or opened. The receiver is not nearly as accurate for direct-current measurements as a galvanometer, but is easier to handle in some practical measurements not requiring great accuracy. Where the resistance to be measured requires an alternating current, the telephone receiver, properly tuned to the frequency of the supply, is one of the best devices available. The frequency of the supply has to be kept as nearly constant as possible to be within the most effective range of the receiver, and the applied voltage should be as free from harmonics as possible, in order to reduce the sound at the balance position to a minimum.

The necessary alternating current may be obtained from an ordinary lighting circuit with a high resistance in series, or through a step-down transformer. The measurement of the resistance of electrolytes at low frequency is subject to error due to polarization, and only in special cases can 60-cycle current be used for accurate measurements. A frequency of 1000 cycles is very satisfactory, as with it there is generally little or no trouble from polarization. An induction coil with a vibrating interrupter may be used and can be supplied with a direct current from a small battery. Special forms of 1000-cycle generators and oscillators are also on the market for use as a power supply where very accurate measurements are desired. Where iron is present in the generator it is almost impossible to eliminate the third harmonic, but special oscillators are now available that produce a current of the desired frequency, practically free of all harmonics, and the sound in the telephone receiver is reduced practically to zero at the position of balance.

20. Portable Direct-Reading Slide-Wire Bridge. — A convenient portable Wheatstone bridge of the slide-wire type is shown in Fig. 14. It is intended for all-round practical work, particularly for locating faults in the insulation of wiring installations. The slide-wire *a-a* is divided into two parts electrically connected at *b*. This reduces the length of the case, and at the same time the wire is long enough to give sufficient accuracy. The contact *d*, which corresponds to point *d* in Fig. 11, is formed by a " toucher " consisting of a metal blade with a rubber handle. The instrument is entirely self-contained, the galvanometer *g* and a few dry cells *Ba* being mounted in the case. Four standard resistances *R*, usually 1, 10, 100, 1000 ohm, are supplied with the bridge. They are mounted inside the box and the leads are taken to the four sockets *s*. Any one of these may be used, according to the value of the unknown resistance; the desired value is selected by inserting the plug *p* into the corresponding socket.

The scale is calibrated directly in ohms, so that no calculations are necessary. Four sets of figures in different colors are printed on the scale, and the sockets are labeled accordingly. When the plug is in the " blue " socket, blue figures are read on the scale, etc. In this way even an unskilled person can use the bridge with little probability of making a mistake. Uniformly divided scales are also provided with the instrument, and may be attached in place of the direct-reading scales, if so desired.

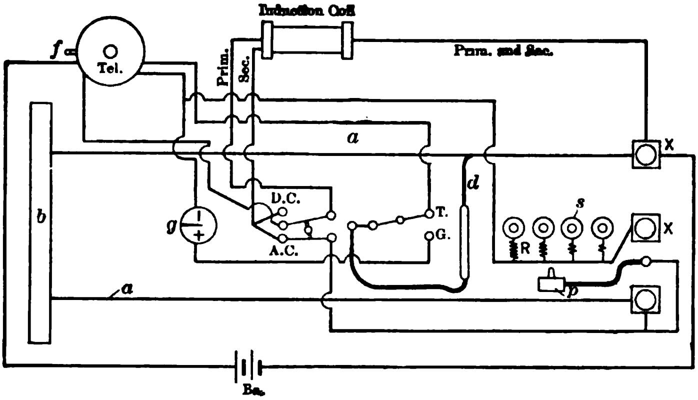


FIG. 14. A portable D. C.-A. C. slide-wire bridge.

An induction coil and a telephone receiver are provided, by means of which the bridge may be used with alternating currents (§ 19). The coil is supplied with current from the same battery as is used in the D.C. measurements. Switches are provided in the instrument for changing to direct or to alternating current at will; also for connecting either the galvanometer or the telephone receiver in the circuit. The battery switch *f* is built in the telephone receiver, so that the operator may close the switch with the same hand with which he is holding the receiver to his ear. With the other hand he touches the slide-wire with the movable contact and thus locates the point where the click in the receiver disappears, or is reduced to a minimum.

Similar portable direct-reading bridges are also built with the slide-wire bent into a circle and mounted on a spool. A contact pointer is fastened to a knob at the center of the circle and the galvanometer balance is obtained by turning the knob.

21. EXPERIMENT 1-I. — Practice with a Portable Slide-Wire Bridge. — For the theory of operation and the description of these instruments see §§ 19 and 20.

(a) Inspect the instrument, read the accompanying directions, and obtain a clear understanding of its connections and operation.

(b) Use the device for measuring a set of calibrated resistances, within as wide a range as possible, paying particular attention to the sensitiveness of the instrument at different values of the balancing resistances. (1) Perform the measurements first with direct current, using in succession the galvanometer and the telephone receiver; then (2) repeat some of the measurements with the induction coil and the telephone receiver.

(c) Determine the resistance of an electrolyte with direct and with alternating current, and note the difference.

(d) Measure the resistance between a line and the ground (resistance of a fault).

Report. Give the numerical results. State your opinion about the instrument, its advantages and shortcomings, the limits of accuracy, the best range, the relative advantages of the use of the telephone receiver over that of the galvanometer, and of direct current over alternating current and vice versa.

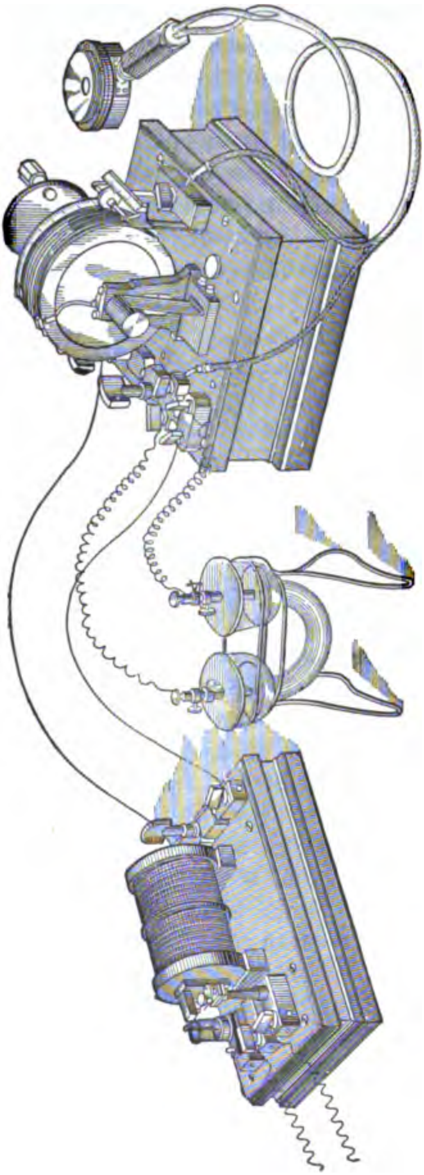


Fig. 15. Resistance measurement of an electrolyte with the Kohlrausch bridge, using alternating current.

22. Kohlrausch Bridge. — A disadvantage of a stretched slide-wire (Fig. 12) is that it cannot be made long enough for accurate measurements without making the instrument itself too large. Kohlrausch solved this difficulty by winding the slide-wire on an insulating cylinder, as shown in Fig. 15, to the right. The cylinder may be rotated by a handle, contact being made by a roller. The balancing resistances are placed in the base of the instrument, and may be connected in the circuit by means of the plugs shown on top of the base.

The Kohlrausch bridge is particularly well adapted for measuring resistances of liquid conductors with alternating currents (see § 19). An induction coil for producing alternating currents is shown to the left; a telephone receiver is substituted for the galvanometer. A vibration galvanometer (§ 43) may also be used.

The liquid under test is put into the U-shaped vessel shown between the bridge and the induction coil, this vessel being provided with platinum electrodes on both ends. The form of the vessel is too complicated to allow the specific resistance of the liquid to be deduced from the dimensions. This, however, is no serious objection; the constant of the vessel may be determined once for all by measuring in it the resistance of a standard liquid, whose specific resistance is known. For example, it is known that the specific resistance of a 25 per cent solution of zinc sulphate is 21.4 ohms per cu. cm. at 18 degrees C. Let the resistance of such a solution, as measured in a Kohlrausch vessel, be 152 ohms. Then the constant of the vessel is $21.4 \div 152 = 0.141$. The resistance of any other liquid measured in this vessel must be multiplied by this coefficient to get its specific resistance.

Different types and sizes of cells are suited to different electrolytes and various concentrations. For example, polarization may occur in some cases unless the electrodes are platinized, and in other cases platinized electrodes appear to act catalytically and assist chemical action. Then again, platinized electrodes may, because of their spongy nature, absorb so much of the electrolyte as to cause errors in measurement when later used on solutions of different nature or concentration. Care must be taken to secure a cell giving a correct current density for the electrolyte employed, and the cell design should take into account other electrical characteristics, such as plate capacity and stem inductance and resistance. The rate of heating and cooling of the cell is of importance, and cells of the pipette type are to be recommended since they not only permit filling with minimum exposure of the electrolyte to the action of air, but also allow the cell to be completely immersed in a bath of constant temperature. An accurate determination of the temperature is necessary, since electrolytes have comparatively high temperature coefficients of conductivity. For more accurate work with high resistances, an oil-bath is therefore employed. The Kohlrausch bridge is better adapted for an accurate determination of the resistance of an electrolyte than the substitution method described in §14.

23. EXPERIMENT 1-J. — Measurement of Resistances with the Kohlrausch Bridge. — For the description of the bridge see the preceding section. The measurements to be performed are similar to those specified in Experiments 1-G, 1-H, and 1-I.

24. Carey-Foster Method of Comparing Resistances. — This is a modification of the ordinary Wheatstone bridge, and is used for an accurate comparison of two nearly equal resistances, as in the determination of the percentage of error of secondary standards. The connections are shown in Fig. 16; the bridge illustrated in Fig. 12 may be used with this method. *P* and *Q* are two resistances to be compared to each other; *R* and *X* are two other resistances, the numerical values of which it is not necessary to know. It is advisable, however, to have *R* and *X* not very different from each other, in order to obtain a balance near the center of the slide-wire.

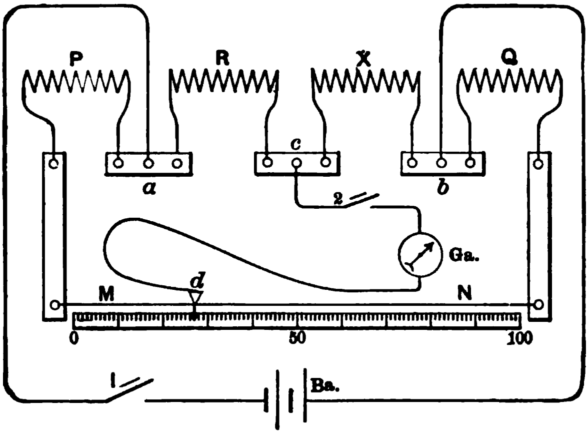


FIG. 16. Carey-Foster bridge.

The bridge is balanced as usual; then the positions of *P* and *Q* are interchanged by means of the commutator shown in Fig. 12; a new balance is obtained by slightly moving the contact *d*. The resistance of the slide-wire between the two positions of the contact is equal to the difference between the resistances *P* and *Q*.

The reason for this is that, the branches *R* and *X* being the same in the two measurements, the total resistance of each of the two other branches must also be the same. If *P* is smaller than *Q*, the difference must be compensated by the corresponding amount of resistance in the slide-wire.

The method presupposes that the resistance of the slide-wire per division of the scale is known. The slide-wire is calibrated by measuring with the bridge the difference in the resistance of two standards *P* and *Q*, which difference has been determined before in some other way.

This is best done by taking two equal resistances P and Q, say of 1 ohm each, and then shunting P by a comparatively high resistance, for example 100 ohms. This reduces the resistance of P to a certain value P'. According to the formula (9) in § 10 we have:

$$\frac{1}{P'} = \frac{1}{1} + \frac{1}{100}; \qquad \text{or} \qquad P' = 0.9901 \text{ ohm}.$$

The difference between Q and P' being known, the slide-wire may be calibrated, at least in the middle portion used for measurements. A balance may be obtained at any desired point of the slide-wire by changing the value of R.

25. EXPERIMENT 1-K. — Comparison of Resistances by Means of the Carey-Foster Bridge. — The method is explained in the preceding section. Special types of Carey-Foster bridge are on the market, with which an accuracy of comparison of over one one-hundredth of a per cent is possible. For ordinary practice with the method, the bridge shown in Fig. 12 is sufficient.

(a) Connect four resistances as indicated in Fig. 16 and practice in comparing their values.

(b) Calibrate the slide-wire as explained above.

(c) Determine the limits of accuracy of the method by varying the difference between P and Q, the ratio $R : X$, and the absolute values of the four resistances.

26. Testing Rail Bonds. — On most electric roads the current for operating cars returns to the power house or substation through the track rails; this saves one conductor. In order to make the use of the rails for this purpose possible they must be "bonded" together, that is, connected by more or less flexible copper conductors as shown in Fig. 17. Rail bonds must be periodically inspected, as they get loose or broken, causing an excessive resistance between the rails. Poor bonding brings with it an excessive drop of voltage and loss of power, electrolytic corrosion of gas and water pipes in the vicinity, telephone troubles, etc.

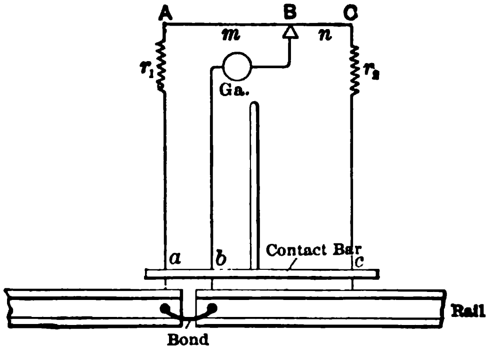


FIG. 17. Diagram of a bond tester.

If rail bonds could be tested while the road is not in operation, the simplest method would be to send a current from the station and to measure the drop across each bond with a low-reading voltmeter. Usually, however, testing must be done with cars running on the same line, so that the current in the rails is widely fluctuating. In order to utilize this current, special devices are used as described below. Where no station current is available a portable storage battery is used, with the leads clamped to the rails adjacent to the bond under test. The resistance of the bond is expressed in feet of solid rail; thus a resistance of 2 feet means that the voltage drop caused by the bond contacts is the same as would be caused by two additional feet of solid rail. Convenient methods of testing bonds are as follows:

(a) If no special bond tester is available, *two ordinary milli-voltmeters* may be used, one connected between *a* and *b* (Fig. 17), the other between *b* and *c*. The milli-voltmeters are read simultaneously, so that the fluctuations in the rail current do not affect the ratio of the readings. This ratio is evidently equal to the ratio of the resistances of the bond *a-b*, and the rail *b-c*. It is advisable to have some protective resistance in series with the milli-voltmeter connected across the bond, since the voltage drop across a loose or broken bond may be many times in excess of the range of the instrument. The protective resistance may be short-circuited with a push button. Good contacts with the rail are obtained by means of file edges or hack-saw blades.

(b) A convenient device for testing rail bonds — the so-called *Roller bond tester* — is shown in Fig. 18, while a diagrammatic view of its connections is represented in Fig. 17. It operates on the principle of a slide-wire Wheatstone bridge. The unknown resistance *a-b* of the bond is compared to a definite length *b-c* of the solid rail by means of the slide-wire *A-C* and the galvanometer connected between *b* and *B*. The slider *B* is moved back and forth until the galvanometer shows zero. Then the ratio of the resistance of the bond to that of the rail is equal to the ratio $(r_1 + m) \div (r_2 + n)$. It is not necessary to calculate this ratio every time; the scale of the instrument (Fig. 18) gives the resistance directly in feet of solid rail.

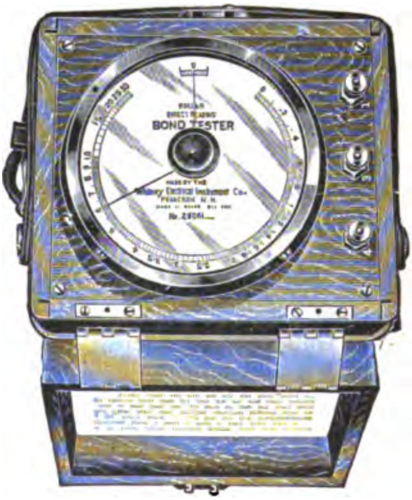


FIG. 18. The Roller bond tester.

The resistances r_1 and r_2 act as an extension of the slide-wire, making the useful part of it longer and therefore more accurate. Without these resistances, the short slide-wire would contain all the values from zero to infinity. Fluctuating currents in the rail do not affect the balance, because fluctuations occur in all the branches of the bridge simultaneously.

One observer is sufficient with this instrument; he holds the contact bar by the upright in one hand, and operates the knob of the slider with the other hand. The instrument itself is suspended from his shoulders. The large scale indicates the resistance the small scale is for galvanometer deflections. In many cases, it is sufficient to know that the resistance of the bond is not beyond a certain limit; then the pointer of the slider is simply set at this limit. On all bonds having resistance below this limit, the galvanometer deflects one way; on defective bonds it deflects the other way, provided, of course, that the direction of the rail current remains the same. This makes the instrument very convenient for rapid testing, even with an unskilled observer.

27. EXPERIMENT 1-L. — Testing Rail Bonds. — The method and the apparatus are described in the preceding article. Become familiar with the device in the laboratory before trying it on an actual track. Two rails should be provided in the laboratory, connected by a variable resistance; this resistance is to represent various states of a bond, from a perfect contact to a broken bond.

(a) Connect the rails to a D.C. circuit, placing a regulating resistance in series with them, to produce fluctuating currents as in practice.

(b) Having set the resistance of the bond very low, measure it first with a steady current, then with fluctuating currents.

(c) Repeat the same test with higher resistances of the bond.

(d) Having become thoroughly familiar with the instrument, measure a number of rail bonds on a street-car line in actual operation.

In performing this experiment, particular attention should be paid to the accuracy of the instrument throughout its range, and to the influence of current fluctuations. Determine the limits of error in the readings due to a poor contact between the rail and the saw blade or the file surface used for making the contact.

28. Plug-Type Wheatstone Bridge. — Series and Decade Arrangement of Coils. — For accurate work, bridges are used in which the three "known" resistance arms R, M, and N (Fig. 10) consist of carefully adjusted resistance coils (Fig. 19). These coils are mounted inside of a box and leads are taken to contacts on its top. Resistances are varied either by inserting plugs, as in Fig. 19, or by dial switches which allow a somewhat quicker adjustment.

The older method of connecting coils is that shown in Fig. 19; the coils are connected in series, and each plug short-circuits a coil. Thus,

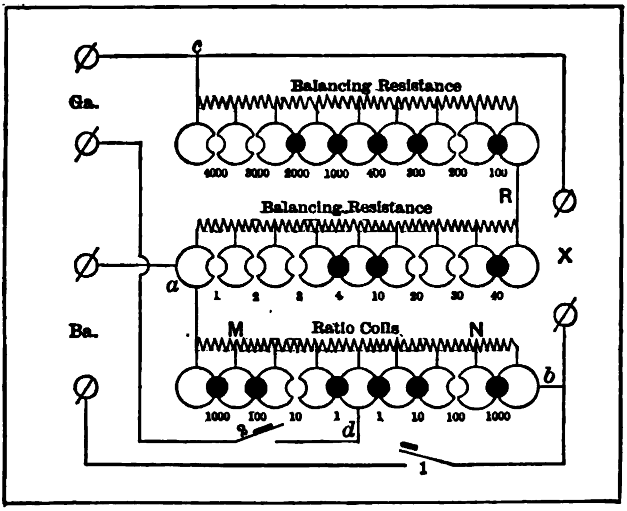


FIG. 19. Diagram of a plug-type Wheatstone bridge.

in the upper row all the coils are short-circuited, except 4000, 3000, and 200 ohm, and the reading is 7200 ohms.

The new or the "decade" method is shown in Fig. 20. Here the

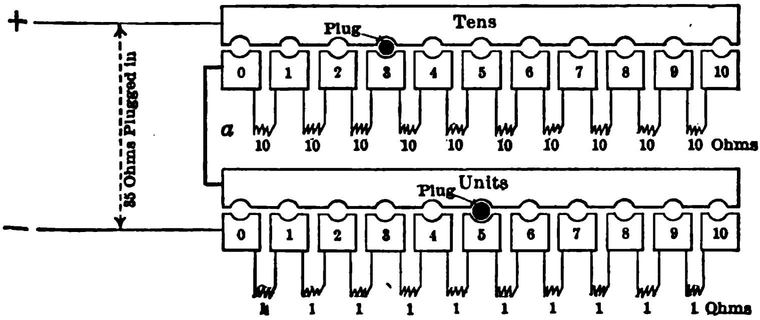


FIG. 20. Decade arrangement of resistances.

resistances which are not in use are simply left out of the circuit. Therefore only one plug is necessary for each decade or row of ten coils.

The advantages of the decade arrangement are:

(1) A smaller number of plugs is required.

(2) Additional resistance between the plugs and the contact blocks is less.

(3) The result is more easily read, no summation being necessary.

(4) There is less possibility of making a mistake or losing a contact plug.

The advantage is made evident by comparing the two upper rows of contacts in the bridge shown in Fig. 19 with the four rows in Fig. 21. In the first case we have to add mentally

$$4000 + 3000 + 200 + 30 + 20 + 3 + 2 + 1 = 7256 \text{ ohms};$$

in the second case we read directly 6192 ohms. Moreover, in the first case 8 plugs are in the circuit, and 8 more are lying idle on the table. In the second case there are but 4 plugs, and they are always in use.

29. Example of a Plug-Type Bridge. — A simple bridge is shown in Fig. 19; the lettering is the same as in Fig. 10. The bridge has three arms: two ratio arms M and N in the lower row, and the balancing resistance R in the two upper rows of coils. The box is provided with three pairs of terminals — for the battery, for the galvanometer and for the unknown resistance X. The battery key 1 and the galvanometer key 2 are also mounted on the box.

Unlike the slide-wire bridge (Fig. 11), plug-type bridges have only a limited number of ratios $M \div N$, — usually in powers of ten. The balancing is done by the third arm R. In the instrument under consideration, the resistance of this arm may be varied from 0 to 10,000 ohms in steps of 1 ohm. Thus, with the ratio $N \div M = 1000 \div 1$, resistances can be measured up to 10,000,000 ohms. On the other hand, by selecting $R = 1$ ohm and $N \div M = 1 \div 1000$, resistances may be measured down to 0.001 ohm. The practical range of the bridge is within considerably narrower limits, because it is not sufficiently accurate near the theoretical limits of its range.

To measure a resistance, connect it to the terminals marked X; select a ratio $N \div M$, say $10 \div 100$. Press the battery key, and then the galvanometer key; vary the resistance R, until the galvanometer remains at zero when its key is pressed. Best results are obtained when the ratio is selected so that balancing can be done with all four places of the resistance R — thousands, hundreds, tens, and units. The right ratio $N \div M$ can always be selected, if the approximate value of X is known in advance; otherwise, the best ratio is found by trials. With the setting shown in Fig. 19 the unknown resistance

$$X = 7256 \frac{100}{10} = 72560 \text{ ohms}.$$

In some bridges used for precision work, some of the coils may be joined in series or in multiple or in any combination of series and multiple. The coils may thus be checked against each other in many combinations. For example, all the 10-ohm coils taken in parallel may be compared with any 1-ohm coil. In such a high-grade bridge the precision of adjustment is one-twentieth of one per cent for the coils of the one-tenth ohm series, and one-fiftieth of one per cent for the other coils of the rheostat. The ratio coils may be certified to be like each other to within one one-hundredth of one per cent.

30. Portable Testing Sets. — The above-described bridges are primarily intended for *stationary* work. There is a demand for self-contained *portable* bridges, for locating faults and for other emergency purposes in the operation of electric plants. The slide-wire bridge described in § 20 is one of this kind. A more accurate instrument, shown in Fig. 21, consists of a Wheatstone bridge with a decade arrangement of resistances. A galvanometer and a battery of small dry cells are mounted in the same box, so as to make the instrument self-contained. Some features of this set are as follows:

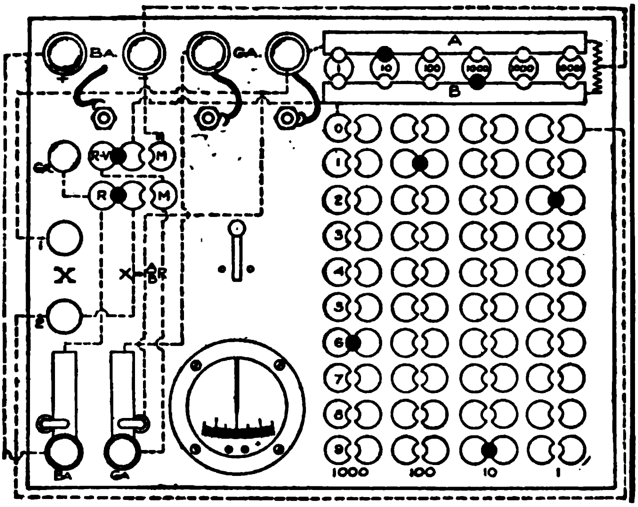


FIG. 21. The diagram of connections of a portable testing set.

(1) The terminal marked "Gr." (ground) and the extra blocks *MM* enable the bridge to be used for locating faults according to either the Murray or Varley loop method (see index to Vol. II).

(2) The instrument may be used as an ordinary resistance box.

(3) The galvanometer may be used separately, or in series with a resistance, for any desired purpose.

(4) If the battery is exhausted, or not strong enough for a certain

purpose, the flexible leads may be disconnected and an outside battery connected to the bridge.

(5) An outside galvanometer may be used if desired, by disconnecting the flexible galvanometer leads.

Explicit directions are supplied with the instrument, so that any one with a very elementary knowledge of electricity can use it.

31. EXPERIMENT 1-M. — Measurement of Resistances with a Plug- or Dial-Type Wheatstone Bridge. — A few types of bridges are described in §§ 28 to 30. The problems which may be solved with these bridges are the same as are enumerated in the preceding experiments with simpler bridges. If possible, the student should connect the coils of the bridge itself in various combinations, so as to check them against each other. The purpose of the exercise is not only to learn how to perform measurements, but also to study the bridge itself, and to ascertain the limits of its accuracy, the best range, the needed sensitiveness of the galvanometer, the best battery voltage, the influence of temperature, the disturbing effect of thermoelectric forces, the best galvanometer resistance, and the effect of the relative position of the galvanometer and battery on the accuracy of the measurements.

32. Measurement of Very Small Resistances. — The Wheatstone bridge is not suitable for measuring small resistances, say below 0.1 ohm, because the resistance of the contacts at *a*, *b*, *c* and *d* (Fig. 10) may constitute an appreciable part of the resistances X and R themselves. The drop-of-potential method (§ 2) gives satisfactory results with resistances considerably below 0.1 ohm, if care is taken to have the potential leads properly applied, so as to eliminate the contact resistance (Fig. 38). For an accurate comparison of very low resistances the potentiometer is used (§ 73).

Lord Kelvin combined the Wheatstone bridge and the drop-of-potential method into a scheme which is exceedingly accurate for measuring low resistances, say down to 0.0001 ohm. The diagram of connections of this so-called Kelvin double bridge is shown in Fig. 22.

The resistances R and X to be compared are connected in series with a low-voltage battery *Ba.*, and some regulating resistance not shown in the figure. The connections to the galvanometer are taken from the points E, F, G, and H, separate from the main terminals. This is a matter of paramount importance, because the contact resistances at A, B, C, and D should not enter into the result. Four ratio coils, m, n, m', and n', are used instead of two ratio coils, M and N, of the ordinary Wheatstone bridge (Fig. 10).

The unknown resistance X is measured by adjusting either R or the ratio coils, until the galvanometer gives no deflection. Both pairs of ratio coils are adjusted simultaneously, so that the relation

$$m \div n = m' \div n' \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (13)$$

is preserved all the time. When a balance is obtained the following relation holds true

$$R \div X = m \div n = m' \div n' \quad . \; . \; . \; . \; . \; . \quad (14)$$

from which X can be calculated.

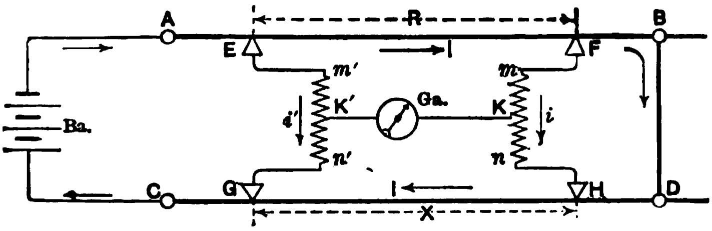


Fig. 22. Principle of the Kelvin double bridge.

Proof: When no current flows through the galvanometer, the points K' and K are at the same potential. Therefore the voltage drop from E to K' is the same as the voltage drop EFK. With the notation in the sketch we have

$$i'm' = IR + im.$$

The same holds true with respect to the point G; hence,

$$i'n' = IX + in.$$

From these two equations we get

$$R : X = (i'm' - im) \div (i'n' - in).$$

Substituting for m' its value from eq. (13), we obtain after a simple transformation the relation (14).

There are two types of Kelvin double bridge: In one type a set of a few standard resistances R is supplied with the bridge, and the adjustment is made by the ratio coils. In the other type the ratio coils are made to give only a limited number of combinations, but the standard resistance R is made adjustable, so as to give practically any value of resistance within certain limits. Such a variable standard low resistance is shown diagrammatically in Fig. 23.

AB is a heavy piece of resistance metal of uniform cross-section and uniform resistance per unit of length; CD is another piece of resistance metal of smaller cross-section, and the two are joined together by a heavy copper bar *AC* into which both are silver-soldered; *LL* are the current terminals and *PP* are the potential terminals. The resistance of *AB* between the marks 0 and 100 on the scale *S* is 0.001 ohm. From the point 1 on *CD* to 0 on *AB* the resistance is also 0.001 ohm, from 2 to 0 it is 0.002 etc., and from *D* to 0 it is 0.01 ohm. The slider *M* moves along the resistance *AB*, and its position is read on the scale *S* which is divided into 100 equal parts and can be read by means of a vernier to thousandths. Subdivided in this way, the resistance between the tap-off points *PP* may have any value from 0.001 to 0.01 ohm in steps of 0.000001 ohm. Using a ratio of 1 to 10 in the bridge, resistances from 0.1 ohm to 0.01 ohm may be measured in steps of 0.00001 ohm, and by using the inverse ratio, all values from 0.001 ohm to 0.0001 ohm in steps of 0.0000001 ohm.

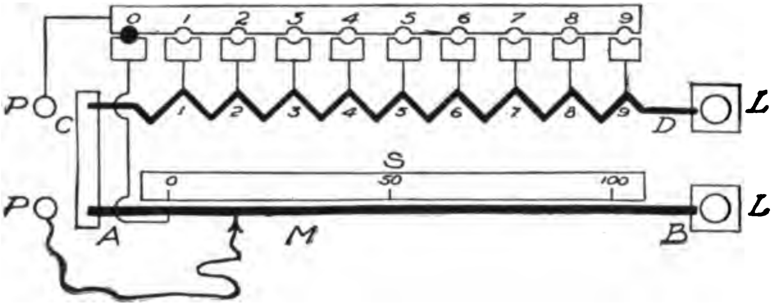


FIG. 23. A variable low resistance, for use as a standard in connection with the Kelvin double bridge.

It will be noted that there are no contacts in the main circuit between the terminals *LL*. The only contacts are at the tap-off points to the potential terminals *PP*. The resistance of these contacts is negligible, as they are in series with the ratio coils, which have a resistance of several hundred ohms.

33. EXPERIMENT 1-N. — Measurement of Low Resistances with the Kelvin Double Bridge. — See § 32.

34. Hoopes Conductivity Bridge. — One of the important practical applications of the Kelvin double bridge is that of determining the conductivity of bars of copper and aluminum. This is done regularly in factories manufacturing wire for electrical purposes; wire is also tested by large consumers before accepting a consignment. It is desirable in such cases to modify the standard Kelvin bridge described in the preceding articles, so as to fulfil the following requirements:

(1) The apparatus must be direct-reading in percentage of conductivity of some agreed pure metal, instead of giving results in ohms.

(2) The apparatus must be adapted for testing a large number of similar samples in a short time.

(3) It must be easily handled by a person who has but little electrical knowledge.

A bridge which satisfies these conditions was designed by Mr. Wm. Hoopes (Figs. 24 and 25). When Figs. 22 and 24 are compared it will be seen that the electrical connections in the Hoopes bridge are identical with those in the regular Kelvin bridge. *AB* is a standard wire made of the same material as the samples to be tested, in order to have the same temperature coefficient; *CD* is a sample under test. The operations for determining percentage of conductivity are very simple. The sample wire is cut off to the standard length of 25 inches in a special cutting-off machine, and accurately weighed. Then it is clamped in the position *CD* and the contact *F* set at a place corresponding to the weight of the sample. The contact *H* is moved back and forth until the galvanometer shows zero; percentage of conductivity is then read off directly on the lower scale.

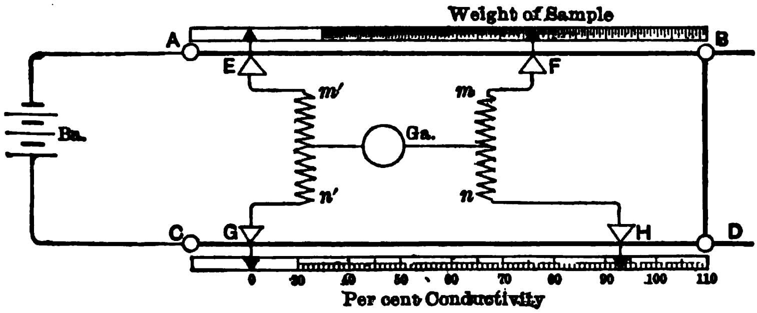


FIG. 24. Diagram of the Hoopes conductivity bridge.

The theory of the method is easily understood from the fact that a sample of metal wire or bar of a given length and weight, has a definite and known resistance if its conductivity is 100 per cent. Therefore, the scale of the standard wire may be made so as to balance a sample of identical composition when *H* is on the division 100 of the lower scale. Then if the sample under test has a lower conductivity, a smaller length of it balances the standard wire, and the slide *H* has to be moved to the left. The decrease in length is directly proportional to the decrease in conductivity. Therefore the lower scale may be calibrated directly in per cent.

The upper scale could be calibrated in circular mils, instead of weights; but it is easier to determine the weight of a piece of wire 25 inches long, than its average cross-section. A single standard wire covers, with some overlap, a range of three numbers of the Brown and Sharpe gauge; if a wider range is required, several standards are used. Each standard, with its scale, is mounted on a separate piece of hard rubber, and can be easily clamped in place.

The details of the bridge may be seen in Fig. 25. The standard conductor is shown in the rear, and its two contacts are provided with knife edges. One contact is left at the zero position and the other can move along the bar; its position is indicated on the scale. A special locking device prevents the contact from being moved along the standard except when the knife edge is raised. This is done to prevent wear on the standard. The sample wire is clamped in front and the instrument has two knife-edge contacts bearing on it, one fixed at the zero position, the other movable at the opposite end. Wires which are not perfectly straight can be stretched by means of the clamp shown at the right-hand end. The long rod projecting from the right-hand end is used to give large movements to the contact knife edge, and the knurled knob in front operates a rack and pinion for small adjustments.

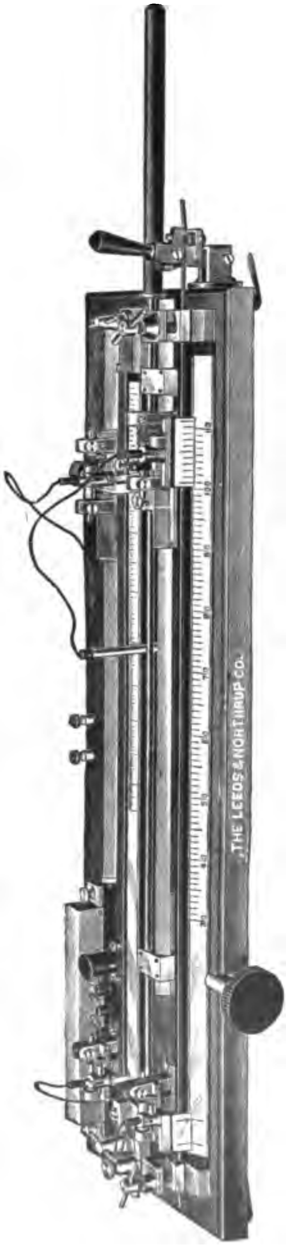


FIG. 25. The Hoopes conductivity bridge.

35. EXPERIMENT 1-O. — Determination of Conductivity and of Temperature Coefficient of Wires with Kelvin or Hoopes Double Bridge. — See §§ 32 and 34.

36. Insulation Measurements and Localization of Faults. — For practical measurements of insulation resistance of electrical apparatus and of lines, for localization of faults, and for disruptive tests on insulation see Vol. II. Faults in direct-current machinery are described in §§ 306 to 311 of this volume.

CHAPTER II

AMMETERS AND VOLTMETERS — CONSTRUCTION

37. In dealing with electric power, two of the most important quantities to be measured are *current*, and *difference of potential.* Technical instruments for measuring electric current are commonly called *ammeters*, because the practical unit of electric current is the " ampere." Instruments for measuring differences of potential, or electric pressures, are called *voltmeters*, since the practical unit of difference of potential is the " volt."

An ammeter is connected in series with the circuit in which the current is to be measured (Fig. 26); a voltmeter is connected at two points of the circuit, between which it is desired to determine the difference of potential. There is no fundamental difference in the construction of ammeters and voltmeters; with the exception of electrostatic voltmeters all usual voltmeters are in reality ammeters calibrated in volts. In other words, a voltmeter measures the current passing through it, which current is proportional to the voltage at the terminals of the instrument, and the scale is marked directly in volts.

To illustrate, let an ammeter of high resistance, say 1000 ohms, be connected *across* a line, and let the indication be 0.1 ampere. According to Ohm's law, it takes 100 volts to cause a current of 0.1 ampere to flow through a resistance of 1000 ohms; therefore the voltage of the line is 100 volts. To use the instrument as a voltmeter, the scale may be conveniently changed so as to have the division " 100 v. " correspond to what was formerly " 0.1 amp."

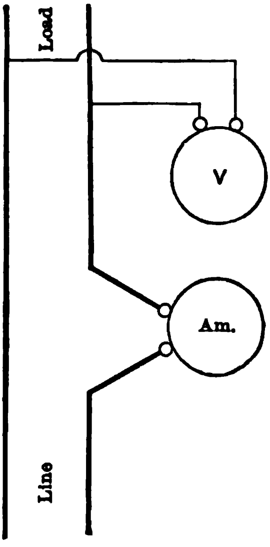


FIG. 26. Standard connections for a voltmeter and an ammeter.

The resistance of a voltmeter must be sufficiently high in order to make its power consumption as small as possible. It will be seen from Fig. 26 that the current consumed in a voltmeter returns to the generator and is lost insofar as the load is concerned. The resistance of the instrument windings is usually insufficient, and for this reason a "multiplier" or high resistance *R* (Fig. 27) is connected in series with the instrument. The multiplier may be mounted within the instrument case or may be external to it. By varying the resistance of the multiplier the range of a voltmeter may be varied at will. Let, for example, a current of 0.01 ampere be required for the full-scale deflection of a certain voltmeter movement. Then, if the range of the instrument is to be 150 volts, the total resistance of the voltmeter circuit, including the winding and the multiplier, must be 150/0.01 = 15,000 ohms. In order to use the same instrument up to 300 volts, an additional 15,000-ohm multiplier must be provided in series, and the scale readings multiplied by 2.

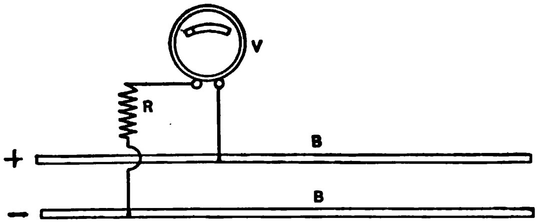


FIG. 27. The use of a voltmeter multiplier.

On alternating-current circuits a potential transformer (§ 54) is frequently used in place of a multiplier. With an electrostatic voltmeter (§ 53) a resistance-type multiplier cannot be used, and the total voltage is reduced by means of condensers in series.

38. Classification of Ammeters and Voltmeters. — Ammeters and voltmeters are built on various principles, enumerated below; each system has its advantages and shortcomings, and a field of application of its own. Some instruments may be used with both direct and alternating currents; others are suitable for one kind of current only. The types in practical use at present are:

(1) Movable-iron type instruments in which the exciting coil is stationary and a piece of iron is allowed to take a position of equilibrium in the magnetic field due to this coil.

(2) Constant magnetic field instruments characterized by a stationary permanent magnet, or an electromagnet excited with an auxiliary direct current. The current to be measured flows through a light movable coil (d'Arsonval type) or through stretched wires placed in the magnetic field.

(3) Electro-dynamometer type instruments based upon the mechanical torque or force between two coils through which the current to be measured is passed.

(4) Hot-wire instruments in which the current to be measured heats a wire and thus changes its length.

(5) Thermo-electric type instruments in which the current to be

measured heats the thermo-junction of a secondary circuit. The secondary current is measured with a galvanometer.

(6) Induction-type instruments based upon the principle of split-phase alternating field.

(7) Electrostatic voltmeters based on the mechanical attraction between two metal bodies at different electric potential.

These types are described more in detail below.

MOVABLE-IRON INSTRUMENTS

39. Description of the Three Types.—(A) One of the oldest instruments of this type is shown in Fig. 28. *C* is a stationary coil which carries the current to be measured. *P* is a soft-iron plunger so suspended that it can move freely up and down. The shaft which supports the plunger also carries the counter-weight *Q* and the pointer *N*. When a current flows through the coil, the plunger is drawn in, against the increasing leverage of the weight *Q*. The corresponding deflection is shown by the pointer on the scale. When the circuit is opened, the counter-weight brings the moving system back to zero.

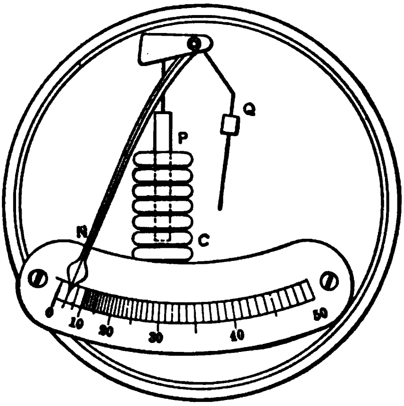


Fig. 28. Plunger-type iron-core instrument.

In the form shown in Fig. 28, the instrument is used only as a current indicator where no particular accuracy is expected. The disadvantages of this type of construction are as follows: (a) The gravity control is inconvenient because it requires a careful setting of the instrument. (b) The moving part is heavy and causes considerable pivot friction. (c) The scale is short and very much suppressed at lower divisions. (d) The energy consumption in the instrument itself is considerable.

Such an instrument may be used on alternating as well as on direct current, because the force of attraction between the plunger and the coil does not depend on the direction of the current. In instruments intended for alternating-current circuits the plunger is laminated, or made of iron wires; this is done to prevent the formation of eddy currents and subsequent heating (§ 210). The spool on which the coil is wound must be made of an insulating material, or, if made of metal, must be subdivided so that no secondary currents can be induced in it.

The calibration of the same instrument is somewhat different on alternating and on direct current, because of the influence of hysteresis, eddy currents, and saturation in the plunger. The calibration also depends to some extent on the frequency and on the wave-form of the alternating current.

(B) A much better instrument of the same type, the so-called *Thomson inclined-coil ammeter*, is shown in Fig. 29. The coil is placed at about 45 degrees to the direction of the shaft; a piece of soft iron (iron vane) is mounted on the shaft in an inclined position. When the coil is energized, it produces a magnetic flux, as indicated by the arrows; the vane tends to move so as to become parallel to the lines of force. In doing so it turns the shaft, and the deflection is shown on the scale. The motion is opposed by a spiral spring, the counterweight serving to balance the moving part.

This instrument is more compact than the above-described plunger-type instrument; the moving part is lighter, and the friction is much reduced, making the instrument more sensitive. Moreover, the spring control is more positive than the gravity control used in the plunger instrument. Thomson inclined-coil instruments are made practically "dead-beat" by means of an aluminum vane fastened to the moving part. Frictional resistance of the air to movements of this vane, absorbs a considerable portion of the energy of oscillations, and the pointer assumes its final deflection after very little swinging to and fro.

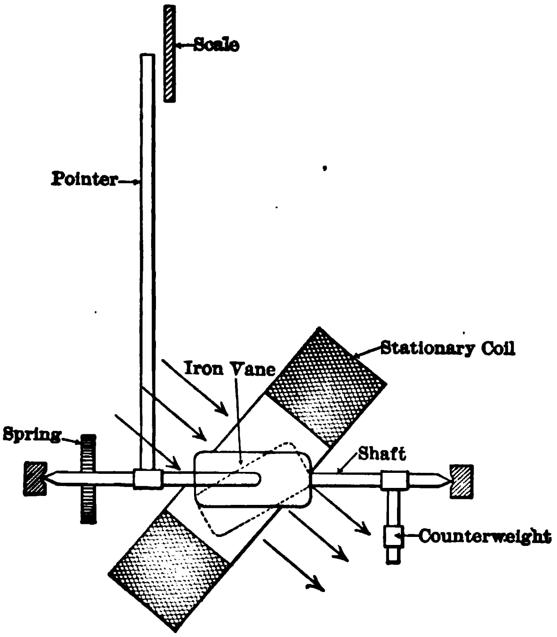


FIG. 29. A cross-section of the Thomson inclined-coil ammeter.

(C) In the instrument shown in Figs. 30 and 31 a movable iron piece and a fixed iron piece are used. When a current flows through the field coil these pieces are magnetized in the same direction and therefore repel each other. Spiral springs are used for the controlling force, and the moving system is effectively damped by means of an aluminum vane attached to the moving part and swinging in a practically enclosed box. Because of the use of two iron pieces the mechanical force is increased and it is possible to reduce the amount of iron and consequently the core loss. The indication of the instrument is said to be practically independent of the wave-form of the alternating current and of its frequency within wide limits.

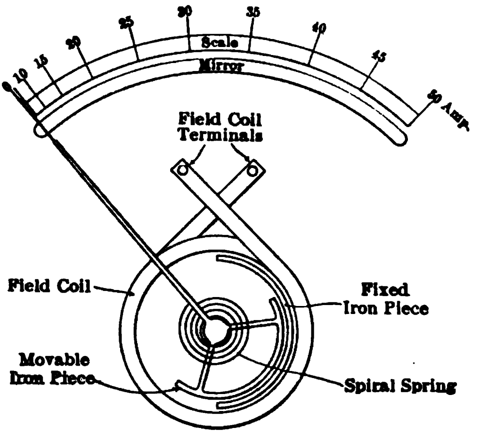


FIG. 30. The Weston iron-vane ammeter.

40. Theory of Iron-Core Instruments. — (a) *Direct-Current.* The relationship between the position of the iron plunger or vane and the mechanical pull or torque exerted by it is rather a complicated one, as may be seen from the curves shown in Fig. 142. The skill of the designer consists in choosing such dimensions and proportions as will produce a scale of desired characteristics. For some purposes a uniform scale is desired, for others an extended scale on part of the range and a suppressed scale for the rest of the range is preferable.

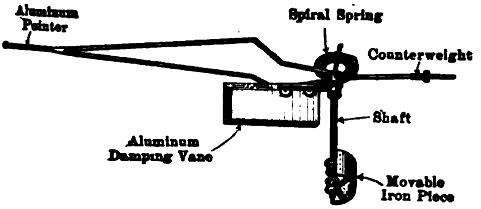


FIG. 31. The movable part of the ammeter shown in Fig. 30.

(b) *Alternating Currents.* Disregarding hysteresis, the indication of a movable-iron instrument is independent of the direction of the current through the exciting coil, the iron core being affected equally by the magnetomotive force of either direction. Therefore, when an alternating current is sent through the coil, the indication of the instrument is nearly the same as if every second half-wave were reversed and the alternating current converted into a pulsating uni-directional current. During each alternation the mechanical force upon the moving part of the instrument varies between zero and a maximum, but with usual frequencies the inertia of the moving part is sufficiently large to prevent it from following the instantaneous variations of the current. The pointer therefore assumes the position corresponding to the average value of the mechanical force.

In any given position of the iron plunger or vane, the flux density in it is approximately proportional to the exciting current, since in an instrument of this kind the flux density is always well below the knee of the saturation curve (Fig. 135). The density of the lines of force in the air is also proportional to the exciting current. But the mechanical attraction or repulsion between the external flux and that in the iron part is proportional to the product of the corresponding flux densities; hence, in this case it is proportional to the *square* of the exciting current. Thus we have:

$$\text{instantaneous pull} = ki^2 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

where i is the instantaneous value of the alternating current and k is a coefficient of proportionality. Since the moving element assumes the position corresponding to the average pull we have

$$\text{average pull} = k\frac{1}{T}\int_0^T i^2 dt = kI^2_{eff} \quad . \; . \; . \; . \; . \; . \quad (2)$$

where dt is an infinitesimal element of time and T is the duration of one cycle of alternating current. In this expression

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2 dt} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

is called the *effective* value of the alternating current, and is defined as the square root of the mean of the squares of the instantaneous values. For direct current the effective value is the same as the actual value. Thus, theoretically, a soft-iron instrument calibrated with direct current should indicate the effective values of alternating currents. In reality this is not quite true because of the disturbing effect of hysteresis and of eddy currents (§§ 209 and 210). A good instrument calibrated with direct current is correct within a few per cent with alternating currents of moderate frequency and of reasonably good wave-form. See Note on p. 78.

With a given position of the plunger, the pull is proportional to the square of the current; hence, if the coefficient of proportionality were approximately constant, the deflections of the pointer would be proportional to the square of the current. In a properly designed instrument the coefficient of proportionality, k, may be made smaller for the positions of the moving part at higher currents, and the scale thus made approximately uniform for a considerable part of the range.

41. Polarized-Vane Ammeter. — A simple direct-current ammeter which comprises a permanent magnet in its construction, is shown in Fig. 32. It consists of a movable soft-iron vane *v* which is polarized by a stationary permanent magnet *NS*, and of a stationary current coil *C* which causes a deflection of the polarized vane when a current flows through the coil. The coil surrounds a stationary iron core *I*, one extremity of which faces the soft-iron vane. The vane is fastened to a pivoted staff mounted between bearings. Just above the vane and on the same staff is also mounted an aluminum cylinder *a*, which rotates between the poles of the permanent magnet and acts as a damping device.

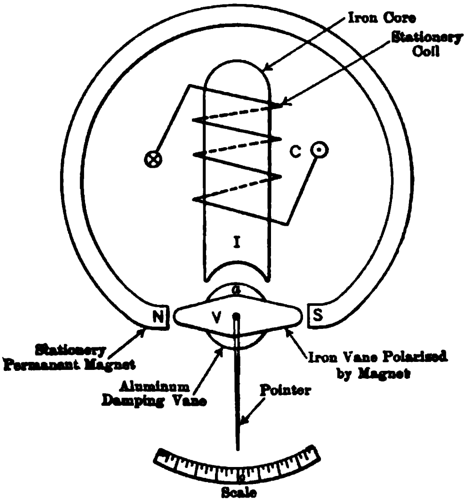


FIG. 32. A polarized-vane ammeter.

When no current flows through the coil, the permanent magnet causes the vane to take a definite position which corresponds to the zero reading on the scale. When a current flows through the coil, the iron core which it surrounds becomes magnetized. The end facing the polarized vane will attract one end of the latter and repel the other, thus causing a deflection of the vane. The polarity of the iron core depends upon the direction of the current, and the magnetic force depends upon the magnitude of the current and thus determines the amount by which the polarized vane is deflected. The permanent magnet serves as the restoring force. This type of instrument is widely used for indicating battery charge and discharge on gasoline automobiles, and for this reason the zero is usually in the center of the scale.

PERMANENT-MAGNET AND D.C.-ELECTROMAGNET TYPE INSTRUMENTS

42. Moving-Coil d'Arsonval Type Instruments. — The principle upon which these instruments are constructed will be seen from Figs. 33 and 34. The magnetic circuit of the instrument consists of a well-aged permanent magnet *M* with pole-pieces *N* and *S* made of soft iron. The soft-iron cylinder *I* completes the magnetic circuit, leaving two narrow air-gaps. A light coil *C* of fine wire is pivoted or suspended in such a way that it can swing freely in these air-gaps. When a direct current is passed through the coil, the latter tends to move in such a way as to cut the radial lines of force in the air-gap, according to the fundamental law of electromagnetism. The deflection is shown on the scale by the pointer *P*. The purpose of the soft-iron pole-pieces and the cylinder is to provide a uniform radial flux distribution in the air-gaps. This makes it possible to obtain a perfectly uniform scale which is one of the advantages of this type of instrument. The mechanical torque between the coil and the magnetic flux is proportional to the current and to the flux density at the place where the coil is located. If this density is uniform, the torque is proportional to the current, so that with a proper restoring force it is possible to have a uniform scale.

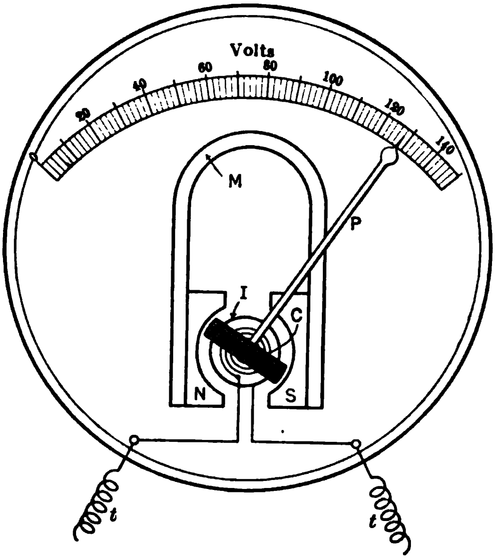


FIG. 33. Arrangement of parts in a moving-coil instrument.

Instruments of this type are constructed for various purposes, from a delicate *galvanometer*, carrying currents of the order of microamperes to a rugged recording ammeter whose coil is designed, say, for 0.1 ampere. In a galvanometer the coil is usually supported between two stretched vertical wires, the torsion of which acts as the restoring force. The same wires serve also as the leads for the current. The *bifilar* suspension used in some galvanometers consists in supporting the coil by means of two wires from the top. When the coil is deflected the wires are forced out of their vertical position, slightly lifting the coil. Gravity then acts as the restoring force.

In ordinary ammeters and voltmeters, and in less delicate galvanometers, the movement of the coil is opposed by two spiral springs, one on top and the other on the bottom (Fig 34). The springs are coiled in opposite directions so that one of them is twisted while the other is untwisted during the movement of the coil. This compensates for a possible non-uniformity in the material of the springs. In most instruments, the same springs are used for leading the current into and out of the coil. In some cases separate leads are used, independent of the springs; such leads must be very soft so as to offer practically no opposition to the movement of the coil. Without the controlling force of the springs, the needle would be deflected to the end of the scale with any current.

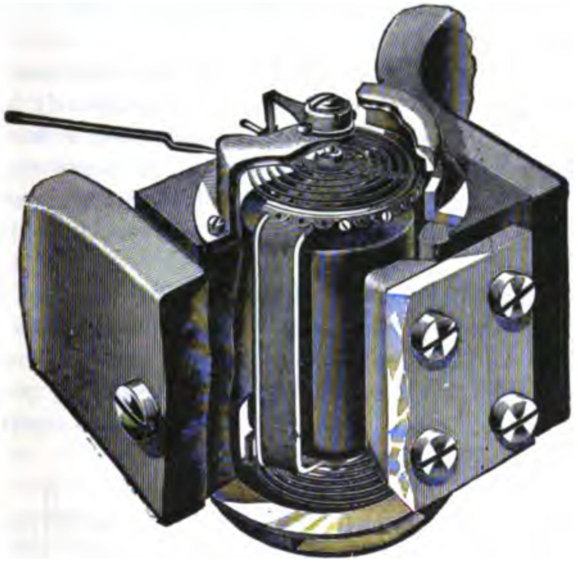


FIG. 34. Construction of a d'Arsonval type instrument.

It is easy to see that an instrument with permanent magnets can be used for direct current only, because the direction of deflection depends upon the direction of the current. When an instrument of this type is connected to an alternating-current circuit, the needle merely trembles at its zero position without giving any deflection. This is because it receives opposite impulses in such rapid succession that it has no time to move in either direction. For the use of this principle in the " vibration galvanometer " see § 43

Moving-coil instruments are easily made " dead-beat " by winding the moving coil on an aluminum frame. Eddy currents induced in the frame during the movement of the coil effectively check any tendency to swing about the point of equilibrium. In some makes of instruments of this type, it is claimed that the damping is so adjusted that the pointer passes a little beyond the final position and immediately returns to it. The purpose of this arrangement is to make sure that the movement is not impeded by friction.

The mechanical support of the coil deserves particular attention. The pivots are made of the best-grade hardened steel and are mounted in sapphire jewels to reduce friction. The instrument must be carefully handled, since shocks dull the pivots and may damage the jewels, thus increasing the friction and reducing the accuracy of calibration. In one make of instruments, resilient jewel mountings are used, so that when the instrument is dropped or subjected to a blow, the moving coil and the jewels move together until the coil strikes the iron cylinder *I*, where the shock is absorbed.

The moving coil carries only very small currents, usually not over 0.02 ampere; the resistance of the coil is comparatively low. Therefore, an instrument used as a voltmeter is provided with a high-resistance multiplier (Fig. 27). When used as an ammeter, a moving-coil instrument is provided with a so-called *shunt* of low resistance which carries all but a small fraction of the current to be measured (§ 47).

Single Air-Gap Construction. The magnetic circuit in Fig. 33 has two air-gaps. The parts may also be so arranged as to have but a single air-gap. In this case the pole-pieces *N* and *S* are made to overlap each other, and the lines of force in the air-gap are parallel to the axis of rotation of the coil. The central cylinder is omitted and the coil embraces one of the pole-pieces. Only one side of the coil is active in producing the torque, instead of two, but the construction is somewhat simplified. Moreover, the air-gap is between two flat surfaces, instead of between two cylindrical surfaces as in the double air-gap construction. The single-gap type is used but little.

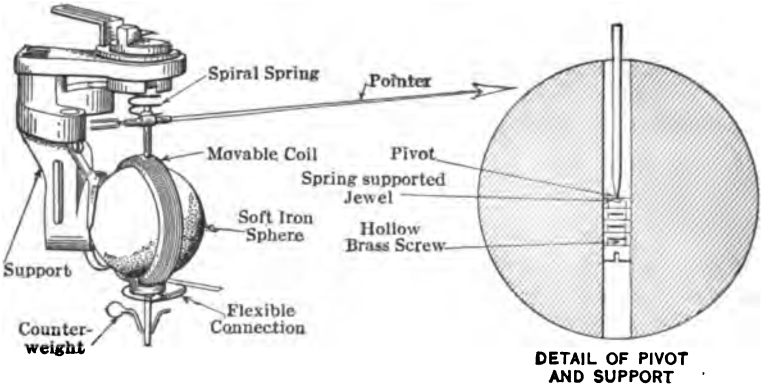


FIG. 35. Unipivot type d'Arsonval instrument.

Unipivot Construction. The movement shown in Fig. 35 differs from that shown in Fig. 34 in that the movable coil is supported on only one pivot, which is located at the center of the soft-iron sphere. The counter-weight shown in the figure is so adjusted as to make this point the actual center of gravity. The coil therefore swings freely in all directions, and the instrument may be used in places subject to vibrations. The spiral spring shown on top furnishes the restoring force and also serves as one of the leads to the coil. The other lead is a very fine volute spiral attached to the bottom of the coil and marked "flexible connection." Instruments of this type are available in ranges from very delicate galvanometers responsive to a few microamperes to ordinary portable ammeters and voltmeters of usual commercial rating.

43. Vibration Galvanometer. — The vibration galvanometer is an instrument for the detection of small sinusoidal alternating currents of a particular frequency, and is used in many cases in which the telephone receiver was formerly employed. The instrument is particularly useful in alternating-current "null" methods, for example, in various arrangements of the Wheatstone bridge for measuring resistances with alternating currents (§ 19) and for comparing inductances and capacities.

The usual vibration galvanometer is of the d'Arsonval type (§ 42), with a moving coil between the two poles of a permanent magnet. The coil is quite narrow and is held in position between two flat suspension strips. These strips are stretched taut so that the *natural frequency* of vibration of the moving system is of the order of magnitude of, say, 60 complete oscillations per second. The mechanical tuning to a particular frequency is usually accomplished in two ways. For rough adjustment a fret can be moved along one of the suspensions, thus changing its effective length. To secure the fine adjustment the tension of the suspension strips is altered.

When an alternating current is passing through the coil it is alternately deflected to the right and to the left with each half-wave; in other words, the coil is set in vibration. When the moving system is tuned to the particular frequency of the alternating current, that is, is in *resonance* with it, the sensitiveness of the instrument is at a maximum, and the amplitude of vibration with a given current is the greatest. The sensitivity drops very rapidly when the instrument and the current are "out of tune," even by a fraction of one per cent. For this reason it is of importance to keep the frequency of the supply very nearly constant, and it is often necessary to have a separate generator driven by a motor which is provided with a particularly sensitive speed governor.

A small mirror is mounted on the coil, and a stationary beam of light is reflected by this mirror to a scale. When no current is passing through the coil a sharply defined straight image of the lamp filament is seen on the screen. As the coil vibrates, the image becomes extended into a band of light. The width of the band is practically proportional to the magnitude of the current. To give an idea of the current range of the instrument, it may be stated that a well-known standard vibration galvanometer has a sensitivity of 40 mm. per microampere at 60 cycles; it can be tuned to frequencies of from 50 to 80 cycles.

A higher harmonic in the current affects the deflection to an imperceptible extent, so that the instrument may be said to possess a highly selective sensitivity. For this reason it is superior to a telephone receiver as a detector. In Wheatstone-bridge work the noise in the telephone receiver under balanced conditions can only be reduced to a minimum unless the current is purely sinusoidal; on the other hand, the vibrations of the galvanometer coil can be practically stopped, giving a much more sharply defined reading.

The advantages of a direct-current galvanometer over a "tuned" instrument are quite apparent, and for this reason various methods have been proposed and used whereby an ordinary d'Arsonval galvanometer may be employed with small alternating currents. This usually involves a rectification of the galvanometer current. A synchronous commutator has been used successfully for such rectification, and also crystal detectors.

44. Peak-Reading Voltmeter. — In some cases the wave-form of an alternating voltage is badly distorted and only the instantaneous maximum value is of interest. Such a case arises, for example, in testing high-voltage cables where the voltage becomes distorted because of the high capacitance of the cables and where at the same time it is important not to subject the insulation, even for an instant, to a voltage in excess of that specified.

An instrument which indicates only the recurring maximum value of an alternating current or voltage consists of two small parallel wires or strips upon which a tiny mirror is mounted. These wires are placed in the field of a strong electromagnet excited with direct current. When an alternating current flows through the wires, they oscillate, and the amplitude of the oscillation is proportional to the instantaneous maximum of the current. A beam of light from a straight-filament lamp is reflected from the mirror of the oscillating system to a ground-glass scale. When the wires are at rest the image of the filament appears on the scale as a narrow vertical line. When the system is oscillating the line of light spreads out into a band with the maximum point sharply defined, and the width of the band is proportional to the amplitude of the current through the strips. The scale may be calibrated directly in volts or in amperes.

The instrument is practically a simplified oscillograph (Vol. II) in which the motion along the time axis is omitted. In another type of peak or crest voltmeter, a part of the high voltage is rectified and measured with a direct-current voltmeter.

45. Thomson D. C. Astatic Instruments. — The magnetic field in an instrument of this type is produced by an electromagnet which must be excited from some source of direct current. A permanent magnet may also be used.

Two moving coils are mounted diametrically opposite each other upon an aluminum disk and are placed in the magnetic field parallel to the shaft. Two small pieces of soft iron are rigidly mounted on the shaft, and are held in the zero position by the components of the magnetic field perpendicular to the shaft. This magnetic attraction thus acts as the restraining force in place of the usual springs. The moving torque is produced by the interaction of the direct current through the coils of the moving element with the lines of force of the stationary electromagnet.

The current is conducted into the moving coils by two small silver-alloy spirals, which exert very little force as springs. The actuating and restraining forces are both proportional to the field strength of the electromagnet, so that the exciting current need not be kept exactly constant. This is of importance in switchboard service where the stationary coils are excited from the bus-bars, the voltage between which may vary at times by several per cent.

To compensate for the effect of external magnetic fields, two sets of magnetic poles are used, arranged astatically; that is, a stray field which increases the torque on one side of the armature decreases it by the same amount on the other side. In addition, the mechanism is magnetically shielded by an iron case. The damping of the moving part is produced by eddy currents induced in the aluminum disk on which the movable coils are mounted.

46. Accurate Measurement of a Small E.m.f. — There are cases in which a small e.m.f., say below one volt, down to a few millionths of a volt, must be measured without drawing any current from the source of the e.m.f. to be measured. With such a requirement an ordinary d'Arsonval type milli-voltmeter or galvanometer is not applicable, and a potentiometer, which is described in Chapter III, may be used. However, a potentiometer is a delicate and expensive instrument, and requires the use of a standard cell which must be handled with great care. Where extreme accuracy is not required, the following " microvolter " method of measurement, proposed by Dr. E. F. Northrup, may be used (Fig. 36). It is based on the potentiometer principle but requires no standard cell. The main circuit consists of a battery *Ba* and a rheostat *R*. An accurately adjusted low resistance *r* of negligible temperature coefficient is shunted around part of *R*, in series with a milli-ammeter *Ma*. The unknown voltage *E* is connected across *r* in series with a sensitive galvanometer *G*. The current through *r* is varied by means of the sliding contact on *R* until the galvanometer shows zero deflection. Let the milli-ammeter current under these conditions be *i* amperes. Then we have $E = ir$, and from this expression the unknown e.m.f., *E*, may be computed. For a given *r* the milli-ammeter may be calibrated directly in microvolts, making the above computation unnecessary.

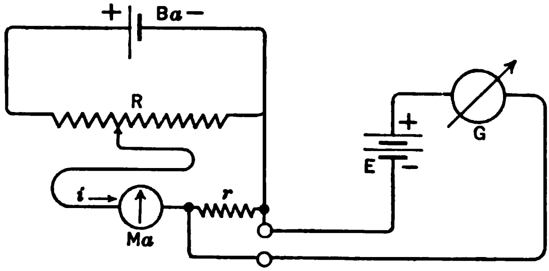


FIG. 36. The microvolter method of measurement of small e.m.f's.

For voltages of the order of 50 milli-volts and higher, the foregoing scheme may be simplified by the use of the same indicating instrument as galvanometer and as milli-ammeter. The diagram of connections, which Dr. Northrup calls the "milli-volter" principle, is shown in Fig. 37. The main circuit is through the battery *Ba*, a variable resistance *R*, a standard resistance *r* and the resistance marked *Cu*. The latter is made of copper and is equal to that of the moving coil of the galvanometer *G*. In position 1, the unknown e.m.f., *E*, is connected across the standard resistance *r* in series with the galvanometer. The resistance *R* is adjusted until the galvanometer deflection is zero, so that the voltage drop across *r* is equal to *E*. The connections are then

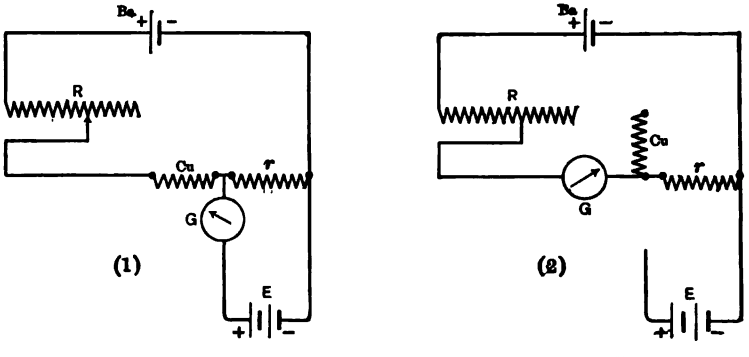


FIG. 37. The milli-volter method of measurement of small e.m.f's.

quickly changed to position 2, in which the galvanometer is used as a direct-reading ammeter. If the current and the resistance r are known, the e.m.f., E, may be computed. For a given r the galvanometer may be calibrated directly in milli-volts. In the second position the resistance Cu is eliminated, since the movable coil of the galvanometer takes its place, making the total resistance of the main circuit equal to that in position 1.

The instruments built on these two principles have been used mainly for temperature measurements by means of thermo-couples, but they are also suitable for the calibration of ammeters and shunts and for accurate measurements of small e.m.f.'s.

47. Ammeter Shunts. — A moving-coil d'Arsonval type instrument, when used as an ammeter, usually has to be provided with a so-called *shunt*, of low resistance (Figs. 38 and 39), which carries the main current to be measured. The instrument itself is connected across the terminals of the shunt and measures the milli-volt drop across it. In an instrument of this type it is impracticable to build a coil which would carry more than a small fraction of one ampere. Thus, an ammeter of the moving-coil type, when used to measure even a moderate current, is simply a milli-voltmeter connected across a shunt.

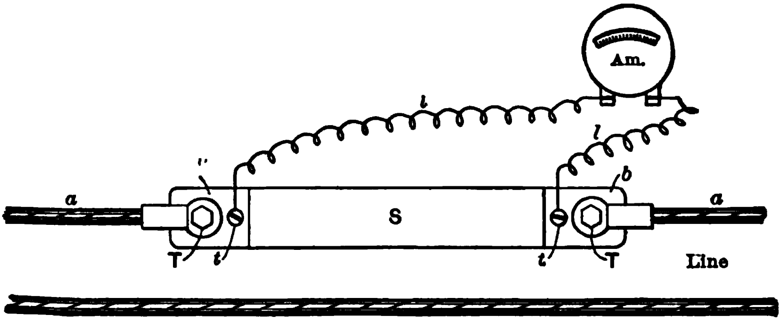


FIG. 38. Current measurement with a milli-voltmeter across a shunt.

Let, for example, the resistance of a shunt be 0.001 ohm, and let the full-scale deflection of the instrument be 50 milli-volts, or 0.050 volt. According to Ohm's law, the line current is

$$i = \frac{0.050}{0.001} = 50 \text{ amperes.}$$

The current through the instrument itself is but a few milli-amperes and can be neglected.

In commercial instruments the shunt and the milli-voltmeter are usually calibrated together, and the scale reads directly in amperes instead of in milli-volts. In ammeters of moderate capacity the shunt may be placed inside the case, so as to make the instrument self-contained. In larger instruments, the shunt is separate (Fig. 39) and is connected to the milli-voltmeter by flexible leads.

Ammeter shunts are made of strips of manganin, German silver, or other material of high specific resistance and low temperature coefficient. The strips *S* (Fig. 38) are sweated into heavy brass blocks *bb*. Two separate pairs of terminals are provided: Large terminals *TT* for the line connection, and small terminals *tt* for connecting the shunt to the milli-voltmeter. The latter terminals are placed so as to measure the drop across the body *S* of the shunt, independent of the distribution of current in the blocks *bb*. This distribution may vary with an uncertain contact at the main terminals *TT*.

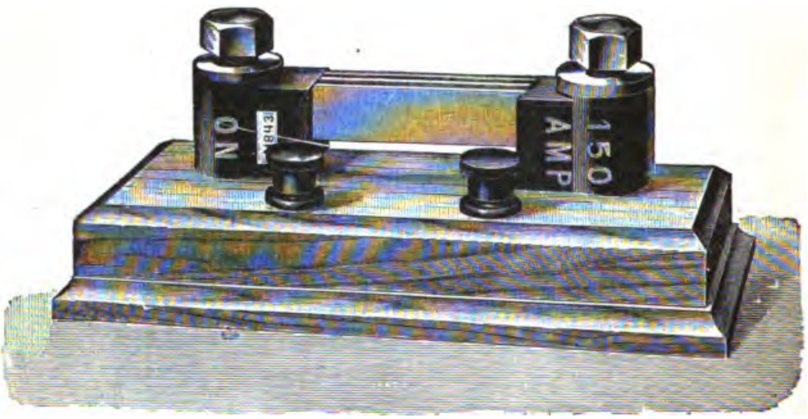


Fig. 39. An ammeter shunt.

One common mistake which the beginner is apt to make when using a moving-coil instrument is to forget to connect a multiplier or a shunt. This invariably results in the burning out either of the moving coil (Fig. 34) or of the spiral springs. Sometimes the pointer is bent or broken.

ELECTRO-DYNAMOMETER-TYPE INSTRUMENTS

48. Electro-dynamometer Voltmeter. — An instrument of this type (Fig. 40) consists of two stationary coils *SS* and a movable coil *M*. All the coils are connected electrically in series. Spiral springs are used as the controlling force for the movable coil and hold it at a certain angle with the stationary coils when no current is flowing. When a current is passing through the coils, the moving coil tends to place itself in a plane parallel to that of the stationary coils, and the resulting deflection is read on the scale. The general make-up of the instrument is the same as in Fig. 34, except that stationary coils are substituted for permanent magnets.

The instrument can be used either with direct or with alternating current. The coils being connected in series, the current reverses simultaneously in both, so that the force of attraction between the two does not change its sign. In fact, electro-dynamometer-type instruments may be calibrated with direct current and used on alternating-current circuits.

Damping is accomplished by means of an air damper consisting of an aluminum vane. The entire set of coils is mounted in a double-closed iron shield, which protects it from external magnetic fields and electrostatic influences. For very accurate measurements, reverse readings with direct current should be made, to eliminate the effect of any slight residual magnetism in the shield. In a double or astatic electrodynamometer (Fig. 60) the directions of the currents in the two elements may be so chosen as to neutralize the effect of a *uniform* external field, such as that of terrestrial magnetism. There is no assurance, however, that the influence of a non-uniform local field will also be neutralized.

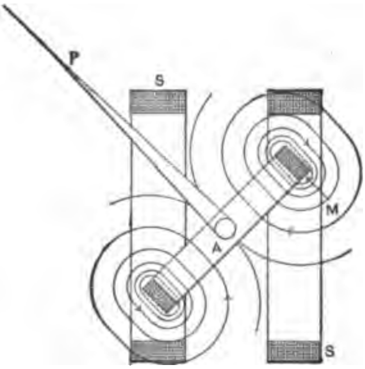


Fig. 40. Arrangement of coils in a dynamometer-type voltmeter.

Voltmeters of this type usually have such a large non-inductive resistance (multiplier, § 37) in series with the coils that at a given voltage the instrument takes practically the same effective current within a wide range of commercial frequencies. However, when using the instrument at high frequencies or for low voltages at which the resistance of the multiplier is reduced, one must be careful to calibrate the instrument accordingly. The unavoidable inductance of the coils may in such cases play an appreciable part and make the calibration depend upon the frequency of the voltage to be measured.

Let a certain alternating current flow through the voltmeter and let the moving part take a position of equilibrium, at which the electromagnetic torque between the coils balances the spring torsion. If an instantaneous value of the current is i, then the electromagnetic torque at that instant is equal to ki^2, where k is an instrument constant *for that particular relative position of the coils.* The torque varies periodically between zero and a maximum, and the average value is equal to the spring torque. In other words

$$\text{Spring torsion} = k.\ \text{ave}\,(i^2) \quad . \; . \; . \; . \; . \; . \; . \quad (4)$$

But, by definition, *the square of the effective value I of an alternating current is equal to the mean of the squares of the instantaneous values over a cycle.* Hence, equation (4) may also be written as

$$\text{Spring torsion} = kI^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

Now let a direct current be sent through the instrument, of such a value, i_{dc}, that the deflection is the same as before. We then have

$$\text{Spring torsion} = ki^2_{dc} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

Comparing eqs. (5) and (6), we see that $I = i_{dc}$. In other words, a dynamometer-type instrument calibrated with direct current indicates true effective values of alternating current, irrespective of waveform. In application to a voltmeter this conclusion presupposes that the ohmic resistance of the instrument is so high as compared to its reactance (§ 132) that the current taken by the instrument is practically independent of the frequency and is simply proportional to the applied voltage.

The force of torsion of the spring is nearly proportional to the angle of deflection. Hence, if k were constant for different angles, the deflection would be proportional to the square of the current flowing through the instrument, that is, proportional to the square of the voltage. This follows directly from eq. (5). The corresponding scale would be very much suppressed at the lower range and open at the upper range. Sometimes such a scale is desirable, but if a more uniform scale is preferred, the instrument may be so designed as to make about two-thirds of the scale in the upper range more nearly uniform.

49. Electro-dynamometer Ammeters. — The principal difficulty in constructing an ammeter on the same principle as the voltmeter described in the preceding section, consists in devising a satisfactory method of conducting the current to be measured, or a fraction of it, to and from the moving coil. This problem has been only partly solved, and the two instruments described below, while quite accurate and valuable in standardization work, are not used to any extent for practical purposes.

(a) *Siemens electro-dynamometer.* This is the original ammeter of this type and is illustrated in Figs. 41 and 42. *BB* are the terminals of the instrument; the current flows from the left terminal through the stationary coil SS to the upper mercury cup C; thence through the movable coil MM and the lower cup C to the right terminal B. The coil MM is suspended from the top of the instrument by a cocoon thread (not shown in Fig. 41), and is controlled by the spiral spring t operated by the torsion knob K. The zero of the scale is between the stops qq. When a current flows through the instrument, the pointer P strikes against the right stop q. The knob K is then turned to the left, until P comes back to zero. The index I shows the angle of torsion on the dial D.

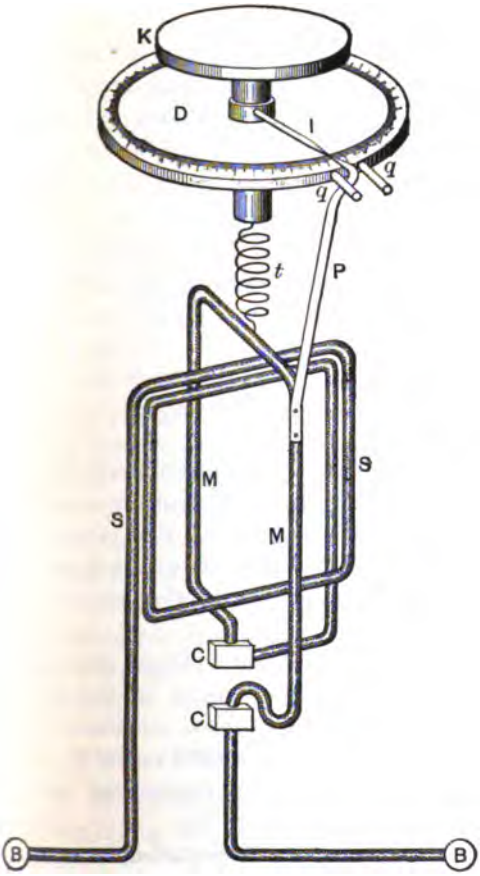


FIG. 41. Arrangement of parts in the Siemens electro-dynamometer.

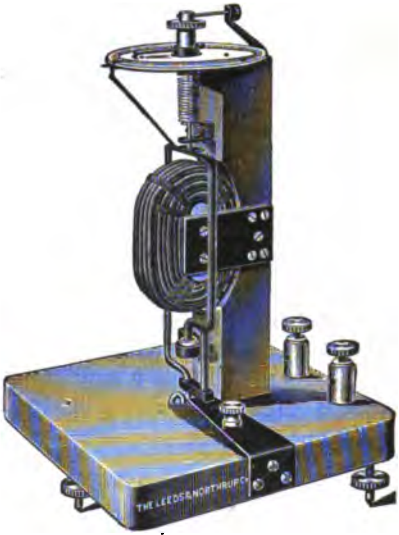


FIG. 42. An electro-dynamometer ammeter.

This instrument is not direct-reading, and the presence of mercury necessitates leveling and makes the instrument inconvenient to handle except for special purposes. The moving coil is practically undamped, so that readings are slow and require some skill, especially with fluctuating currents. When calibrating or using it with direct current, the operator must neutralize the influence of the terrestrial magnetism upon the moving coil by placing the instrument in such a position that the plane of the moving coil is perpendicular to the magnetic meridian. To compensate for the influence of stray fields, readings should be taken with the current flowing through the instrument in both directions and the results averaged.

The theory of the electro-dynamometer voltmeter given in the preceding section is applicable here. Since the torsion-head reading is always taken with the same relative position of the stationary and moving coils, the coefficient k is an absolute constant of the instrument. If the angle of torsion is α, we have

$$\text{Spring torsion} = C\alpha \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

where C is the spring constant. Equating formulæ (5), (6) and (7), we get

$$kI^2 = ki^2_{dc} = C\alpha$$

or

$$I = i_{dc} = \sqrt{\alpha} \cdot \sqrt{C/k} = D\sqrt{\alpha} \quad . \; . \; . \; . \; . \; . \quad (8)$$

where $D = \sqrt{C/k}$ is the calibration constant of the instrument. We see from eq. (8) that in the Siemens dynamometer the current is proportional to the square root of the angle of torsion. The constant D can be determined by comparison with a standard direct-current instrument, and the same constant used when measuring effective values of alternating currents.

The definition of the effective value of current or voltage, given in § 48, holds true whether the quantity varies according to the sine law, or in a more complicated manner. *For sinusoidal quantities the effective value is equal to the amplitude divided by the square root of* 2. A proof of this proposition will be found in almost any elementary textbook on alternating currents. See also the Note on p. 78.

(b) *Weston direct-reading ammeter.* This type of electro-dynamometer ammeter, is similar in its construction to the voltmeter shown in Fig. 40. The stationary coils carry the whole current to be measured, while the movable coil is connected across a shunt (Figs. 38 and 39), and the current passing through the coil is proportional to the voltage drop in the shunt. In this manner the necessity for carrying heavy currents through mercury cups is obviated. In order to be absolutely accurate at all frequencies within a certain range, both the shunt and the moving coil should be altogether non-inductive. In practice this is not possible, but a very close approximation of this condition is obtained by making the shunt practically non-inductive and keeping the resistance of the moving-coil circuit high as compared to its reactance.

In standard laboratory instruments of this type, the self-inductance of the movable coil circuit is practically compensated for by the use of condensers. For an instrument without condensers, indications are guaranteed to be correct within one-fourth of one per cent of full scale value, both for direct current and for alternating currents of any frequency up to 133 cycles per second. For a compensated instrument the error can be made less than one-tenth of one per cent. Instruments of this type may also be compensated for frequencies of 500 cycles and over.

Because of the necessity of making the resistance of the moving-coil circuit comparatively high, the voltage drop across the shunt and the power consumption of the instrument must also be much higher than in a d'Arsonval-type instrument. For this reason these instruments are more suitable for calibration and testing purposes than for switchboard service. Damping, and freedom from the effects of external magnetic fields, are obtained as in the direct-reading voltmeter described above. These instruments are also made as milli-ammeters for ranges lower than one ampere. The field and movable coils are then connected in series, and the total current is made to flow through both.

(c) *Transformer-type instrument.* Soft flexible leads can be designed to conduct currents up to about one ampere into the moving coil, by making the instrument somewhat larger and not very accurate. Beyond this value a current transformer (§ 54) may be used, and the range of the ammeter thus increased indefinitely. The recording instrument shown in Fig. 60, when designed as an ammeter, is used with a current transformer.

OTHER TYPES OF INDICATING AMMETERS AND VOLTMETERS

50. Hot-Wire Instruments. — A typical hot-wire instrument is shown in Fig. 43. The current to be measured, or some part of it, passes through the leads *tt* and the stretched platinum-iridium or platinum-silver wire AB and heats the latter so that it sags appreciably. The resulting downward motion of the point C is transmitted to the pointer P. The deflection is magnified by means of the wire CD, the point F of which is under the tension of the spring KH, through a silk thread wound on the pulley G. When C moves downward, F and H move to the left, and the silk thread turns the pulley. The pointer P, mounted on the same shaft with G, shows the deflection on the scale. An instrument of this construction can be used equally well with direct or alternating current. A hot-wire ammeter for large currents is used with a shunt (Fig. 38); the maximum current through the hot wire itself is usually limited to a few amperes or, in special instruments for radio work, to a few hundredths of an ampere. The scale of a hot-wire instrument is not uniform because the heat energy put into the wire increases as the square of the current. In some hot-wire instruments used for high-frequency measurements the scale is calibrated in squares of the current; this gives nearly uniform scale divisions.

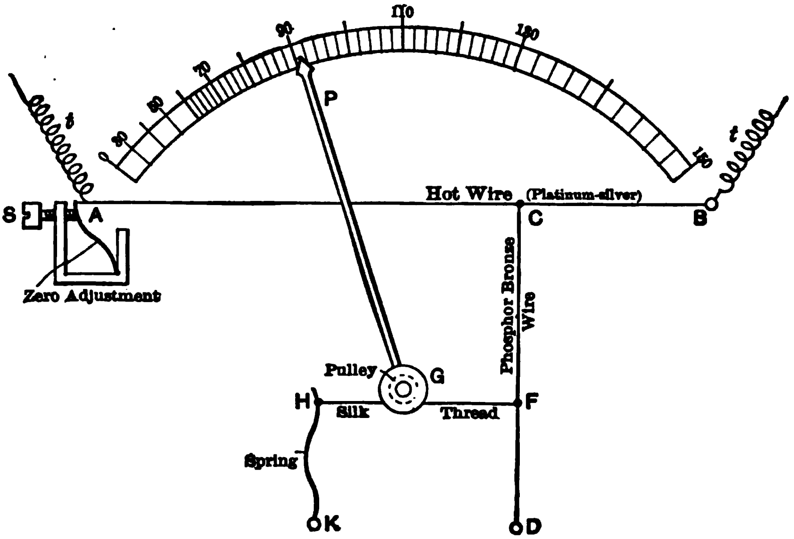


FIG. 43. The principle of a hot-wire instrument.

The instrument shown in Fig. 43 is made "dead-beat" as follows: An aluminum disk is mounted on the same shaft with the pulley G and placed between the poles of a permanent magnet. Eddy currents induced in the disk during its motion effectually damp the vibrations of the needle.

The base of the instrument, between points A and B, is made of an alloy whose coefficient of thermal expansion is equal to that of the hot wire itself. This is done in order to compensate for changes in room temperature which should not deflect the pointer from zero. Nevertheless, a hot-wire instrument easily gets "off zero," and for this reason a zero adjustment screw S is provided at A.

Let r be the resistance of the hot wire and i an instantaneous value of a variable periodic current flowing through it. The total amount of energy converted into heat in the wire during a cycle, corresponding to an interval of time T, is

$$W = \int_0^T (i^2 r)\,dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

With a rapidly fluctuating current, the final temperature of the wire corresponds to the *average* heat input per second. In other words, the temperature of the wire, with alternating or fluctuating current, corresponds to the average amount of power

$$P_{ave} = \frac{1}{T}\int_0^T (i^2r)\,dt = r\,.\,\frac{1}{T}\int_0^T i^2\,.\,dt = r\,I^2 \quad . \quad . \quad . \quad (10)$$

where I is the effective value of the alternating current (§ 48). A constant direct current, i_{dc}, in order to give the same scale deflection, must develop the same amount of Joulean heat in the hot wire. In other words

$$P_{ave} = ri^2_{dc} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Hence, $I = i_{dc}$, and for this reason a hot-wire instrument, calibrated with direct current, indicates true effective values of an alternating current of any wave shape.

51. Thermo-electric Ammeter. — The original instrument based on this principle, and known as the Duddell thermo-galvanometer, is shown in Fig. 44. The direct or alternating current to be measured is sent through the resistor or "heater" shown at the bottom of the figure, and causes its temperature to rise, thus warming a thermo-junction or thermo-couple attached to the end of a movable coil suspended in the air-gap of a permanent magnet NS. A small direct current flows in the coil and causes it to be deflected by the permanent magnet against the torsion of the quartz fiber Q. The sensibility of the instrument depends upon the resistance of the heater and upon its distance from the thermal junction. The instrument can be calibrated with direct current and used with alternating currents, for the same reason as the hot-wire meter described in the preceding section. This thermo-galvanometer has become of some importance in measuring high-frequency A.C. currents, notably in radio work, telephone transmission, X-ray tubes, etc. The instrument is also suitable for use, in some cases, as an intermediate D.C. to A.C. standard.

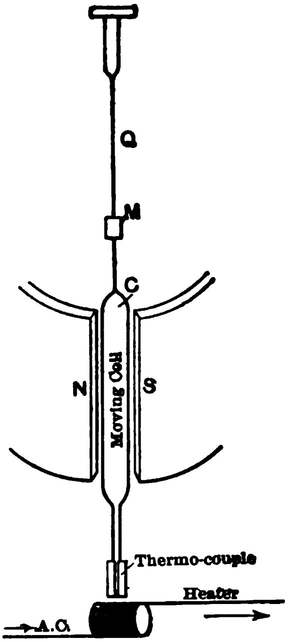


Fig. 44. The Duddell thermo-galvanometer.

52. Induction-Type Ammeters and Voltmeters. — These instruments (Fig. 45) are suitable for alternating current only, and are based upon the principle of the revolving magnetic field produced by two alternating currents out of phase with each other; the principle is the same as in the split-phase induction motor (§ 492). The torque in an ammeter or voltmeter shown in Fig. 45 may be explained in exactly the same manner as the starting torque in a single-phase induction motor

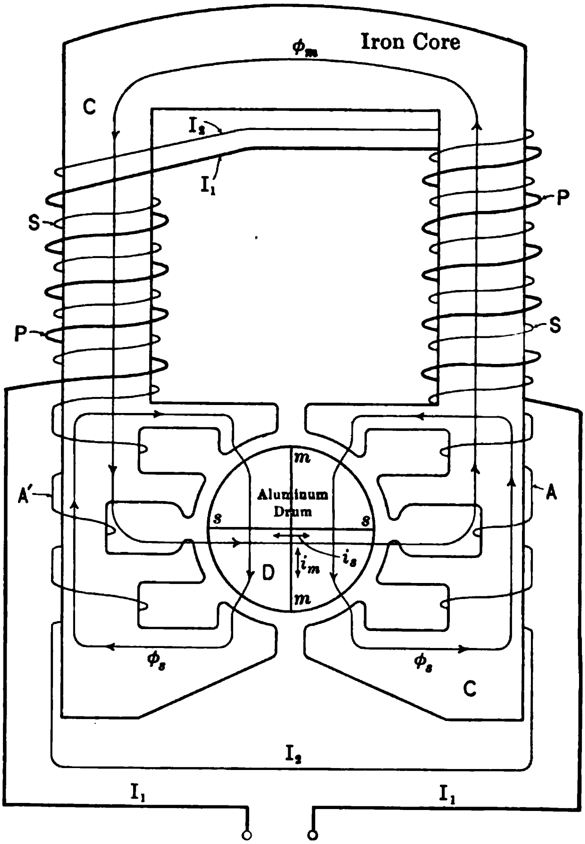


FIG. 45. An induction-type ammeter or voltmeter.

with an auxiliary phase winding. The laminated iron core CC is shaped like the magnetic circuit of a bipolar dynamo, and a hollow aluminum drum D serves as an armature. This drum is pivoted on jewels between the poles, and an indicating pointer is mounted on the same shaft. The restraining force is furnished by phosphor-bronze spiral springs, as in most other types of instruments (Fig. 34). The current I_1 to be measured flows through the primary winding PP. A secondary winding, SS, is placed on the same core and furnishes an induced secondary current I_2 to a pole winding AA'. This arrangement of the windings PP and SS is similar to that in a current transformer (§ 54). The magnetic fluxes are shown by the lines ϕ_m and ϕ_s and are also indicated vectorially in Fig. 46. The main flux ϕ_m is due to the combined magnetomotive forces of both the primary and the secondary windings, and its direction in the aluminum armature is substantially horizontal, that is, parallel to ss. The secondary flux ϕ_s is due to the pole winding AA', and its lines of force run in the drum substantially in the vertical direction, parallel to mm. According to Faraday's law, these two fluxes induce in the aluminum drum secondary currents in such a direction as to oppose the fluxes. Because of the unavoidable leakage reactance of the drum, these currents lag in time phase behind the corresponding induced voltages.

The drum currents, i_m, due to the main flux, flow substantially in the planes parallel to mm, while the currents, i_s, due to the secondary flux, flow in planes parallel to ss. The torque of the instrument is due to the algebraic sum of the torques produced by the interaction of $\phi_m \,.\, i_s$ and $\phi_s \,.\, i_m$, taking into account the phase angles (both in time and in space) between the currents and the fluxes. The flux ϕ_m produces no torque with the current i_m which it induces. The action is simply a repulsion along the lines of force. On the other hand, the currents i_s tend to cut across the lines of force of the flux ϕ_s and give a torque. Similarly, ϕ_s gives no torque with its own secondary currents i_s, but only with the currents i_m.

For those who do not care to follow the theory in detail, it is sufficient to understand that the magnetic circuit and the windings of the instrument produce two fluxes, differing from each other both in *time* phase and in *space* phase. This is a necessary and sufficient condition for the production of a revolving or gliding magnetic field (§ 472). The gliding field, while imperfect in comparison to that in a polyphase machine, is sufficient to induce in the aluminum drum secondary currents as in the squirrel-cage rotor of an induction motor. Because of these currents the drum tends to follow the field. The resisting springs limit the motion of the drum to an angle at which their torsion balances the electromagnetic torque.

Figure 46 is drawn under the assumption of an equal number of turns in the P and S windings, so that the currents are added directly instead of the magnetomotive forces. The main flux, ϕ_m, is due to the geometric sum of the currents I_1 and I_2; in other words it corresponds to the magnetizing component I_0 of the primary current. The secondary flux ϕ_s is due to the current I_2 only, and is displaced in time phase with respect to ϕ_m by nearly 90°. The construction of the instrument is such that ϕ_m and ϕ_s are displaced in space by 90°, so that the above mentioned condition for the production of a revolving field is fulfilled.

An instrument of this type may be so designed as to be practically independent of the effect of the frequency within rather wide limits. It is claimed that the maximum difference in readings for any two frequencies between 25 and 60 cycles is one-half of one per cent of the full scale deflection. To correct for the changing resistance of the drum with temperature, the secondary coil circuit is wound partly with copper and partly with wire of low temperature coefficient. When the temperature of the drum increases it is necessary to have a somewhat larger flux for the same torque. The flux actually becomes larger because the increased resistance of the secondary circuit necessitates a larger induced e.m.f. and consequently a larger ϕ_m.

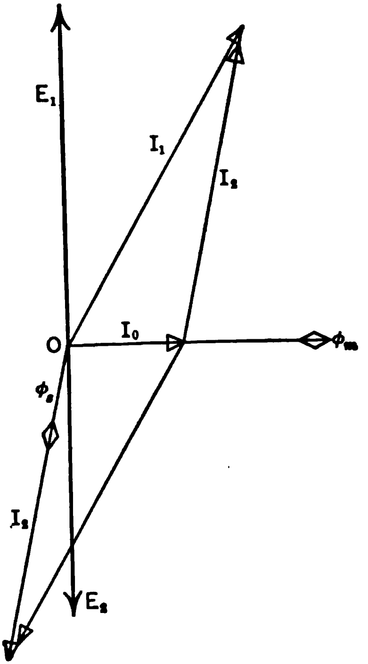


FIG. 46. A vector diagram of the currents and fluxes in the instrument shown in Fig. 45.

In a voltmeter of this type the primary coil is wound with many turns of fine wire, and an external non-inductive multiplier is used, of such high resistance that the indications are practically independent of the frequency, as in the case of the ammeter. Instruments of this type are heavily damped electromagnetically, so that the pointer does not overswing and cause unnecessary wear on the bearings. Other advantages of these instruments are: a long open scale, freedom from error due to external fields, a high torque, and the absence of moving connections.

53. Electrostatic Voltmeters. — The operation of this type of instruments is based on electrostatic attraction between two metal bodies carrying opposite electric charges. One of these bodies is usually stationary and the other movable (Figs. 47 and 48). They are insulated from each other and are connected to the opposite sides of the line, so that they are charged with equal and opposite quantities of electricity. The movable and stationary parts are mutually attracted and the deflection of the movable part is shown on the scale. In general, the scale is suppressed at lower voltages since the attraction increases as the square of the voltage. An instrument may be so designed as to give a practically uniform scale throughout the upper range.

The voltmeters shown in Figs. 47 and 48 have a gravity control; that is, the electrostatic force of attraction must overcome the unbalanced part of the weight of the moving part. For this reason these instruments must be carefully leveled before use. Protective resistance rods, *bb*, are provided in series with the instrument to prevent damage in case of a high-voltage arc-over. The resistance of these rods is of the order of magnitude of one-tenth of a megohm per kilovolt rating of the instrument.

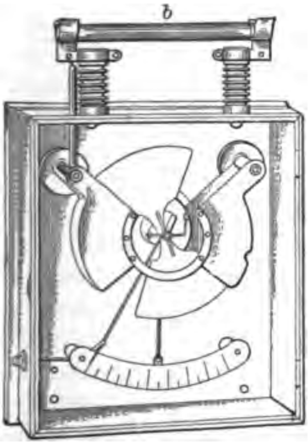


Fig. 47. The vane-type electrostatic voltmeter.

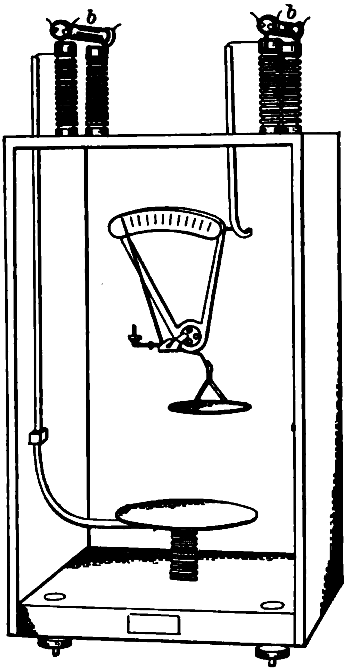


Fig. 48. The pan-type electrostatic voltmeter.

The vane type (Fig. 47) is suitable for potentials of from 3000 to 10,000 volts. The pan-type (Fig. 48) is made for potentials up to 50,000 volts. In still another type, the movable plate is in the form of a round flat cup and is enclosed in a cylindrical glass case filled with oil. The restoring force in this type is furnished by a spring instead of by gravity, the instrument being built for potentials as high as 150,000 volts.

In the voltmeter shown in Fig. 49, the stationary part consists of two curved metal plates *BB* connected to the line, either directly or through the condensers *CC*. The moving element consists of two suspended hollow cylinders *MM*. When an electric pressure is applied between the plates *BB*, the moving element becomes charged by induction, and its cylinders tend to approach the stationary plates. They can do this by moving counter-clockwise, the shape of the stationary plates being such that this movement reduces the distance between them and the moving element. A spiral spring is used as the controlling force, and the deflection is read on the scale. The action of this voltmeter may also be explained on the basis of a general law of electrostatics that a charged system tends to assume a position of maximum capacitance (permittance) or minimum elastance.

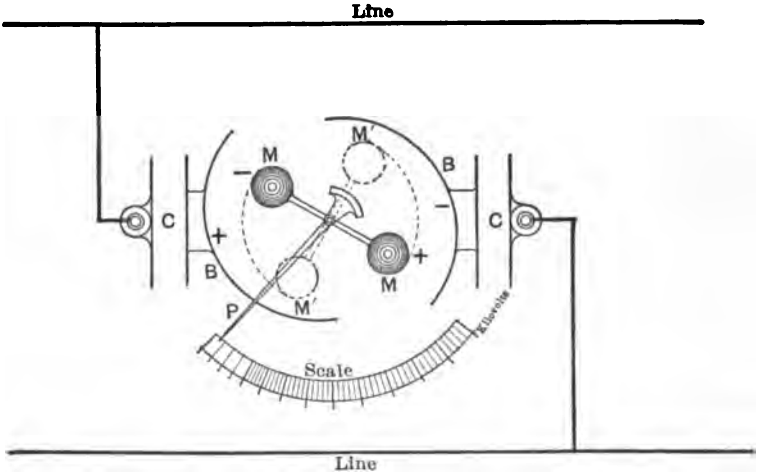


Fig. 49. An electrostatic voltmeter for high voltages.

The plates *BB* and the cylinders *MM* are enclosed in an iron tank filled with oil. The tank shields the voltmeter from the influence of external electrostatic fields. The minimum safe distance between the stationary and the moving parts is greatly reduced by the use of oil, and the actuating forces are greatly increased, not only because of the smaller distance between the parts, but also because of a greater specific inductive capacity of oil over air. Moreover, the oil acts as a damper and makes the instrument nearly "dead-beat." The oil also buoys up the moving element, removing practically all weight from the bearings. This greatly reduces the friction and makes the instrument much more sensitive.

With low voltages the necessary area of the plates, in order to have an appreciable attraction, becomes quite large. For this reason, electrostatic voltmeters are mostly used for high-tension work. In a low-voltage static voltmeter designed by Lord Kelvin the required surface is obtained by using several small plates in parallel; this instrument is known as the *multi-cellular* voltmeter.

The voltage range of an electrostatic voltmeter may be greatly increased by the use of condensers in series with it. It is known from electrostatic theory that when two or more condensers are connected in series the total voltage is divided in the ratio of the reciprocals of the capacitances (Vol. II). The reciprocal of a capacitance is sometimes called the *elastance* of the condenser, and it may be said that the voltage is divided in proportion to the elastances of the individual condensers in series. Thus, condensers play the same part for an electrostatic voltmeter as multiplier resistances used with other types of voltmeters. The smaller the capacitance of a condenser (that is, the higher its elastance) the smaller the voltage to which the voltmeter itself is subjected.

In the voltmeter shown in Fig. 49 the terminals are of the condenser type. They consist of concentric cylindrical layers, alternately insulating and conducting, which form in effect a large number of condensers in series. Some of these layers of the condenser terminal may be short-circuited in order to obtain a full scale deflection at one-half or at one-quarter voltage. This voltmeter is made for potentials up to 200,000 volts, and may have one insulated terminal only, the other curved plate B being connected directly to the tank and the ground.

54. Instrument Transformers. — On an alternating-current circuit an ammeter is often connected to the line indirectly, through a so-called *current transformer* (Fig. 50). Such a transformer consists of a laminated iron core provided with two windings. The primary winding is connected to the line and the secondary to the ammeter. The ratio of the currents in these two windings is very nearly as the inverse ratio of the numbers of turns. Let the primary winding have four turns and carry a current of 100 amperes, and let it be desired to use a 5-ampere ammeter. The secondary winding must then have 80 turns, since $100 \times 4 = 5 \times 80$. The advantages of using a current transformer are as follows: (1) All ammeters may be made for one range, say 5 amperes, and used for any current measurement with a transformer of suitable ratio; the ammeter scale is usually marked to read the true line amperes. (2) On a high-tension circuit it is much safer and more convenient to have the instrument itself insulated from the line. For further details regarding current transformers see Chapter XVIII.

A voltmeter is practically always connected to a high-tension alternating-current line through a so-called *potential* (or shunt) *transformer*

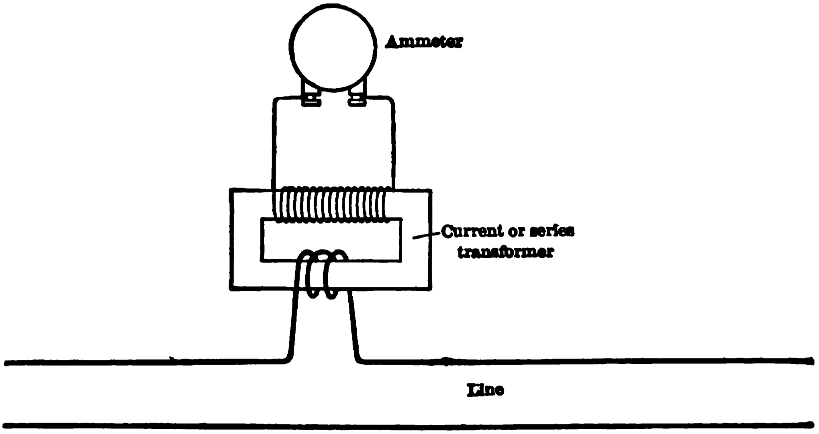


FIG. 50. An ammeter connected to an alternating-current circuit through a current transformer.

(Fig. 51). Such a transformer also consists of a laminated iron core and two windings. The ratio of the voltages is very nearly equal to

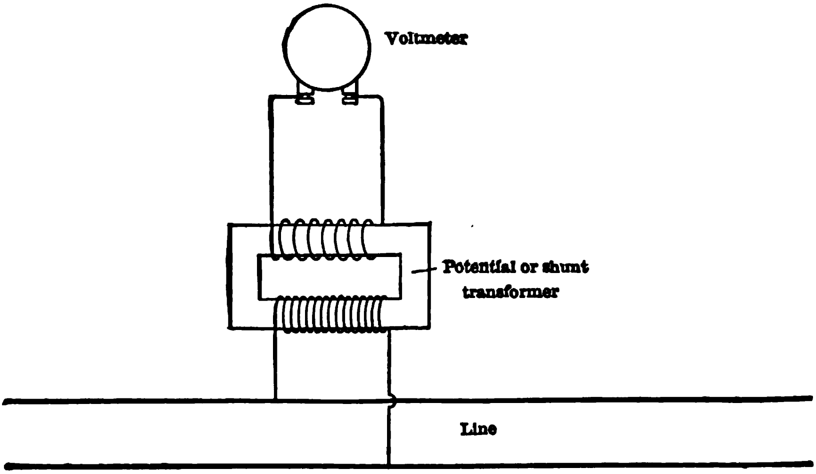


FIG. 51. A voltmeter connected to an alternating-current circuit through a potential transformer.

that of the numbers of turns. For example, if a 110-volt voltmeter is to be used on a 2200-volt line, the ratio of the numbers of turns must equal 20 to 1. The advantages of a shunt transformer are: (1) All voltmeters may be made for one range only, say 110 volts, and may be used for any desired voltage by choosing a suitable transformer. The voltmeter scale may be marked so as to read the line voltage directly. (2) It is more convenient and much safer to have the measuring instruments insulated from the high-tension line. For details of the theory and methods of calibration of potential transformers see Chapter XVIII.

A separate low-tension winding, called the *tertiary* winding, is sometimes provided in a high-tension power transformer for the measurement of voltage, and a voltmeter is connected directly to this winding. This does away with a separate shunt transformer and its errors. If the numbers of turns in the primary and the tertiary windings are known, it is easy to compute the factor by which the voltmeter reading must be multiplied in order to obtain the primary voltage.

55. Requirements for Good Ammeters and Voltmeters. — The principal requirements for a good measuring instrument are:

(1) Permanency of calibration;

(2) Insensibility to stray magnetic fields;

(3) " Dead-beat " quality, *i.e.*, the instrument should not swing too long before giving a definite indication;

(4) Suitable character and legibility of the scale.

(5) A high ratio of the deflecting torque to that of friction.

All these requirements are met to a larger or smaller degree in modern ammeters and voltmeters. However, the stricter the requirements, the more expensive the instrument becomes; it is therefore advisable not to demand greater accuracy than a specific case may require. The following remarks refer to the foregoing requirements:

(1) *The calibration* may be affected by a change of form or relative position of parts inside the instrument; by an increased friction in pivots; by ageing of springs or of iron; by permanent magnets losing part of their magnetism.

(2) Some of the above-described types of instruments are inherently less affected by *external stray fields* than others, and all of them can be sufficiently protected by being inclosed in a steel case. Hot-wire instruments are practically unaffected by stray fields, since their action is not based on a magnetic effect; induction instruments are affected very little, since their air-gap is small and their own field quite strong. Electromagnetic instruments are affected the most, and should never be placed near dynamos, or on a switchboard within a loop of bus-bars and cables carrying strong currents. Electro-dynamometer-type instruments are affected by terrestrial magnetism when used on direct current; therefore in accurate measurements it is essen-

tial to reverse the current in the instrument and to take the average of two readings.

(3) *The "dead-beat" quality* is always desirable in an instrument, though usually it can be attained only by the addition of an extra damping device. An aluminum vane is quite frequently used for this purpose, the damping being attained either by the resistance of the air to the movement of the vane, or by placing the vane between the poles of a permanent magnet, so as to induce eddy currents in it. Oil damping is used in a few instruments. Moving-coil instruments are easily made "dead-beat" by making the frame of the movable coil of aluminum. Eddy currents induced in it by the permanent magnet are sufficient to make the instrument dead-beat. (The word "aperiodic" is sometimes used instead of the expression "dead-beat.")

(4) Different types of instruments have a somewhat *different character of scale.* The scale of moving-coil instruments is usually perfectly regular, all divisions from zero to full scale being equal (Fig. 33). On the other hand, hot-wire instruments have rather an irregular scale, divisions being crowded on the lower part of the scale (Fig. 43), so that the instrument can be read with fair accuracy at not less than 40 per cent of its full range. Sometimes the scale is suppressed on purpose in its lower part so as to have larger divisions in the useful range of the instrument. In instruments based on the hot-wire or dynamometer principle, the scale is of necessity irregular, since the deflecting force is proportional to the square of the current. A *regular scale is by no means always desirable in switchboard service,* though it is very convenient in testing and for experimental purposes, since it increases the useful range of the instrument.

(5) The friction torque in a measuring instrument must be kept as low as possible by using high-grade sapphire jewels and highly polished pivots of hard steel. The absolute value of the friction itself is not a reliable guide to the sensitiveness of an instrument, but in a good instrument the friction torque must be a very small fraction of the actuating torque. In other words, a high-torque instrument may have a somewhat higher friction. The useful torque may be measured by holding the instrument with the axis of the moving element in a horizontal position and placing a small weight upon the needle. The weight is shifted until the desired deflection is obtained when the needle is horizontal. The torque is then equal to the product of the weight and its distance from the pivot. See also § 115. Two competitive instruments may be compared on the basis of gram-centimeters of torque for full-scale deflection, although the instrument with a higher torque is not necessarily superior in all other respects.

56. EXPERIMENT 2-A. — Study of Ammeters. — The purpose of the experiment is to learn the construction, good qualities, and limitations of the principal types of instruments described in §§ 39 to 53. This should help the student to handle instruments properly in his future work, and to select intelligently the right instruments for a given purpose. The instrument to be investigated must be connected into a circuit; if necessary, another instrument, used as a standard, must be also connected into the same circuit, in order to observe the comparative behavior of the two. There are many points in regard to which two instruments of the same range and of different type may be compared to each other; the following are among the most important:

(1) Are there any such defects in the construction that the instrument cannot possibly have a permanent calibration?

(2) Is the instrument "dead-beat," and if not, how can it be made dead-beat in the most convenient way? The degree to which this quality is present can be measured by suddenly closing the circuit on a current that gives a certain per cent of full-scale deflection, say 60 or 70 per cent, and by observing the time which it takes for the pointer to become steady at this point. This method of defining the "dead-beat" degree is entirely arbitrary, but if the same procedure is used on *all* the instruments under test, the results are comparable among themselves.

(3) If the instrument is absolutely dead-beat (aperiodic), note how long it takes the pointer to assume its final deflection. Also see to what degree the moving part is capable of following rapid fluctuations of current. This point is particularly important in hot-wire instruments; they are somewhat sluggish on fluctuating loads, because it takes a certain length of time for a wire to change its temperature. Characterize this sluggishness numerically, by varying the current up and down at a certain speed and by a certain percentage; observe the behavior of the instrument under test, as compared to that of the standard instrument.

(4) See if the pointer always returns to zero, or at least to the same point, when the circuit is opened; if not, investigate the cause. In some instruments a special screw, accessible from the outside, is provided for setting the pointer at zero; it is denoted by S in Fig. 43.

(5) Can the instrument be used indifferently in a vertical and horizontal position? Is the moving part sufficiently well balanced, so that small differences in the position of the instrument do not affect the calibration?

(6) What kind of force is used to deflect the pointer from zero position, and what is the resisting force?

(7) Explain the character of the scale (regular, crowded on one side, etc.) by the character of the moving and resisting forces. What changes should be made in the instrument to get a more regular scale, or one that is still more suppressed on one side?

(8) Is the instrument conspicuously bulky and heavy, as a result of a particular construction adopted? What are the principal materials used, and what, in your opinion, are the most important and expensive operations in manufacturing the instrument?

(9) Is the calibration affected by the temperature, and if so, how much? Is there in the instrument any compensation for the influence of temperature? Some alternating-current voltmeters have a small thermometer on the face of the instrument, and an adjustable rheostat in series with the multiplier.

(10) If the instrument has a shunt, determine if the same has a negligible temperature coefficient, in other words, if the calibration of the instrument changes because of the heating of the shunt.

(11) Determine in how far the instrument is affected by stray magnetic fields and a proximity of large iron masses. Does the iron case shield the instrument entirely from these influences, and does it not introduce an error by itself?

(12) Is the instrument influenced by terrestrial magnetism? Is there any means of eliminating this influence, for instance, by reversing the current and taking an average of the two readings? Is there any position of the instrument with respect to the meridian, such that the above influence becomes a minimum?

(13) Does the instrument show an appreciable hysteresis effect, so that indications on increasing and decreasing currents are different?

(14) Is the calibration the same on D.C. as on A.C., and if not, what is the percentage difference and the cause of the difference?

(15) Is the calibration affected by the wave-form of alternating current?

(16) What is the energy consumption in the instrument itself, and how does it compare with that of other competitive instruments of the same range?

(17) What is the absolute value of the resisting torque (in gram-centimeters) exerted by the spring or by some other restraining device at full-scale deflection? What is the friction torque of the instrument? Inspect the jewels and the pivots with a magnifying glass and report their condition.

Report. Describe the general arrangement of the test, the construction of the instruments used, and give the calibration curves. Also answer, for each instrument studied, the above 17 questions. Do not repeat the questions; simply refer to them by number.

57. EXPERIMENT 2-B. — Study of Voltmeters. — The experiment is performed in the same way as experiment 2-A.

58. Polyphase Boards. — In many practical cases, especially in polyphase work, it is necessary to measure currents and voltages in more than one part of the circuit. At the same time it is not always possible or convenient to have a sufficient number of ammeters and voltmeters. In such cases a so-called polyphase board may be used, by means of which one ammeter and one voltmeter may be connected in succession to several independent circuits or parts of a circuit. The polyphase boards described below may be used with any number of circuits up to four.

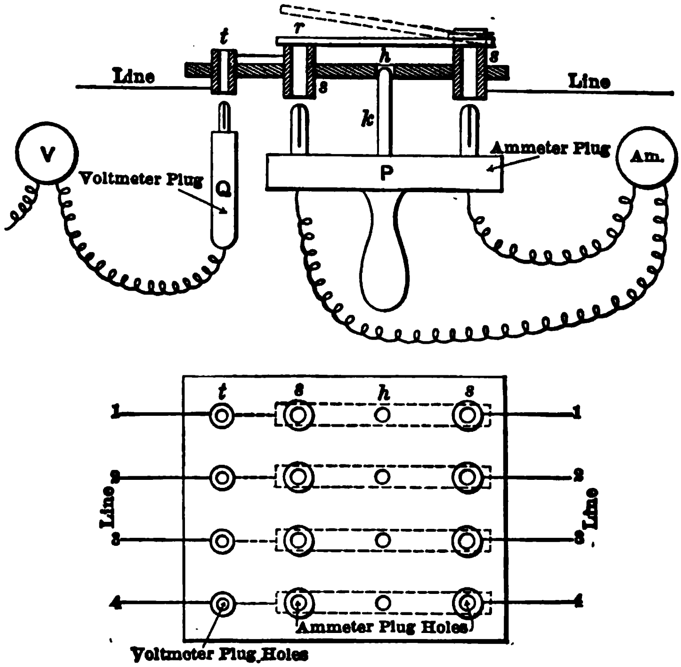


FIG. 52. A plugging arrangement for using one ammeter, one voltmeter, and one wattmeter in several circuits, without interrupting the main current.

(a) *Plug-type board.* The polyphase board shown in Fig. 52 is very convenient for laboratory purposes. The line wires, in which it is desired to measure currents, are connected to the terminals *t* and *s* of the board. When the plug *P* is out, the circuit of each wire is closed from *t* to the left-hand socket *s* and through the spring *r* to the right-hand socket *s*. If it is desired to measure current, say in line 1, the plug *P* is inserted in the corresponding receptacles *ss* as far as it will go. The pin *k* enters the hole *h* and opens the spring contact *r*, as shown by the dotted lines. But before the circuit is opened at *r*, it is shunted between the receptacles *ss* through the ammeter *Am*. When the plug is taken out, the spring closes the circuit at *r* before the ammeter circuit is opened. In this way the line circuit is never completely opened, and there is no sparking at *r* or fluctuation of the load.

The receptacles *t* are intended for two voltmeter plugs *Q*. By inserting these plugs into any two receptacles, the voltage may be measured between any two lines or phases. The same polyphase board accommodates, if necessary, a wattmeter. The current winding of the wattmeter is connected in series with the ammeter and the potential winding in parallel with the voltmeter. Inserting the plugs *P* and *Q* automatically connects the ammeter, the voltmeter and the wattmeter in the desired phases.

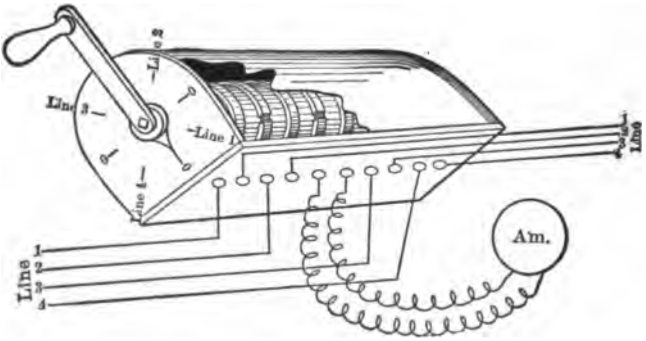


FIG. 53. A controller for connecting an ammeter and a wattmeter into several lines in succession, without opening the main circuit.

(b) *Controller-type board.* Another convenient device for the same purpose is shown in Fig. 53. It is intended for transferring an ammeter and the current winding of a wattmeter from one circuit to another. A separate switch must be used for transferring the voltmeter, as with the polyphase boards shown in Figs. 54 and 55. The device is an ordinary drum-type controller; the lines and the ammeter leads are connected to stationary fingers. The revolving drum is provided with copper contacts which establish the required connections between any of the lines and the ammeter. When the pointer on the handle shows "line 1," the ammeter measures the current in this line; when the handle is in one of the "O" positions, all the lines are connected direct, and the ammeter is out of the circuit. This type of polyphase board is very convenient to handle, especially when readings must be taken in rapid succession.

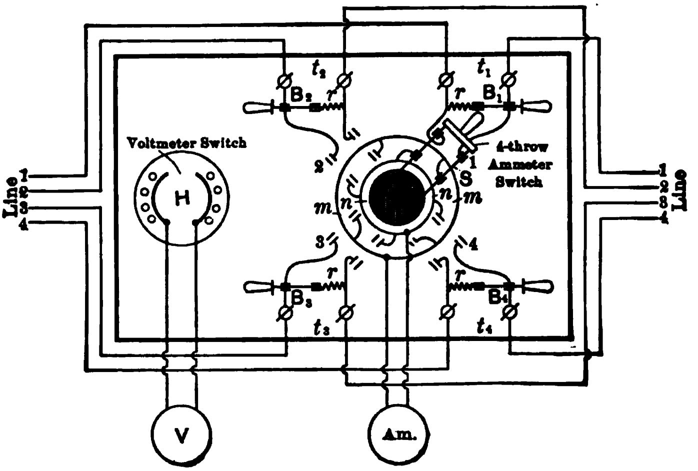


FIG. 54. A polyphase board with a multi-throw switch for currents.

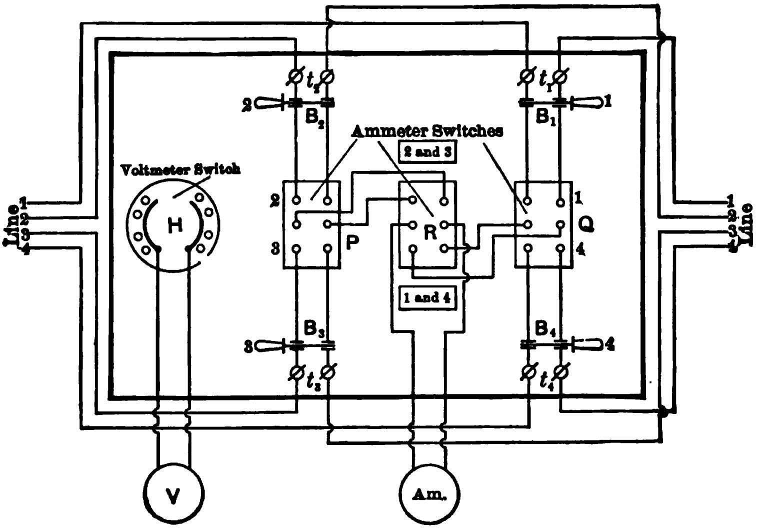


FIG. 55. A polyphase board with three double-pole double-throw switches for currents.

(c) *Boards with knife-switches.* The boards shown in Figs. 54 and 55 are not so convenient in operation as the two devices described above, but they can be built with less difficulty, by using ordinary knife-switches. The ammeter and the voltmeter switches are entirely separate. The lines are connected to four pairs of terminals t_1, t_2, t_3, and t_4. When the switches B_1, B_2, B_3, and B_4 are closed, the currents flow directly through the lines, outside the ammeter. To read amperes in phase 1, throw the central radial switch S (Fig. 54) to position 1, and open the switch B_1. The current from the left-hand terminal, t_1, flows through the left-hand blade of S, connection n, ammeter Am, connection m, and the right-hand blade S to the right-hand terminal t_1, and to the line. Resistances r in series with the switches B are intended to compensate for the resistance of the ammeter and the leads, so that the total resistance of the line remains the same, whether the switch B is open or closed.

The polyphase board shown in Fig. 55 differs from that in Fig. 54 in that the connections are made by means of three ordinary double-pole double-throw switches P, Q, and R, instead of a special radial switch S.

RECORDING OR GRAPHIC INSTRUMENTS

59. It is often of importance to have a continuous record of the performance of electrical machinery, partly as data for economic and engineering calculations, partly as a check on station attendants, machine-tool operators, etc. Recording or curve-drawing ammeters, voltmeters, wattmeters, etc., are used for this purpose. The record is traced automatically on a strip of coördinate paper by a pen fastened to the end of the pointer of the instrument. The paper is moved at a constant speed by a clock mechanism.

The principle of action of almost any of the indicating instruments described in §§ 39 to 53 may be used in the construction of a graphic instrument, but much skill is required in providing the necessary force for overcoming the pen friction, without impairing the accuracy of the instrument. There are three methods of accomplishing this end:

(a) The instrument is so designed as to have a high torque (§ 60).

(b) The instrument acts merely as a relay for an independent source of power which moves the pen (§ 61).

(c) The record is made by means of an electric spark or beam of light, involving no friction (§ 62).

60. High-Torque Recording Instruments. — A Bristol recording ammeter is shown in Fig. 56. The current to be recorded flows through the stationary coil A. The magnetic field produced by the coil attracts the iron disk B which rests on knife-edge supports C and D. The supports have a lateral spring action, which opposes the movement of the

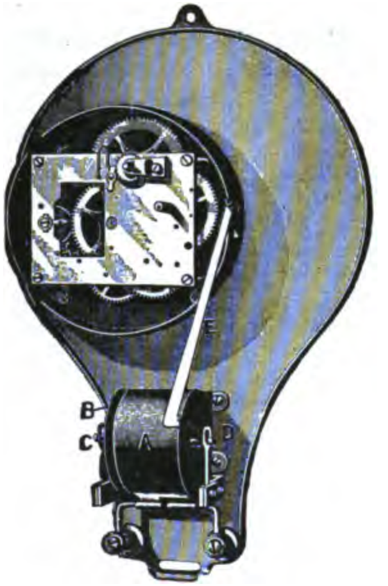


FIG. 56. The Bristol recording ammeter.

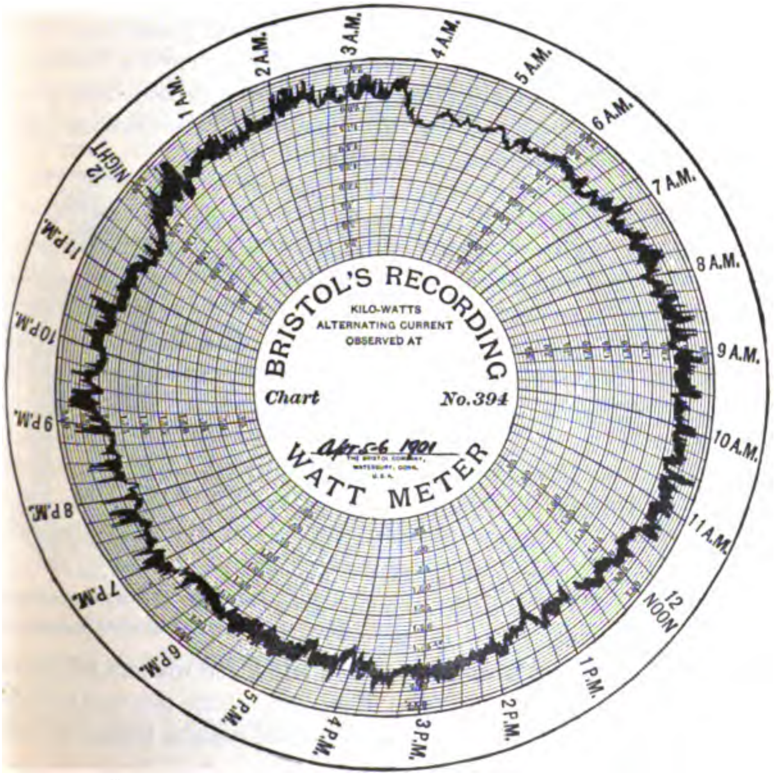


FIG. 57. A 24-hour record taken on a fluctuating load.

disk. The motion of the disk *B* is transmitted to the recording pen-arm *E*, which traces a record on a sheet of coördinate paper, ruled as shown in Fig. 57. The paper is driven by the clock mechanism shown in the upper part of the instrument. One revolution of the paper may be had in twenty-four hours, six hours or even one hour, as desired. A radius-averaging instrument, similar to a planimeter, is available for records of the type shown in Fig. 57. With such a device the average value of the current, voltage, etc. over a desired period of time may be readily obtained.

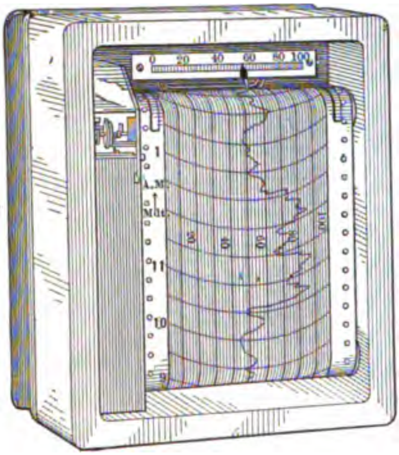


FIG. 58. An Esterline recording instrument.

In the recording instruments of the type shown in Fig. 58 (Esterline), a continuous record is obtained on a roll of paper driven by the clock shown to the left. The coördinate paper has straight abscissæ but curved ordinates. The latter feature is not objectionable for most purposes and greatly simplifies the construction of the instrument. Curve-drawing instruments are available, however, in which the pen is made to move on a straight line, so that the record is obtained in rectangular coördinates (Fig. 61). A scale is provided on top so that the meter may be also used as an indicating instrument.

Direct-current ammeters and voltmeters of this type have the usual d'Arsonval movement (§ 42) shown in Fig. 59, except that it has to be designed for a much higher torque on account of pen friction. A stationary inkwell is attached to the meter element; the pointer consists of an alloy tube, terminating in a glass writing pen at the outer end. The inner end of the tube is bent downward and submerged in the ink. The pointer is flexibly supported and fitted with an adjustable counter-weight, by means of which the pressure of the pen on the paper may be adjusted. The flow of ink through the pointer is due to capil-

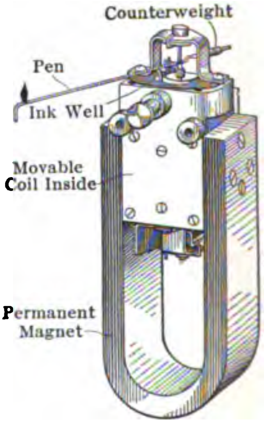


FIG. 59. A d'Arsonval-type recording instrument.

lary action, and is adjusted to the rate at which the ink is used by the pen. It is impossible for the ink to feed too rapidly and blot the record.

For alternating current ammeters and voltmeters (also for wattmeters, see Chapter IV), the electro-dynamometer-type movement is used (Fig. 60). The double element makes the instrument astatic, that is, insensitive to external fields, and the useful torque is doubled. No pivots or jewels are used and the moving element is carried on a steel suspension wire. For damping, an aluminum vane is attached at the bottom of the movable part. This vane is immersed in the oil chamber shown at the bottom of Fig. 60.

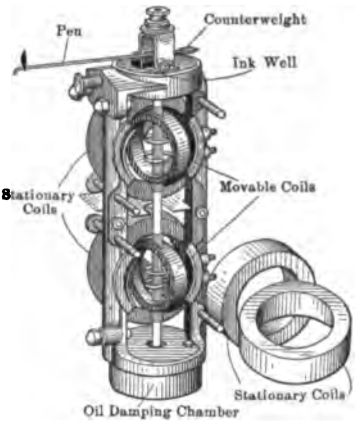


Fig. 60. An astatic electro-dynamometer used as a recording instrument.

The usual rate of paper feed in Esterline instruments is from 0.75 inch to 12 inches per hour; for recording rapid fluctuations a special clock drive may be provided, with speeds as high as 12 inches per minute. On account of the considerable friction pull on the pen, no great accuracy can be expected at such a high rate of feed. A dot record, instead of a continuous record, may also be obtained, when extremely high sensitivity and the smallest possible power consumption are required, as in recording temperatures, electrolytic voltages, etc. In this case the pen is counter-weighted so as to rest normally above the chart, and an electromagnet, closed intermittently by the clock, draws the pen down, making a succession of dots (from 2 to 120 dots per minute).

61. Curve-drawing Instruments on the Relay Principle.—The above-described recording instruments must be designed with a comparatively large torque, so that the pen friction will not impair the accuracy of indications. This necessitates considerable power consumption, and even then leaves the accuracy at small deflections somewhat indefinite. Therefore, in some graphic instruments the energy necessary to move the pen and to overcome its friction against the paper, is supplied from an independent source. The energy of the indicated quantity merely closes one of the two relay contacts, which contact energizes a solenoid or a small motor and makes it move the pen in one direction or the other.

A relay-type curve-drawing voltmeter with the movement based on the Kelvin-balance principle (§ 81) is shown in Fig. 61. The details of connections are shown in Fig. 62. The instrument has four stationary coils and two movable coils between them, arranged exactly as in the Kelvin balance. The movable part is provided on the left side with a contact arm which closes one or the other of the two contacts between which it swings. The auxiliary source of direct current is represented schematically by a battery. The two solenoids, *CC*, which operate the pen, are connected in parallel, and each is connected to one of the above-mentioned contacts. The condensers *KK* and the resistance *R* are for the purpose of reducing sparking at the contacts. The leverage *MM* of the pen is connected by the spiral spring *S* to the movable part of the voltmeter.

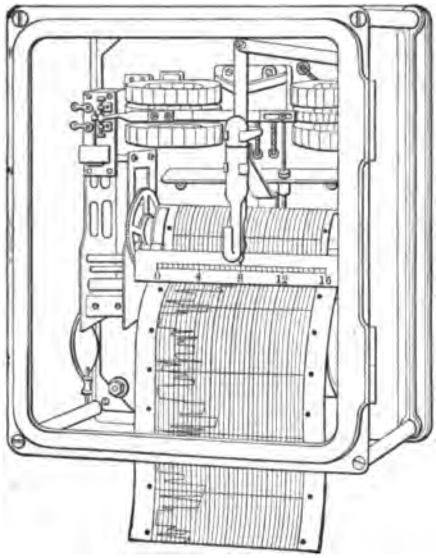


FIG. 61. A Kelvin-balance relay-type graphic voltmeter.

The six coils constituting the voltmeter itself are all connected in series across the circuit in which it is desired to record the voltage; *VR* is the multiplier. Let the position of the pen at a certain moment correspond exactly to the voltage of the line, and let both contacts be open. If now the line voltage drops, the spring *S* will overcome the attraction between the movable and the stationary coils and will close the upper contact. A current then flows from the battery into the left solenoid *C*, a pull is exerted on the plunger *P*, and the pen begins to move to the left, tracing a record. The movement of the pen reduces the tension of the spring *S*; when the pen arrives at the correct point, the electromagnetic pull between

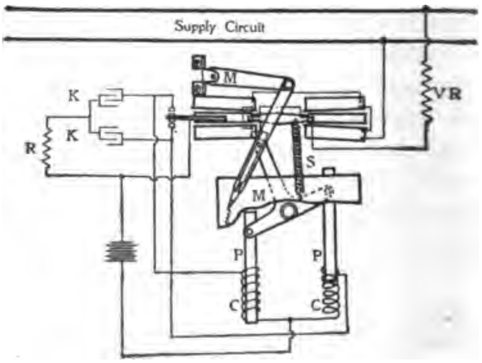


FIG. 62. A diagram of connections in the recording voltmeter shown in Fig. 61.

the stationary and the moving coils of the voltmeter becomes just sufficient to overcome the tension of the spring, and the contact is broken. The pen remains in this position until a new change in voltage occurs.

In reality the pen follows small variations in voltage almost instantaneously. The pen is made of glass and is provided with a reservoir of a sufficient size to hold ink for a continuous record of two months. The quickness of motion of the pen is regulated by two dash-pots connected to the moving coils. The sensitiveness of the record may also be varied by adjusting the distance between the contacts. The rate of feed of paper may be varied within certain limits.

In some more recent instruments of the relay type, the pen carriage is mounted on a nut which can move on a threaded shaft. The shaft is driven by a small electric motor which takes the place of the solenoid magnet, so that the straight-line motion is obtained directly. A complete line of such instruments is available, comprising ammeters, voltmeters, wattmeters, power-factor meters, frequency meters, etc. In all these instruments the clock mechanism which drives the paper roll is made electromagnetically self-winding, and requires no winding by hand whatever.

62. Recording Instruments for Rapidly Fluctuating Currents or Voltages. — The recording instruments described above are not suitable for accurately recording rapidly fluctuating currents or voltages which sometimes occur in practice or in research work. Special instruments have been built for such cases; among these the following may be mentioned:

(a) A dotted record is obtained with an electromagnet energized with an interrupted current (see the end of § 60).

(b) An ordinary indicating instrument is used and its indications are followed as accurately as possible by the observer with a second indicating pointer. This pointer is connected mechanically to the recording pen or stylus, which traces a curve on a moving paper roll or on a smoked cylinder.

(c) A continuous electric spark is made to play between the indicating pointer and a metal support over which the paper passes. Such a discharge may be easily produced by means of a small induction coil and an interrupter. The spark makes a fine burned line on the paper, without the pointer touching it.

(d) A small mirror may be attached to the movable part of an indicating instrument, as in a reflecting galvanometer, and a record of its movements obtained on a roll of sensitized paper by a beam of light. Such an arrangement resembles an oscillograph (Vol. II).

63. EXPERIMENT 2-C. — Study of Recording Instruments. — Different types of recording instruments are described in §§ 59 to 62. The points to be investigated are as follows:

(1) Construction of the indicating and recording parts.

(2) Promptness of response to sudden fluctuations of current or voltage.

(3) Pen friction and its influence on the sensitiveness of the instrument.

(4) Character of the scale of the record.

Report. State your findings and give your opinion of the good points and shortcomings of the instrument. Explain how you would compute the average and the effective values of the current or voltage, if the record is plotted to curved coördinates as in Fig. 57.

NOTE. The ratio between the amplitude and the effective value of a sinusoidal alternating current or voltage may be deduced as follows: The square of the effective value is equal to the average of the squares of the instantaneous values over a cycle. Let a quantity, a, vary with the time, or with some other independent variable, x, according to the sine law. Then

$$a = A_m \sin x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (12)$$

where A_m is the amplitude (maximum value) of a. According to the foregoing definition, we have for the effective value, A, of the variable a:

$$A^2 = A_m^2 \operatorname{ave} (\sin^2 x) \Big|_0^{2\pi} \quad . \quad . \quad . \quad . \quad . \qquad (13)$$

But

$$\operatorname{ave} (\sin^2 x) \Big|_0^{2\pi} = \operatorname{ave} (\cos^2 x) \Big|_0^{2\pi} . \quad . \quad . \quad . \quad . \qquad (14)$$

because sin x and cos x pass through the same range of values over a cycle. Hence

$$A^2 = \tfrac{1}{2} A_m^2 \Big[\operatorname{ave} (\sin^2 x + \cos^2 x) \Big]_0^{2\pi} = \tfrac{1}{2} A_m^2 . \quad . \quad . \qquad (15)$$

and consequently

$$A = A_m / \sqrt{2} . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (16)$$

CHAPTER III

AMMETERS AND VOLTMETERS — CALIBRATION

64. Primary and Secondary Standards. — The absolute values of the ampere, volt, and ohm are established by an international agreement, and primary standards of these quantities are maintained by the respective governments — in this country by the Bureau of Standards, Washington, D. C. Whatever the theoretical reasons for the selection of certain values as units, for practical purposes these units are as arbitrary as the standard inch or the pound. The industry merely demands that the units be of a convenient size and easily reproducible with sufficient accuracy.

It is not necessary to have primary standards of all the three quantities — the volt, the ampere, and the ohm. According to Ohm's law, when two of these are known, the third is represented by their product or ratio. The unit of resistance, or the international ohm, is officially defined as being represented by the resistance of a column of mercury of a certain length and mass, and at a specified temperature. The international ampere is defined as the current which deposits in one second of time a specified quantity of silver out of a solution of nitrate of silver, under certain definite conditions. As a consequence, the international volt is the e.m.f. which, being applied at the terminals of a standard ohm, produces in it a current equal to one ampere. For details regarding these units see the publications of the Bureau of Standards or an electrical handbook.

The mercury ohm and the electrochemical ampere are hardly suitable for practical purposes. Secondary standards are therefore used in practice, calibrated by means of these. Such secondary standards are: Resistances made of wire or strip, and a standard electrochemical cell (§ 72) of a known e.m.f. Current is determined either as the ratio of volts to ohms, or by means of a standard ampere balance (§ 81).

The calibration of ammeters and voltmeters with a standard cell requires rather delicate and complicated arrangements and allows of an accuracy not needed for many practical purposes. In places where many instruments must be calibrated in a comparatively short time, and where extreme accuracy is not required, it is customary to use good ammeters and voltmeters as intermediate standards. They are checked from time to time with the primary standards kept in the laboratory. We shall first describe simple calibrations by means of intermediate standards, and then give the methods of checking the standard instruments themselves.

65. Calibration of D.C. Ammeters with a Standard Milli-Voltmeter and a Shunt. — Standard D.C. instruments are invariably of the moving-coil type, because of the accuracy and sensitiveness of this construction. As is explained in § 47, an ammeter based on this principle is in reality a milli-voltmeter which measures the voltage drop across a shunt (Fig. 38). Therefore, a standard milli-voltmeter and a set of standardized shunts covering the required range of currents are all that is necessary for calibrating a direct-current ammeter (Fig. 63). The ammeter or ammeters to be calibrated are connected to a steady source of direct current in series with a standard shunt and the regulating resistance *R*. A standard milli-voltmeter is connected across the terminals of the shunt. The current is adjusted to a desired value, and the ammeters and the milli-voltmeter are read simultaneously. The current is then changed, new readings are taken, etc.

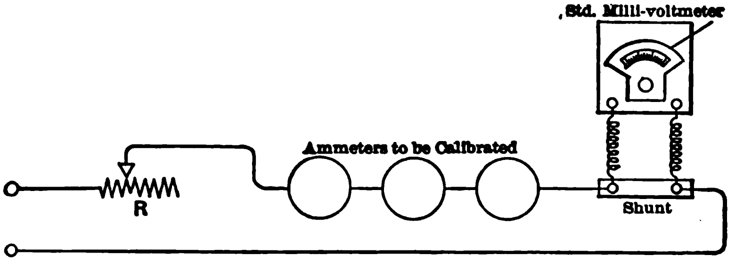


Fig. 63. Calibration of ammeters with a standard shunt and a standard milli-voltmeter.

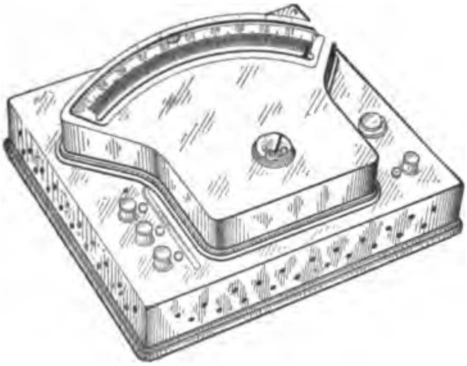


Fig. 64. A standard semi-portable voltmeter (or milli-voltmeter).

A standard milli-voltmeter for accurate calibration is shown in Fig. 64. It is similar to ordinary portable instruments based on the d'Arsonval principle (§ 42), but is several times larger and has a scale shown in Fig. 65. With this scale, parts of divisions may be read much more accurately than with an ordinary scale, such, for example, as in Fig. 33. The scale is provided with a mirror. When readings are being taken, the observer's eye must be in such a position that the end of the pointer is seen to cover its image in the mirror. The calibration of the instrument depends to a slight degree upon its temperature; a thermometer is provided in the base of the instrument so that the necessary correction may be taken into account.

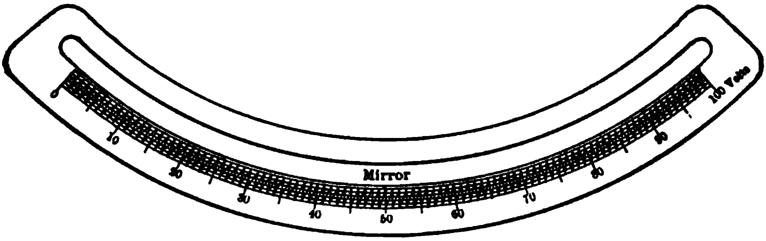


FIG. 65. The precision scale used in standard instruments.

The results of the calibration are given in the form of a table; or a curve is plotted giving true amperes vs. actual readings as abscissæ. Some prefer to plot calibration curves in the form shown in Fig. 66; instead of true readings, corrections only are given, in scale divisions or in per cent.

If a wide range of amperes is to be covered, one shunt is not sufficient. Suppose, for example, that the milli-voltmeter gives a full scale deflection with a current of 1000 amperes flowing through the shunt. With 100 amperes the deflection will be only 10 per cent of the scale; it is hardly advisable to go below this, as the accuracy of the readings is impaired. Thus, below 100 amperes another shunt of ten times the resistance is required; it will give a full-scale deflection at 100 amperes and may be used down to 10 amperes. The next shunt will serve from 10 to 1 amperes, etc.

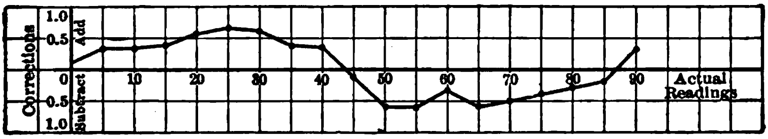


FIG. 66. Calibration curve of an instrument.

Not only ammeters, but milli-voltmeters and shunts may be calibrated in a similar way. To calibrate a shunt, connect it in series with a standard shunt (Fig. 63) and measure the voltage drop with a calibrated milli-voltmeter, first across the standard shunt, and then across the shunt to be calibrated. If the line current is constant, the ratio of the readings will give the ratio of the resistances of the two shunts. An ammeter should be kept in the circuit in order that the observer may be sure that the current remains unchanged between readings. It is well to use a double-throw switch with mercury contacts when changing the milli-voltmeter from one shunt to the other; ordinary knife-switches have an appreciable contact resistance which may vitiate the results.

To calibrate a milli-voltmeter, connect it across a shunt in parallel with a standard milli-voltmeter and take simultaneous readings on both instruments.

66. EXPERIMENT 3-A. — Calibration of D.C. Ammeters with a Milli-Voltmeter and a Shunt. — The method is explained in the preceding section. Connect one or more ammeters as in Fig. 63 and calibrate them throughout their range. Then calibrate a shunt; find its temperature correction, if any. Calibrate a milli-voltmeter; see if the length of the leads and the presence of a switch in the circuit influence the indications.

Report some of the results in the form of curves showing true amperes to actual readings as abscissæ; other results in the form of correction curves, as in Fig. 66.

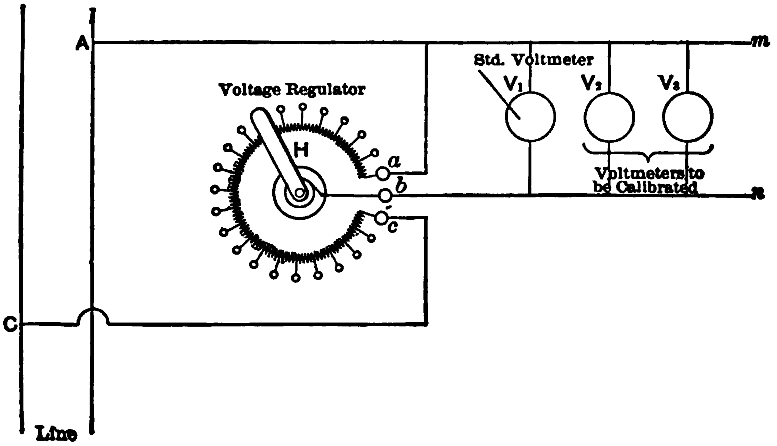


FIG. 67. Calibration of voltmeters.

67. Calibration of D.C. Voltmeters with a Standard Voltmeter. — The arrangement of the instruments is shown in Fig. 67; the voltmeters V_1, V_2, V_3, etc. are connected in parallel between two bus-bars *A-m* and *b-n*. The terminals *a* and *c* of a high-resistance rheostat are connected across the source of supply; by means of the sliding arm H, connected to b, any part of the total line voltage may be applied at the terminals of the voltmeters. When H is in position a, the pressure between m and n is 0. When H is in position c, the total line voltage is applied between m and n. One of the voltmeters is a standard instrument with which the others are to be compared. An ordinary embedded-type field rheostat is convenient for use as a voltage regulator. It usually has only two terminals, a and b; it is easy to solder a third terminal c.

68. EXPERIMENT 3-B. — Calibration of D.C. Voltmeters with a Standard Voltmeter. — The general procedure is explained in § 67; the diagram of connections is shown in Fig. 67. Calibrate a voltmeter with one or more different multipliers. Find out if the instrument itself or the multipliers have an appreciable temperature coefficient. Plot calibration curves as specified in § 66.

69. Calibration of an Alternating-Current Ammeter with an Intermediate D.C.-A.C. Standard. — A hot-wire instrument (§ 50) may be calibrated with direct-current (Fig. 63) and used with alternating currents. The same is true of an electro-dynamometer type ammeter (§ 49) provided that the precaution is observed in regard to terrestrial magnetism and stray fields.

A movable-iron ammeter (§ 39) or an induction-type instrument (§ 52) must be calibrated with a current of the frequency for which the instrument is intended to be used, and of approximately the same wave-form. A precision electro-dynamometer type instrument or a Kelvin balance (§ 87) may be used as such a standard. If neither is available, the instrument may be calibrated with a direct-current standard, using an intermediate or transfer instrument (Fig. 68). A

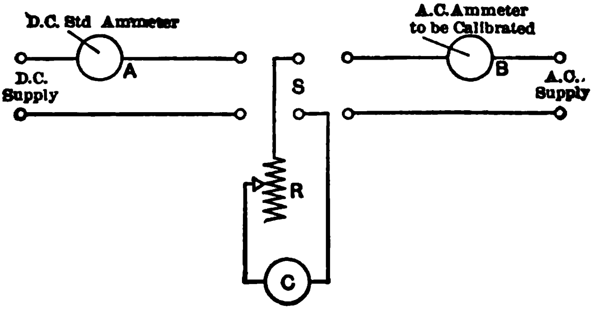


FIG. 68. Calibration of an alternating-current ammeter by means of an intermediate A.C.-D.C. instrument.

hot-wire or a thermo-electric instrument (§§ 50 and 51) is especially well adapted for such service, or an electro-dynamometer can be employed which may not be good enough as a primary standard. The double-pole double-throw switch *S* is first thrown to the right and the intermediate standard *C* is read simultaneously with the instrument *B* to be calibrated. Then the switch is thrown to the left and the current is adjusted by means of the rheostat *R* until the transfer instrument *B* reads the same as before. Then the indication of the standard ammeter *A* is the correct current for the previously noted reading of *B*.

70. EXPERIMENT 3-C. — Calibration of an A.C. Ammeter with an Intermediate D.C.-A.C. Standard. — The method is described in the preceding section and the connections are shown in Fig. 68. Begin the readings with the highest current which all three instruments can carry and reduce the current in steps to zero. Plot the results as shown in Fig. 66.

71. EXPERIMENT 3-D. — Calibration of an A.C. Voltmeter with an Intermediate D.C.-A.C. Standard. — The arrangement of the test is similar to that described in § 69, except that the three instruments are in this case connected in parallel, as in Fig. 67. It is left to the student to devise a convenient diagram of connections, taking Fig. 68 as a guide. Begin the readings with the highest voltage which the three instruments can carry and reduce it in steps to zero. Plot the results as shown in Fig. 66.

72. Standard Cell. — Standard voltmeters and milli-voltmeters used for calibration, as described in §§ 65 and 67, are sufficiently accurate for most practical purposes, provided they are periodically compared to a primary standard of e.m.f. At the present writing, the Weston cadmium cell, which is a primary cell of very accurately known electromotive force, is used as such a standard. In calibrating an instrument by means of a standard cell, a potentiometer (§ 73) is used, because the standard cell must only balance the voltage across a part of the main circuit, and must not be allowed to deliver anything but a very minute current. The construction and chemical composition of the cadmium cell are shown in Fig. 69.

There are two types of cadmium cell. The so-called *saturated form* (or Weston Normal cell) contains solid crystals of cadmium sulphate in the solution. The *unsaturated form* (or Weston Standard cell) contains cadmium sulphate solution saturated at 4° C. The saturated cell is used as the international standard the world over, and its electromotive force is accepted as 1.01830 volts at 20° C. Unfortunately, it has an appreciable temperature coefficient and for this reason is hardly ever used for practical purposes. The unsaturated form has a practically constant e.m.f. within the usual range of temperature met with in practice, and for this reason is generally employed, even in accurate scientific research.

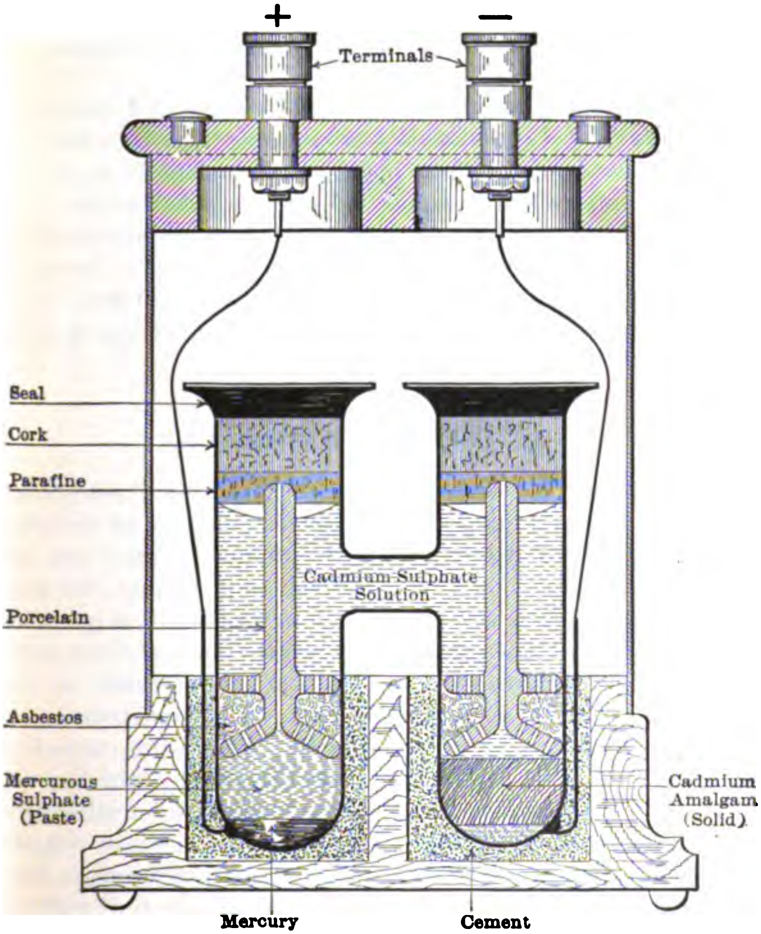


FIG. 69. Cross-section of a Weston standard cell.

Each cell is provided by its maker with a certificate giving its exact electromotive force, and it may also be sent to the Bureau of Standards for certification. A standard cell may be made in a comparatively simple manner by anyone who possesses some skill and experience; explicit directions for setting up may be obtained from the Bureau of Standards.

Minor improvements have been introduced in the construction of the cell from time to time. For leading-in wires, tungsten has been found to be an acceptable substitute for platinum; a harder and less soluble glass has been substituted as the material for the container, etc. For these reasons it is important to use, for each individual cell, the particular value of the electromotive force found by comparison with a certified cell.

The two important precautions to be observed with a standard cell are as follows: (1) In preliminary measurements, never close its circuit on a resistance of less than 10,000 ohms, preferably more. Even a minute current of one-tenth of a milli-ampere polarizes the cell, and it is doubtful if it ever sufficiently recovers to be used again as a reliable standard. (2) Never subject the cell to a temperature below 4° C. or above 40° C. It may take days and even weeks before its electromotive force at the usual room temperature again reaches the normal constant value.

DIRECT-CURRENT POTENTIOMETERS

73. Principle of the Potentiometer. — The term " potentiometer principle " is generally applied to an electrical diagram of connections (Fig. 70) in which a variable voltage is obtained between two points of the main circuit carrying a comparatively large current, the derived circuit carrying only a small current. More specifically, a potentiometer is a piece of electrical apparatus by means of which direct currents and voltages can be measured to a high degree of precision, by means of a standard cell (§ 72) as the known e.m.f. The name potentiometer has been extended to devices for measuring alternating currents and voltages (§ 84) on a similar principle, even though no standard cell is used there. The potentiometer has a wide application for calibration of ammeters, voltmeters, resistances, etc. used as secondary standards. With a potentiometer, voltages from one-millionth of a volt to several thousand volts can be accurately compared to the e.m.f. of a standard cell, and currents measured from a small fraction of an ampere to many thousand amperes.

The ordinary potentiometer connections are shown in Fig. 70; for a clear understanding of the principle involved the student should keep in mind two points:

(a) The standard cell balances the unknown voltage, but is not used as a source of current.

(b) The " zero " method is used, so that it is not necessary to know the galvanometer constant.

A slide-wire *ST* is connected in series with one or two storage cells *Ba* and a regulating rheostat *K*; a constant current is thus established in *ST*. Any desired voltage drop may be had between the contacts *C* and *D* by moving them along the slide-wire, or by regulating the rheostat *K*. The contact *C* is for coarse regulation, *D* for fine regulation. The drop between *C* and *D* is balanced in succession against the standard cell and against the unknown e.m.f. A balance is recognized when the galvanometer needle returns to zero. The ratio of the lengths *C-D* in the two cases is equal to the ratio of the e.m.f.'s under comparison, provided the current in *ST* remains constant. By means of the double-throw switch *L* the connections are conveniently changed from the standard cell to the unknown e.m.f., and back.

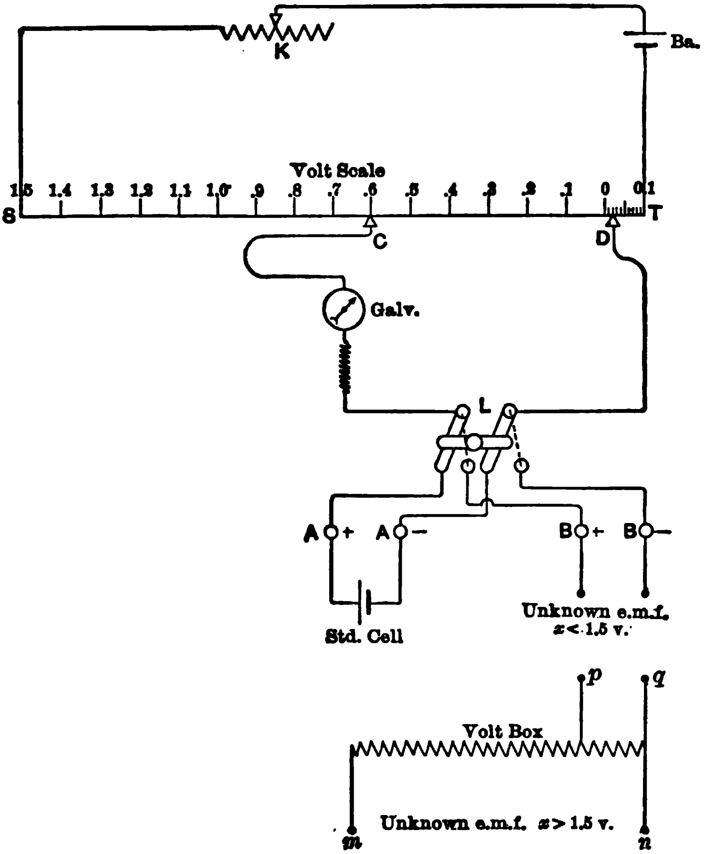


Fig. 70. The potentiometer principle of comparing voltages.

In order to make the arrangement direct-reading, the contacts *C* and

D are set beforehand so as to balance the voltage of the standard cell at the proper divisions of the scale — in case of the Weston cadmium cell at about 1.0183 volts. (See § 72.) The exact setting depends upon the value of the e.m.f. of the cell used, and this e.m.f. must be known. The switch *L* is thrown to the left, and the rheostat *K* adjusted until the galvanometer returns to zero. Then the switch is thrown to the right, and the galvanometer balance again obtained by shifting *C* and *D*. With the position shown in the sketch, the unknown voltage is 0.620 volt.

The potentiometer scale usually has a range of 1.5 volts so that only voltages below this limit can be directly compared to that of a standard cell. For voltages above 1.5 volts, a multiplier, or the so-called "voltbox," is used, shown at the bottom of the sketch. It consists of a high resistance with one or more taps at a known part of it, say 1/10, 1/100, etc. The unknown e.m.f. is connected across the terminals *m* and *n*; the terminals *p* and *q* are connected to the terminals *BB* of the potentiometer. Let, for example, the unknown voltage be about 110 volts and the resistance between *p* and *q* be 1/100 of the total resistance of the volt-box. Then the voltage drop across *pq* is only about 1.1 volts and can be directly compared to the e.m.f. of the standard cell. Multiplying the result by 100, we get the actual unknown voltage.

74. EXPERIMENT 3-E. — Study of a Simple Potentiometer. — Before attempting to use an accurate and delicate potentiometer, it is desired that the student make clear to himself the working principle of the potentiometer, according to the simple diagram shown in Fig. 70. A German-silver or manganin wire stretched along a uniform scale may be used as the slide-wire; ordinary voltmeter leads with knife-edge terminals will do for contacts *C* and *D*. It is not necessary to use a standard cell; any dry cell whose e.m.f. has been previously determined will serve the purpose.

(a) Connect the circuits as shown in Fig. 70 and practice in comparing two e.m.f.'s small enough to be within the range of the slide-wire.

(b) Arrange the connections for measuring amperes by means of a standard shunt (Fig. 63); use the potentiometer in place of a millivoltmeter.

(c) Compare two resistances by measuring the drop at their terminals (Fig. 2).

(d) Finally improvise a volt-box and make clear to yourself the method of measuring voltages beyond the range of the slide-wire.

75. Leeds and Northrup Potentiometer. — A convenient form of potentiometer for ordinary engineering work is the Leeds and Northrup instrument (Fig. 71). The connections are shown in Fig. 72; the same notation is used as in Fig. 70. The larger portion, *SO*, of the slide-wire is replaced by 15 five-ohm resistance coils in series, with their terminals connected to contact buttons. This arrangement makes possible a considerable resistance without increasing the size of the instrument. The dial and the handle corresponding to the contact *C* are visible in Fig. 71 behind the switches. The part of the slide-wire between *O* and *T* is wound on a marble cylinder and is about 16 feet long, its resistance being exactly 5 ohms. The contact *D* is mounted inside an aluminum hood and moves up and down with the hood as it turns on a heavy screw projecting from the center of the marble cylinder. The slide-wire makes 10 turns on the cylinder, so that each turn corresponds to 0.01 volt. It is easy to read on the circular scale at least 1/100 of a turn, so that voltages can be accurately read down to 0.0001 volt or 1/10 milli-volt.

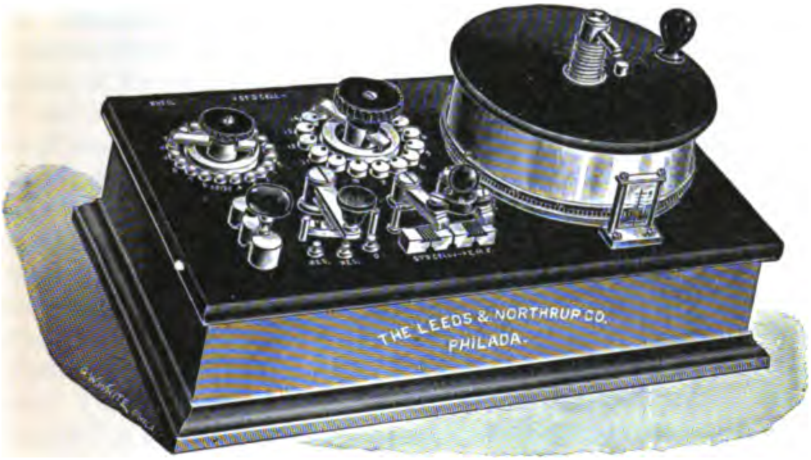


FIG. 71. A Leeds & Northrup potentiometer.

For measuring very low voltages a 10 times greater accuracy may be obtained by introducing the shunt marked 10 : 1; this is done by changing the plug shown at the left, from the hole 1 to that marked 0.1; the plug and the receptacles are clearly seen in Fig. 71. The resistance of the shunt and that of *R* bear such a ratio that when the shunt is applied, the voltage drop across the circuit *SOT* is reduced 10 times, provided the battery current remains the same. This gives the potentiometer a range of from 0.15 volt to 0.00001 volt, or 1/100 milli-volt. The connections to the standard cell and to the unknown e.m.f. are similar to those shown in Fig. 70. The switch-key *V* in the galvanometer circuit has three positions; in the two positions to the left a protective resistance is connected in series with the galvanometer. This is to prevent violent deflections of the galvanometer before the balance is obtained, and also to protect the standard cell against excessive currents which might polarize it.

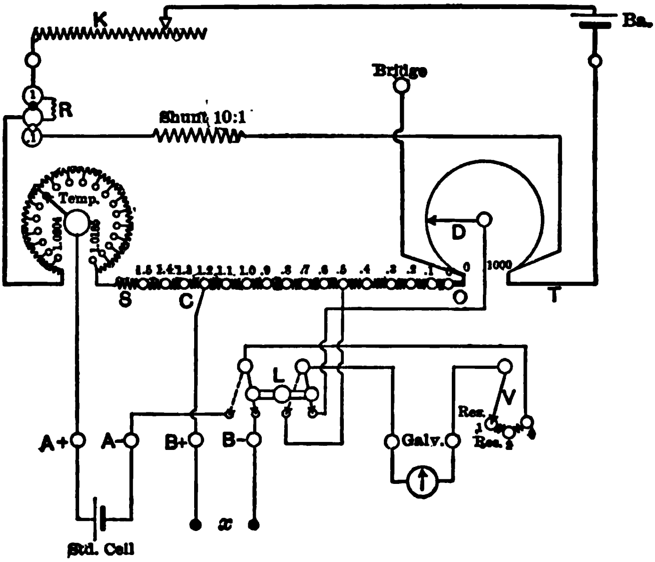


FIG. 72. Diagram of connections of a Leeds & Northrup potentiometer.

The connections are such that the contacts *C* and *D* are used for obtaining balance with the unknown e.m.f. only, but not when adjusting the current — this being done by means of the resistance *K*. It will be seen from the sketch that the standard cell is connected between the point 5 of the potentiometer resistance and the sliding contact on the dial to the left, marked "Temp." (temperature). The e.m.f. of the Weston cell being about 1.02 volts, the resistances connected to this dial permit the operator to set the potentiometer exactly at the certified e.m.f. of the cell, taking into account small variations depending on temperature or any other causes (§ 72).

When switch *L* is thrown to the left, the balance is obtained by regulating *K*, *with any position of C and D.* The advantage of such an arrangement is as follows: Let the resistance *K* be so adjusted as to give a balance with the standard cell, the connections being as in Fig. 70. Let now the switch *L* be thrown to the right, and a new balance obtained by moving *C* and *D*. In the meanwhile the battery current might have changed; in order to check this, it would be necessary to set *C* and *D* back into the position corresponding to the e.m.f. of the standard cell, and to throw the switch *L* to the left. This would mean setting *C* and *D* anew every time, and losing the previous setting. With the circuits shown in Fig. 72 the setting of *C* and *D* may be left intact, and the battery current checked by throwing *L* to the left and pressing the galvanometer button.

76. Checking Potentiometer Resistances. — Provision is made in the potentiometer shown in Fig. 72 for checking the resistances of the dial and of the slide-wire. The coils in the series *SO* are 5 ohms each and are connected to brass blocks with reamed holes (Fig. 71). A pair of flexible cords is furnished, with taper plug terminals to fit the reamed holes. The resistances of these coils can be measured with an ordinary Wheatstone bridge and thus compared with each other to a high degree of accuracy, even if the bridge is not very accurate. For potentiometer work the essential point is that they should be like each other, not that they should be accurately of any particular value. In the same way the resistance of the extended wire can be compared with the resistances of the coils in *SO* and should be found exactly equal to them. The heavy connector leading to the binding post marked " Bridge " is provided for this purpose in order that the extended wire may be used by itself as a Kohlrausch bridge (Fig. 15).

The calibration of the extended wire *OT* on the cylinder may be checked against the dial resistances *SO* by converting the potentiometer itself into a Wheatstone bridge. To do this, short-circuit the two outside upper binding posts with a heavy copper connector, not smaller than No. 10 B. & S.; also short-circuit the terminals *BB* and throw *L* to the right. Connect the battery between the terminals marked " Ba " and "Bridge." The contacts *C* and *D* form one junction of the two arms on each side of the bridge with the galvanometer between, while the posts " Bridge " and " Ba " form the second junction with the battery between. On one side there are 16 equal resistances of 5 ohms each, the resistance of the temperature dial being supplemented with an extra resistance, so as to make it exactly 5 ohms. If the extended wire be read in its 1000th parts, each of the 16 coils of the potentiometer circuit will be equal to $\frac{1}{16}$ of 1000, or 62.5 divisions when balanced against the wire.

77. Directions for Using Leeds and Northrup Potentiometer. — Because of the extended use of the above-described potentiometer, it has been deemed advisable to reprint here the explicit directions for its use, as given by the makers.

Setting up. — All connections must be made as indicated by the stamping on the potentiometer. Particular attention must be given to the polarity of the standard cell, battery and e.m.f., the corresponding + and − signs being marked.

If used with a wall galvanometer having a telescope and scale, it will be found convenient to place the potentiometer so that the telescope is directly over the glass index of the extended wire, thus permitting the observer to read the galvanometer deflections and potentiometer settings without change of position.

Potentiometer current. — A medium-sized storage cell will be found advantageous, producing a steady current. Errors of measurement are frequently due to an unsteady source of e.m.f.

Setting for standard cell. — Set the standard cell switch to correspond with the certified e.m.f. of the standard cell, as given in its certificate. *Place plug in hole* 1, *and see that it is always in this position when checking against the standard cell.* Place the double-throw switch at Std. Cell. The switch-key should be at the left of the three contacts marked RES., RES. and O., so as to include the greatest extra resistance in series with the galvanometer. This precaution is always to be taken before the final balance is obtained, to prevent any violent deflections of the galvanometer.

Adjust the regulating rheostat until the galvanometer shows no deflection. The final galvanometer reading should be taken with the switch-key at zero, so as to remove the series resistance and thus increase the sensibility. The middle "RES." contact can be used as an intermediate point of contact as the galvanometer balance is approached.

Measurement of unknown e.m.f. — The potentiometer gives direct readings for voltages up to and including 1.6 volts; therefore no voltage exceeding this amount must be connected to the e.m.f. posts of the instrument. For voltages higher than the direct range of the instrument, a volt-box or multiplier must be used.

The standard cell balance having been obtained as described under "Setting for Standard Cell," proceed as follows:

Place the double-throw switch in position E.M.F. and the galvanometer key to the left of the contacts so as to again include the highest series resistance. The balance for the unknown e.m.f. is now obtained by manipulating the tenths switch and rotating the contact on the extended potentiometer wire. The final position of the two contacts in conjunction with the position of the plug at left of instrument indicates the voltage under test, as described later.

Plug at 1 *or* 0.1. — Plug at 1 gives readings for voltage direct from settings of tenths switch and extended wire contact.

Plug at 0.1 shunts the potentiometer circuit so that the voltage measured is one-tenth. Therefore, the readings taken from the settings of the tenths switch and slide-wire contact must be divided by 10.

To balance galvanometer for unknown e.m.f. —

Place plug in hole 1 for voltages up to 1.6.
Place plug in hole 0.1 for voltages up to 0.16.

Rotate the tenths switch (having the galvanometer key at first RES.) until a condition of balance is obtained exactly or approximately. To secure an exact balance, rotate the contact on the extended wire. The unknown e.m.f. can now be read directly from the position of the tenths switch and the extended wire contact if plug is at 1, or by dividing by 10 if plug is at 0.1.

Example. — A balance was obtained with the tenths switch at 1.3, the extended wire contact at 176, and the plug at 1. The voltage under test, therefore, is 1.3176. If the plug at 0.1 had been used, the same reading would have indicated 0.13176.

To ascertain if current in the potentiometer circuit has altered during a measurement, it is only necessary to plug in at 1, place the double-throw switch on Std. Cell and close the galvanometer key. No deflection indicates that current is constant. If previous balance does not exist, the regulating rheostat must again be adjusted until the galvanometer shows no deflection.

To measure voltages from 1.6 *to* 16. — Voltages up to 16 volts may be measured by using a greater voltage across the *Ba* posts. For this purpose a battery of about 20 volts should be used. Insert the large plug at 0.1 and throw the switch to Standard Cell. Then balance the galvanometer by means of the regulating rheostat. When the rheostat has been set to secure a balance, insert the large plug at 1, set the switch on E.M.F., and read the voltage in the usual manner. Multiply the instrument reading by 10.

78. EXPERIMENT 3-F. — Calibration of a Voltmeter with a Potentiometer and Standard Cell. — The theory and the general method of measurements are given in §§ 73 and 74. For a description of a practical potentiometer and the instructions for its use, see §§ 75 to 77. Study carefully the connections of the available potentiometer; read the instructions furnished by the maker. By means of the potentiometer calibrate a D.C. voltmeter or measure an unknown voltage below 1.6 volts. Measure one or two voltages in the range between 1.6 and 16 volts, and finally a voltage above 16, using a volt-box. Pay particular attention to the sensitiveness and accuracy obtainable, and to the factors which influence these. If possible, check the resistances of the potentiometer itself, as explained in § 76; or devise another method more suitable for the apparatus at hand.

79. EXPERIMENT 3-G. — Calibration of an Ammeter with a Potentiometer and a Standard Resistance. — The experiment is similar to that described in § 66, except that the voltage drop across the shunt (Fig. 63) is measured with a potentiometer instead of with a milli-voltmeter. For the use of the potentiometer see §§ 73 to 78.

80. EXPERIMENT 3-H. — Comparison of two Resistances by means of a Potentiometer. — The method is the same as in Fig. 2, except that a potentiometer is used instead of an ordinary voltmeter or milli-voltmeter. For the theory and a description of the potentiometer see §§ 73 to 78.

81. Deflection Potentiometer. — The so-called "null" type potentiometer described above is suitable only for the measurement of steady currents and voltages, because it takes some time to obtain a balance. Moreover, the accuracy obtainable is often much greater than is required for many practical purposes. A need has been felt for some time for a potentiometer-type instrument of an accuracy intermediate between that of a null-type potentiometer and of the best indicating voltmeters and milli-voltmeters of the direct-deflection laboratory type. This problem has been solved in the so-called deflection potentiometer developed by Dr. H. B. Brooks of the Bureau of Standards, and shown in Figs. 73 to 75. In this instrument an approximate balance is obtained as in the ordinary potentiometer (§ 73) while the last one or two significant figures in the value of the unknown voltage or current are given by the deflection of the same galvanometer which indicates the approximate balance. This instrument has the following advantages over the null-type potentiometer: its setting requires much less time, and small fluctuations in the unknown current or e.m.f. can be directly followed on the galvanometer pointer without requiring new settings. At the same time it is more accurate than the best standard indicating instruments, and the unknown quantities are determined directly in terms of a standard cell and accurately adjusted resistances.

The particular instrument shown in Fig. 73 can be used (a) to measure voltages within the direct range from zero to 1.5 volts to within 0.0001 volt without the use of a multiplier, (b) in connection with a volt-multiplier for measurements of e.m.f.'s higher than 1.5 volts, and (c) to measure current by means of shunts, the instrument when so connected reading directly in amperes.

The volt-ranges obtained by means of the multiplier, viz., 3, 15, 30, 150, 300, and 750, together with that of the potentiometer direct, 1.5 volts, have been chosen with a view to facilitating the checking of indicating voltmeters having scales usually found in commercial use.

RANGES OF E.M.F. MEASUREMENTS

Maximum E.M.F. in Volts	Multiplying Factor*	Resistance in Ohms across which the unknown E.M.F. is connected	E.M.F. per Small Division of Galvanometer Scale
1.5	0.01	40	0.001†
3.0	0.02	80	0.002
15.0	0.1	400	0.01
30.0	0.2	800	0.02
150.0	1.0	4000	0.1
300.0	2.0	8000	0.2
750.0	5.0	20000	0.5

* The multiplying factor is that number by which the potentiometer readings as given on the instrument should be multiplied to reduce to volts.

† By estimating parts of a division, the values in this column can be read to about one-tenth of that given.

Current is measured by reading the drop across current-carrying shunts supplied with the potentiometer. These shunts are so constructed that the reading is either some direct decimal of the current value, or is modified by a multiplying factor of 5. There are seven standard sizes designed for use with the instrument. The current capacity and multiplying factor of each are given in the following table.

SHUNTS FOR USE WITH POTENTIOMETER

Maximum Potentiometer Reading	Maximum Ampere Capacity	Multiplying Factor	Resistance in Ohms	Amperes per Small Division of Galvanometer Scale
150	1.5	0.01	1.00	0.001
120	6.0	0.05	0.20	0.005
150	15.0	0.1	0.10	0.01
120	60.0	0.5	0.02	0.05
150	150.0	1.0	0.01	0.1
50	250.0	5.0	0.002	0.5
50	500.0	10.0	0.001	1.0

The potentiometer resistance is composed of thirty coils, the outer ring of switch contact points around the main dial *A* forming the 30 drop points. These are numbered 0, 5, 10, 15, etc., up to 150. The figures on the galvanometer scale (Fig. 74) are in the same units, and indicate up to three more or less than the number on which the main dial switch rests. In this way any value between the main dial points can be read by inspection of the galvanometer deflection, one small division on the galvanometer scale being equal to one-tenth of a main dial unit.

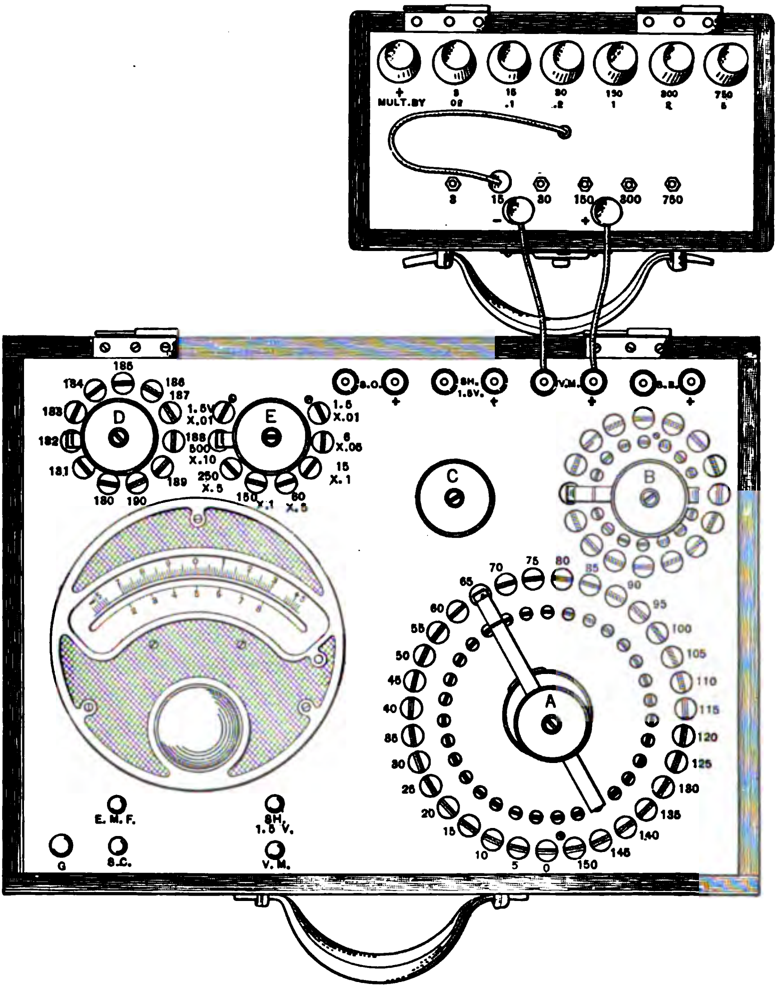


FIG. 73. The Brooks deflection potentiometer, General Electric Co.'s model.

There are two scales marked on the galvanometer. In the upper one, zero is located at the center. At the right of zero each consecutive ten small divisions are marked 1, 2, and 3 respectively, the corresponding locations at the left being marked 9, 8, and 7. On the lower scale the figure 5 is located at the center, the figures at the right and left being respectively 6, 7, 8 and 4, 3, 2. If the point on which the main dial switch stops ends in zero, the upper scale is read; if with a figure 5, the lower. It will be seen that if the needle is deflected toward the right, the total reading is always more than that indicated by the main dial switch; if to the left, it is less.

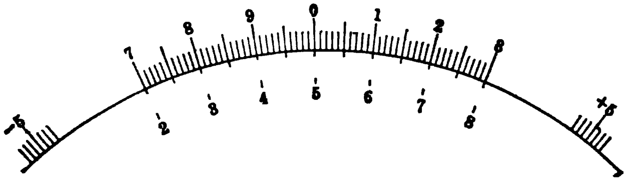


Fig. 74. The galvanometer scale of the deflection potentiometer.

For example, if the reading on the main dial is 55 and the galvanometer reads on the lower scale 4.71 (the last figure estimated as one-tenth of a small division), the total reading would be 54.71. But if the main dial read 60 and the needle were deflected to the same position, the upper scale would be read, viz., 9.71, the total reading being 59.71. If in the two examples cited the galvanometer had been deflected to the right the total reading would have been more than indicated by the main dial switch.

82. Operating Instructions for the General Electric Type of Deflection Potentiometer. — The following specific instructions are given by the makers of this type of instrument and are here reproduced in order to give a better idea of the possibilities of the deflection potentiometer.

Storage cell. — Connect a storage cell to posts marked " S.B. " observing the polarity signs. The e.m.f. of this cell must not exceed 2.1 or be less than 1.9 volts. If the potential is not within this range, it will be impossible to adjust the current to the correct value. The use of a lead cell between these limits assures a practically steady current. If the e.m.f. of the cell is too high, discharge it through a suitable resistance until the voltage is within the limits given above. Do not insert outside resistance in the circuit as this will introduce an error in the instrument readings.

Resistance of leads. — The total resistance of any two leads connecting the storage cell, volt-multiplier, or current-carrying shunts, should not exceed 0.05 ohm. (Five feet of No. 16 B. & S. lamp cord for each lead will be well within this limit.)

Standard cell. — Connect a standard cell to the posts marked " S.C." A Weston standard cell is used to determine the current flowing through the potentiometer coils. The e.m.f. of this cell, when in good condition, is always between 1.0180 and 1.0190 volts. The eleven points of the standard cell dial " D " are numbered consecutively from 180 to 190.

Galvanometer zero adjustment. — Place the switch of the standard cell dial on the point corresponding to the e.m.f. of the cell. Make sure that the galvanometer needle is at zero; if it is not, adjust by means of the small screw at the right of the scale opening. Push down the button marked " S.C." Press down the galvanometer key " G " and adjust the current by means of the coarse and fine rheostats " B " and " C " until the needle is brought exactly to the zero mark. This adjustment of current through the potentiometer coils should be made frequently in order to compensate for slight changes in the e.m.f. of the storage cell.

Measurement of e.m.f. not exceeding 1.5 *volts.* — Attach the terminals of the circuit to be tested to posts marked " 1.5 V.," place switch " E " on point marked " 1.5 V.," and connect the galvanometer through the high protective resistance by pressing the button marked " G " part way down.

Galvanometer locking device. — Lock in this position by turning the button so that the arrow points toward the back of the potentiometer. Push the buttons marked " E.M.F." and " 1.5 V." and rotate the main dial switch until the needle shows the least deflection; press the galvanometer key down as far as possible and read the value as above described, multiplying the reading by 0.01 in order to reduce to volts. This multiplying factor, 0.01, is written at the point on dial " E " at which the switch is set. If no balance can be obtained, it is probably due to the unknown e.m.f. leads being reversed.

Measurement of e.m.f. higher than 1.5 *volts.* — The volt-multiplier must be used if an e.m.f. higher than 1.5 volts is to be measured. Connect the two short posts on the multiplier to the potentiometer posts marked " V.M." and the positive terminal of the circuit under test to the line post on the volt-multiplier box marked " +," the selection of the other post depending on the e.m.f. to be measured. Place the flexible lead of the volt-multiplier on the point bearing the number corresponding to the terminal used. Push the button marked " V.M." and proceed to obtain a reading as above described, multiplying this by the factor given on the volt-multiplier.

Current measurements. — Select a shunt having the smallest rated capacity that will carry the desired current and connect the drop terminals on the shunt to the posts on the potentiometer marked "SH." Connect the current terminals of the shunt in series with the line carrying the current to be measured. Place the switch "E" on the point corresponding to the rated capacity of the shunt. Push the button marked "SH." and proceed to obtain a balance as above described. Multiply the reading obtained by the factor given on the switch "E" under the shunt rating at the switch point used. The result gives the value directly in amperes.

Checking galvanometer coils. — To check the accuracy of the galvanometer, connect up in the way before described for measuring voltages within the range of the instrument, viz., 1.5 volts. Supply the e.m.f. to the posts marked "1.5 V." by connecting to the drop terminals of a shunt having not more than 0.1 ohm resistance. Adjust the current through the potentiometer coils as before; push the buttons "E.M.F." and "1.5 V." and regulate the current in the shunt until the galvanometer shows no deflection when the main dial switch is on any convenient point. When it is accurately balanced, move the main dial switch to the next point in either direction and note the deflection. If the galvanometer is reading correctly, the needle should be over either the point marked "− 5" or "+ 5" depending upon which way the switch was moved. A deviation of two-tenths of a small division from the points −5 or +5 means a maximum error, when making measurements in the regular way, of only one-tenth of a small division at full-scale deflection (2.5), or 1/100 of one per cent when the main dial switch reads 100.

Checking potentiometer coils. — Proceed as in checking the galvanometer, except that a balance should be secured on each of the 30 points of the main dial, beginning with point 5, by adjusting the current through the shunt, moving the switch at each balance one point back and noting the deflection of the needle. The deflection should be the same on all of the coils if they are in good condition. These coils are adjusted to within one part in 10,000 before the instrument leaves the factory. The check above described is only approximate, not better than to within one part in 500, but serves to show if any coil has suffered serious damage.

83. Theory of the Deflection Potentiometer. — The principle of the Brooks deflection potentiometer described above is shown in Fig. 75. E is the unknown voltage to be measured, and is above 1.5 volts. CD is the resistance of the volt-box, the galvanometer circuit being tapped off at points C and F. The total resistance of the volt-box is R, and the point F corresponds to one p-th part of it from the left. Therefore the resistances CF and FD are marked R/p and $R(p-1)/p$ respectively. AB is the main potentiometer resistance fed from the storage battery e_1; it corresponds to the slide-wire ST in Fig. 70. The current in the main potentiometer circuit is regulated by means of the resistances r_3 and r_6, corresponding to the dials B and C in Fig. 73. When the contact points N and P are moved to the right the current i_1 in AB is increased, and vice versa. The galvanometer tap-off point, H, corresponds to point C in Fig. 70. It will be seen further that simultaneously with the change in the position of point H the resistance r_4 of the galvanometer circuit is varied at point L. The variable points H and L are obtained in Fig. 73 by means of the two rows of buttons controlled by the handle A.

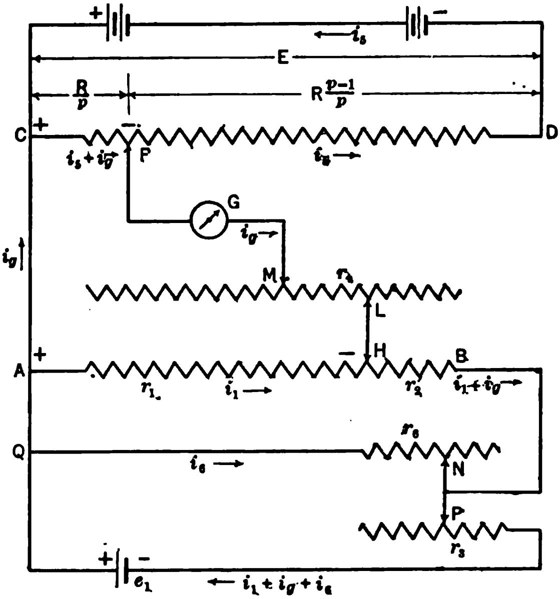


FIG. 75. The principle of the deflection potentiometer.

The purpose of this seemingly complicated arrangement is to keep the total resistance of the galvanometer circuit constant, independent of the positions of the sliders HL and NP, as is required by the theory indicated below. *The galvanometer circuit is understood here to comprise all the resistances in Fig.* 75, *connected as shown, and considering the galvanometer itself as if it were a source of voltage, with the e.m.f.'s* e_1 *and* E *removed and a short-circuit put in place of each.* Beginning with the galvanometer, the part $GMLH$ of the galvanometer circuit has no branches; between H and A the "galvanometer circuit" consists of the branch r_1 in parallel with the series-parallel combination $HBNPQ$. The latter consists of r_2 in series with the parallel combination of r_3 and r_6. Between A and C there are no branches, and from C to F there are two branches in parallel, namely $CF = R/p$ and $DF = R(p-1)/p$. The circuit is completed from F to G by a single conductor. The resistances r_3 and r_6 are in parallel in the galvanometer circuit as defined above. They are so adjusted that when the slide NP is moved to the right or to the left, the combined resistance of r_3 and r_6 in parallel remains constant. Similarly, when HL is moved to the right or to the left, the resistance r_4 compensates for any change in the combined resistance of r_1, r_2, r_3, and r_6, considering the latter combination from the point of view of the galvanometer circuit.

The galvanometer deflections must be correct in the sense of giving the last decimals of the unknown voltage or current (a) when a different tap F is taken on the volt-box; (b) when the volt-box is dispensed with for e.m.f. measurements below 1.5 volts, and (c) when an ammeter shunt is inserted between C and F for current measurements. A different ballast resistance is introduced in each case into the galvanometer circuit to keep its total resistance constant. This is obtained automatically on the dial switch E (Fig. 73) so that no attention on the part of the operator is required beyond setting the switch to correspond with the particular ammeter shunt or voltage ratio used.

(1) *An e.m.f. less than* 1.5 *volts.* — Consider the simplest case of measuring an e.m.f., E, below 1.5 volts, and assume this e.m.f. to be due to a cell of negligible resistance, connected directly between the points C and F, with the volt-box omitted. Let an approximate balance be obtained at H, as explained in the preceding section, and let there be a small current i_g in the galvanometer circuit. The problem is to find E from the potentiometer setting and from the value of the galvanometer current i_g. A current i_g in the direction indicated in the sketch means that the difference of potential C-F is less than A-H, the points A and C being practically at the same potential. Consider the unknown voltage E to consist of two component parts, E_1 and ϵ, and let E_1 acting alone give a perfect balance ($i_g = 0$) with the setting at the same point H. We can now apply the principle of superposition of currents in a network of conductors in which two or more e.m.f.'s are applied, viz., that *the actual current in a conductor is equal to the sum of the currents produced in the same conductor by the individual e.m.f.'s* or *their partial combinations acting separately.* Let the e.m.f.'s e_1 and E_1 be applied simultaneously as the first partial combination. The e.m.f. E_1 by supposition is such that in combination with e_1 it gives $i_g = 0$, and the setting at H reads the value of E_1 directly. As the other partial e.m.f. take ϵ, acting separately, with e_1 replaced by a short-circuit. If the equivalent resistance of the galvanometer circuit, as described above, is R_g, then

$$\epsilon = i_g R_g \quad \dots\dots\dots\dots \quad (1)$$

so that the galvanometer current is a measure of the unbalanced part of the unknown voltage E, and with the assumed direction of i_g the correction ϵ must be subtracted from E. The resistance R_g must be a constant for all settings of HL and NP, in order that the galvanometer may be calibrated directly in decimal fractions of a volt, as is the main potentiometer dial. Then only can a galvanometer reading be added directly to a dial reading, as is stated in the directions for the use of the deflection potentiometer. Thus the potentiometer setting measures E_1, and the galvanometer current indicates the remainder of E.

If the source of e.m.f., E, has a considerable resistance ρ, so that the voltage drop ρi_g has an appreciable value, this resistance must be considered as a part of the resistance of the galvanometer circuit, and the ballast resistance reduced accordingly. Since ρ itself may be variable and may affect the value of E, it is safer not to use the direct deflection principle for such measurements but to apply a null potentiometer. A good galvanometer requires a current of only 10 microamperes per scale division, so that the internal resistance of the source E must be quite high indeed, or E itself very small, in order to affect appreciably the accuracy of the direct-deflection principle.

(2) *An e.m.f. greater than* 1.5 *volts.* — Now, let the unknown voltage E be applied through a volt-box, as shown in Fig. 75, and let it be again required to determine E in terms of the setting at H, of the volt-box ratio p, and of the galvanometer current i_g. In order to do this, imagine a small e.m.f., ϵ, to be inserted in the galvanometer circuit, either in the part AC or in the part FM, in such a direction as to oppose the current i_g. Adjust ϵ to such a value that it satisfies equation (1), that is, produces a current $-i_g$, and apply again the principle of superposition, assuming the voltages E, e_1, and ϵ to act simultaneously. Since the first two together produce the current i_g and the third one the current $-i_g$, the combination will give a zero galvanometer current. Thus, with this combination the voltage drops CF and AH are exactly equal to each other and the potentiometer setting at H reads the value which is lower than the true voltage CF by the known amount ϵ. But in the actual measurement ϵ is read on the galvanometer directly, in the form of the current i_g, so that the true voltage CF is again equal to the algebraic sum of the setting at H and the galvanometer reading.

It must be clearly understood that the deflection potentiometer does not read the actual voltage CF, but that which would exist if the galvanometer current were equal to zero. Only under such conditions does the volt-box subdivide the applied voltage in the correct ratio p of the resistances. In other words, with the galvanometer properly calibrated as a voltmeter and with its resistance kept constant, the deflection gives the true electrostatic difference of potential between the points F and H, as if the galvanometer circuit were broken between these points, and as if the circuits CDE and ABe_1Q had only one common connection AC with zero current flowing through it. The potentiometer thus measures that value of the voltage drop CF which when multiplied by p gives the correct value of the voltage E across the volt-box CD. Any internal resistance in E or any resistance in series with the volt-box has nothing to do with this result. The applied e.m.f., E, may be increased to $E + E'$, and the part E' consumed in some resistance, leaving again part E across the volt-box. Thus, the given voltage source $E + E'$ of appreciable resistance may be replaced by an e.m.f., E, of zero resistance, and the reasoning given above will apply. The resistance of the galvanometer circuit, external to the potentiometer, must comprise only R/p and $R(p - 1)/p$ in parallel, but no external or internal resistance of the unknown e.m.f., E.

(3) *Voltage drop across an ammeter shunt.*—Now, let the resistance CF be that of an ammeter shunt. Assume a counter-e.m.f., $\epsilon_1 = R_s i_g$, to be inserted between C and F in series with the ammeter shunt of resistance R_s, and a counter-e.m.f., $\epsilon = R'_g i_g$, to be inserted between F and M. Here R'_g is the resistance of the galvanometer circuit from C through the potentiometer to F; that is, R'_g is different from the former value R_g in that it does not include any of the resistances of the circuit CDE. With these two additional e.m.f.'s in the circuit, the current in CF becomes equal to i_6 and the current in FD remains equal to i_5. The galvanometer current i_g is reduced to zero. Thus, the voltage drop iR_s in the shunt is smaller than the potentiometer voltage V between A and H by the amount $\epsilon_1 + \epsilon_g$; in other words,

$$iR_s = V - i_g(R'_g + R_s) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Thus, in order that the galvanometer may indicate as a voltmeter the resistance of its circuit must be equal to $R'_g + R_s$, and the resistance of the part FD or the internal resistance of E does not enter into the problem. With the ballast resistance properly adjusted, galvanometer deflections represent the true corrections to the approximate settings at H, and with a simple conversion coefficient the instrument reads directly in amperes.

Another way of explaining the meaning of the galvanometer deflection when measuring a current with a shunt is as follows: Let the applied e.m.f., E, be considered as made up of a part E_1 which will give $i_g = O$, and of a residual ϵ_3. If E_1 act alone, it will send a current V/R_s through R_s, where V is the difference of potential corresponding to the null setting on *AB*. If e_3 act alone, with e_1 replaced by a short circuit, it will send the current i_g through the branch *CAHF*, and also the current $(R'_g/R_s)i_g$ through R_s. The sum of these two currents resulting from e_3, plus the current caused by E_1, gives the actual current i_5 at the time of observation; that is,

$$\begin{aligned} i_5 &= \frac{V}{R_s} + i_g + \frac{R'_g}{R_s} \cdot i_g \\ &= \frac{1}{R_s}\left[V + (R_s + R'_g)\, i_g\right] \\ &= \frac{1}{R_s}(V + R_g\, i_g) \quad \ldots\ldots\ldots\ldots \quad (3) \end{aligned}$$

Here R_g consists of R'_g plus the resistance of the shunt. This expression is identical with equation (2) and is not limited in any way by the resistance in the source of current.

ATLERNATING-CURRENT POTENTIOMETERS

84. The difficulties in the way of measuring alternating e.m.f.'s on the potentiometer principle (Fig. 70) are as follows: two continuous e.mf.'s differ from each other in magnitude only, while two alternating e.m.f.'s may differ in magnitude, phase, frequency, and wave-form. Moreover, there is no standard of alternating e.m.f. corresponding to a standard cell for continuous voltages. If the problem be limited to that of comparing two strictly sinusoidal e.m.f.'s of the same frequency, then the known e.m.f. must be adjustable not only in magnitude but in phase as well. There are at least two practical methods of accomplishing this end, viz.:

(a) By means of a phase-shifter (Drysdale, Fig. 76).

(b) By resolving the known e.m.f. into two components in quadrature with each other (Larsen, Fig. 77).

An alternating-current potentiometer is quite useful for the calibration of ammeters, voltmeters, and other alternating-current instruments, for the determination of the ratio and of the phase angle of instrument transformers, for capacity and inductance measurements, for the study of alternating leakage fluxes, for research with artificial transmission or telephone lines, and generally for accurate measurements with alternating currents.

85. Drysdale Potentiometer. — The principle of the Drysdale potentiometer is shown in Fig. 76, and the lettering corresponds to that of the standard direct-current potentiometer shown in Fig. 70. The unknown voltage drop to be measured is that between the terminals

BB, across part of the circuit *MN*, fed from the same alternating-current line *FH* to which the phase-shifting transformer of the potentiometer is connected. The stationary part of this transformer is wound like the stator of a two-phase motor (§ 472) while the movable part has one diametral winding. One phase of the stator, *aa*, is connected directly across the line; the other, *bb*, is connected to the same line in series with a resistance *R* and a condenser. The connections are similar to those in a split-phase motor (§ 492) so that a revolving magnetic field is produced in the air-gap. The phase of the secondary induced e.m.f. depends upon the position of the secondary winding with respect to the two primary windings. The secondary part may be turned into any desired position by means of a worm gear, and the angle of shift from the zero position is indicated on a scale. See also the potential-shifter in § 127b.

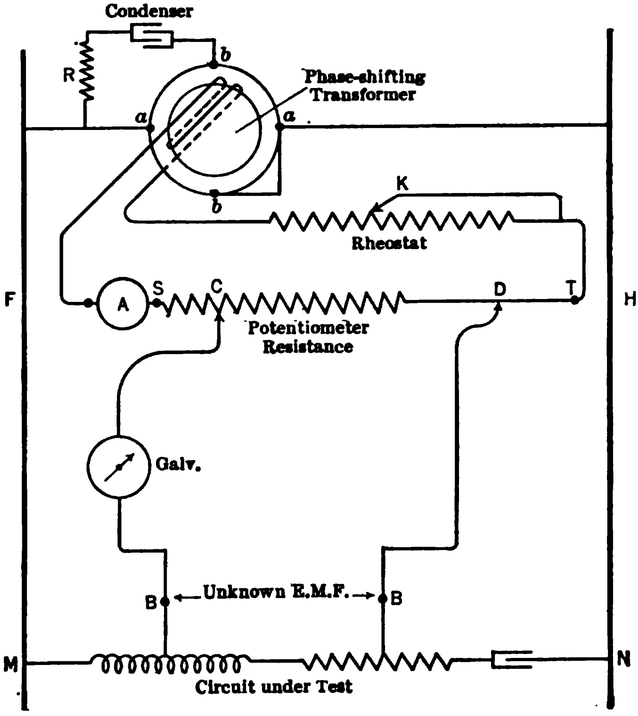


FIG. 76. The principle of the Drysdale A.C. potentiometer.

No standard cell can be used directly, and for this reason the current in the "slide-wire," or in the potentiometer circuit proper, *ST*, is adjusted by means of an alternating-current ammeter *A*. Since this current is but a fraction of an ampere, a dynamometer-type, hot-wire, or thermo-electric instrument should be used. Any of these instruments may be accurately calibrated with direct current (§ 69). The use of this milli-ammeter limits the accuracy of the whole potentiometer because the accuracy of the result cannot be greater than that with which the current in *ST* can be adjusted to a prescribed value. The galvanometer which indicates the equality of the voltages *CD* and *BB*, is a null instrument of the vibration type, tuned accurately to the frequency of the supply (§ 43). A telephone receiver may be used instead, although it is difficult to obtain perfect silence in it on account of unavoidable higher harmonics. A crystal detector (rectifier) with a direct-current galvanometer has also been successfully used.

To measure an alternating voltage, the current in the ammeter *A* is adjusted by means of the rheostat *K* to such a value that the indications on the dials corresponding to the slide-wire *ST* read directly in volts. Then the contact points *C* and *D* and the phase-shifter are adjusted until the galvanometer indication is zero. The unknown voltage is then read directly on the dials and the corresponding phase angle on the phase-shifter. This angle has no particular meaning unless two e.m.f.'s are measured. Then the difference of the two angles represents the electrical phase displacement between the two e.m.f.'s. Non-inductive ammeter shunts are available, by means of which alternating currents can be measured to the same degree of accuracy as alternating voltages. Since it is possible to measure a current, a voltage, and the phase angle between the two, the instrument may also be used for power measurements.

86. Larsen Potentiometer. — This potentiometer, in the form further developed by Dr. A. E. Kennelly and his co-workers, is shown in Fig. 77. It is particularly useful at higher frequencies, say above 1000 cycles per second, when the source of testing current is an oscillator (Vreeland oscillator, pliotron, etc.). In such a case the amount of power necessary to operate the phase-shifting transformer in Drysdale's potentiometer is prohibitive, and it becomes necessary to vary separately the magnitude of each of the two quadrature components of the known e.m.f. In Fig. 77, *O* is the source of alternating power, connected inductively through the coil K_w to the working circuit (circuit under test) and through the coil K_p to the potentiometer circuit. The two important parts of this circuit are a non-inductive resistance *ab* and the reactive coil *bc*, without iron core. The winding *bd* is placed in an inductive relation to *bc*. When the current in the galvanometer *G* and in the winding *bM* is zero, the flux linking with *bM* is due to the current in the winding *bc*; that is, this flux is in phase with the current in the potentiometer circuit. Hence, the voltage induced in *bd*, or in any part of it, is in quadrature with the flux or with the potentiometer current. The voltage drop across *ab*, or across any part of it, is in phase with the potentiometer current. Thus, by means of the sliding contacts *M* and *N* it is possible to impress between points *B* and *D* an e.m.f. of any desired magnitude and phase within the limits of the device. When the galvanometer current is zero, the known and the unknown e.m.f.'s are equal and opposed to each other. The known value is read on the dials, as in any potentiometer.

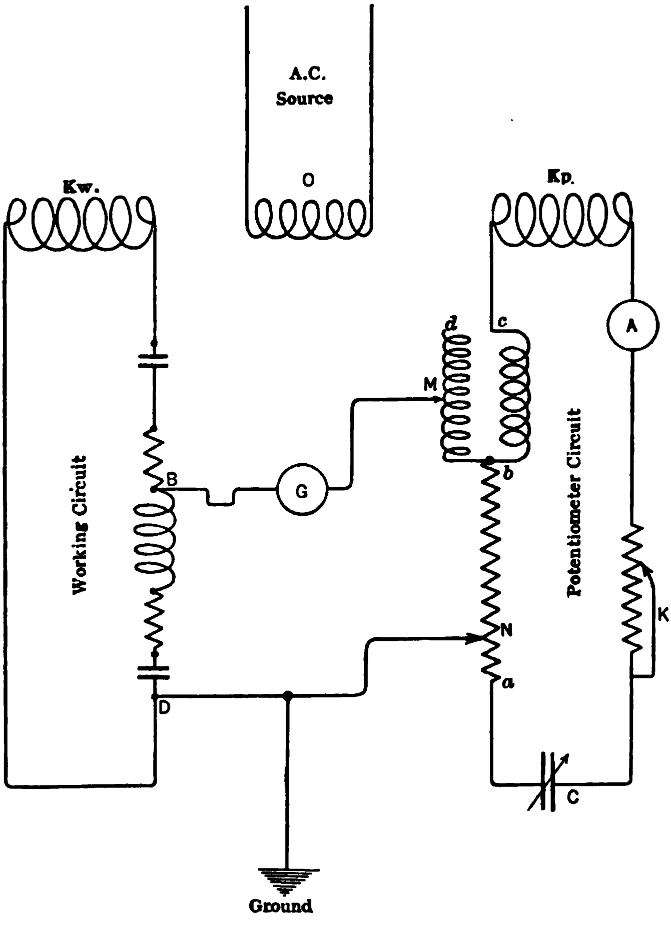


FIG. 77. The Larsen A.C. potentiometer.

Because of the absence of a standard cell, the potentiometer current is adjusted to a certain value on the ammeter *A* by means of the rheostat

K and condenser *C*. The latter serves to change the phase of the current. The ground connection shown in the sketch is advisable at higher frequencies, in order to minimize parasitic currents due to the distributed capacitance of the apparatus. Stray alternating magnetic fields must also be guarded against.

87. The Kelvin Ampere Balance. — When using a potentiometer and a standard cell for the measurement of a current, the latter has to be obtained each time indirectly as the ratio of an e.m.f. to a resistance. A direct secondary standard of current is available in the so-called Kelvin balance (Figs. 78 and 79). It is based on the electro-dynamometer principle (§ 49), but differs from the usual instruments in that the coils lie in parallel planes and the electromagnetic attraction is balanced by means of a weight instead of by a spring. Such an instrument is calibrated either by means of a silver voltameter, or with a potentiometer, using a standard resistance and a standard cell.

A Kelvin balance consists of four stationary coils *A*, and two movable coils *B* suspended by metal strips *C*; these strips serve also as current leads for the coils *B*. When the instrument is used as an ammeter, the six coils are connected in series; a current flowing through the coils produces attractions and repulsions, as marked in Fig. 79. The influence of terrestrial magnetism is neutralized by making the current in the two movable coils flow in opposite directions.

A weight is placed in the trough *D*; with this weight the movable coils are in balance without current when the sliding weight *w* is in its extreme left position; this position corresponds to the zero of the scale. When a current is flowing through the instrument, it is necessary to move *w* to the right in order to keep the movable coils in balance. The value of the current is indicated on the scale by the position of the weight *w*. It will be seen in Fig. 78 that the instrument is provided with two scales: one is the accurate uniform scale; the other, the so-called inspectional scale, is calibrated directly in amperes. The ampere divisions are proportional to the square roots of the distances from the left end of the scale, because the attraction between the coils is proportional to the square root of the current (§ 40). The inspectional scale is accurate enough for ordinary purposes. When a greater accuracy is required the uniform scale is used; the current is then calculated by extracting the square root of the reading and multiplying it by the constant of the instrument.

The weight *w* is slipped into its proper position by means of a self-releasing pendant, hanging from a hook carried by a sliding platform, which is pulled in the two directions by two silk threads passing through holes to the outside of the glass case (Fig. 78). For the fine adjustment a small rider is provided, as in an ordinary chemical balance. The rider is actuated by a fork, the handle of which is visible below the base. The round knob in the base, fastened to the chain, lifts the moving coils off the suspension when the instrument is not in use. Four

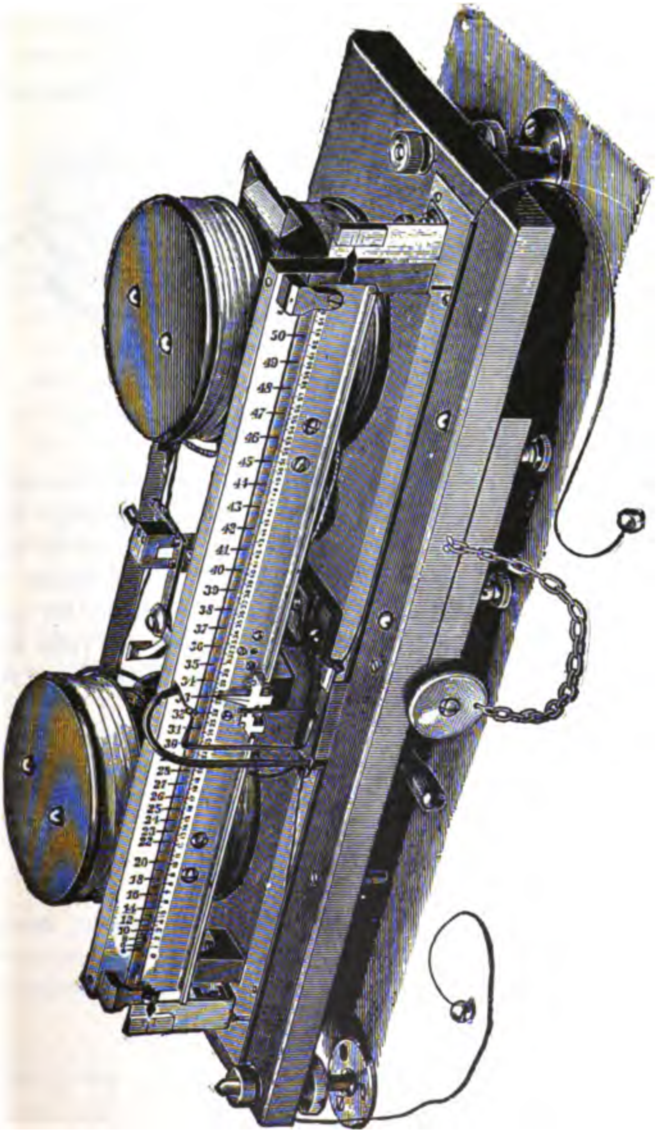


Fig. 78. The Kelvin balance for measuring currents.

pairs of weights (sliding and counterpoise) are supplied with each instrument; of these the carriage of the moving weights and its counterpoise constitute the first pair. These weights are adjusted in the ratios of 1 : 4 : 16 : 64, so that each pair gives a round number of amperes, or half-amperes, or quarter-amperes, or of decimal subdivisions or multiples of these magnitudes of current, on the inspectional scale.

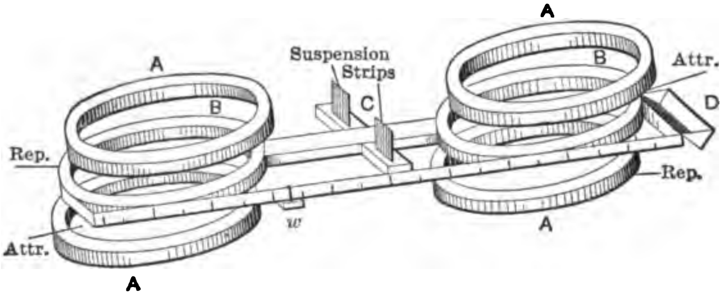


Fig. 79. The principle of the Kelvin balance.

These instruments can be used equally well with direct or alternating currents, and are made in seven sizes covering the range from 0.01 ampere to 2500 amperes. The smallest size may also be used as a voltmeter, when provided with a suitable high resistance in series (multiplier). Balances are built on the same principle for measuring watts; also composite balances for measuring amperes, volts and watts with one and the same instrument. Detailed instructions for using the balance always accompany the instrument.

CHAPTER IV

WATTMETERS AND POWER-FACTOR METERS

88. A WATTMETER is an instrument for measuring electric power delivered to a circuit. The name " wattmeter " comes from the name "watt," which is a practical unit of power, in the volt-ampere-ohm system. There are three types of wattmeters: (a) *Indicating wattmeters*, which show the average value of power or the rate at which energy is consumed over a short interval of time; (b) *Recording*, or graphic, *wattmeters* which trace a curve of such average values on a record sheet; (c) *Watt-hour meters*, sometimes also called integrating wattmeters, which sum up and record the *total energy* delivered during an interval of time. The latter are treated in the next chapter.

To make this distinction clearer, take the case of a 100-volt direct-current circuit, in which the current regularly fluctuates between 50 and 70 amperes every 10 seconds. Imagine a wattmeter of each of the three above types connected to such a circuit, and observe their behavior — say for five minutes. The needle of the indicating wattmeter will fluctuate regularly between the divisions of the scale, corresponding to 5000 and 7000 watts. The recording wattmeter will trace a wavy line between the same values (Fig. 61). The dial of the watt-hour meter will show at the end of five minutes

$$\frac{(7000 + 5000)}{2} \times 5 = 30{,}000 \text{ watt-minutes,}$$

or

$$\frac{30{,}000}{60} = 500 \text{ watt-hours.}$$

89. Power in Alternating-Current Circuits. — The power expended in a direct-current circuit (Fig. 26) is equal to the product of the current delivered times the voltage at which it is supplied. In practical units

$$\text{watts} = \text{amperes} \times \text{volts}$$

or, with the customary notation,

$$p = i \times e \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

Thus, in a direct-current circuit, power may be measured with an ammeter and a voltmeter, unless the load is rapidly fluctuating. In an alternating-current circuit the presence of self-induction and capacitance so modifies the relations that the product of the "effective volts" times "effective amperes" (§ 40) is not equal to the true average watts expended in the circuit. The relation (1) is, however, always true for the corresponding instantaneous values of current and voltage. In other words,

$$dw = p \cdot dt = i \cdot e \cdot dt \quad . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

represents the true electrical energy delivered to an alternating-current circuit during an infinitesimal element of time dt; i and e are instantaneous values of current and voltage. The foregoing expression is true for any wave-shape whatsoever. The average energy per second, that is the average rate at which energy is delivered, or the *power in watts*, is

$$P = (1/T) \int_0^T i \cdot e \cdot dt \quad . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

where T is the time of one cycle of the alternating current.

Assume first that both the current and the voltage vary according to the sine law. We have then

$$i = I_m \operatorname{Sin} 2\pi ft \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

$$e = E_m \operatorname{Sin} (2\pi ft \pm \phi) \quad . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

Here ϕ is the *phase angle* (§ 140) by which the e.m.f. leads or lags behind the current, and $f = 1/T$ is the frequency, or the number of cycles per second (Fig. 109). Substituting the values of i and e from equations (4) and (5) in eq. (3) we obtain

$$P = (I_m E_m / T) \int_0^T \operatorname{Sin} 2\pi ft \cdot \operatorname{Sin} (2\pi ft \pm \phi) \cdot dt$$

or, after integration,

$$P = \tfrac{1}{2} E_m I_m \operatorname{Cos} \phi.$$

Substituting the *effective* values, E and I, of the voltage and current, as shown by the ammeter and voltmeter (see Note on p. 78), we find that the *average real power* is

$$P = EI \operatorname{Cos} \phi \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

Equation (6) shows that the true power delivered to an alternating-current circuit depends not only upon the effective values of the current and voltage, but also upon the phase angle between the two. The expression EI is sometimes called the *apparent power;* the ratio between the true power and the apparent power, or Cos ϕ, is called the *power factor* of the part of the circuit under consideration (see also § 140).

Now, let both the current and the voltage be *non-sinusoidal,* although of the same frequency. Equation (3) still holds true, and P represents the true average power. But in order to define the power factor a further assumption is necessary. For example, let the effective values of the current and voltage, as measured, say with hot-wire instruments, be again denoted by I and E, and let the average true power, measured with a *wattmeter*, be P. Then by analogy with eq. (6), the power factor may be defined as

$$\text{Cos}\,\phi = P/(EI) \quad \ldots\ldots\ldots \quad (7)$$

While eqs. (6) and (7) are identical in form, the angle ϕ in eq. (6) is an actual phase angle, while in eq. (7) it is a fictitious angle which refers to the *equivalent* sine values of current and voltage. The foregoing definition of the phase displacement ϕ with non-sinusoidal currents is not the only possible one. For another definition see the discussion on the measurement of reactive power (§ 102).

It will be seen from formula (3) that in order to measure the true average power in an alternating-current circuit, an indicating instrument is required which automatically integrates the instantaneous products of current and voltage. Such an instrument is called an *indicating wattmeter.* Two types of such wattmeters are described below, the electro-dynamometer type and the induction type.

ELECTRO-DYNAMOMETER-TYPE WATTMETER

90. An indicating wattmeter based on the electro-dynamometer principle (§§ 48 and 49) is shown in Fig. 80. The internal electrical connections are shown in Fig. 81. The instrument has two stationary coils and a moving coil between them, as in the voltmeter shown in Fig. 40. The movable coil is controlled by spiral springs, as in Fig. 34.

The stationary coils consist of a few turns of heavy wire or strip connected in series with the circuit, as in an ammeter. The movable coil consists of many turns of fine wire, and is similar to the movable coil of a d'Arsonval-type voltmeter. It is connected across the circuit, in series with a high non-inductive resistance, called the multiplier. The current through the movable coil is practically proportional to, and in phase with, the voltage. The instantaneous torque between the stationary and the movable coil is proportional to the product of the instantaneous currents in them, and is thus proportional to the product of the current and the terminal voltage. According to equations (1) and (2), the *mechanical impulse* due to this torque (torque times an infinitesimal interval of time dt) is proportional to the instantaneous energy delivered to the circuit. If the movable part of the instrument had no inertia, it would vibrate in accordance with the fluctuations of instantaneous power. But with usual frequencies of alternating currents, these fluctuations follow in such quick succession that the movable element assumes the position corresponding to the *average* impulse; thus it automatically integrates the power over a cycle, according to expression (3).

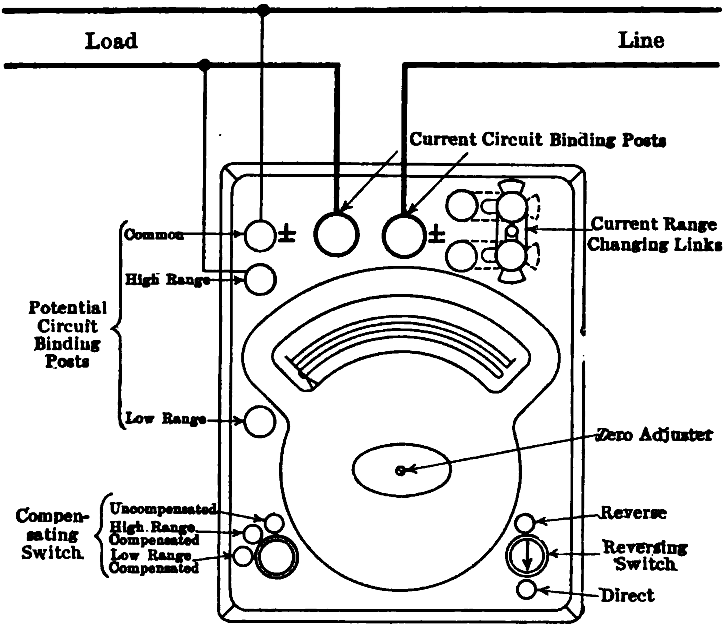


FIG. 80. External connections and a general view of an indicating wattmeter.

With a non-inductive load, that is, when the current and the voltage are in phase, the torque is positive at all instants. On a partly inductive load the product of the instantaneous volts and amperes is negative over a part of the cycle (Fig. 109). Since the average torque in the instrument is counterbalanced by a spiral spring, whose restraining torque is proportional to the deflection, *the indication corresponds to the true average power regardless of the power factor.* The same is true for non-sinusoidal currents and voltages of any wave form.

The scale is divided directly in watts or in kilowatts (1 kilowatt = 1000 watts). The instrument may be calibrated with a standard wattmeter or with a standard direct-current ammeter and voltmeter, and may be used on either direct or alternating current. The current terminals (Fig. 80) are connected in series with the circuit in which it is desired to measure the power. The potential terminals are connected across the line. In some portable precision instruments, the potential circuit should be kept open when no reading is being taken, so as not to overheat the movable coil and the multiplier.

A slight error is introduced by the increase in the resistance of the moving coil with the temperature, since the current through the potential circuit then becomes less with the same applied voltage. In some high-grade wattmeters, this error is compensated by means of a non-inductive shunt around the movable coil. This shunt is made of a material possessing a comparatively high resistance-temperature coefficient, so that with the increase in temperature a larger fraction of the total current flows through the moving coil. At high frequencies the temperature compensating shunt introduces a phase angle of sufficient magnitude to cause a slight error (§ 93). For further details of construction of dynamometer-type wattmeters see § 96.

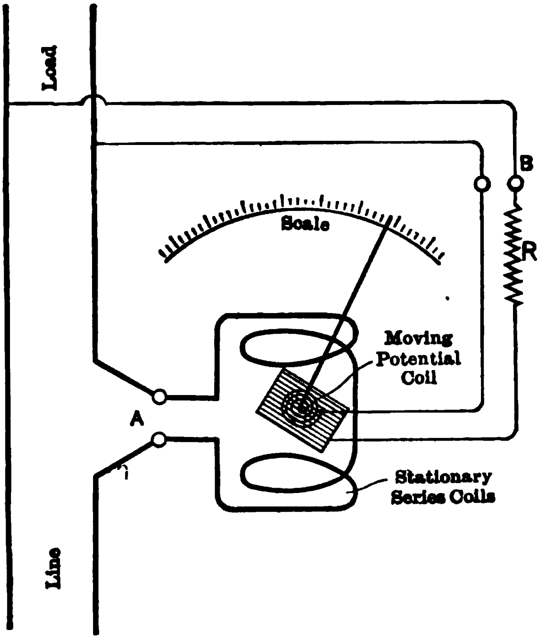


FIG. 81. The arrangement of parts in an indicating wattmeter.

91. Correction for Power Consumption in the Wattmeter Itself.— An indicating wattmeter, such as is shown in Fig. 81, consumes some power in its series and potential circuits. The inaccuracy introduced thereby is in some cases large enough to require a correction. It will be easily seen from Fig. 81 that the current in the series winding of the wattmeter is a sum of the load current and the current consumed in the potential circuit of the wattmeter itself. Therefore, the instrument indicates more power than is actually consumed in the load. The corresponding correction is

$$i^2R = E^2/R \quad \ldots\ldots\ldots\ldots \quad (7a)$$

where i is the current in the potential circuit, R the ohmic resistance of this circuit, and E is the load voltage. In accurate measurements, the resistance of the potential circuit and the load voltage may be ascertained and the above correction computed and subtracted from the wattmeter reading.

Sometimes the potential leads are connected to the "line" side of the current coils, as, for example, in the watt-hour meter shown in Fig. 94. In this case, the current coils carry the true load current, but the voltage across the potential terminals of the instrument is the line voltage and not the load voltage. If the resistance of the series winding is r and the load current is I, the correction for the power consumed in the instrument itself is equal to I^2r.

Before connecting a wattmeter in a circuit, it is advisable to estimate the possible error due to the power consumption in the instrument itself, and to decide whether the potential terminals should be connected across the load or across the line. Consider, for example, a 110-volt supply; at this voltage the current through the potential circuit of a wattmeter is of the order of 21 milli-amperes, and consequently the power consumption is about 2.3 watts. Consider two wattmeters, one rated at 100 amperes and loaded at 100 amperes, the other rated at 1 ampere and loaded at 1 ampere. Assuming the resistance of the series circuit of the first wattmeter to be 0.5 milli-ohm and that of the second 0.7 ohm, we get the results shown in the following table. The first reading in each case refers to the wattmeter connected as in Fig. 80 and the second reading corresponds to the case with the potential coil across the line. A careful study of this table will show the difference in the conditions in the two cases.

POWER CONSUMPTION IN TWO WATTMETERS

Connection	(1) Load Current (Amperes)	(2) Amperes through Current Coil	(3) Load Voltage	(4) Voltage across the Potential Circuit	(5) True Power (Watts)	(6) Instrument Indication (Watts)	(7) Correction	
							E^2/R (Watts)	I^2r (Watts)
Voltage coil across the load..........	100	100.021	110	110	11,000	11002.3	2.3	None
Voltage coil across the line...........	100	100	110	110.05	11,000	11005	None	5.0
Voltage coil across the load..........	1	1.021	110	110	110	112.3	2.3	None
Voltage coil across the line...........	1	1	110	110.70	110	110.70	None	0.70

It is usually preferred to have the wattmeter connected as in Fig. 81, and to correct for the loss of power in the potential winding. The reason is that it is more difficult to measure the resistance r of the series winding, and, moreover, it depends on the contact resistance at the current terminals.

92. Compensation for Power Consumed in the Instrument.—Some wattmeters are provided with a compensating winding, which automatically corrects for the power consumption in the instrument itself, thus simplifying its use. Such a compensated wattmeter is shown in Fig. 82, the compensating coil being connected in series with the potential coil. Without the compensating coil, a small current flows through the series and the potential windings of the wattmeter (Fig. 81) even when the load is zero, and causes the pointer to indicate some power, although in reality the power consumption is zero. The number of ampere-turns in the compensating winding is equal and opposite to that created by the series coils, so that the pointer indicates zero at no load. The compensation is good at all voltages, since the current in the compensating coil is always the same as in the series winding (at no load). The potential circuit, between the terminals C and B, consists of the compensating coil, the movable potential coil, and a high resistance (multiplier) R.

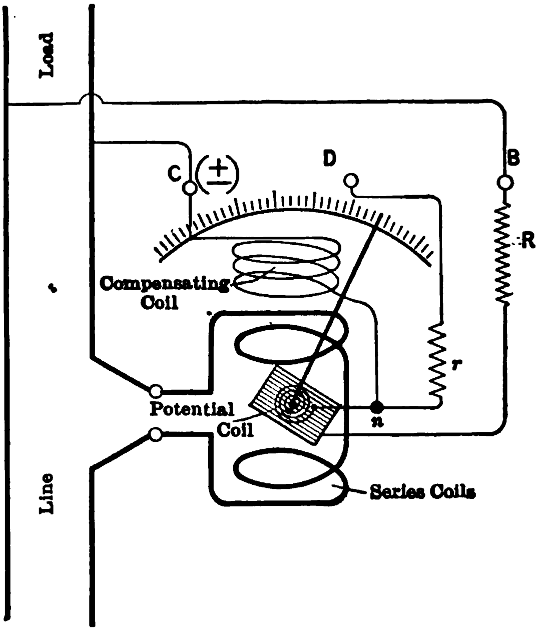


FIG. 82. Compensation for the power consumption in the wattmeter itself.

In cases in which the series winding and the potential winding are supplied from two independent sources, the compensating coil should be left out of the circuit, and the terminal D used, instead of C. The "uncompensated," or "independent," terminal D is connected to the potential coil through a low resistance r equal to the resistance of the compensating coil. This is done in order to have the same total resistance between B and D, as between B and C. In this way the instrument calibrated between one pair of terminals reads correctly between the other pair of terminals.

The uncompensated terminal D should be used, instead of C, when the wattmeter is being calibrated with a separate source of current for the series winding. Such is, for example, the case when a large storage cell is supplying the current, while a battery of small cells gives the required voltage. With an arrangement of this kind, a large wattmeter can be calibrated with a comparatively small expenditure of power. See also § 128.

93. Inaccuracy at Low Power Factors. — Theoretically there should be no inductance in the potential circuit of a dynamometer-type wattmeter, in order that the current in the moving coil be in phase with the line voltage. In reality, the potential coil and the multiplier have a small amount of inductance, so that, strictly speaking, the current in the potential circuit lags slightly behind the applied e.m.f. The inaccuracy thus introduced is usually negligible, especially at high values of power factor, but with very low power factors may become quite noticeable.

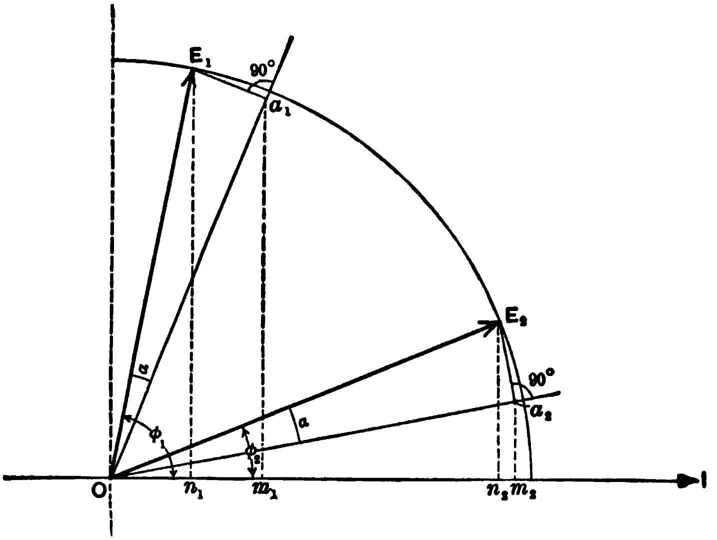


FIG. 83. Vector diagram, showing the influence of inductance in the potential circuit of a wattmeter.

The influence of the power factor is shown in Fig. 83. I is the vector of the current in the series coil. E_1 is the vector of the applied e.m.f. at a low value of the power factor of the load (large phase displacement ϕ_1). If the potential circuit had no inductance, the current in the moving coil of the wattmeter would be in phase with E_1 and the indications of the instrument at a given current I would be proportional to the working component $On_1 = E_1 \cos \phi_1$ of the voltage. In reality, the current in the potential coil is lagging behind E_1 by a small angle α, due to the inductance of the potential circuit. The result is the same as if the applied voltage were reduced to Oa_1, with the working component Om_1. Applying the same reasoning at a high power factor, corresponding to the applied voltage OE_2, the points n_2 and m_2 are obtained. It is easy to see that with the same current I, the same voltage E, and the same angle of lag α in the potential circuit, the percentage of error, $n_1m_1 \div On_1$, is much larger at the low power factor than the percentage of error, $n_2m_2 \div On_2$, at the high power factor. When the value of α for a given instrument is known, readings can be corrected either graphically, as in Fig. 83, or by computation, by means of eq. (9) or eq. (10) given below

To express the error analytically we can write, for any angle ϕ in Fig. 83:

$$Om = E \operatorname{Cos} \alpha \operatorname{Cos} (\phi - \alpha)$$
$$On = E \operatorname{Cos} \phi.$$

Hence, the error in the ratio is

$$\epsilon = (Om - On)/On = \frac{\operatorname{Cos} \alpha \operatorname{Cos} (\phi - \alpha)}{\operatorname{Cos} \phi} - 1 \quad . \; . \; . \quad (8)$$

Expanding, we find

$$\epsilon = \tfrac{1}{2} \operatorname{Sin} 2\alpha \operatorname{Tan} \phi - \operatorname{Sin}^2\alpha \quad . \; . \; . \; . \; . \; . \; . \quad (9)$$

When α is small, say one or two degrees, and ϕ is of considerable magnitude, we can put approximately $\operatorname{Sin} 2\alpha = 2 \operatorname{Sin} \alpha$ and $\operatorname{Sin}^2\alpha = 0$. The preceding expression is then simplified to the following *approximate* one

$$\epsilon = \alpha \operatorname{Tan} \phi \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (10)$$

where α is in radians. For example, when $\operatorname{Cos} \phi = 0.6$, $\operatorname{Sin} \phi = 0.8$ and $\operatorname{Tan} \phi = 4/3$, the error per degree of α is

$$\epsilon = (\pi/180)\ (4/3) = 2.3 \text{ per cent.}$$

Under ordinary circumstances, and with a good instrument, the inaccuracy due to this cause is negligible, but under extreme conditions a correction, as in Fig. 83, may be necessary. When the dielectric loss in a high-grade condenser is being measured, and in other similar cases in which the voltage and the current are practically in quadrature with each other, and the measured power is extremely small for the values of current and voltage involved, special methods of power measurement may be necessary. The reader is advised to consult the Bureau of Standards or makers of accurate measuring instruments. He will also find considerable information in the publications of the principal electro-technical and physical societies.

For corrections due to errors in instrument transformers see § 95.

94. EXPERIMENT 4-A. — Calibration and Study of an Indicating Wattmeter of Electro-Dynamometer Type. — The instrument is described in §§ 90 to 93; it is calibrated with direct current, and may be used on either direct or alternating current.

(1) Connect the wattmeter under test to a direct-current line and provide a suitable load; into the same circuit connect an ammeter and a voltmeter which have been previously standardized. Begin the calibration with the largest load; read volts, amperes, and watts. Gradually reduce the load and take similar readings at each step. In connecting the wattmeter keep in mind the explanations in §§ 91 and 92 and take properly into account the energy consumption in the instrument itself. To see the influence of this factor, use in turn the uncompensated and the compensated potential terminals (Figs. 80 and 82) and note the difference in the indications with the same load. See if the i^2R correction accounts for the discrepancy; try this with a large and with a small load.

(2) Connect the wattmeter in such a way as to have the error for the power consumption in the instrument itself first in one and then in the other form mentioned in § 91. Make clear to yourself the order of magnitude of this error and the best way of connecting the wattmeter with a large and with a small current, and at a high and at a low voltage.

(3) Measure an A.C. load at a constant current and voltage, and at a variable power factor, beginning with the highest. Devise a method for ascertaining whether the current is leading or lagging. Show how to determine whether power is flowing into the load, or the load is pumping power back into the line. Measure a large load at a low power factor by overloading the current and the potential coils of the wattmeter for a short time. Show that the error is much greater than at a high power factor. Devise some other method of measuring the power input into a load of very low power factor.

(4) Make a study of the influence of terrestrial magnetism and of stray fields, both D.C. and A.C., upon the accuracy of the instrument. Try various combinations, such as a stray D.C. field and A.C. currents in the instrument, etc., and make clear to yourself theoretically, which combinations should affect the indications and which should not.

Report. Give a calibration curve in the form shown in Fig. 66. State in detail your findings in regard to the energy consumption in the instrument itself and in regard to the influence of the terrestrial field and stray fields. Give data showing that the true power is less than the apparent power when the load is partly inductive. Figure out the power factor of the load according to eq. (7) in § 89. State the difficulty experienced in measuring a load at a very low power factor and the method used for overcoming this difficulty.

95. Effect of Inaccuracies in Instrument Transformers upon a Wattmeter. — When a correctly calibrated indicating wattmeter or watt-hour meter is connected to a line through very accurate current and potential transformers (§ 54), its indications are simply multiplied by the nominal ratios of these transformers. When greater accuracy is required and when the transformer errors cannot be neglected, the wattmeter should be calibrated and its potential circuit adjusted in combination with the transformers. This is not always possible on account of heavy currents or high voltages involved, and for this reason it may be necessary to correct the wattmeter readings for the effect of errors in the instrument transformers. These errors are discussed in detail in Chapter XVIII. The errors in the ratio are taken directly from the calibration curves of the transformers, and the resultant error in the wattmeter due to this cause is very nearly equal to the sum of these errors. Thus, if the current ratio is 0.2 per cent high and the voltage ratio is 0.5 per cent high, the error in volt-amperes will be

$$(1 + 0.002)\,(1 + 0.005) - 1 = 0.002 + 0.005 + 0.00001$$

or very nearly 0.7 per cent, the last term being negligible.

The wattmeter error due to the phase angle of the transformers depends upon the algebraic difference of the phase angles of the current and voltage transformers respectively. For example, if the angular error due to the current transformer is 1° 40′ leading, and that due to the potential transformer is 30′ leading, the effect is the same as if the phase displacement between the current and the voltage were increased by 1° 10′. Let this *resultant* phase angle error be denoted by α, and let the correct phase angle between the line current and the line voltage in the primary circuit be ϕ, with the current lagging. The secondary phase angle, that is the one in the wattmeter circuit, is $\phi - \alpha$, and the wattmeter reads high. The correction ratio, to be subtracted from unity, is

$$\begin{aligned}\epsilon &= [\mathrm{Cos}\,(\phi - \alpha) - \mathrm{Cos}\,\phi]/\mathrm{Cos}\,\phi \\ &= 2\,\mathrm{Sin}\,(\phi - \tfrac{1}{2}\,\alpha)\,\mathrm{Sin}\,\tfrac{1}{2}\,\alpha/\mathrm{Cos}\,\phi. \quad \dots . \quad (11)\end{aligned}$$

When α is small, say one or two degrees, and ϕ is a much larger angle, the foregoing expression is simplified *approximately* to

$$\epsilon = \text{Sin}\,\alpha\,\text{Tan}\,\phi \qquad (12)$$

or even to

$$\epsilon = \alpha\,\text{Tan}\,\phi \qquad (13)$$

where α must be in radians. See also § 93.

The error in a current transformer is a function of the current itself, and the error due to the resultant phase angle is a function of the power factor ϕ of the load. Thus, the total error due to the inaccuracies in the current and potential transformers is variable and cannot be allowed for in the construction or adjustment of the meter itself. For this reason it is essential to keep the errors in instrument transformers at a minimum.

96. Construction Details of Dynamometer-Type Wattmeters.— Most indicating wattmeters used for practical measurements are of the direct-deflection type described above. For very accurate work, or for calibration purposes, dynamometer-type wattmeters of the torsion type are also used, similar to the Siemens dynamometer shown in Fig. 42, except that the movable coil is in the potential circuit and consists of many turns of fine wire, so that no mercury cups are necessary. The current through the moving coil is so small that it can be easily conveyed by soft metal leads or by the suspension wires.

As a primary watt standard, a Kelvin-type balance instrument is sometimes used, similar to the ampere balance described in § 87, except that the movable coils are wound and used as potential coils of a wattmeter. The attraction and repulsion between two sets of coils, caused by the currents in the coils, is measured by balancing it against a weight or the torsion of a spiral spring. If a weight is used, it is arranged to be moved along a scale, and its distance from the fulcrum, when equilibrium is established, is a measure of the torque between the sets of coils. If a spring is used, it is put in tension by means of a knob carrying a pointer that moves over a circular scale. The scale in either case is marked in watts. The weight or the spring is adjusted to bring a pointer (attached to the moving coils) to the zero position. The motion of the moving coils is very slight, making the bearing wear negligible; the coils are always in the same position when in balance, eliminating possible errors due to varying magnetic conditions in different positions; the coils are close together, eliminating errors due to external magnetic fields; no iron is used in the magnetic circuit, precluding frequency errors.

For measurement of very small amounts of power, a direct-deflection electro-dynamometer is sometimes used in which the moving coil is provided with a mirror, and the deflection is read by means of a telescope on an illuminated scale, as in a mirror-type galvanometer. In very accurate instruments of this type, no metal is used in the supporting parts so as to avoid eddy currents. The current coils are made of stranded copper to minimize eddy currents.

Accurate wattmeters for calibration purposes are sometimes of the astatic type, that is, with a double movement similar to that shown in Fig. 60. The effect of the external fields is thereby minimized. High-grade magnetic shielding is a more positive protection against irregular external fields and should be used whenever possible. This is of particular importance in measurements at a low power factor at which the moving torque is comparatively small. To increase the deflection a weak controlling spring or metal strip is used for suspension.

When using an indicating wattmeter at a low power factor, it is sometimes necessary to overload its potential or current circuit or both in order to obtain a sufficient deflection. For example, a 10-ampere, 150-volt wattmeter will give a full-scale deflection of 1500 watts at these values of current and voltage on a non-inductive load. At a power factor of 10 per cent the deflection with the same values of current and voltage will be only 150 watts, which may not be sufficiently accurate. For such a measurement a 5-ampere, 75-volt wattmeter is preferable, provided that its coils can stand 100 per cent overload for a short time. The full scale of the latter wattmeter corresponds to 375 watts, so that 150 watts can be read satisfactorily. For this reason the coils of many indicating wattmeters on the market are guaranteed to stand a double current for a short time. Special wattmeters are also made for measurements at low power factor and with a correspondingly higher current consumption in the potential circuit. For the necessary precautions and errors in measuring power at low values of power factor, see §§ 93 and 95 above.

The range of a given A.C. wattmeter may be increased indefinitely by the use of current and potential transformers (§ 54 and Chapter XVIII). As a matter of fact, most industrial wattmeters are of the 5-ampere, 110-volt type, even though the scale may read thousands of kilowatts. For most practical purposes the errors due to these transformers are negligible, but in special cases and for accurate measurements they must be carefully considered (§ 95).

For power measurements in a polyphase circuit, two or more single-phase wattmeters may be used, or one wattmeter transferred from phase to phase, as shown in § 58. A more convenient way is to use a so-called polyphase wattmeter which consists of two movements, such as are described above, mounted on the same shaft so that their indications are automatically added and the sum is read on the scale. It is also possible to mount the two movements side by side and have them connected by a light aluminum rod. The so-called "two-wattmeter" method of polyphase power measurement is explained in § 100.

The stationary coils of wattmeters used for the measurement of very large currents are sometimes made hollow and cooled by circulating water, thereby increasing their current-carrying capacity as much as ten times. This method is resorted to for very accurate measurements, especially at low values of power factor when the errors due to a current transformer are inadmissible.

In high-grade indicating wattmeters, the entire coil system is mounted in a closed iron case which effectually shields it from external magnetic fields and electrostatic influences. In high-tension measurements it is of importance to connect one side of the potential coil near the same point of the circuit at which the current coils are connected, in order to avoid a large difference of potential within the instrument, and an electrostatic action between the coils. A large difference of potential occurs then only in the multiplier resistance outside the instrument.

In most instruments of the electro-dynamometer type the natural damping is insufficient and an additional damping is usually provided by means of a light aluminum vane connected to the moving system and placed in an enclosed box. Graphic or curve-tracing wattmeters are usually of the electro-dynamometer type, and are similar in their general construction to the graphic ammeters and voltmeters described in §§ 59 to 61.

INDUCTION WATTMETERS

97. For alternating-current switchboard service, where no extreme accuracy is required, wattmeters based upon the principle of a revolving magnetic field are sometimes used. The magnetic and electric circuits of such an *induction-type wattmeter* are shown in Fig. 84. The iron core AA consists of thin laminations to reduce eddy currents, and the magnetic circuit is completed through a cylindrical iron core C. In the annular air-gap G between these two cores an aluminum drum is mounted in which secondary currents are induced by the alternating fluxes in the pole-pieces. Because of the reaction of these currents upon the inducing fluxes (§ 98) the drum tends to rotate. Its deflection is controlled by a spiral spring and is indicated on the scale. There are three windings: the series winding through which the main line current flows; the potential winding connected across the line in series with a high reactance; a compensating winding closed upon itself through an adjustable resistance *R*. Very often these instruments are not connected to the line directly, but through potential and current transformers (Chapter XVIII).

With a non-inductive load, the flux due to the shunt winding must be in time quadrature with that due to the series winding, because under those conditions the torque exerted upon the aluminum drum is a maximum. The current in the potential winding cannot be made to lag behind the terminal voltage by exactly 90 degrees, because of the unavoidable resistance both in the winding itself and in the reactor in series with it. For this reason the compensating winding, short-circuited through the resistance *R*, is used. By varying this resistance, the secondary current is adjusted to such a value that the flux in the core, due to the combined magnetomotive force of this current and of that in the potential winding, lags exactly 90 degrees behind the terminal voltage. If a two-phase supply is available, a non-inductive load may be connected to one phase through the series winding of the wattmeter, and the potential circuit to the quadrature phase. Resistance *R* is

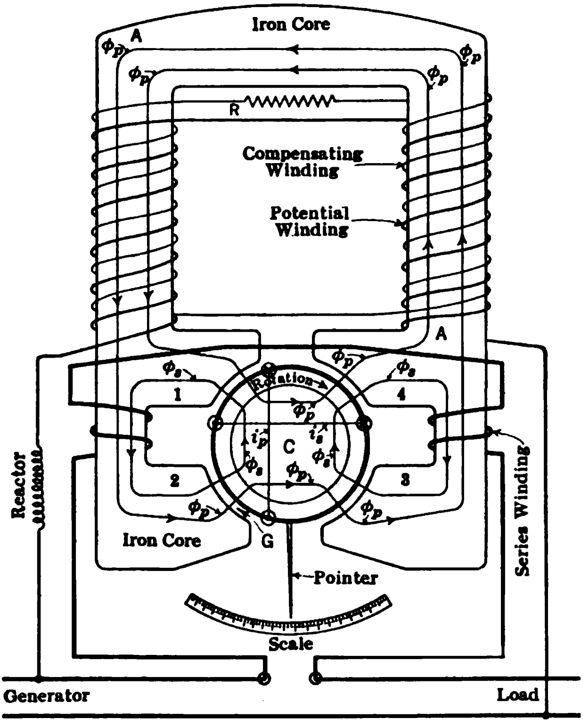


FIG. 84. An induction-type indicating wattmeter.

then adjusted until the pointer shows zero. Under these conditions the wattmeter will give a maximum indication when the potential circuit is connected to the same phase as the load. A wattmeter so adjusted at zero power factor will read nearly correctly at any other value of power factor, because only the components of the two fluxes in time quadrature with each other produce a torque upon the aluminum drum.

The action of the secondary winding in preventing temperature errors is the same as in the ammeter and voltmeter based on the induction principle (§ 52). The voltage circuit consists partly of copper and partly of an alloy having zero temperature coefficient. The amount of each is so proportioned that the resistance drop varies at the same rate as the reactive drop, thus maintaining a constant angle of lag at all temperatures.

The instrument is somewhat affected by the frequency and by the wave shape of the current and voltage, but is accurate enough for switchboard service. Its long scale, extending over nearly 300 degrees, and a rugged construction with no leads going into the moving part, constitute additional advantages. It has to be calibrated by means of a dynamometer-type wattmeter (§ 90), which in turn is standardized on direct current.

For polyphase circuits a similar wattmeter consists of two single-phase movements such as those described above, electrically and mechanically independent, but with the two aluminum drums mounted on the same shaft. The total torque on the shaft is equal to the sum of the torques of the two movements, and the meter automatically adds and indicates the total power in the two phases. An iron shield between the two elements prevents stray fields set up by one element from affecting the torque of the other, and the meter is not affected by any unbalancing of a polyphase circuit. For further details see §100 below and also Vol. II.

98. Theory of Induction-type Wattmeter. — Consider first the case of a non-inductive load, with the compensating resistance R properly adjusted so that the potential flux ϕ_p and the series flux ϕ_s (Figs. 84 and 85) are in time quadrature with each other. Each of these fluxes induces secondary currents in the aluminum armature, and the torque is due to the interaction of the flux ϕ_p with the current i_s, and of ϕ_s with i_p.

Consider first the action of the flux ϕ_s, assuming it to be sinusoidal in time. It induces in the drum an electromotive force $e_s = - d\phi_s/dt$ in a lagging time quadrature with itself, as shown by the e_s curve in Fig. 85. The current due to this electromotive force is lagging behind it by a certain angle, which depends upon the relative resistance and reactance of the paths in the drum. This current may be resolved into a component i_s in phase with e_s, and another in quadrature with e_s. The latter is not shown in the figure, because it does not contribute to the useful torque. Similarly, the flux ϕ_p induces a voltage e_p, and of the resulting current only the in-phase component, i_p, is shown as a sine-wave. The same relations are shown vectorially to the right.

We shall assume that at the instant of time marked *mm* the fluxes ϕ_p and ϕ_s have the directions indicated in Fig. 84. These directions are arbitrary and depend upon the direction in which the corresponding coils are wound and connected to the circuit. At the instant under consideration the flux ϕ_s is decreasing, and, therefore, the corresponding secondary current i_s is shown linking with it and so directed as to strengthen it. On the other hand, the flux ϕ_p is increasing, and the induced current i_p is shown linked with it and opposing the flux. By applying the familiar rule for the mechanical force between a flux and a current, it will be found that the action of i_s and ϕ_p is such as to tend to rotate the drum clockwise, and the action of ϕ_s and i_p is in the same direction. It will be seen from Fig. 85 that ϕ_p and i_s pass through zero at the same instant, so that the torque due to these two merely fluctuates between zero and a maximum, but never reverses. The same is true of ϕ_s and i_p. Thus, the direction of the torque deduced for a particular instant is true for any instant; that is, the tangential mechanical effort is cumulative. The drum assumes the position at which the counter-torque of the controlling spring is equal to the average electromagnetic torque. The disregarded quadrature components of the two currents, with the corresponding fluxes, give alternating torques, the average value of each being zero.

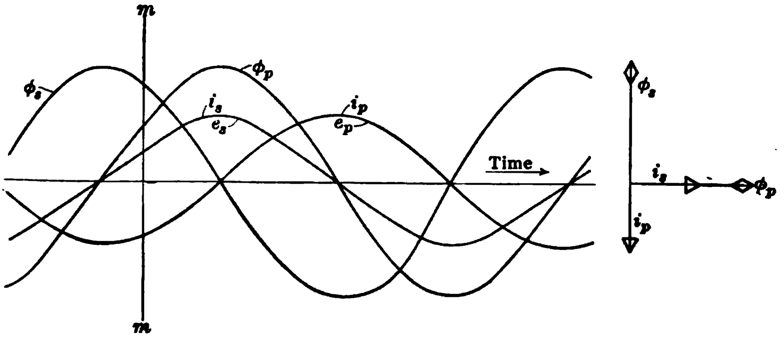


FIG. 85. The currents and the fluxes in the wattmeter shown in Fig. 84.

Now let the load be partly inductive. The main current may be considered as consisting of the component I Cos ϕ in phase with the voltage, and I Sin ϕ in quadrature with it. The component I Cos ϕ acts in the wattmeter as a non-inductive current, and therefore produces upon the moving element a torque proportional to the true power EI Cos ϕ. It remains to be shown that the quadrature component I Sin ϕ contributes no torque. For this purpose the sine-waves and the vectors of ϕ_s, e_s, and i_s in Fig. 85 must be shifted by 90 degrees in one or the other direction, according to whether the current is leading or lagging. Then by repeating the reasoning given above it may be shown that the individual torques either give the average value equal to zero or else the two torques oppose each other. Thus, theoretically at least, the instrument calibrated at unity power factor reads the true power at any other phase angle.

Should the impressed frequency be increased, the current and the flux in the potential circuit would be reduced and the compensation would no longer be correct. The currents induced by both fluxes in the aluminum drum would also be different. Thus, generally speaking, the calibration depends upon the frequency. It is possible, however, so to proportion the instrument that the calibration would remain sensibly constant within the range of frequency variations encountered in power plant operation. A non-sinusoidal applied voltage or current may be considered as a superposition of a fundamental sinusoidal quantity and of various harmonics of higher frequencies. In as much as the calibration of the instrument is affected by the frequency, it cannot give correct readings on a circuit of any arbitrary wave-form, as is approximated by the dynamometer wattmeter. However, by proper design the instrument may be made to read with a sufficient degree of accuracy within the usual range of wave-form, or it may be specially calibrated for a particular wave-form.

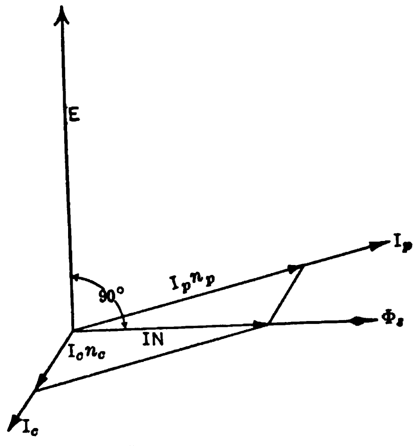


FIG. 86. The corrective action of the compensating winding in the wattmeter shown in Fig. 84.

The diagram in Fig. 86 explains the corrective action of the compensating winding. E is the vector of the applied voltage and I_p is the current in the potential winding which lags behind the voltage by less than 90 degrees. The current in the compensating winding is I_c

The corresponding numbers of turns are n_p and n_c. The combined magnetomotive force, IN, of the two windings is obtained as the diagonal of the parallelogram constructed on I_pn_p and I_cn_c. The flux ϕ_s is due to IN. It will be seen that by properly selecting the proportions, the flux ϕ_s can be made to lag behind E by exactly 90 degrees, even though I_p is not in quadrature with E. It is of advantage to have the secondary load R consist of a non-inductive resistance so as to have the current I_c as nearly as possible in phase opposition with E and thus to affect the primary excitation I_pn_p by as large an angle as possible. The diagram in Fig. 86 is an ordinary transformer diagram (Fig. 252).

99. EXPERIMENT 4-B. — Calibration and Study of an Induction-Type Wattmeter. — The instrument is described in §§ 97 and 98. Provide a suitable load whose magnitude and power factor may be varied at will. A phantom load (§ 128) and a phase-shifting transformer (§ 127) will be found convenient. Have an ammeter, a voltmeter, and a standardized dynamometer-type wattmeter in the circuit. For greater accuracy it may be desirable to have two standard wattmeters, one for the higher, and one for the lower range of the instrument to be calibrated. A power-factor meter (§ 104) would save time in making load adjustments. The source of power must be nearly sinusoidal and its frequency must be kept constant. Arrange the instruments in the circuit in such a way that their own power consumption will have the least influence upon the results and that a correction can be made for this power consumption (§§ 91 and 92). (1) If a two-phase circuit is available check the phase compensation and set it right. Otherwise adjust the instrument to read correctly at the lowest possible power factor and also at unity power factor. Show experimentally that the influence of this adjustment is much greater at low values of power factor. (2) Adjust the load so as to have the highest safe value of current and the lowest possible power factor. Read all the instruments. Reduce the current in steps, keeping the phase angle nearly constant, and take similar readings at each step. (3) Take similar runs at one or two higher values of power factor and finally at non-inductive load. (4) Investigate the effect of frequency and of wave-form upon the calibration of the instrument. (5) Note discrepancies in the construction, if any, between the instrument tested and the one described in the text above.

Report. (1) Give the diagram of connections used and the form of the data sheet. (2) Plot curves of percentage of error against true watts as abscissæ. (3) Describe the adjustment of the compensating resistance and give data showing its influence upon the accuracy of the meter under different load conditions. (4) Give data showing the influence of the frequency and wave-form upon the instrument error. (5) Referring to Figs. 84 and 85, explain in detail the reason for which reversing either the current or the potential leads makes the armature deflect in the opposite direction. (6) Describe differences, if any, between the instrument tested and the one shown in the text.

100. The Two-Wattmeter Method. — Let a source of electric power (Fig. 87), direct or alternating, be connected to a steady load by means of three conductors, and let it be required to measure the average power delivered to the load. This can be done by means of two wattmeters, A and B, connected as shown in the figure. Conductor 2 is taken to be the "common return wire" for the other two. Wattmeter A measures the power between the conductors 1 and 2, and wattmeter B that between 3 and 2. The total power delivered to the load is equal to the algebraic sum of the two readings. If the source of power delivers direct current and the load is constant, the wattmeter readings represent the actual energy delivered to the load per unit time. With alternating or variable currents and voltages, the wattmeter readings give the *average* power or the average rate at which the energy is delivered.

The method of connections shown in Fig. 87 is usually referred to as the *two-wattmeter method,* and it gives the correct result for the total power with any kind of currents and voltages, balanced or unbalanced.

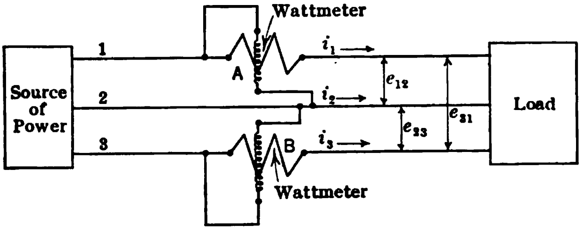


FIG. 87. The two-wattmeter method.

The only evident limitation is that each wattmeter in itself must be sufficiently accurate for that particular kind of current and frequency.

It is immaterial which of the three conductors is chosen as the return conductor. While the individual readings will be different, the algebraic sum is the same. If the load consists of pure non-inductive resistances, and if both wattmeters, A and B, are so connected as to read in the positive direction, the total power is equal to the arithmetical sum of the readings. However, when the load is partly inductive, or contains a counter-e.m.f., it is possible for one of the readings to become negative. Then it is necessary to reverse the terminals of that particular wattmeter and to take the difference of the two readings as the total average power delivered to the load. If the negative reading is greater than the positive, then the power is being "pumped back" to the assumed source of supply. This is possible, for example, with two interconnected power stations.

For this reason, when using the two-wattmeter method on unbalanced or inductive loads, or between two sources of power, it is of importance to determine in advance the polarity of the terminals of the instruments. This can be done by connecting each wattmeter to a resistance load in such a way as to obtain a positive deflection of the pointer. If with the same terminal sequence the meter reads positive on an unknown load, its reading is to be taken as positive, indicating that power is actually delivered to the load, and not returned to the source. Otherwise the terminals are reversed to read positive, and the reading itself is considered negative.

In practice two separate single-phase wattmeters are seldom used. A polyphase wattmeter which contains two wattmeter movements on the same shaft is preferable, because the reading gives the total power directly, provided that the terminals are properly chosen, as explained above. If the torque of one movement is positive and that of the other is negative, the instrument automatically registers the difference of the two, provided that the positive indication is greater. If a polyphase wattmeter is not available, a single-phase instrument is used with a polyphase board (§ 58) so arranged as to read the power in the leads 1-2 and 3-2 in succession.

The two-wattmeter method is largely used for measuring power in three-phase circuits, and when the load is partly inductive the two wattmeter readings are different from each other, even though the currents and the voltages are perfectly balanced. With sinusoidal currents and voltages, when the power factor of the load is 50 per cent, one of the readings becomes zero, and, at lower values of power factor, becomes negative. This is of importance in measuring the input of an induction motor at no load, when the power factor may be quite low. The proper sign of the two readings must be carefully determined. For further details regarding the measurement of power in polyphase circuits see Vol. II. The information given above and the theory given in the next section are sufficient for the tests on alternators and induction motors treated in this volume, and for the connections of watthour meters considered in the next chapter.

101. Theory of the Two-Wattmeter Method. — The connections within the source of power (Fig. 87) and within the load are immaterial.

Since electricity behaves like an incompressible fluid, the sum of the instantaneous line currents must be zero, that is,

$$i_1 + i_2 + i_3 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

provided that they are considered positive in the same direction, as indicated by the arrowheads. For example, if at a certain instant the currents in conductors 1 and 3 are 9 and 5 amperes respectively, flowing to the right, the current in the conductor 2 is 14 amperes flowing to the left, and we have the identity $9 + 5 - 14 = 0$, no matter what the connections in the load or in the generator may be. Equation (14) is true whether the currents are constant or vary with the time.

The voltages between the wires are denoted by two subscripts, the first subscript referring to the wire assumed to be at a higher potential. Thus, if at a certain instant the potential of conductor 1 is below that of 2 by 50 volts, $e_{12} = -50$ v. The positive electricity, which constitutes the current i_1, in reaching conductor 2 has "dropped" or performed work corresponding to the difference of potential e_{12}. Hence, the impulse communicated to the movable part of the wattmeter A in an element of time dt is $i_1 e_{12}\, dt$. The wattmeter movement automatically integrates such impulses and thus registers the *average* power between the conductors 1 and 2. With alternating currents it is possible for the sum of negative instantaneous impulses $i_1 e_{12} dt$ to be greater than that of the positive ones, and the wattmeter reading is then negative. The same is true of the wattmeter B. If the total average power delivered to the load is P and the time of one cycle is T, then we have for the total energy delivered during one cycle:

$$PT = \int_0^T (i_1 e_{12} + i_3 e_{32})\, dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

It is of importance to note that in this equation the subscripts of the voltage between the conductors 3 and 2 are marked in that order, while in the sketch the same voltage is marked with the subscripts 2–3. The latter marking corresponds to the *cyclic* order and leads to the equation

$$e_{12} + e_{23} + e_{31} = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

For example, if at a certain instant $e_{12} = 50$ volts and $e_{23} = 60$ volts, then $e_{31} = -110$ volts, while $e_{13} = +110$ volts. This can be readily visualized by thinking of a zero-center voltmeter applied to measure these voltages, with one terminal always used for the first subscript and the other for the second subscript.

Now, let the conductor 3 be chosen as the return wire, and let the series coils be connected in the line conductors 1 and 2. The expression under the integral sign in equation (15) must now be changed to $(i_2e_{23} + i_1e_{13})$, and it is necessary to show that the two expressions are identical. Using eqs. (14) and (16), we get

$$\begin{aligned} i_2e_{23} + i_1e_{13} &= -(i_1 + i_3)e_{23} + i_1(e_{12} + e_{23}) \\ &= i_1e_{12} - i_3e_{23} = i_1e_{12} + i_3e_{32}. \end{aligned}$$

The two expressions are thus identical, and the same total power is obtained no matter which of the three conductors is selected for the common potential terminal.

The foregoing theory and method of connection may be extended to any number n of conductors, instead of three. The required number of wattmeters is $n-1$, and one of the conductors is considered as the common return and is used for the potential terminal of all the meters. A three-phase system with an additional return conductor, or with the neutral points grounded, is equivalent to a four-wire system, and three wattmeters are necessary to measure its total power. A mistake is sometimes made in using only two wattmeters under such conditions.

102. Reactive-Component Wattmeter. — Let a dynamometer-type or induction-type wattmeter (§§ 90 and 97) be connected in the usual way to measure the true power in a partly inductive single-phase circuit, with the current and voltage strictly sinusoidal. Let the instrument reading in the usual notation be

$$P = IE \operatorname{Cos} \phi \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (17)$$

See Fig. 88 and equation (6) in § 89. Now, let the same wattmeter be connected in the following way: Its current coils are to be in series with the main circuit, but its potential circuit is to be connected to a source of alternating voltage identical with the former voltage but displaced with respect to it in time by 90 electrical degrees. The current through the potential winding is thereby displaced by 90 degrees, and is now in phase with the component I Sin ϕ of the load current. Thus the reading of the instrument is

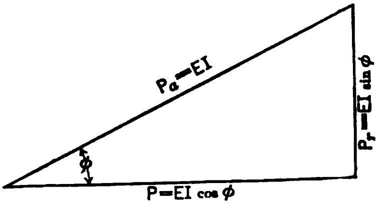


FIG. 88. True, reactive, and apparent power.

$$P_r = IE \operatorname{Sin} \phi \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (18)$$

The quantity P_r is known as the *reactive power*, and is a measure of the rate at which the magnetic or electrostatic energy is surging between the load and the source of power. The value of Sin ϕ is called the *reactive factor*.

With P and P_r represented as two sides of a right-angle triangle, the hypotenuse gives the *apparent power* or volt-amperes

$$P_a = EI \quad \text{. . . . .} \quad (19)$$

The value of P_a can be determined directly as a product of the ammeter and voltmeter readings.

From the triangle (Fig. 88), we have

$$\mathrm{Tan}\,\phi = P_r/P \quad \text{. . . . .} \quad (20)$$

Therefore, the phase angle ϕ and consequently the power factor, $\mathrm{Cos}\,\phi$, may be determined from the two wattmeter readings, P and P_r. The apparent power may also be computed as

$$P_a = P/\mathrm{Cos}\,\phi = P_r/\mathrm{Sin}\,\phi \quad \text{. . . . . . .} \quad (21)$$

With a *non-sinusoidal* current and voltage, a wattmeter connected in the usual way still gives the true average power. If, however, we again connect it as before, with its potential winding quadratured, the new reading has to be *defined* as the reactive power. Thus we again get the same right-angle triangle, from which the values of ϕ and P_a may be readily computed. It must be clearly understood, however, that these are merely *defined* values and not unique physical values, as is the true power.

With non-sinusoidal currents at least two other definitions of the quantities P_r, P_a, and ϕ are possible; (a) To take P_a as the product of the actual ammeter and voltmeter readings, and to construct a right-angle triangle upon the values of P and P_a; (b) To take $\mathrm{Cos}\,\phi$ from the indication of a reliable power-factor meter (§ 104) and to construct a right-angle triangle using the experimental values of P and ϕ. These two procedures give different values for the remaining parts of the triangle. Thus, with non-sinusoidal currents, the exact values of the reactive power, power factor, and apparent power depend upon the adopted definitions and upon the measuring instruments used. With sinusoidal currents there is but one set of values, and with currents which depart but moderately from a sine law, the discrepancies are not great enough to impair the usefulness of the concept of reactive power for practical purposes. See also § 109 below.

A wattmeter connected to measure the reactive component of power gives a switchboard attendant useful information about the condition of his circuits and about the amount of idle circulating current. In some cases he is in a position to reduce the reactive lagging current or even to change it to a leading component. For such cases the reactive wattmeter is provided with a scale having its zero in the center, the two sides being marked "leading" and "lagging." For polyphase connections of reactive power meters, see Vol. II.

103. EXPERIMENT 4-C. — Study of an Indicating Reactive-Power Meter. — The theory is explained in the preceding section. Provide a source of two-phase power, nearly sinusoidal, from an alternator, or a rotary convertor, or by transforming to two-phase from a three-phase supply (see phase transformation in Vol. II). Take an ordinary indicating wattmeter and provide a load such that the current can be kept constant while the power factor is varied at will. Connect a double-throw switch in the potential circuit of the wattmeter, to enable it to be connected at will to either phase. The provisions must be such that the voltage in the auxiliary phase is kept strictly equal to and in exact quadrature with that in the loaded phase. Have an ammeter and a voltmeter in the circuit, and if possible a power-factor or phase-angle meter (§ 104). (a) Begin with a nearly non-inductive load; read volts, amperes, power factor, watts, and reactive watts. Reduce the power factor of the load in steps, keeping the voltage and the total current constant, and take similar readings at each step. (b) If possible, repeat the run, using a badly distorted source of voltage, or a load that distorts the current wave, such as an arc lamp or a rectifier. (c) Show that the sign of deflection of the reactive-power meter is reversed when changing from a leading to a lagging current.

Report. (1) For a few sets of readings, construct the triangle shown in Fig. 88, using different combinations of observed quantities, for example, EI and P, ϕ and P_r, P and P_r, etc. With a sine-wave voltage and current, the result should be nearly the same. (2) Give a few triangles or computed values to show the discrepancies with non-sinusoidal waves. (3) Explain theoretically, by drawing A.C. waves, that the indication of a reactive-power meter is reversed when the current changes from a lagging to a leading one.

POWER-FACTOR METERS

104. Some operating features of alternating-current power plants and transmission lines depend essentially upon the *power factor* of the load (§ 89). If the switchboard were provided with all the necessary ammeters, voltmeters, and wattmeters, and if changes in the load were sufficiently slow, the power factor could be computed but it would be a tedious process to figure out the power factor periodically. With the usual fluctuating load, such a calculation is impossible, and, therefore, it is convenient to have an indicating or graphic instrument which shows the power factor *directly* without computation. Two types of power factor meters are used in practice, viz., the *iron-vane* type and the *electro-dynamometer* type. These instruments are built both for single-phase and for polyphase circuits, but in a polyphase circuit their indications have a definite meaning only when the voltages and the load are perfectly balanced. At this writing there is no universally accepted definition of power factor for an unbalanced load.

105. Iron-Vane-Type Power-Factor Meter. — The principle of this power-factor meter is shown in Figs. 89 and 90. In a three-phase meter the stationary part consists of a laminated iron core A, provided with a regular three-phase winding, (Fig. 301) connected to the line through the current transformers T. The movable part consists of a soft-iron vane V, of the shape shown in Fig. 90, and magnetized by a stationary coil P, connected across one of the phases of the circuit, either directly or through a potential transformer. The currents in the polyphase winding of the instrument, being induced through the current transformers, are practically in phase with the line currents. The current in the coil P bears a definite phase relation to the line voltage. Therefore the position of the vane will depend *on the phase relation* between the line current and the line voltage, or, which is the same, *on the power factor* of the load. The instrument is calibrated directly in percentage of power factor, the calibration extending over all the four quadrants of the scale. This is necessary because currents may be leading or lagging, and the phase angle between them and the voltage will be less or more than 90 degrees, according to whether the power is being transmitted in one or the other direction.

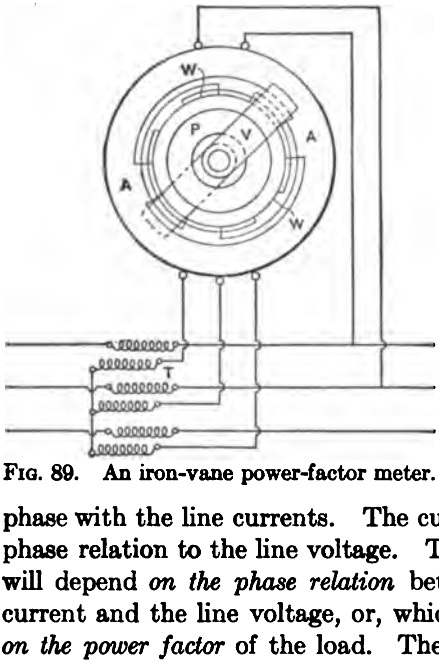


Fig. 89. An iron-vane power-factor meter.

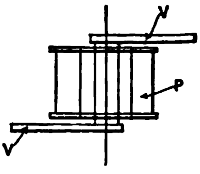


Fig. 90. The potential coil and the movable vane of the power-factor meter shown in Fig. 89.

In order to understand the action of this meter, the reader should make clear to himself that the stationary winding produces a rotating magnetic field (§ 472). This revolving field may be thought of as being due to two fictitious alternating fields, at right angles in space and in time quadrature with each other (§ 491). For one of the fictitious exciting coils the space position, as well as the corresponding time phase, is arbitrary, and we chose it so that the current will be in phase with that in the potential coil *P*. The iron vane will then orient itself in the direction of this field, since the force of attraction between two alternating fields in phase with each other is a maximum. The average force of attraction between the iron vane and the second alternating field is nil, the two fields being in time quadrature with each other; this force alternately becomes an attraction and an equal repulsion. If the time phase between the line current and the voltage should change, the position of the two fictitious coils in space would also change, and the iron vane would assume a new position.

For two-phase circuits the instrument is provided with two windings in space quadrature, instead of three. For a single-phase circuit an instrument is used with two windings, also in quadrature, connected as in a split-phase induction motor (§ 472). One of the windings is connected across the line in series with a high inductance and the other in series with a high resistance, so that a time-phase displacement between the two currents is obtained, as is necessary for the production of a revolving magnetic field. The above-described power-factor meter is by no means an accurate measuring instrument; its indications are affected by the magnitude of the line current and by the line voltage. But it is useful for a comparative judgment of the variations of power factor during the day, and for regulating the machines so as to maintain approximately a desired power factor.

106. EXPERIMENT 4-D. — Study and Calibration of an Iron-Vane Power-Factor Meter. — The instrument is described in the preceding section. Provide a three-phase load (see § 447), and have an ammeter, a voltmeter, and a wattmeter connected in the circuit by means of a polyphase board (§ 58). Connect the power-factor meter as in Fig. 89 and apply the largest safe inductive load. Read the four instruments and gradually increase the power factor of the load, keeping the same total current, until the load becomes non-inductive. Repeat the same test with two or three smaller values of current. If an over-excited synchronous motor is available, the calibration may be extended to the quadrants corresponding to leading current.

Before or after the calibration, investigate a few features of the construction of the instrument, as follows:

(1) Open the potential circuit and observe the vane rotate under the influence of the field produced by the series winding alone. Reverse two of the series leads, whereupon the vane will revolve in the opposite direction.

(2) Reverse the potential leads; the pointer will turn by 180 degrees. This corresponds to the change from output to input, in other words, from alternator to synchronous motor.

(3) The instrument is intended to be read with balanced load only; unbalance the load and see what effect this has on indications of the power-factor meter.

(4) Connect the meter to a single-phase supply circuit through a split-phase arrangement, like that shown in Fig. 315. Take a few calibration points at different values of power factor and of line current.

(5) Investigate the effect of the frequency, of the line voltage, and of the load current upon the accuracy of the instrument.

Report. (a) Give the calibration curve of the instrument at the rated current, voltage, and frequency. (b) Give the findings in regard to the errors due to each of the three factors. Explain the cause of these errors. (c) Give the findings with the unbalanced load. (d) Report the operation of the instrument on the single-phase system.

107. Electro-Dynamometer-Type Power-Factor Meter. — One of the dynamometer-type power-factor meters (Weston) is shown in Fig. 91, in application to the single-phase, two-phase, and three-phase circuits respectively. It consists of two stationary current coils SS_1 and of two movable potential coils MM_1, the latter being fastened together at right angles to each other. The movable coils are pivoted to rotate freely and have no control springs.

In the *single-phase meter*, Fig. 91(*a*), the movable voltage-coil M is connected in series with a resistor R across the circuit, and its current is therefore practically in phase with the voltage. The other voltage coil, M_1, is connected in series with a condenser C across the circuit, and its current is practically 90 degrees out of phase with the voltage. At 100 per cent power factor, the current in M is, therefore, in phase with the current in the current coils S and S_1, and the torque between M and the fixed coils is a maximum. This torque tends to make M parallel to the fixed coils. Under the same condition the current in M_1 is 90 degrees out of phase with the current in S and S_1, and its average torque is zero, because a maximum current in M_1 coincides with zero current in S and S_1, and vice versa.

As the power factor changes from 100 per cent to some other value, the current in M, being in phase with the voltage, becomes more and more out of phase with the current in SS_1, and its torque decreases. The current in M_1 comes more and more into phase with the main current, and its torque increases. The direction of the torque depends on whether the current is leading or lagging. The coils are so connected that the two torques oppose each other. The moving system therefore assumes a definite position for every value of the power factor of the circuit, the position being such that the two torques are equal and opposite. Instead of the condenser *C*, the current phase in one of the movable coils may be shifted by means of an inductance. Whether inductance or capacitance is used, the instrument is affected by frequency changes and must be recalibrated for a different frequency. Otherwise the values of *R* and *C* (or *L*) must be properly adjusted.

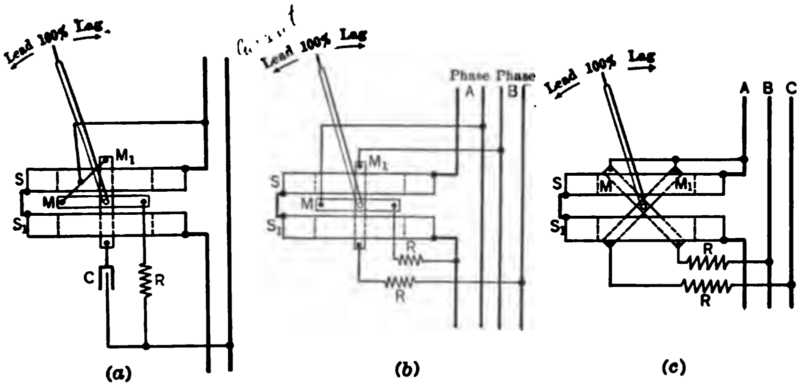


Fig. 91. An electro-dynamometer-type power-factor meter.

The *two-phase meter* is shown at (*b*). In this case the 90-degree phase relationship between the currents in the two voltage coils is obtained by connecting the two coils to the two phases of the circuit. In other respects the principle of operation is the same as that of the single-phase meter.

Fig. 91(*c*) shows a diagram of the *three-phase meter*. In this case one voltage coil is connected across the phase *AB* and the other across *AC*, the current coils being in the phase *A*. At 100 per cent power factor the voltage (or the current) in one of the potential phases leads the main current by 30 degrees, and in the other phase it lags by 30 degrees, so that the torques of the two coils balance each other. A change in power factor increases the lead of one phase voltage and decreases the lag of the other, or vice versa, causing a relative change in the torques of the two coils and making the movable system assume a new position of equilibrium at which the two torques again balance each other. A dynamometer-type power-factor meter possesses a much higher accuracy than the one with an iron vane (§ 105), and if carefully constructed, should give an accuracy of at least one per cent, from about one-fifth of its current rating up.

108. EXPERIMENT 4-E. — Study of Electro-Dynamometer-Type Power-Factor Meter. — The purpose of the experiment is to make the student familiar with the power-factor meter described in the preceding section. (1) Provide a single-phase load whose power-factor may be varied at will by the addition of inductance or capacity, and connect into the circuit an ammeter, a voltmeter, a wattmeter, and the power-factor meter under test. Set the load at nearly 100 per cent power factor and change it in steps to as low a value as possible, keeping the current and the voltage nearly constant. Also produce a partly leading current, if feasible. At each setting read the four instruments. Take a few readings at a slightly lower and at a slightly higher frequency, and also at frequencies much above and below the rated value for the power-factor meter. Be sure that the other instruments give reliable readings at such frequencies. Connect an adjustable resistance in each potential circuit, and by adjusting these try to bring the calibration back to that corresponding to the scale of the instrument.

(2) Take a few similar readings on a two-phase load and on a three-phase load, both balanced and unbalanced.

(3) Adjust the single-phase load to be exactly of unity power factor, and disconnect the moving coil in series with the condenser. The indication of the instrument should remain unchanged, thus proving that the current in M_1 is in phase quadrature with the main current. Show similarly that at any other power factor, the resultant torque on the moving system is equal to the differential action of the two coils.

(4) Show that, in the three-phase meter, both moving coils exert equal and opposite torques at unity power factor when the planes of the movable coils are at an angle of 45 degrees with the stationary coils. Disconnect one of the movable coils and read the indication on the scale. Do the same with the other coil, having the first coil connected. Repeat this experiment at one or two other values of the load power factor.

Report. (1) Give the calibration curve of the single-phase instrument at the rated frequency. Explain how and why the frequency of the supply affects the calibration. Explain the effect of the additional resistance, and whether this resistance should be added to the capacitive or resistance circuit, for a higher and for a lower frequency than the rated frequency. (2) Give the results of the calibration of the meter on a two-phase and on a three-phase circuit with a balanced load. Explain the meaning of the readings with the unbalanced load. (3) Give the results of the tests with one of the potential coils disconnected, on both the single-phase and the three-phase circuit. Show that the observed behavior of the instrument could be predicted theoretically.

109. Reactive-Factor Meter. — Besides the power factor (§ 89), which for sinusoidal quantities is defined as the cosine of the angle between the current and the voltage, the so-called *reactive factor*, which is the sine of the same angle (§ 102), is sometimes used. A reactive-factor meter can be made out of the above described power-factor meter by simply changing the scale to read the sines of the corresponding angles instead of the cosines. In some reactive factor meters the functions of the outer and the inner windings (Fig. 89) are reversed, the inner coil being connected through a current transformer to one of the line conductors, and the outer polyphase winding connected across the line through two V-connected potential transformers.

The advantage of a reactive-factor meter over a power-factor meter is that the former shows the idle component of the current most emphatically. Thus, with a power factor of 95 per cent, the reactive factor is 31.2 per cent (and not 5 per cent), because

$$\overline{0.950}^2 + \overline{0.312}^2 = 1.000.$$

Similarly, with a power factor of 98 per cent, the reactive factor is 19.7 per cent (and not 2 per cent). This large indication will induce the attendant to correct the waste more quickly, and therefore makes the meter particularly useful in securing the proper operation of rotary converters, where a slight drop in power factor may cause considerable heating of the armature. In this respect the meter plays a part similar to that of a wattmeter connected to measure the reactive power (§ 102).

With non-sinusoidal currents and voltages the equivalent phase-displacement angle ϕ has a different value according to whether it is computed from eq. (17) or eq. (18), in § 102. Let these values be denoted by ϕ_1 and ϕ_2 respectively; the theory shows that $\phi_1 > \phi_2$ if the reactive power P_r be measured with a dynamometer-type wattmeter. Thus, while with sinusoidal currents and voltages we have

$$\text{Cos}^2\,\phi + \text{Sin}^2\,\phi = 1 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (22)$$

here we must write

$$\text{Cos}^2\,\phi_1 + \text{Sin}^2\,\phi_2 < 1 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (23)$$

Assume that we have an accurate power-factor meter and an accurate reactive-component meter, both carefully calibrated with sinusoidal currents. According to eq. (22), the sum of the squares of their indications on a circuit with sinusoidal currents and voltages, should be 100 per cent squared. If the same instruments be used on a circuit with an irregular wave-form, the sum of the squares of their indications, according to eq. (23), will be less than 100 per cent squared.

CHAPTER V

WATT-HOUR METERS

110. The Watt-Hour as a Unit of Consumption of Electrical Energy. — A certain amount of coal is required to generate electrical energy at the rate of one watt during one hour. The energy used by the consumer depends upon the number of watts consumed, and upon the interval of time during which the power has been used. For these reasons it is proper to charge for electric energy on the basis of watt-hour or kilowatt-hour consumption (§ 88). A 50-watt incandescent lamp uses, in one hour, 50 watt-hours of electrical energy. Two lamps, used half an hour each, or 4 lamps used a quarter of an hour each, also consume together the same amount of energy. Within certain limits it does not make much difference to the company supplying the power, during how long a period of time a certain amount of energy is consumed. If the rate is ten cents per kilowatt-hour, the cost to the consumer is in either case

$$10 \times \frac{50}{1000} = 0.5 \text{ cent.}$$

When larger amounts of energy are consumed, the kilowatt-hour consumption may be only one of the factors in the total charge for the electric service. The character of the demand, that is, steady, fluctuating, with high peaks, long hours, short hours, at off-peak times for other loads, etc., determines the cost of service, and in many cases is taken into consideration in the monthly bill (§ 129). Still other possible items in the bill for the electric service may be the rating of the installed machinery, a charge for the installation itself (lines, transformers, etc.), a penalty for low power-factor, a penalty for phase unbalancing, etc. All these factors are outside the scope of this chapter, which is concerned only with the measurement of the actual watt-hours consumed.

The reason for measuring electrical energy in watt-hours, instead of simply in watts, is that one watt represents a definite amount of energy liberated or absorbed *in one second;* in other words, it is only a *rate* of expenditure of energy. This follows from the definition of the watt as the product of "volt × ampere," where the ampere is a quantity of electricity *per second.* On the other hand, the energy contained in one lb. of coal is measured in B.T.U.'s, calories, joules, etc., all of which are units independent of time. Therefore, watts must be multiplied by time in order to reduce them to the same physical meaning and dimensions as the heat units.

An instrument which measures the total electrical energy consumed in a circuit during a certain period of time is called a *watt-hour meter*. Sometimes it is referred to simply as an "electric meter." The monthly bill of a small consumer is usually computed on the basis of the reading of the watt-hour meter on his premises.

The various watt-hour meters on the market are so constructed that the instantaneous speed is proportional to the rate of consumption of energy. Therefore, the *totalized* number of revolutions over a certain interval of time is proportional to the total energy consumed during that time. For this purpose the meter shaft is connected by means of a worm gear to a counting or totalizing gear train with dials, shown in Fig. 92. The speed of the meter and the gear ratio are usually so adjusted as to make the meter direct-reading, or at least to have a simple multiplier, such as 10 or 100. With the position of the hands shown in the figure, the reading is 18,255 kw.-hours. If the reading one month later is, for example, 18,490 kw.-hours, the consumption during the month was 235 kw.-hours, and at a rate of, say, 10 cents per kw.-hour, the monthly bill will be $23.50.

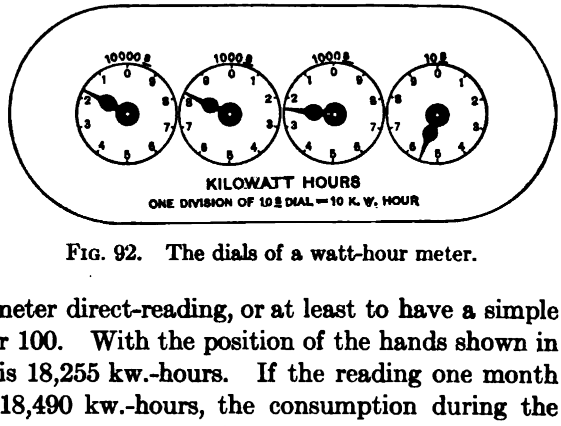


FIG. 92. The dials of a watt-hour meter.

111 Types of Watt-Hour Meters. — Watt-hour meters, in use at present, are of the motor type exclusively, a small motor being arranged to revolve at a speed proportional to the rate at which energy is passing through it. The number of revolutions is recorded on a dial; if the constant of the meter is known, this number of revolutions can be reduced to kilowatt-hours. In most meters the shaft of the motor is geared to the recording mechanism in such a way that the meter reads directly in kilowatt-hours.

Two types of *indicating* wattmeters are described in the preceding chapter, viz., the dynamometer type and the induction type. Corresponding to these two types are two types of watt-hour meters — *commutator meters*, used on direct-current circuits, and *induction-type* meters, used on alternating-current circuits. A third type is the mercury-motor meter described in § 117, which was produced for the purpose of eliminating the commutator. On a D.C. circuit in which the voltage is kept practically constant, watts are proportional to amperes, so that it is sometimes sufficient to measure ampere-hours instead of watt-hours (§ 119).

COMMUTATOR-TYPE WATT-HOUR METER

112. The Thomson commutator meter (Figs 93 and 94) consists of a small direct-current vertical-shaft motor, without iron in its magnetic circuit. The armature *A* is connected across the line, in series with a high resistance *R*, the value of which depends upon the voltage of the supply. In the regular operation of the meter the armature runs so slowly that its counter-e.m.f. is negligible in comparison with the line voltage. Consequently, the current through the armature is practically constant and is equal to the line voltage divided by the resistance *R*, plus that of the armature itself.

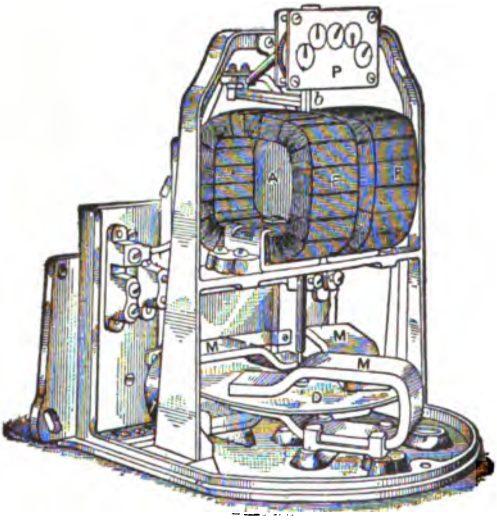


FIG. 93. A general view of the Thomson watt-hour meter.

The stationary field coils *FF* are in series with the line, and the current flowing through them is equal to the line current. Thus, the torque between the field and the armature is proportional to the product "line current times line voltage," or to the power delivered. In this respect the device is similar to the indicating wattmeter shown in Fig. 81.

A motor connected in this manner would run away unless it were loaded. To make its speed proportional to the power to be measured, the meter is provided with a Foucault current brake (Fig. 93). This brake consists of an aluminum disk *D* mounted on the armature shaft, and placed between the poles of the permanent magnets *MM*. The rotation of the disk in the field of the magnets causes eddy currents to be induced in it. The counter torque of these currents balances the driving torque of the motor element. The speed of rotation is usually adjusted by moving the brake magnets in and out. In some meters an iron disk is mounted near the pole-pieces, and shunts some of the flux around the air-gap. By shifting this disk the useful flux is adjusted so that the meter runs at the correct speed.

To understand the operation of the meter and the action of this brake, assume first that the meter has no friction, and that, at a certain load current I and line voltage E, the meter runs at a certain speed s, determined by the counter-torque of the eddy currents in the brake. At low speeds the eddy currents are proportional to the speed. The retarding torque between these currents and the permanent magnets is proportional to the value of the currents themselves, that is, proportional to the speed. Thus, the torque due to the eddy currents is Bs, where B is a coefficient of proportionality. The driving torque is proportional to EI, and we have the relationship

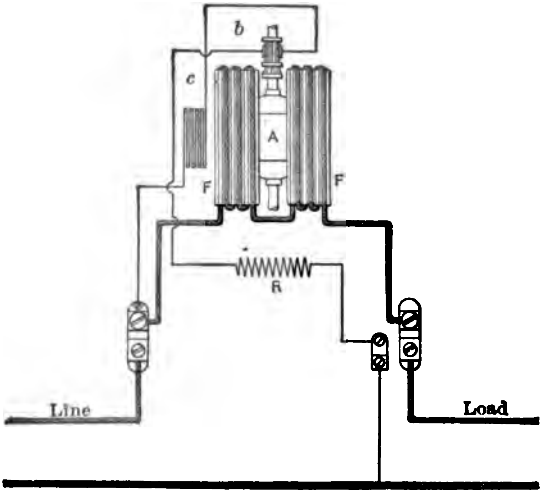


FIG. 94. The electrical connections in a commutator-type watt-hour meter.

$$kEI = Bs \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

In this expression k is the constant of the instrument, and kEI is the driving electromagnetic torque, equal to the brake torque. It will thus be seen that the instantaneous speed of the meter is proportional to the power EI to be measured. The unavoidable bearing and brush friction would somewhat modify this simple relationship, were it not for the fact that all modern meters are provided with a friction compensating device which is described in the next section. With this compensation the speed of the meter is practically proportional to the load, down to a small per cent of its rating. This means that the total number of revolutions of the meter during a given interval of time, as shown on the dial P, is proportional to the total electrical energy that has passed through the meter. This is true irrespective of whether the load is constant or fluctuating, since the armature adapts itself quickly to changes in load, and even at a rapidly fluctuating load, rotates at a speed corresponding to the average load.

In some more recent meters the revolving armature is made spherical. This gives a more advantageous utilization of the materials, and secures a higher torque with a lower watt loss in the meter itself. Some large-capacity meters are provided with two armatures on the same shaft, connected astatically. Such an arrangement minimizes the effect of stray magnetic fields, since any stray field which tends to weaken the torque of one armature strengthens that of the other.

A magnetic shield is provided between the stationary field coils and the permanent magnets *M*. In the earlier unprotected meters it was found that a strong electromagnetic field due to a severe short-circuit was sufficient to affect the strength of the permanent magnets and thus to throw the meter out of calibration.

Most direct-current meters of the above-described type cannot be used with ammeter shunts, and the whole line current must flow through the meter. This presents serious difficulties with currents ranging up into several thousand amperes. By a proper modification of the design, meters are produced in which the stationary element is designed for 5 amperes at full load, and the meter is connected across a shunt, like an ammeter.

113. Compensation for Friction. — The unavoidable friction in the bearings and between the commutator and the brushes produces a retarding torque, which would cause the meter to run slow on light loads. This would represent an appreciable loss to the company supplying the power because a great majority of meters run on light load most of the time. For example, an uncompensated 10-ampere meter might not start at all, or might run very slowly with only one 0.25-ampere lamp in the circuit. Thus a customer could leave such a lamp in the circuit permanently without an adequate return to the company.

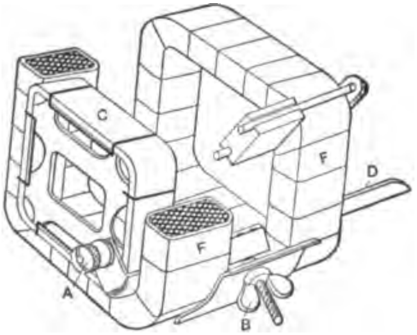


FIG. 95. Compensation for friction in a Thomson watt-hour meter.

To remedy this, a compensating coil *c* (Figs. 94 and 95) is provided in the armature circuit; the field produced by this coil gives an additional torque just sufficient to balance the friction of the meter. As it is impossible to predetermine the exact amount of friction, especially since it varies with the pressure of the brushes on the commutator, with the wear of the jewel supporting the shaft, etc., this compensating coil is made adjustable. A possible arrangement is shown in detail in Fig. 95. *FF* are the main field coils, *C* is the compensating coil. Its distance from the armature can be varied by means of the screws *A* and *B* and the guide *D*.

In some meters the compensating coil is stationary, but is provided with several taps connected to the buttons of a dial. By means of a small lever more or less turns of the compensating coil may be introduced into the circuit, and in this way the amount of compensation may be varied at will. Another method of adjusting the amount of compensation is by means of a variable shunt connected in parallel with the compensating coil. In some meters the compensating coil shown in Fig. 95 is made to move up and down, instead of to and from the armature.

A meter which is *over-compensated* is liable to creep at no-load, registering energy when none is consumed. The well-established policy on the part of all reputable operating companies is to furnish each customer with a meter accurately calibrated and adjusted against creeping at no-load. In spite of this, a meter is occasionally found which creeps at no-load, due to one or more of the following causes: (a) vibration which reduces the friction, (b) a stray magnetic field, and (c) too high a voltage. Sometimes the potential circuit is connected by mistake on the load side of the meter, when the friction compensation has been adjusted with the potential circuit connected on the service side. With the former connection the compensating current flows through the main or series coil, and the resultant ampere-turns increase the compensating torque; this also may cause the meter to creep.

Even with the most careful adjustment in the shop, there is a possibility of creeping, and for this reason a simple anti-creep device is often used. The most common method is to force a small U-shaped piece of iron wire or " clip " over the edge of the brake disk. This clip is attracted by the drag magnets and tends to remain stationary in a position where it is closest to one of the poles. When the meter is rotating under load, the accelerating force, as the wire approaches a magnet, is equal to the retarding force as it leaves the magnet, so that the effect on the accuracy of the meter is negligible. In placing the clip, care should be taken not to injure the disk, and to locate the clip so that it is not likely to hit the magnets.

In some meters a piece of iron wire is attached to the disk near the shaft, giving the same effect. The effect of the clip is varied by changing its length and position. To make this adjustment properly requires good judgment. The tester must take account of the character of the installation and of the location, and set the clip so that it will not prevent the meter from starting on a light load, and yet so that the meter is not likely to creep under the worst conditions which may exist at some later time. The minimum starting current should be determined after adjusting the clip.

The *starting current* of a meter is defined as the smallest current required at normal voltage to start the moving element from rest, and to cause it to rotate continuously through one or more revolutions past the anti-creeping device. When the meter has an anti-creeping clip, the starting current should be taken when the clip is nearest to a magnet pole. If a low-reading ammeter is not available, the starting current may be estimated in terms of the total watt rating of the test lamps required to start the meter.

114. Calibration of Commutator-Type Watt-Hour Meters. — The calibration of a watt-hour meter comprises (a) an adjustment of the brushes and the bearings to give the best operating conditions; (b) an adjustment of the compensating device so that the meter does not creep at no-load and yet starts with a very small load-current; (c) an adjustment of the brake magnets to make the meter run at the proper speed corresponding to the gear ratio of the recording dial. In some cases parts (a) and (b) are omitted in order to determine the percentage of error of the meter " as found." Thus, in the case of a complaint on the part of a customer, the percentage of "slow" or "fast" of the meter is determined before any adjustments whatever are made. Should the meter be found incorrect, it is carefully inspected and adjusted, and then calibrated to read correctly. The three practical methods of calibration are described below.

(1) *Comparison with a good indicating ammeter and voltmeter* (*or indicating wattmeter*). The D.C. watt-hour meter under test is connected to a steady load, and the input into this load is measured in the usual way with a moving-coil ammeter and voltmeter (§ 42). A calibrated indicating wattmeter (§ 90) may be used in place of these two instruments. The number of revolutions of the meter shaft during a certain interval of time is determined with a stop-watch. From these data and the constant of the meter, that is the number of watt-seconds per revolution, the percentage of error may be readily computed. The connections must be such that the watt-hour meter does not measure the potential loss in the standard instruments, or the instruments measure the potential loss of the watt-hour meter.

Let, for example, a watt-hour meter be tested at 200 amperes and 120 volts, and let the constant marked on the meter be 10 watt-hours per revolution of the disk. Let the disk be found to make 30 revolutions in 46 seconds. Then the true energy that has passed through the meter is

$$200 \times 120 \times 46/3600 = 306.7 \text{ watt-hours.}$$

The energy indicated by the watt-hour meter is

$$10 \times 30 = 300 \text{ watt-hours.}$$

Thus, the meter ran *slow*

$$(306.7 - 300)/306.7 = 2.2 \text{ per cent,}$$

and this represents its percentage of error.

(2) *Comparison with a standard watt-hour meter* (*portable standard*). The disadvantage of the preceding method is the necessity of keeping the load nearly constant, and of using one or two indicating instruments and a stop-watch. For this reason a standard portable watt-hour meter is being used to an increasing extent. It is connected to the same circuit as the meter under test, and its number of revolutions (and fractions thereof) is determined for an interval of time, during which the meter under test makes a predetermined number of revolutions. If the constants of the two meters are known, the percentage of error of the meter under test can be readily determined.

Let the meter to be tested have a constant of 0.4 watt-hour per revolution of the disk, and let the corresponding constant of the standard meter be 0.5. Let the meter to be tested make 45 revolutions while the standard meter makes 35.2. The corresponding amounts of energy indicated by the two meters are $0.4 \times 45 = 18.0$ and $0.5 \times 35.2 = 17.6$ watt-hours. Thus, the meter under test runs *fast* by

$$(18.0 - 17.6)/17.6 = 2.3 \text{ per cent.}$$

The portable standard meter usually has a number of stationary coils for different ranges of current, and it has multiplier resistances for at least two different voltages. This permits the use of the standard meter at a large percentage of its rated current, for which the meter is particularly accurate. The temperature of the potential circuit affects somewhat the speed of rotation, and for this reason a meter should not be tested until its potential circuit has been connected for at least twenty minutes. The standard meter is sometimes provided with "heating connections," so as to bring the potential circuit quickly to the working temperature.

The rotating standard should be set in a place free from vibration and stray magnetic fields. It should be kept away from iron masses, motors, electrical apparatus, and conductors carrying heavy currents. The stationary coils should be placed in the plane of the magnetic meridian, and several comparative tests should be made with the current and potential connections reversed, so as to determine the possible effects of a stray field.

Rotating standards are not made much in excess of 100-ampere current capacity. To calibrate a meter of larger capacity, a known fraction of the total current is passed through the standard meter (Fig. 96). The arrangement is based on the Wheatstone-bridge principle. *A* and *B* are two manganin shunts, the drop across *A* being about 0.1 volt and that across *B* about 0.4 volt. The third branch of the bridge is formed by a small shunt *C* of the capacity of the standard meter, and the fourth branch consists of the standard meter in series with an adjustable resistance *D* which is set so as to reduce the current in the galvanometer *G* to zero. Let, for example, the resistances *A* and *C* be such that the voltage drop across either is the same when there is a current of 2000 amperes in *A* and 40 amperes in *C*. In other words, the galvanometer *G* will show no deflection when the current through *C* is 2 per cent of that in *A*; the total line current is then 51 times that passing through the standard meter.

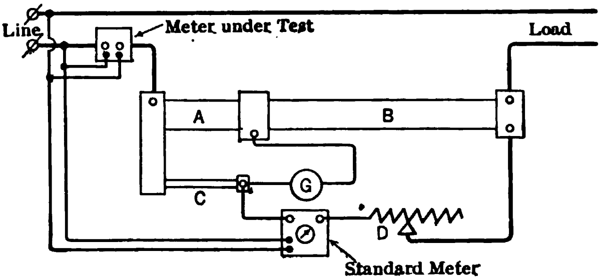


FIG. 96. A shunt for calibrating large-capacity meters.

(3) *Calibration with a standardized load resistance.* A resistance made of some metal of zero temperature coefficient is carefully calibrated in the laboratory by accurately determining the wattage consumption for all voltages within the working range of the meters to be tested; the results are tabulated or plotted as a curve. Such a resistance is then used as an artificial load for the meter under test, and the voltage at its terminals is measured with a voltmeter. The table or the curve will then give the true watts. The number of revolutions of the meter disk per second, multiplied by the meter constant, will give the meter watts. The percentage of error, if any, may be computed from these data.

Care must be exercised in regard to the connections to the voltmeter, which must indicate not only the voltage applied to the meter, but also to the cable terminals of the resistance. As the internal resistance of a continuous-current voltmeter is very high, the current taken by the voltmeter may be ignored for all loads in excess of 1 ampere; for loads less than 1 ampere, the voltmeter should either be connected to the load side of the watt-hour meter or a correction made for the current taken by the voltmeter. For loads in excess of 1 ampere, the drop in the field coils of the watt-hour-meter also enters as a factor, depending upon the point of connection of the potential circuit of the meter. If the potential circuit is connected to the load side of the watt-hour meter, the voltmeter should also be connected at this point, and then no error will be introduced. If the potential circuit is connected to the service side of the watt-hour meter, and the voltmeter is also connected at that point, allowance should be made for the drop in the watt-hour-meter field coils when computing the wattage consumed by the resistances.

115. Torque and Friction. — A high torque per unit weight of the moving element is considered to be a desirable feature in almost any electrical measuring instrument, because the disturbing influence of the pivot friction is less and the meter registers more accurately at light loads. Special instruments have been devised by means of which the torque on the shaft of a meter can be directly measured in gram-centimeters, and two or more meters of different makes may be compared.

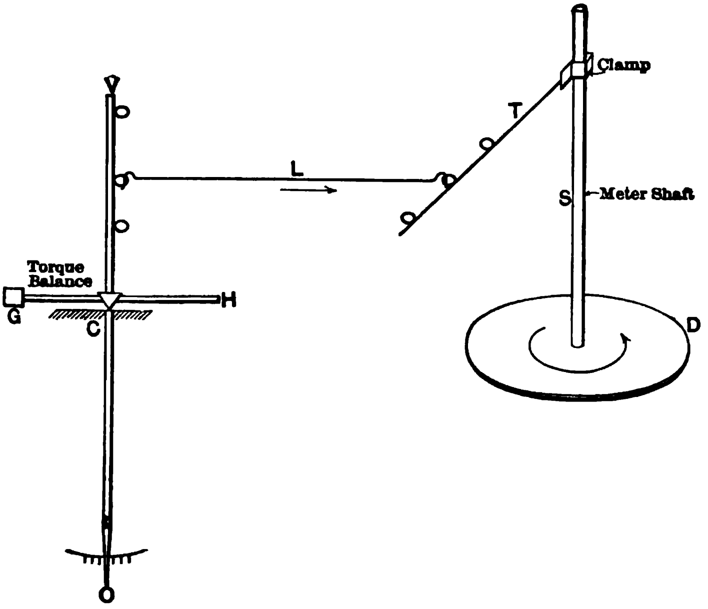


FIG. 97. A torque balance for testing watt-hour meters.

The two torque-measuring devices described below may be used for testing either indicating instruments or watt-hour meters.

(a) *Torque balance.* The instrument is shown diagrammatically in Fig. 97. A light arm T is clamped to the shaft S of the meter under test, and is connected by a link L to a sensitive balance GH. The balance has a knife-edge support at C; the position of equilibrium is indicated by the pointer CO. A definite weight G is attached to the balance, causing it to tip toward the left. Then the meter is loaded electrically until the pull on the link L brings the pointer O back to zero. The critical load is measured on an indicating wattmeter; the mechanical torque is calculated from the weight G and the leverage $TLVC$. From these data the torque is calculated per watt of load and per ounce weight of the revolving part of the meter, supported by the lower bearing. This gives a basis for the comparison of competitive meters of similar construction. The rods V and T are provided with several loops, so as to vary the torque in steps; two or more weights are supplied with the instrument, to cover a wide range of meters to be tested.

(b) *Pendulum device.* The essential feature of this device (Fig. 98) is a scale S on a concave spherical surface turned from a brass casting to a radius of curvature of 1 meter. The bob is supported from an adjustable arm, and is so arranged that the point of support P is at the center of curvature of the spherical scale. A silk fiber D is used as the suspension and is wound on a friction pin A, for convenience in adjusting its ength. The fiber passes through a V-shaped notch in the end of a brass strip B, which is capable of a small horizontal adjustment to aid in the initial centering. The whole is mounted on an ordinary clamp stand, the tripod being fitted with leveling screws. The clamp C allows the scale and bob F to be lowered as a unit, for convenience in applying to an instrument.

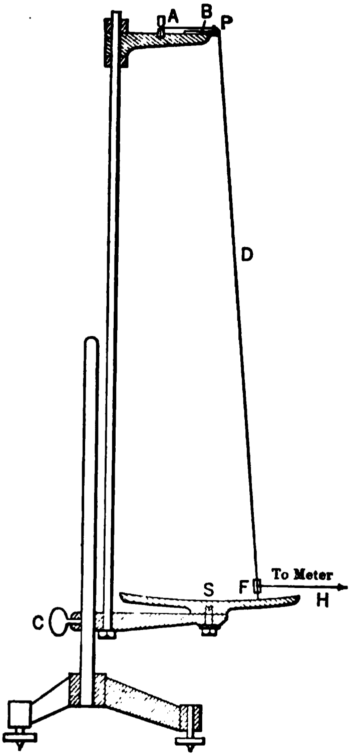


FIG. 98. A pendulum device for measuring the torque of a meter.

The bob consists of a small, hollow brass cylinder with a fine sewing-needle passed through it perpendicular to the axis. The silk fiber *H*, by which the horizontal force to be measured is transmitted to the bob, is attached to the needle and passes out along the axis of the hollow cylinder. The point of attachment is made at the center of mass of the bob, which, for convenience, is marked by a file scratch. The mass of the bob alone is adjusted to 0.5 gram, but in order to change the range, concentric cylinders, each cut in halves, are made to fit snugly over the inner cylinder. These extra cylinders make the total mass 1, 2, 5, 10, and 20 grams respectively, giving a range of 40 to 1 for full-scale deflection. A radial saw-cut in each half-cylinder allows these extra weights to be slipped into place without disconnecting either of the silk fibers.

The graduations on the scale consist of concentric circles, the radii of which are proportional to the trigonometric tangents of the angles of deflection of the pendulum, so that

$$\text{horizontal force} = \text{weight of bob} \times 0.001 \times \text{reading in divisions.}$$

In measuring the torque of a deflection instrument — for example, a voltmeter — the horizontal thread is fastened to the pointer at a convenient distance from the pivot, the whole pendulum system adjusted to the proper height, and the voltmeter moved horizontally until the desired deflection is produced. We then have

$$\text{torque} = \text{arm} \times \text{weight} \times 0.001 \times \text{reading in divisions.}$$

In the case of a watt-hour meter it is only necessary to attach the thread to the edge of the disk, apply current and voltage to the meter, and allow the thread to wind upon the disk as far as it will.

(c) Meters can also be compared by determining their so-called *friction-torque ratio.* For this purpose the friction compensator is adjusted at no-load so that the meter is just balanced, i.e., so that it will creep under the slightest vibration. When this is done, a load is applied equivalent to the full capacity of the meter, and the speed of the revolving part is measured. Then the friction compensator is disconnected and the speed measured again at the same load. The percentage of decrease in speed is proportional to the friction-torque ratio; thus, 2 per cent decrease in speed would mean a friction-torque ratio of 1 : 50; 1 per cent, 1 : 100, etc. The lower this ratio, the better is the meter, at least in so far as the disturbing influence of its friction is concerned.

116. EXPERIMENT 5-A. — Calibration of a Commutator-Type Watt-Hour Meter.— The purpose of the experiment is to study the performance of a commutator-type meter described in §§ 112 and 113, and to obtain experience in the methods of calibration indicated in § 114.

(1) Adjust the bearings and the brush tension on the meter, disconnect the compensating winding and make the anti-creeping device ineffective, if present. Connect the meter to a load at the normal voltage, and determine the minimum starting current at which the disk positively rotates. Be sure that the current required for the potential circuit does not flow through the field coils. Connect the compensating winding and adjust it as accurately as possible, to prevent the meter from creeping at no-load. Again determine the starting current.

(2) With the final adjustment so obtained, try to make the meter creep at no-load by subjecting it to vibration or stray fields, or by raising the supply voltage. Adjust the anti-creep device to make the meter reasonably immune against creeping due to the foregoing causes. Determine the starting current with the meter finally adjusted and ready for service.

(3) Calibrate the meter by one or more of the methods described in § 114. Begin with an overload of about 25 per cent and reduce the load in steps to zero, taking readings at each step. Adjust the permanent magnets so as to make the meter read correctly at full load. Check at a few other loads.

(4) Determine the torque per watt per unit weight of the moving element, and the friction-torque ratio, as explained in § 115.

Report. (a) Describe the findings in regard to the light load and anti-creeping adjustments, and give the value of the starting current. (b) State the influence of jarring, external magnetic fields, brush tension, etc. (c) Give the calibration curve of the meter using percentage of load as abscissæ and percentage of " slow " or " fast " as ordinates. (d) Give the value of the specific torque and the friction-torque ratio of the meter. (e) Mention the features of construction, the observed behavior of the meter, a method of testing, etc., different from those described in the text.

117. Mercury Watt-Hour Meter.— The above described commutator meter is widely used on D.C. circuits with satisfactory results, but it has the following disadvantages: (1) The commutator and the brushes are liable to get out of order. (2) Because there is no iron in the magnetic circuit, a large number of turns is required on both windings to produce a sufficient torque. (3) The moving part being comparatively heavy, the friction is considerable and varies somewhat during the life of the meter. (4) A commutator meter is not well adapted for operation with a shunt, and this limitation makes it inconvenient and bulky with heavy currents. (5) The meter is to some extent affected by external fields, even with a double astatic armature. (6) The pivot and the jewels are likely to give trouble, especially in locations subjected to vibration.

It is claimed that these objectionable features are largely eliminated in the D.C. watt-hour meter of the mercury-motor type, shown in Figs. 99, 100 and 101. It is essentially a homopolar electric motor with mercury contacts. The armature consists of the copper disk *A* floated in the mercury chamber *F* (Fig. 101) between the poles of an electromagnet *Y*. *EE* are the contact ears to the chamber, and the line current flows through the mercury and through the copper disk. This disk is slotted, as shown in Fig. 100, so as to guide the current at right angles across the field, thus rendering it more effective in producing the torque. The exciting winding *S* (Fig. 101) of the electromagnet is connected across the line, and a few series turns *C* are provided to compensate for the armature reaction (§ 358).

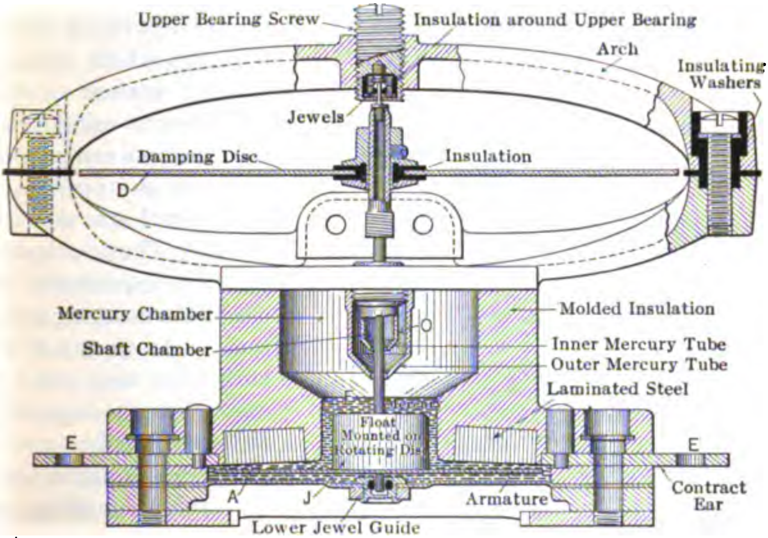


FIG. 99. A cross-section of a mercury watt-hour meter.

According to the fundamental law of electromagnetism, a conductor carrying an electric current in a magnetic field tends to move at right angles to both the current and the field. The direction of the current in the disk is radial, and the lines of force cross the disk essentially in a direction parallel to its shaft because of the magnetic return plate above the disk (marked "Laminated Steel" in Fig. 99). Thus, the mechanical force upon the disk is tangential and is proportional to the load current. The field strength varies only with the applied voltage and is therefore nearly constant. The brake or damping disk *D* is placed in the field of permanent magnets and performs the same function as in any other watt-hour meter; that is, it acts as a load and causes the meter to run at a speed proportional to the electromagnetic torque (§ 112).

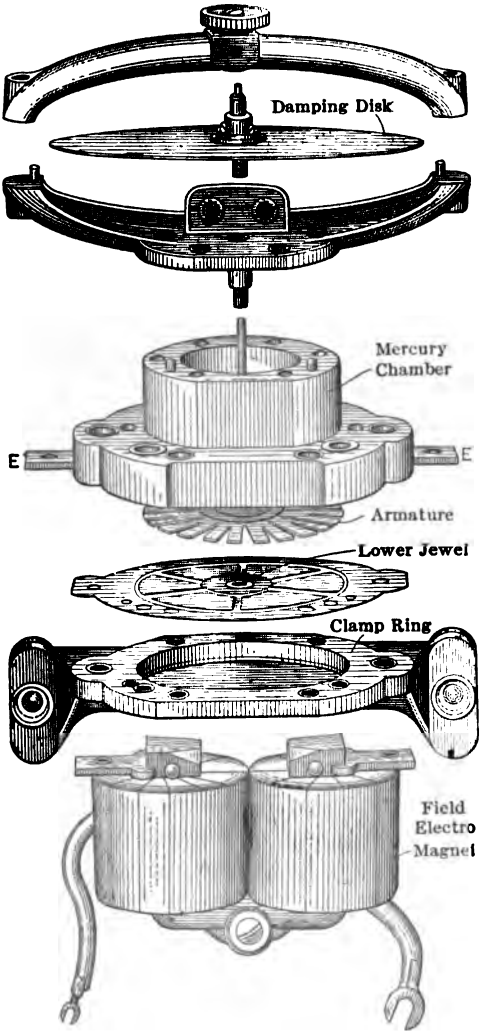


FIG. 100. The principal parts of the mercury meter shown in Fig. 99.

The armature disk *A* is provided with a float *J* (Fig. 99) which gives the moving system a slight buoyancy upwards, so that the main bearing of the meter is on top, and the lower jewel acts merely as a guide. The double sleeve or tube *O* prevents mercury from escaping in shipment. Any spray of mercury that may enter the outer tube is caught in the inner tube, and as the meter is again put in an upright position this mercury falls immediately back into the regular chamber.

On account of the presence of iron with variable permeability, the accuracy of the mercury meter is somewhat affected by fluctuations in the line voltage. A reduction in the line voltage causes the meter to run slightly fast and an increase in voltage above normal causes it to run slow. This factor must be considered in an installation with a poor voltage regulation. The meter is not affected appreciably by changes in temperature. An increase in temperature will increase the resistance of the damping disk, thus reducing the eddy currents for a given speed, and, therefore, reducing the counter-torque; consequently the meter will tend to speed up. However, a similar change occurs in the resistance of the potential winding, resulting in a decrease in the motor field and a corresponding decrease in the motor torque. The combined effect is that the speed remains practically unchanged.

On account of the inherently low starting torque of mercury meters, there has been a slight tendency to creep on over-voltage. To offset this, an anti-creeping device is used which consists of a very small piece of irregular-shaped iron placed between the float *J* and the armature disk *A*. The effect is to stop the meter when the position of the iron is such that the greatest field flux passes through it.

A small amount of dross is sometimes found in the mercury chamber, and is due to oxidation of the copper and the formation of copper-mercury amalgam. This trouble has not been found to be serious, although some foreign companies platinize the surface of the armature and then bake it in japan in order to reduce the quantity of copper-mercury amalgam. It is also of importance to eliminate all ammonia in the manufacture of the molding materials from which the mercury chambers are usually made. Any trace of ammonia readily forms an ammonia-mercury amalgam which is a heavy plastic substance and soon affects the accuracy of the meter. A method has been found of chemically treating condensite or bakelite mercury chambers so as to reduce greatly the formation of dross.

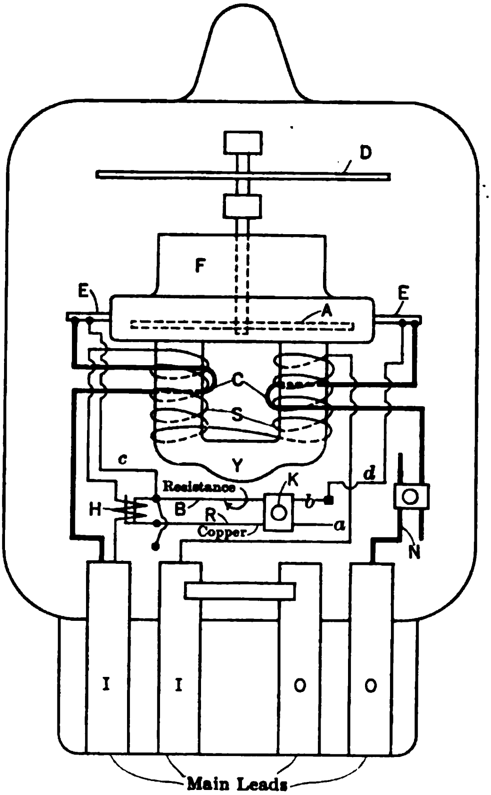


Fig. 101. The electric circuits of the meter shown in Figs. 99 and 100.

The friction compensation is accomplished by means of the thermo-couple *ab* (Fig. 101) which is heated by the coil *H* in the potential circuit. The local circuit of the thermo-couple is closed through a resistance wire *B*, the adjustable clamp *K*, and the copper wire *R*. The leads *c* and *d* from this circuit are connected to the contact ears *EE* of the mercury chamber. At no-load, or at light loads, the thermo-couple circuit furnishes an armature current sufficient to overcome the friction.

118. EXPERIMENT 5-B. — Calibration of a Mercury Watt-Hour Meter.— The procedure is similar to that of experiment 5-A. Vary the line voltage to a considerable extent, and study the effect of poor voltage regulation on the accuracy of the meter. Report results as in experiment 5-A.

119. Ampere-Hour Meters. — In a direct-current circuit in which the voltage is kept practically constant, watts are proportional to amperes, and it is sufficient to measure ampere-hours, instead of watt-hours. Multiplying the former by the actual or agreed voltage of the supply gives the energy consumption in watt-hours. Electrolytic meters are to some extent used in such cases, especially abroad. The current flowing through the meter decomposes a certain chemical contained in it. The amount of material decomposed is proportional to coulombs or to ampere-hours which pass through the meter; the scale can be made direct-reading.

An ampere-hour meter may also be built on the same principle as the commutator meter shown in Fig. 94. In this case the stationary field is supplied by a permanent magnet, and the current through the armature is made proportional to the line current by means of an ammeter shunt. The mercury-motor meter shown in Fig. 101 is used extensively as an ampere-hour meter, the shunt electromagnet being replaced by a permanent magnet. In this case the torque is proportional only to the current through the armature and does not depend upon the line voltage.

There is also a small commutator-type direct-current ampere-hour meter of the general form well known in Europe for a number of years, in which three flat coils, suitably disposed between two aluminum disks, rotate in the field of two powerful permanent magnets. The armature is connected across a suitable shunt of 35 to 40 milli-volts drop. The demand for such meters has been principally in foreign countries and for very small consumers, in rated capacities of 1, 2 and 5 amperes, sometimes larger. The meter reads in ampere-hours, and, if desired, the dial can be arranged to read in kilowatt-hours at a given voltage.

Ampere-hour meters are used to a considerable extent in storage-battery operation, in determining the economy of electric railway cars, in electro-plating, etc.

INDUCTION-TYPE WATT-HOUR METERS

120. Watt-hour meters used on alternating-current circuits operate on the principle of the split-phase induction motor (§ 492), and in this respect are similar to the corresponding type of indicating wattmeters (§ 97). A representative magnetic circuit of an induction-type watt-hour meter is shown in Figs. 102 and 103. Other watt-hour meters on the market differ from it in minor details only. The magnetic circuit is built up of soft-steel laminations to suppress eddy currents. The series or current winding is placed on the pole-pieces *a*, *b*, and the potential winding (shunt coil) on the pole-piece *c*. The revolving part, or armature, of the meter consists of a corrugated aluminum disk placed partly in the air-gap between the poles. The shaft which supports the disk is geared to the recording train, as in the D.C. meter shown in Fig. 93. The alternating magnetic fluxes excited by the wind-

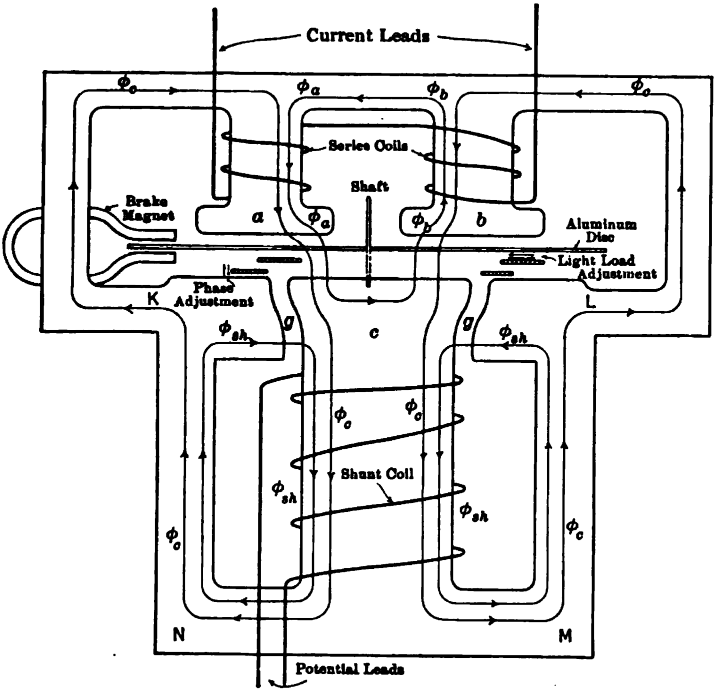


FIG. 102. An induction-type watt-hour meter.

ings on the pole-pieces induce secondary eddy currents in the disk, and the reaction between these currents and the fluxes sets the disk in rotation.

It is shown in § 122 that the torque is a maximum when the flux excited by the shunt winding lags by 90° in time behind that in the series winding. This condition must obtain at unity power factor, because the meter must then revolve at its highest speed. But at unity power factor the voltage across the potential winding is in phase with the line current; therefore the flux in the pole *c* must be *lagged* behind the applied voltage by exactly 90 degrees. For this purpose the lower part, *KLMN*, of the magnetic circuit is shaped like the core of a choke coil, with small air-gaps *gg* on top. Most of the flux excited by the shunt coil takes the path marked ϕ_{sh}, and only a small part, ϕ_c, acts upon the aluminum disk. The portion ϕ_{sh} serves to reduce the shunt current by its reactive or choking action and to keep it as nearly as possible in quadrature with the voltage.

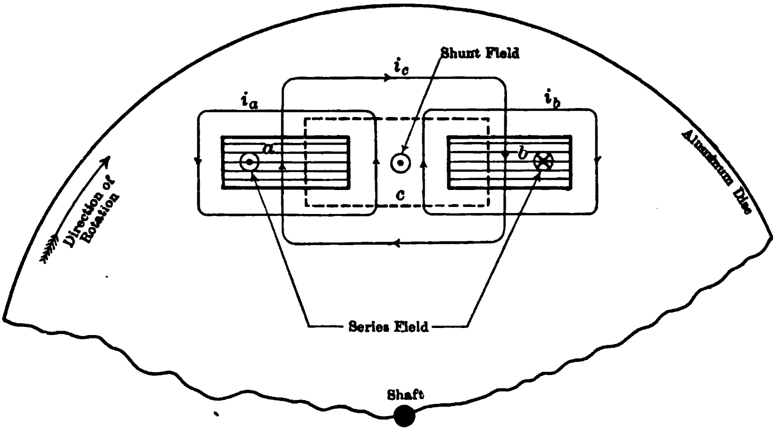


FIG. 103. The plan view of the meter shown in Fig. 102.

The resistance of the shunt coil is kept low, so that the potential circuit is highly inductive and the current lags by nearly 90 degrees behind the applied voltage. However, it is impossible to displace the current by fully 90 degrees because of unavoidable resistance and core loss. The exact quadrature lagging is accomplished by means of a short-circuited secondary coil the action of which is similar to that described in §§ 97 and 98. Such a " lagging " or " phase-adjustment " coil is shown schematically in Fig. 102 above the top of pole *c*. In some meters it consists of a few turns of wire short-circuited through an adjustable resistance. In others it consists of a single turn punched out of copper plate and mounted so that its position may be varied up and down by means of an adjusting screw.

The speed of the meter at full non-inductive load depends upon the position of the brake magnets (§ 112), one of which is shown schematically to the left in Fig. 102. These magnets act on the same aluminum disk as the A.C. field, and retard the motion of the disk by induced eddy currents. The speed of the meter is adjusted either by moving the magnets in and out, or by shunting part of their flux through a piece of iron.

On account of the unavoidable friction, the meter requires a light-load adjustment (§ 113). Otherwise it would run slow at light loads and would not start until the load torque was high enough to overcome the friction. The friction compensating device usually consists of a short-circuited turn of flat copper shown in the main air-gap in Fig. 102 just below the aluminum disk. When this turn is concentric with the pole *c*, the secondary currents induced in it only reduce the magnitude of the flux ϕ_c. But when this turn is shifted to one side, it acts as a "shading coil," and its action becomes somewhat similar to that of the poles *a* and *b*. The currents induced in the aluminum disk are shifted both in time and in space position so as to exert a small electromagnetic effort upon the disk. This effort may be adjusted to compensate the friction, and to make the meter start on a very small fraction of the rated load. Creeping at no load is sometimes prevented by punching two small slots on the periphery of the aluminum disk. When one of these slots comes under the shunt pole, the induced currents are so modified that the meter stops. These slots have no appreciable effect when the series coils are also excited. See also § 113 in regard to light-load adjustment and creeping.

It is well to remember that the shaft of an A.C. meter is constantly vibrating because of the alternating action upon the aluminum disk. This subjects the supporting jewel to a type of wear absent in a D.C. meter. A light moving element is therefore more important in an A.C. meter. In some meters a steel ball is used in place of a jewel for the lower bearing, thus substituting rolling friction for the ordinary jewel friction A flexible or yielding top bearing is usually necessary to prevent the noise due to vibration.

Reactive-energy meter A watt-hour meter may be used to measure the total reactive energy instead of the true energy. For this purpose, its potential circuit is connected to a source of voltage displaced by 90 degrees with respect to the load voltage. This subject is treated in § 102 in application to indicating wattmeters, and the theory and the connections are directly applicable to watt-hour meters.

121. Polyphase Watt-Hour Meters. — A two-phase or three-phase watt-hour meter is obtained by combining two single-phase meters described above. The two armatures are mounted on the same shaft and act on the same recording train, thus automatically adding together the indications of the two component meters. In some polyphase meters the two field elements act on the same aluminum disk. The brake magnet also acts on the same disk, thus making the whole construction light and compact.

The method of connecting a polyphase meter into a three-phase circuit is shown in Fig. 104; the scheme of connections is based on the fact that one of the three line wires, *B* for example, may be considered as a common return for the two other line wires *A* and *C*. (See the two-wattmeter method, § 100.) The three upper connections belong to the upper component meter, the three lower connections to the lower meter. There are no electrical connections between the two component meters inside the case. The upper meter registers the energy between the wires *A* and *B*; the lower meter, that between *C* and *B*. Three terminals are used instead of four, because one of the current terminals serves at the same time as a potential terminal. For the general theory of power measurement in polyphase systems see Vol. II.

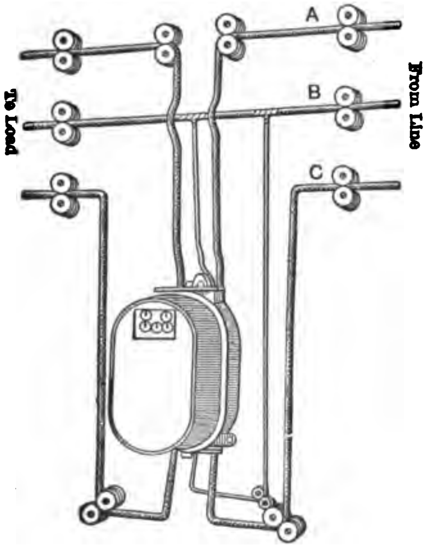


Fig. 104. Connections between a three-phase line and a polyphase watt-hour meter.

Induction type watt-hour meters are built only for moderate currents and voltages. For heavy currents and for high voltages they are connected to the line through current and potential transformers (see Chapter XVIII).

122. Theory of Induction Watt-Hour Meter. — It is customary to dismiss the theory of this type of instruments (Figs. 102 and 103) by stating that a gliding magnetic flux is produced in the air-gap due to the combined action of the series and shunt fields, as in a split-phase induction motor. The aluminum disk is said to follow this gliding flux because of induced eddy currents, as in the familiar Arago experiment.

The difficulty with this explanation is that the consecutive poles do not follow alternately around a cylindrical air-gap, as in a motor. There are only two series poles and one shunt pole, active in the production of the gliding field, and it is not clear what becomes of this assumed gliding flux after it has traveled, say from *a* to *b*. It cannot go any farther; and, should it return, a counter-torque would be exerted upon the aluminum disk. Another explanation is therefore offered below, based upon a consideration of the actual currents and fluxes in the air-gap.

In Fig. 105 the sine-waves ϕ_a and ϕ_b represent instantaneous values of the fluxes produced by the windings on the poles *a* and *b*. The vectors of these fluxes are shown in Fig. 106. An instantaneous flux is considered positive when it is directed upward in the air-gap. Hence ϕ_a is positive when ϕ_b is negative and vice versa. In a properly adjusted meter, with a non-inductive load, the flux ϕ_c, due to the winding on pole *c*, lags 90 degrees behind ϕ_a, or behind ϕ_b, depending upon the desired direction of rotation of the armature. It is shown in the figures lagging behind ϕ_a. The transformer action causes these three fluxes to induce secondary currents in the disk. These currents surround the projections of the corresponding poles and are denoted by i_a, i_b, and i_c respectively, in Fig. 103. Current i_a is so located with respect to the flux ϕ_a that their mutual mechanical action has no component in the plane of the disk. However, current i_a flows partly in the field ϕ_c, and there is a tangential mechanical force between the two. Similarly, there is an interaction between the current i_b and the field ϕ_c, and between the current i_c and the fields ϕ_a and ϕ_b. All these torques are in the plane of the disk, and it remains only to be shown that the actions are cumulative and therefore must result in the rotation of the disk.

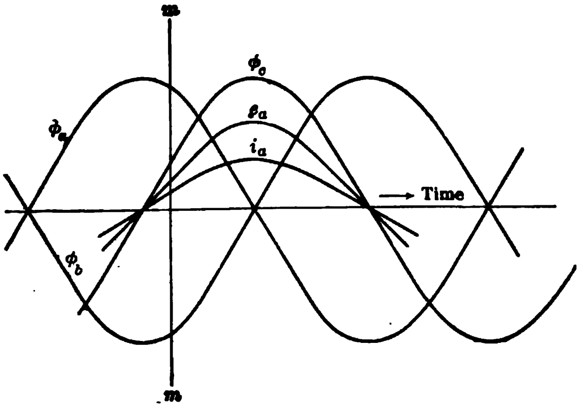


FIG. 105. The fluxes and the currents in the meter shown in Figs. 102 and 103.

The voltage e_a, induced by ϕ_a, lags behind ϕ_a by 90 degrees, and is therefore shown in phase with ϕ_c. The total current I_a (Fig. 106 A), due to e_a, lags behind the latter by a certain angle β, depending upon the ratio of the reactance of the path to its resistance (§ 140). This current, I_a, may be resolved into the component i_a in phase with e_a and i'_a in quadrature with it. Since ϕ_c and i_a are in time phase with one another, the mechanical force between the two is always in the same direction, varying from a positive maximum to zero. The current component i'_a is in quadrature with ϕ_c, and therefore the instantaneous force is alternately positive and negative, the average value being equal to zero. Thus, for our purposes it is only necessary to consider the components i_a, i_b, and i_c of the currents which are in phase with the corresponding induced voltages.

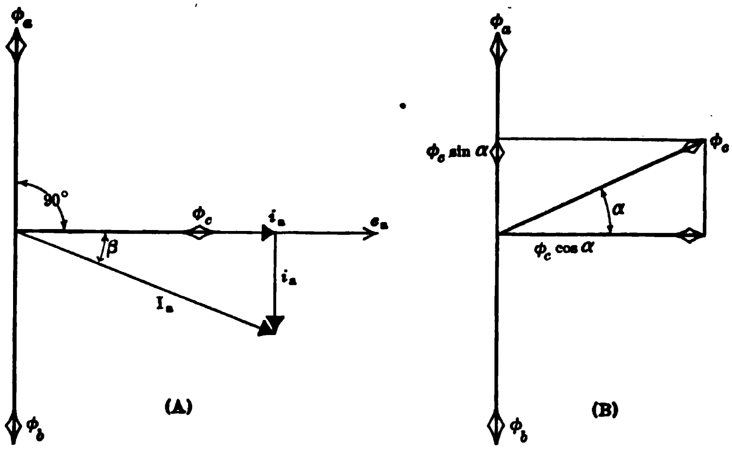


FIG. 106. The vectors of the quantities shown in Fig. 105.

To show that the electromagnetic forces are cumulative, consider an instant of time such as is represented by *mm* (Fig. 105). The three fluxes have the directions shown in Fig. 103 by two dots and a cross within small circles. The absolute values of the fluxes ϕ_a and ϕ_b are decreasing; that of ϕ_c is increasing. Hence, according to Faraday's law of induction, the three currents have the instantaneous directions shown by the arrowheads in Fig. 103. Applying the well-known rule for the direction of the mechanical force between a flux and a current, it will be found that all four combinations cause the disk to rotate from left to right. Since we have shown before that the direction of each force keeps the same sign throughout a complete cycle, the cumulative action is thus proved for any instant.

Now let the line current lag behind the applied voltage by an angle α (Fig. 106 B). This is the same as if the applied voltage, and consequently the flux ϕ_c, were advanced by the same angle α with respect to their former phase position, and the fluxes ϕ_a and ϕ_b remained intact. The flux ϕ_c may be resolved into two components in time: ϕ_c Cos α in quadrature with ϕ_a, and ϕ_c Sin α in phase with ϕ_a. The discussion given above for the flux ϕ_c may be applied to the component ϕ_c Cos α, because it occupies the same space and time position as the total flux ϕ_c occupied at unity power factor. Thus the torque due to this component of the flux is reduced in the ratio of Cos α : 1 as compared to its former value. The component ϕ_c Sin α simply increases the flux ϕ_a and reduces ϕ_b by the same amount. This affects the values of the corresponding currents i_a and i_b (Fig. 103). But since one is reduced by the same amount by which the other is increased, their total torque with ϕ_c Cos α remains the same as if the component ϕ_c Sin α did not exist. Thus, the net result is that the tractive effort upon the disk is reduced in the ratio of Cos α : 1. An ideal meter should read correctly at any value of power factor, if it is correctly adjusted at unity power factor.

Two assumptions are made in the foregoing theory which do not hold true in an ordinary single-phase motor designed for delivering power: (1) The reaction of the disk currents upon the fields is neglected. (2) The rotational e.m.f'.s and the corresponding currents in the disk are neglected. These assumptions are justified in a meter, because the disk currents are small and the speed of rotation is very low.

123. Calibration of Induction-Type Watt-Hour Meters. — The points to be covered and the procedure are in general the same as in the case of a commutator-type meter (§ 114). The following instructions cover only those points which have not been taken up before, or where the procedure differs from that followed with a D.C. meter. The preliminary adjustments include the bearings, the friction compensation, the full-load speed, and the phase lag (§ 120). An inaccurate lagging of the potential phase is of little effect at a large non-inductive load, but becomes of increasing importance as the load decreases and becomes more inductive, that is, as the meter speed is reduced. After the brake magnets are adjusted so that the meter runs correctly near unity power factor, the lagging coil is adjusted so that the meter reads nearly correct at a power factor of about 50 per cent Then the reading at 100 per cent power factor should be checked again, and the two adjustments set by several trials until the meter has as nearly as possible the same constant at both values of power factor. If a two-phase source of supply is available, with the phases exactly in quadrature, a further check on the phase adjustment is possible at zero power factor, by connecting the current leads to a non-inductive load on one phase, and using the other phase for the potential circuit. See also §§ 127 and 128 regarding phase-shifters and a phantom load.

Of the three methods of calibration described in § 114, a direct comparison with a standard watt-hour meter (rotating standard) is the most convenient. The standard watt-hour meter is usually also of the induction type and its number of watt-hours per revolution is known. Accurate current and potential transformers may be used to bring the rotating standard and the meter under test to within approximately the same range, although a standard wattmeter usually has several ranges, which are obtained by properly interconnecting its coils.

The indicating-wattmeter method, with a stop-watch, is also used to a considerable extent, especially when the voltage and the frequency are steady. When a standardized load resistance is used, care must be taken to have it wound non-inductively, or else its true power consumption and the power factor must be determined by separate tests.

The constant of an induction-type watt-hour meter is affected by the wave-form and frequency of the supply. Care must be taken, therefore, to see that these are standard, or else the deviations must be known. A good meter is affected but little by reasonably small departures from the sine-wave and from the rated frequency, but a meter of a new or unknown type should be carefully tested in this respect, especially at low values of power factor.

124. EXPERIMENT 5-C. — Calibration of an Induction-Type Watt-Hour Meter. — The meter is described in §§ 120 to 122. The experiment is performed in the same order as Experiment 5-A, keeping in mind the difference in the construction of an A.C. meter and the necessity of considering not only the magnitude of the load but its power factor and frequency as well. These points are covered in § 123. If a phase-shifter is available (§ 127), make use of it in part of the calibration, and make clear to yourself its advantages over an inductive load. A phase-shifter may be improvised in the laboratory to show its mode of operation, even though its accuracy may not be sufficient for actual use. Provide a phantom load (§ 128), and compare the ease of manipulation, the accuracy, and the energy consumption with those when an actual load is used.

125. Some Causes of Inaccuracy. — In the 1916 edition of the Electrical Meterman's Handbook, published by the National Electric Light Association, the following table is given on page 414 in regard to possible troubles in an induction-type watt-hour meter and their effect upon the operating characteristics. This table will be found useful in the practical management of meters. Moreover, it will give a thoughtful student an opportunity to verify his understanding of the theory. Given a trouble, say a short-circuit in the potential coil, he should be able to predict theoretically its effect upon the performance of the meter.

Condition Found	Possible Causes
Slow. Full load and light load....................	Short circuit in current or potential coils; loose magnets.
Slow. Principally at light load..................	Defective bearing; other excessive friction; loose light-load adjustment.
Fast. Full load and light load....................	Weakened or loose magnets; short-circuit in series reactance of potential circuit (non-inductive load).
Fast. Principally at light load..................	Loose or defective light-load adjustment; vibration; short-circuit in current coils.

A frequent cause of trouble with polyphase meters, especially when used with instrument transformers (Chapter XVIII), is wrong connections. A clear understanding of polyphase connections (Vol. II) is the only safe guide in cases not covered by standard diagrams furnished by the maker.

126. EXPERIMENT 5-D. — Troubles in Induction-Type Watt-Hour Meters. — The principal troubles are enumerated in the preceding article. A meter should be provided in which the coils may be partially short-circuited at will, bearings, magnets, and compensating coils put out of adjustment, etc. The student can then check the points covered in the foregoing table, as well as provide other combinations of troubles. If a polyphase meter is available, the effect of wrong connections should be studied, with and without instrument transformers, especially the effect of reversed leads.

Report. Give the findings in each case of trouble, and explain theoretically that the observed behavior of the meter could be predicted from its construction and mode of operation.

127. Potential Phase-Shifters. — A watt-hour meter which may be used on partly inductive loads must be tested for its accuracy of calibration at different values of power factor. It is rather inconvenient to provide such an adjustable load in the laboratory, and somewhat tedious to set it at a desired value of current and power factor. It is much more convenient always to load the meter on a lamp bank or a grid resistance at practically unity power factor, and to supply the potential windings with a current properly shifted in phase. For example, let it be required to test a meter at a phase displacement of 30 degrees between the voltage and the current, the latter lagging. It is much simpler to advance the voltage by 30 degrees than to lag the current by the same amount. There are two types of devices used for shifting the potential phase in instrument testing, particularly in the calibration of watt-hour meters. Either of them may be conveniently combined with the artificial load described in the next section.

(a) *The two-component potential shifter.* The principle upon which this device is built may be understood from Fig. 107. Two transformers or auto-transformers (§§ 379 and 395) *A* and *B* are connected to phases I and II, respectively, of a two- or three-phase supply. Each transformer is provided with a number of taps. By properly cross-connecting these taps, a voltage of any desired magnitude and time phase may be obtained. In the sketch the taps *n* and *p* are shown connected together, and the potential leads to the meter are taken from the taps *m* and *q*. The theory of this method may be explained by reference

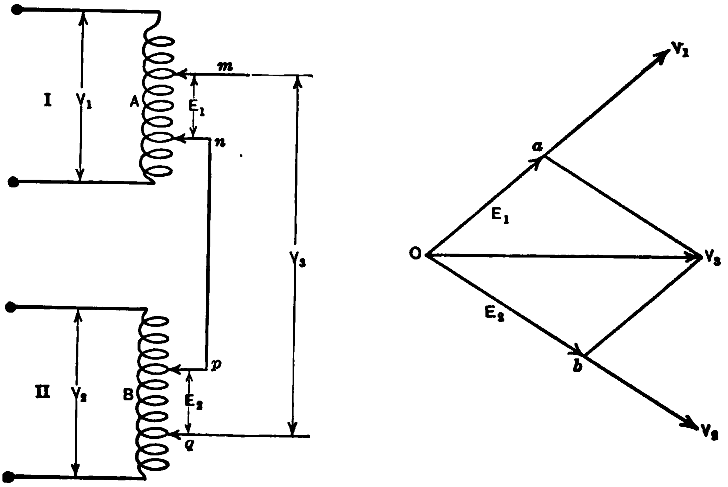


FIG. 107. A two-component potential phase shifter.

to the vector diagram. OV_1 and OV_2 are the vectors of the applied voltages I and II, assumed to be at a known phase angle to each other, such as 60°, 90°, or 120°.

The voltage between the taps m and n is a certain fraction of the total voltage OV_1 and is shown as the vector $E_1 = Oa$ in phase with OV_1. Similarly, the voltage $Ob = E_2$ between p and q is a fraction of the voltage OV_2. The secondary potential difference V_3 between m and q is a geometric sum of the voltages E_1 and E_2, and is therefore equal to the diagonal OV_3 of the parallelogram constructed on Oa and Ob. By giving different positive and negative values to E_1 and E_2, the resultant secondary voltage V_3 may be adjusted to any desired magnitude or phase position with respect to V_1 and V_2.

In a convenient practical device of this kind the connections are such that the phase of the secondary voltage is shifted by simply turning a handle on a dial or inserting a plug in one of a number of holes. Moreover, the magnitude of the voltage remains constant and only its time-phase is shifted. A scale is provided which indicates either angles or the corresponding values of power factor. If the meter load is slightly inductive, the scale itself is shifted so as still to have the correct 100 per cent point when the voltage and the current are in phase with each other. When the phase displacement between the current and the voltage is exactly 90 degrees, an indicating wattmeter connected in the circuit shows zero. It must be remembered that this device requires a two-phase or three-phase source of supply even for testing single-phase meters, but it does away with the necessity of providing an adjustable inductive load.

(b) *The revolving-field potential shifter.* This device is described in § 85 in application to an A.C. potentiometer. The potential shifter is shown there with a split-phase stator, because only a single-phase supply is supposed to be available. With a two-phase supply, the windings *aa* and *bb* are connected directly to the two phases, so that the condenser and resistance R are unnecessary. With a three-phase supply, the stator must be wound accordingly, or else the three-phase system must be converted into a two-phase system, by one of the known methods (see Phase Transformation, Vol. II). The point on the dial corresponding to zero power factor is determined by an indicating wattmeter which shows zero at this point.

128. Artificial or Phantom Load. — The regular testing of many watt-hour meters under full-load conditions results in the wasting of considerable amounts of energy, requires bulky resistances, and causes excessive heat losses which sometimes make the meter room rather uncomfortable. For this reason it is preferable to supply the load current for the meter through a separate transformer, at a considerably reduced voltage, say at about 10 volts for a 110-volt meter. The potential leads of the meter are, of course, connected to a source of the rated voltage. With such an arrangement it is possible to have the same current and voltage relations in the meter as if it were measuring an actual load, at the same time saving about 90 per cent of the energy which otherwise would be required for calibration. Such an artificial or phantom load may be conveniently used in combination with one of the potential phase shifters described in the preceding section.

A phantom load usually consists of a constant-potential transformer with low-voltage secondary, resistors connected to the secondary, and switches for varying the load. The primary of the transformer is connected to the same source of supply as the potential winding of the watt-hour meter. The secondary circuit of the load also contains the series winding of the watt-hour meter, an ammeter, a wattmeter, etc. With a 10 to 1 transformer ratio only 1 ampere is drawn from the supply for every 10 amperes that flow through the meter. The phase relationship between the current and the voltage may be adjusted by means of a potential phase shifter. Phantom load combinations are also available in portable form for testing meters on consumers' premises.

129. Maximum-Demand Indicators. — The actual cost of supplying a certain number of kilowatt-hours of energy to a consumer generally depends on the rate at which he consumes this energy (§ 110). A consumer who uses 1 kilowatt regularly for eight hours a day is more desirable to the operating company than another who uses 4 kilowatts for two hours. The amount of energy per month may be the same in both cases, but the first consumer requires less capacity of the generating apparatus and a smaller transmission line than the second one; he also causes smaller fluctuations of the load. Therefore the company may give him a better rate per kilowatt-hour. This principle of " discrimination " in charging for electrical energy is used by many operating companies. The consumer is charged so much per kilowatt-hour of actually consumed energy, and then an extra charge is made for each kilowatt of his *maximum demand.*

This demand or peak is usually agreed to be the maximum amount of energy consumed in any consecutive thirty minutes during the month, or in any consecutive fifteen minutes, etc., according to the local conditions. This amount of energy, divided by the time, is taken as the maximum demand in kilowatts, although it only gives the average rate of energy consumption during the peak period. For example, if the total energy consumption during a half hour of heaviest load is 1250 kw.-hours, the demand is taken to be 2500 kw. This method of reckoning does not penalize the customer for very short peaks.

An instrument for measuring the maximum demand during a month, or other agreed period of time, is called a maximum-demand indicator. Some of them are attachments to watt-hour meters; some are modified watt-hour meters; and some are based on the thermal principle. It would lead too far to describe these numerous devices in this book. The following description of one of the oldest demand indicators will suffice to illustrate the general principle. Much information on the subject will be found in various publications of the National Electric Light Association.

130. The Wright Maximum-Demand Indicator.—This device is shown schematically in Fig. 108. It records the maximum current of certain duration which has passed through it at any time since it was last set. A liquid is hermetically sealed in a glass tube having a bulb at each end. A band of resistance metal is placed around the left-hand bulb *A*. The current to be measured passes through this band, heats the air in the bulb, and the expansion of the air forces the liquid up into the right-hand side of the tube, causing it to overflow into the middle or indicating tube. The liquid deposited in the indicating tube remains there until the indicator is reset. The glass tube is carried on a backing, so hinged that the meter can be reset by tipping the tube and allowing the liquid to run out of the indicating tube into the side tubes. It is the difference in temperature of the air in the two bulbs that causes the indicator to register. Any change in the temperature of the external air causes equal air expansion in both bulbs, and hence does not affect the reading.

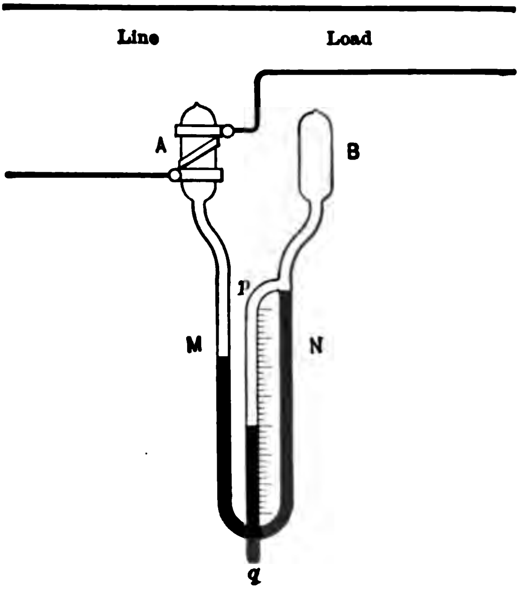


Fig. 108. Wright maximum-demand indicator.

The instrument is purposely made slow-acting; standard indicators are so designed that if the maximum load lasts only five minutes, the meter will register about 80 per cent; if ten minutes, 95 per cent, and the full 100 per cent is registered when the load has continued about forty minutes. In this way the customer is not penalized for short overloads which do not inconvenience the power supply station.

CHAPTER VI

REACTANCE AND RESISTANCE IN A.C. CIRCUITS

131. Physical Concept of Inductance. — When an electric current exists in a conductor, a magnetic field is created around it. As long as the current remains constant, this field does not react in any way upon the electric circuit. But if the current varies, the flux also will change; in doing so it induces in the conductor an electromotive force. In other words, the magnetic field reacts upon the circuit and either affects the rate of change of the current, or else requires an additional applied voltage to counteract its effect. This induced e.m.f., in accordance with the fundamental law of electromagnetic induction, has such a direction as to oppose the change in current, and consequently to retard the change in flux. The action is very much the same as if the magnetic field had some kind of inherent *inertia*.

The following analogue has been given to illustrate this phenomenon. Imagine the magnetic field surrounding a conductor as consisting of whirls in ether driven by the current. The axes of rotation of these whirls may be assumed to coincide with the direction of the lines of force. Assume these whirls to possess some inertia. As long as the current is steady, the whirls are spinning at the same speed, and their inertia does not affect the circuit. If, however, the applied e.m.f. be reduced, tending to reduce the current, the whirls by virtue of their inertia tend to spin at the same speed, and thus oppose the decrease of current. The current gradually decreases, and the field returns some of its stored energy to the source of the e.m.f. On the contrary, should the current be increasing, the inertia of the whirls opposes its rise. The applied e.m.f. has to perform some additional work in accelerating the whirls, and the stored energy of the magnetic field is increased by the same amount.

The opposing action of the whirls, or of the magnetic field, is stronger, the greater the flux and the quicker the changes in the applied electromotive force, that is, the greater its frequency. This is in some respects analogous to the reaction of a heavy fly-wheel driven at a non-uniform speed — the reaction is proportional to the inertia of the wheel and to the rate at which it is accelerated.

Whatever the explanation of the phenomenon, the observed fact is that an electrical conductor opposes any change in the magnitude or direction of a current in it, by generating a counter-e.m.f. The value, ϵ, of this counter-e.m.f. is expressed by the relationship

$$\epsilon = -L\, di/dt \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

where i is the instantaneous current, di/dt is its rate of change with the time, and L is a coefficient of proportionality, called the *inductance*, or the coefficient of self-induction, of the part of the circuit under consideration. The minus sign is necessary if the direction of the induced e.m.f. is considered positive in the same sense as the current. When the current increases, the reaction of the magnetic flux induces an e.m.f. which opposes this increase. In other words, ϵ is negative when di/dt is positive, and vice versa. The minus sign makes the quantities on both sides of the equation of the same sign. The practical unit of inductance in the ampere-ohm system is called the henry. *An electrical device or circuit is said to have an inductance of one henry if one volt of counter-e.m.f. is induced in it when the current changes at a rate of one ampere per second.*

The inductance of a conductor changes with a change in the distribution of the current through its cross-section. Thus at a low frequency the current in a straight cylindrical conductor is distributed uniformly over its cross-section. At high frequencies, however, there is a decided tendency for the current to exist only in the outer parts, this phenomenon being known as the *skin effect.* The flux linkages within the conductor are thus reduced and the inductance becomes smaller, although the shape of the conductor is the same. Therefore, the inductance of a conductor depends not only upon its shape but also upon the distribution of current in it.

132. Difference between Inductance and Ohmic Resistance. — The presence of inductance in an alternating-current circuit usually necessitates an increase in the applied e.m.f. in order to maintain the same current as without the inductance. An increase in resistance has a similar effect. Thus, it may at first seem that, from a practical standpoint, resistance and inductance produce the same effect on the electrical relations in the circuit, and do not need to be distinguished. There is, however, a very fundamental difference between the two, as a consideration of the following points will show:

(1) An ohmic resistance is noticeable to the same extent whether the current is steady or variable; the drop caused by it is at any moment proportional to the *instantaneous value* of the current. An inductance becomes apparent only when the current is varying; the induced e.m.f., or the resulting voltage drop, being proportional to the

rate of change of the current, and not to its absolute value; see eq. (1) above.

(2) The ohmic resistance and the accompanying heating of a metallic conductor are said to be due to collisions of streaming electrons, which constitute the electric current, against the molecules of the metal. Inductance, on the other hand, is supposed to be caused by the inertia of the magnetic flux surrounding the conductor.

(3) The energy spent in overcoming an ohmic resistance is converted into heat, and is lost electrically. The electric energy spent in overcoming an inductance, when the current increases, is stored in the form of electromagnetic energy of the field, and is given back to the circuit, when the current decreases, through the medium of induced e.m.f.

(4) A wire, whether straight or wound into a coil, has the same ohmic resistance, while its inductance in the second case is increased many times because of the concentration of the magnetic flux. Thus, generally speaking, the ohmic resistance of a cylindrical conductor does not depend upon the shape of its geometric axis, while its inductance is essentially determined by the shape into which the conductor is bent.

133. Inductance and Reactance. — In many practical problems inductance is treated in connection with sinusoidal alternating currents of a constant frequency. It is convenient, therefore, to combine the frequency factor with the value of inductance; this simply means a change in the unit and in the physical dimension of the quantity which expresses the inertia of the magnetic field. The new physical quantity is called the *reactance* of a circuit or of a device. It will be shown that reactance, like ordinary resistance, may be expressed in ohms.

The following considerations lead to the concept of reactance. According to the foregoing definition of inductance, the applied voltage necessary for overcoming the effect of an inductance of L henrys is

$$e = L\,di/dt \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1a)$$

where the voltage $e = -\epsilon$ may be properly called the instantaneous inductive drop of potential in the conductor or circuit of inductance L. The voltage e is at any instant equal and opposite to the counter-e.m.f. ϵ induced in the conductor by the varying magnetic flux. It is more convenient in the following discussion to consider the voltage e rather than ϵ.

Let the current vary according to the sine law, Fig. 109, at a frequency of f cycles per second. The instantaneous values of the current may be expressed by the formula

$$i = I_m \mathrm{Sin}\, 2\pi f t \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

where I_m is the amplitude of the wave, and t is time in seconds. Substituting the value of i from eq. (2) in eq. (1a), we obtain the following value for the inductive drop:

$$e = 2\pi fLI_m \text{Cos}\, 2\pi ft \quad \dots\dots\dots\dots \quad (3)$$

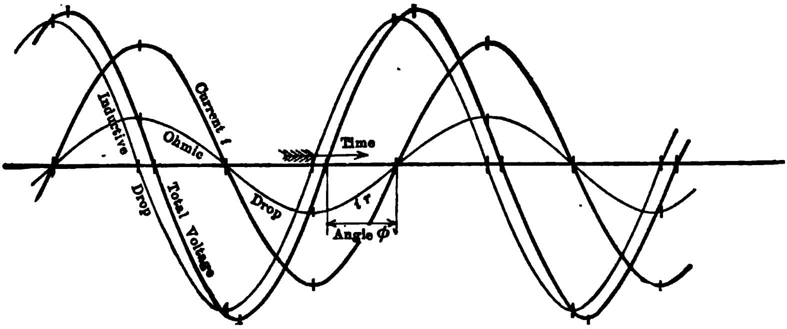


FIG. 109. Instantaneous values of current and voltage in an alternating-current circuit containing resistance and inductance.

This result interpreted means that a sinusoidal current through a pure inductance sets up a sinusoidal counter-e.m.f. of the same frequency and requires an applied voltage of the amplitude $E_m = 2\pi fLI_m$.

Denoting $$2\pi fL = x \quad \dots\dots\dots\dots \quad (4)$$

we get $E_m = xI_m$. The same relation holds true for the effective values, I and E, of current and voltage (see §§ 48 to 50), because for a sinusoidal quantity the effective value is equal to the amplitude divided by the square root of 2 (see Note on page 78). Thus we obtain

$$E = xI \quad \dots\dots\dots\dots \quad (5)$$

The quantity x is called the *reactance* of the circuit or electrical device under consideration. Equation (5) shows that a reactance, being the ratio of a voltage to a current, may be expressed in ohms like an ohmic resistance. It will be seen from equation (4) that reactance is a composite concept; its value depends on the inductance of the device and on the frequency of the supply. In practice the frequency usually remains constant, and hence the reactance also remains constant (save when the magnetic conditions surrounding the conductor are changed).

The current i and the voltage e (marked "Inductive Drop" in Fig. 109) do not reach their maxima simultaneously; the current passes through zero when the e.m.f. reaches its maximum, and vice versa. This is in accord with eqs. (2) and (3), for the current is a sine function while the voltage varies according to a cosine function.

Since Sin $2\pi ft$ = Cos $(90° - 2\pi ft)$, the two waves are said to be displaced in time phase by 90 electrical degrees with respect to each other. The current lags behind the voltage, because the latter reaches its maxima at the instants of time corresponding to $2\pi ft = 0$, 2π, 4π, etc., while the current reaches its highest positive values when $2\pi ft = 0.25\pi$, 2.25π, 4.25π, etc. The curves shown in Fig. 109 may be obtained experimentally with an oscillograph (See Vol. II).

134. Experimental Determination of Reactance.—An experimental study of reactance is made somewhat complicated by the presence of unavoidable ohmic resistance. The influence of the resistance may be eliminated as is shown in § 138 below, or it may be reduced to a negligible amount. The latter conditions may sometimes be obtained by using a coil of comparatively low resistance and of high inductance and testing it at a high frequency and low current. The ohmic drop is then small while the reactive drop may be relatively quite high.

The reactance of such a coil of low resistance, *AB* (Fig. 110), may be determined experimentally, by measuring the current flowing through the coil and the A.C. voltage at its terminals. According to eq. (5)

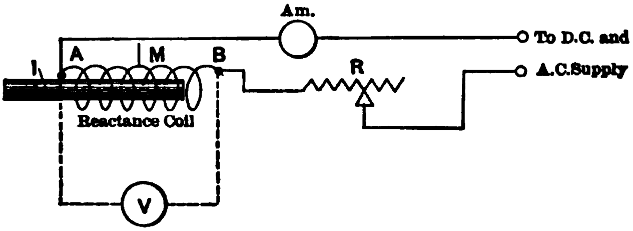


FIG. 110. Resistance and reactance in series.

the reactance is equal to the ratio of the voltage to the current. The current is measured on the ammeter *Am*, the voltage on the voltmeter *V* connected across the terminals of the coil. The rheostat *R* is for the purpose of regulating the current.

An iron core *I* is shown within the coil; the presence of iron intensifies the flux produced by the coil, and thus greatly increases its reactance. By moving the core in and out the reactance may be varied within wide limits. This method is often used for regulating current in an A.C. circuit.

Four factors influence the accuracy of determination of reactance by this method: The ohmic resistance of the coil, the iron loss in the core (hysteresis and eddy currents), the saturation of the iron core, and higher harmonics in the supply voltage. These factors may either be kept down by a suitable choice of conditions, or their influence may be determined and corrected for, as is explained below.

135. Factors which Affect the Value of Reactance. — The reactance of a coil with an iron core depends on the following factors:

(a) Frequency of the supply (Fig. 111).
(b) Position of the core (Fig. 112).
(c) Intensity of current (Fig. 113).
(d) Number of turns in the coil.

In these diagrams the student is asked to pay attention, for the time being, only to the curves marked "Reactance x." It will be seen from these curves that for a given coil:

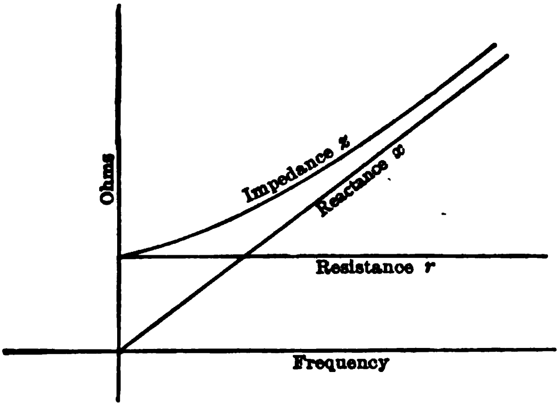


FIG. 111. Influence of frequency on reactance and impedance (resistance and inductance are kept constant).

(a) The reactance is directly proportional to the frequency of the supply, in accordance with the definition of reactance; see eq. (4).

(b) The reactance depends essentially upon the presence and the position of the iron core, or plunger. The reactance decreases as the plunger is *drawn out* of the coil; the lowest limiting value of the reactance is that of the coil without iron.

(c) With iron, the value of the reactance, determined as the ratio of reactive volts to amperes, depends upon the intensity of the current flowing through the coil. This is because the magnetic flux is *not* proportional to the current, the iron approaching its saturation limit (§ 165). In this case the current wave is not sinusoidal, even when the voltage varies according to the sine law. Strictly speaking, the theory given above does not hold true when eq. (2) ceases to be applicable, but it is possible to speak of the reactance of a coil as defined by eq. (5). In this case the reactance reaches its maximum at some point within the range of maximum permeability of iron, in other words, where the flux per ampere of current is a maximum. Without iron, there is no reason why the reactance should depend on the value of the current, as both the flux in the coil and the applied voltage are proportional to the current.

(d) The inductance, as well as the reactance, of a given coil is proportional to the square of the number of turns in series. Consider

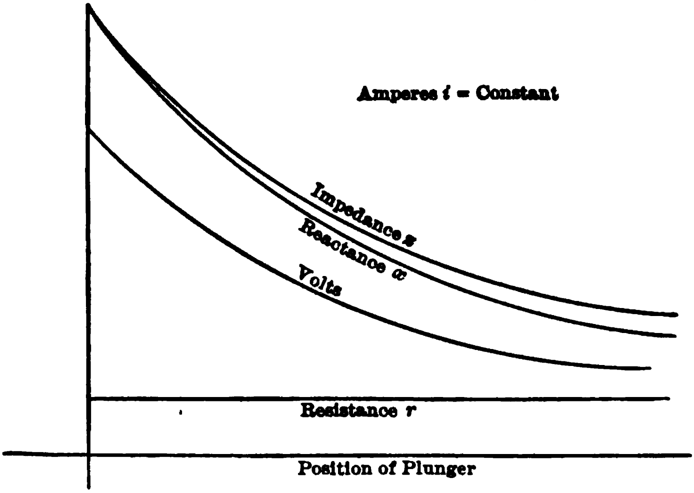


FIG. 112. Influence of the position of the plunger (Fig. 110) on the reactance and impedance of the circuit.

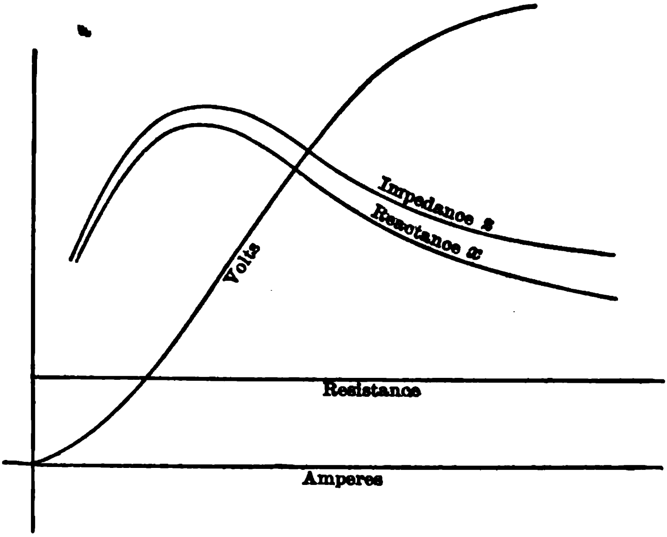


FIG. 113. Influence of the magnitude of current on the reactance of a coil (effect of saturation in iron).

two coils of the same outside dimensions and with the same space available for winding. Let one of the coils be wound with a finer wire, so as to accommodate m times more turns in the same space; assume that the currents in the two coils are adjusted so as to produce equal magnetic fluxes. Let the induced voltage and the current in the first coil be E and I respectively — those in the second coil E' and I'. Since the flux in the second coil is the same as in the first, I' must be $= I \div m$, in order to have the same number of exciting ampere-turns. The flux in the second coil is interlinked with m times as many turns as that in the first coil; consequently $E' = mE$. Thus, the reactance of the second coil

$$x' = \frac{E'}{I'} = \frac{mE}{I \div m} = \frac{m^2E}{I} = m^2x,$$

where x is the reactance of the first coil. This proves the above proposition.

136. EXPERIMENT 6-A. — Study of a Reactance Coil without Iron. — The purpose of this experiment is to investigate the influence of the frequency and of the number of turns in a coil upon its reactance, as discussed in the preceding article. The connections are shown in Fig. 110; a coil should be used with a considerable number of turns of heavy wire in order to have an appreciable inductance with a negligible resistance. (a) First investigate the influence of the frequency. Connect the circuit to the terminals of an A.C. generator, the speed of which may be varied at will. Bring the speed up to the highest safe limit and keep the current down as much as possible by means of rheostat R and by cutting down the field excitation of the alternator. Read the voltage across the coil, the current in it, and the speed of the alternator. Increase the current in three or four steps and take similar readings at each step, keeping the speed constant. Reduce the speed in steps to the lowest practicable limit, and at each step take a few readings with different values of current and voltage.

(b) To investigate the influence of the number of turns, it is convenient to have a coil wound with two or more wires in parallel. By using separate sections of the winding, or combining them in series and in parallel, different numbers of active turns are obtained. The experiment may be performed at a constant high frequency; the reactance should be determined at low, medium and high values of current.

(c) In cases where inductance is harmful, for example in high-grade resistance units, coils are wound *non-inductively*, that is, so as to produce practically no magnetic flux. This is accomplished by connecting the two halves of a winding in such a way that each tends to produce a magnetic flux in the opposite direction to that of the other. Try a non-inductive arrangement by sending a current through the coil with two sections of the winding connected in opposition. If the coil has only one winding, take the middle tap *M* (Fig. 110) as one terminal, and use the points *A* and *B* connected together as the other terminal. Before closing the circuit, insert enough resistance *R* to prevent an inrush of current. The voltage drop across the coil will be found to be small, and consequently its reactance will be low.

Report. Compute the values of reactance from eq. (5) and plot them to frequency as abscissæ; figure out the inductance of the coil (in henrys) according to formula (4). Give the data and the results, showing that the inductance increases as the square of the number of turns. Describe the experiment with the coil wound non-inductively. Point out and explain any inaccuracies in the results due to the influence of the ohmic resistance.

137. EXPERIMENT 6-B. — Study of a Reactance Coil with an Iron Core. — The factors to be investigated are enumerated in § 135. (a) The influence of the frequency and of the number of turns may be investigated as in experiment 6-A. However, the current must be kept constant for each set of readings, since the value of the reactance depends somewhat upon the current, due to the presence of the iron core. This part of the experiment may be omitted, if experiment 6-A has been previously performed.

(b) To investigate the influence of the position of the plunger, first remove it (Fig. 110), and raise the current in the coil to its maximum safe value. Then gradually move the plunger in, at the same time cutting the resistance *R* out of the circuit, so as to keep the current constant (Fig. 112).

At each position read the volts across the coil, the current, the position of the plunger, and the frequency. Take similar sets of readings at two or three smaller values of current. Take another set of curves, keeping the voltage across the coil constant, and allowing the current to drop as the plunger is inserted into the coil.

(c) To investigate the influence of the intensity of current upon the reactance (Fig. 113), set the plunger at the position of maximum flux and take readings of volts and amperes, increasing the current in steps. Repeat the test with two or three different positions of the plunger.

Report. Plot curves showing variation in the reactance of the coil with the frequency, with the position of the plunger, and with the intensity of the current, as in Figs. 111, 112, and 113. Explain the character of the curves theoretically, and point out the inaccuracies due to the ohmic resistance.

138. Impedance, or Combination of Reactance and Resistance.—Reactance coils have been considered so far as devoid of ohmic resistance, or at least having a negligible resistance. In many practical cases, however, the ohmic resistance of a circuit is of equal, if not of greater importance, than its reactance. It is necessary, therefore, to investigate the combined effect of a reactance and a resistance in series.

Experience and theory show that a reactance x and a resistance r, connected in series, as in Fig. 114, cannot be added arithmetically, even though both are expressed in ohms. They must be added *geometrically*, at right angles, as in Fig. 115. For example, let a reactance of 4 ohms be connected in series with a resistance of 3 ohms. To force a current of 10 amperes through the reactance alone, an e.m.f. of 40 volts is necessary; to force the same current through the resistance alone, 30 volts are required. But when the two are connected in series, only 50 volts are necessary, instead of 40 + 30 = 70 volts. In other words, the combined effect of 4 ohms and 3 ohms is in this case equivalent to 5 ohms, and not to 7 ohms.

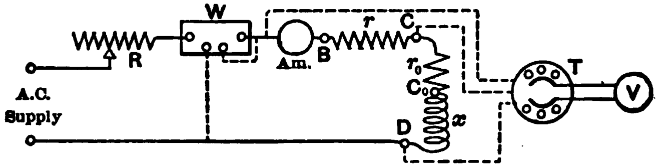


FIG. 114. Diagram of connections for investigating electrical relations in a circuit containing resistance and reactance in series.

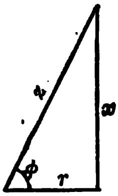


FIG. 115. Geometrical addition of a resistance and a reactance, the result being an impedance.

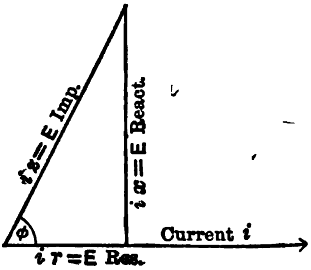


FIG. 116. The triangle of voltage drop, corresponding to Fig. 115.

This peculiar relation can be explained by means of the equations established in §133. When a coil has some ohmic resistance r, in addition to an inductance L, the e.m.f. which must be applied at its terminals, in order to produce a current i, according to eq. (1a), is

$$e = L\frac{di}{dt} + ir \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

If the current wave is sinusoidal, then, substituting the value of i from eq. (2) and remembering the relation (4), we obtain

$$e = xI_m \text{ Cos } 2\pi ft + rI_m \text{ Sin } 2\pi ft \quad . \; . \; . \; . \; . \; . \; . \quad (7)$$

This shows that the applied voltage at any instant is a sum of two voltages, each varying as a sine-wave (Fig. 109). The wave of the inductive drop is the same as before; the wave of the ohmic drop, ir, is in phase with the current, both being represented by the same sine function. The total voltage is also represented by a sine-wave, for the sum of two sine-waves of the same frequency is also a sine-wave.

It will be seen from Fig. 109 that the amplitude of the wave marked "total voltage" is less than the sum of the amplitudes of the two component waves. Its phase position is also intermediate between the two. We may express this voltage as

$$e = zI_m \text{ Sin } (2\pi ft + \phi) \quad . \; . \; . \; . \; . \; . \; . \; . \quad (8)$$

where z and ϕ are two new quantities which determine the magnitude and the position of the resultant e. Equating (7) and (8) we obtain

$$z \text{ Sin } (2\pi ft + \phi) = x \text{ Cos } 2\pi ft + r \text{ Sin } 2\pi ft \quad . \; . \; . \; . \quad (9)$$

This relation holds true at any instant t; applying it first for $ft = 0$, and then for $ft = \frac{1}{4}$, we obtain

$$\left.\begin{aligned} z \text{ Sin } \phi &= x \\ z \text{ Cos } \phi &= r \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (10)$$

whence

$$\left.\begin{aligned} z &= \sqrt{x^2 + r^2} \\ \tan \phi &= x \div r \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (11)$$

For given values of x and r, the values of z and ϕ may either be calculated from eq. (11) or constructed as in Fig. 115. The quantity z is called the *impedance* of a piece of apparatus or of a circuit, and is expressed in ohms. It is equal to the quadrature geometrical sum, and not to the arithmetical sum, of the component resistance and reactance in series.

Returning now to the above numerical example, we see that a 4-ohm reactance and a 3-ohm resistance act together as an impedance $z = \sqrt{4^2 + 3^2} = 5$ ohms. Equation (8) may be represented symbolically thus:

$$E = zI \, (\phi) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (12)$$

where (ϕ) means that the current wave lags behind that of the applied voltage by the angle ϕ (Fig. 109). This equation, compared with eq.

(5), clearly indicates the influence of an ohmic resistance in series with a reactance.

The analytical relationship represented by eqs. (7) and (8) is shown graphically in Fig. 116, where I is the effective value of the current, $I = I_m/\sqrt{2}$. The two legs of the right triangle represent the vectors Ir and Ix of the ohmic and reactive drop respectively. The resultant vector, or the hypotenuse Iz, represents the total applied voltage. The current vector I is in phase with the ohmic drop. The triangles in Figs. 115 and 116 are similar, and the latter is obtained from the former by multiplying all of its sides by the effective value, I, of the current. It is well to keep in mind, however, that *the triangle in Fig.* 116 *is a vector diagram of quantities which vary according to the sine law with the time, while the triangle in Fig.* 115 *is a combination of constant scalar quantities, r, x, and z.*

When a coil has an appreciable ohmic resistance, its reactance is determined as follows: Let an alternating voltage E be applied to the terminals of the coil and let I be the current in the coil. Then the ratio E/I gives the impedance z of the coil. The ohmic resistance r is measured with direct current. In the first eq. (11) r and z are now known, and we have

$$x = \sqrt{z^2 - r^2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (11a)$$

At high frequencies the effective ohmic resistance (§ 149) may be considerably higher than the resistance measured with direct current. In this case a wattmeter should be used when measuring E and I, and the effective resistance computed from the relationship $P = I^2r$, where P is the wattmeter reading.

139. EXPERIMENT 6-C. — Study of a Reactance Coil with an Appreciable Ohmic Resistance. — The purpose of the experiment is: (1) To prove that a resistance and a reactance in series must be added geometrically; (2) To investigate the influence of the factors enumerated in § 135, as modified by the presence of resistance. The diagram of connections is shown in Fig. 114, except that the wattmeter W is not needed. The non-inductive resistance r is connected in series with a reactive coil x. The latter must be so chosen as to have as small an ohmic resistance as possible; this unavoidable resistance is denoted by r_0. If possible, use a high-frequency source, say a 500-cycle alternator. By means of the switch T, the voltmeter V may be made to measure the voltage either across the non-inductive resistance or across the reactive coil alone, or across the whole impedance.

(1) Remove the iron core from the reactive coil, send a considerable

alternating current through x and r, and read the current and the three voltages. Take similar readings at smaller values of current, keeping the frequency constant. (2) Take a similar set of readings at a constant current and resistance, varying the frequency. (3) Take a similar set of readings at a constant current and frequency and with a variable resistance r. (4) Repeat a few readings with the iron core in the coil. (5) Take a few readings with different numbers of effective turns. (6) Measure the resistances r and r_0 with direct current.

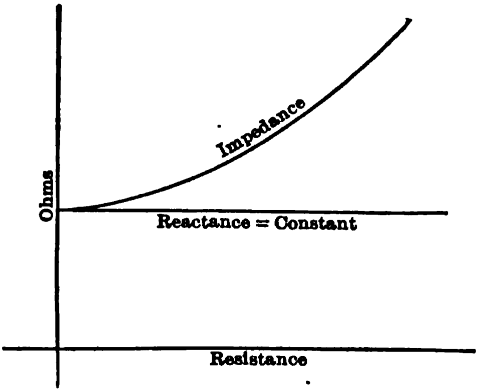


FIG. 117. Influence of variable resistance on the value of an impedance.

Report. (a) From the first set of readings plot the straight lines between total volts and amperes, and partial volts and amperes. Compute z as the ratio of the total volts to amperes. Compute $r + r_0$ from the direct-current measurements, and determine x as in Fig. 115. Draw the diagram shown in Fig. 121 to indicate the influence of r_0. Draw a triangle of voltages and show that the angle opposite the total voltage is obtuse because of the presence of resistance r_0. (b) Show that the values of the true reactance of the coil are proportional to the frequency, while the actual readings of volts across the coil, without being corrected for r_0, increase more slowly than the frequency; give a theoretical reason. (c) From the third set of readings plot a curve as in Fig. 117, and the diagram shown in Fig. 118. (d) Explain theoretically the reason for the observed difference in the voltage readings across the coil with the iron core in and out, at the same current and frequency. Also explain the observed effect of the change in the number of turns.

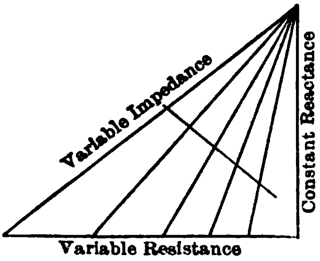


FIG. 118. Explanatory diagram to the curves shown in Fig. 117.

140. Phase Angle and Power Factor. — The phase angle ϕ, Fig. 116, is of great importance in determining the average power spent in a circuit (§ 89). The actual power varies from moment to moment with the instantaneous values of current and voltage. When inductance is present, the power supplied by the source becomes at times negative; this happens during the parts of the cycle when the inductance gives back the stored electromagnetic energy (see §§ 131 and 132). For many practical purposes, only the average value of power, during one complete cycle of alternating current, is of importance.

Let e be an instantaneous value of the voltage and i the corresponding value of the current, as in § 138. Then the average power during an interval of time T corresponding to one cycle is

$$P = (1/T)\int_0^T ei\,dt. \quad \dots\dots\dots \quad (13)$$

Substituting the value e from eq. (6), we get

$$P = \frac{L}{T}\int_0^T idi + \frac{r}{T}\int_0^T i^2dt,$$

or

$$P = \frac{L}{T}\left(\frac{i^2}{2}\right)_0^T + rI^2 \quad \dots\dots\dots \quad (14)$$

where I is the effective value of the current (§ 48). The first term on the right side of eq. (14) is equal to zero, because i is the same for any two moments differing by T. This shows that with periodically fluctuating currents, no power is permanently lost or gained in an inductance; it is merely stored during a part of each cycle and returned to the circuit during another part of the cycle. The power lost in the resistance is represented by the familiar expression $r\ I^2$ and is not returned to the source of supply in the form of electrical energy.

Now let the current and the voltage vary according to the sine law, the current being expressed by eq. (2). If the voltage leads the current by an angle ϕ (Fig. 109), its equation is

$$e = E_m \text{ Sin } (2\pi ft + \phi) \quad \dots\dots\dots \quad (15)$$

Substituting these values in eq. (13) and integrating, we obtain

$$P = \tfrac{1}{2}\ E_m I_m \text{ Cos } \phi$$

or, using the effective values in place of the amplitudes,

$$P = EI \text{ Cos } \phi \quad \dots\dots\dots \quad (16)$$

Referring to Fig. 116, this means that the average power is equal to the product of the voltage and the "in-phase" component, I Cos ϕ, of the current; or else, the power is also equal to the current times the "in-phase" component, E Cos ϕ, of the voltage.

The value Cos ϕ is called the *power factor* of the circuit. The component of the current in phase with the voltage is known as the power or energy component, that in quadrature with it as the reactive or wattless component. Similar names are sometimes applied to the components of the voltage in phase and in quadrature with the current.

The average power given by eq. (16) is also known as the *real power* (true watts). The expression EI is called the *apparent power*, or total volt-amperes; the expression EI Sin ϕ is called the *reactive power*, or the reactive volt-amperes.

141. EXPERIMENT 6-D. — Power Relations with Resistance and Reactance in Series. — The purpose of the experiment is to make clear the relations deduced in § 140. The connections are shown in Fig. 114. A voltmeter switch T is used, by means of which the voltmeter may be made to indicate at will either the total voltage, the voltage across the inductance, or that across the resistance. The resistance r_0 is meant to represent the small ohmic resistance unavoidable in the reactance coil x.

Select by trials such values of r and x as will give about the same drop across BC as across CD; read amperes, volts, and watts. The voltmeter connection is shown to include the drop in the ammeter; this is advisable in order to have the same total voltage as that impressed on the potential coil of the wattmeter. The resistance of the ammeter must thus be included in the value of r which is measured at the close of the experiment by the drop-of-potential method, using direct current. Take several sets of readings, varying the applied voltage by means of the rheostat R. Repeat the same test with different values of r and x.

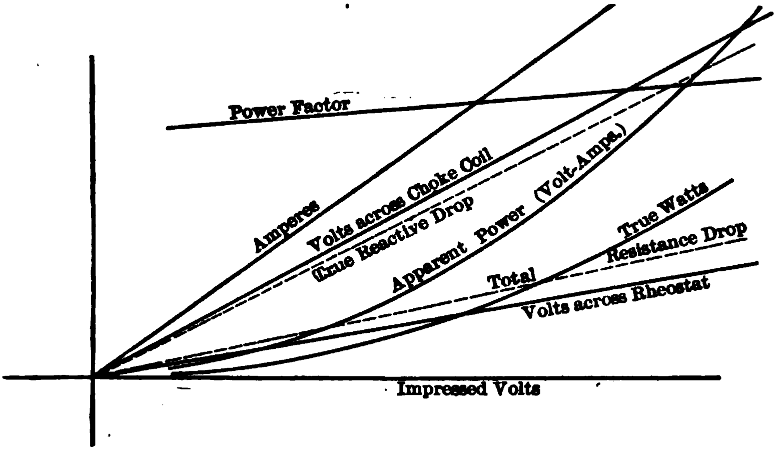


FIG. 119. Variations in amperes, watts, volts, etc., in the circuit shown in Fig. 114, with changes in the impressed voltage.

Report. (1) Plot amperes, watts, and component volts to total volts as abscissæ (Fig. 119). Explain the shape of the curves from theoretical considerations.

(2) Plot corrected values of pure resistance drop and pure reactance drop, as shown by dotted lines. This is done by adding the Ir_0 drop to Ir and subtracting the total ohmic drop geometrically from the applied voltage, according to Fig. 116.

(3) For a few points selected from the curves, show that the power read on the wattmeter checks with the calculated expression $(r + r_0)\cdot I^2$.

(4) Check the values of Cos ϕ calculated as the ratio of true watts to apparent watts with those determined from the triangle of voltages (Fig. 116).

142. Impedances in Series. — The relations deduced in §§ 138 to 141 may now be extended to the case of two or more resistances and reactances connected in series (Fig. 120). Such conditions arise in practice, for example, when both the line and the current-consuming devices possess resistance and reactance, and it is desired to calculate the generator e.m.f. and the power factor at the generator bus-bars.

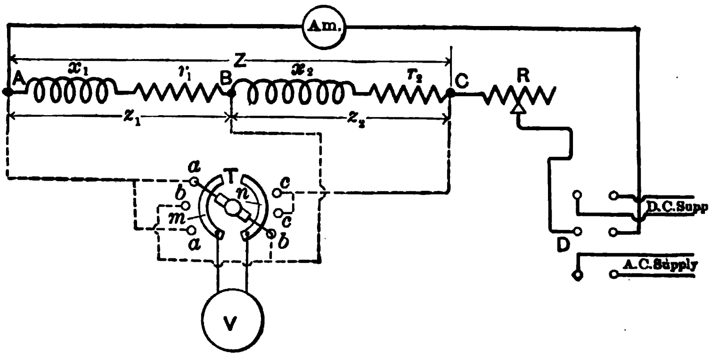


FIG. 120. Impedances in series.

The relations are shown in Fig. 121, the diagram being constructed for a given current i (horizontal vector). In this figure, and in the remainder of the chapter, both the instantaneous and the effective values are denoted by small letters. It is understood that in a vector diagram a current notation always means the effective value. The total voltage drop across AC (Fig. 120) does not depend on the order in which the resistances and the reactances are connected. Therefore, we may substitute an equivalent resistance, $R = r_1 + r_2$, for the two separate resistances r_1 and r_2. In a similar way an equivalent reactance, $X = x_1 + x_2$, may be introduced. This gives the triangle

APC analogous to that shown in Fig. 115; *AC* is the combined, or equivalent, impedance *Z* of the circuit. Multiplying *Z* by *i*, according to eq. (12), we obtain the total voltage *AC'*. The triangle *AMB* gives the resistance, the reactance, and the impedance between the points *A* and *B* of the circuit; the triangle *BNC* gives the same values for the part of the circuit between *B* and *C*. The triangle *APC* is a combination of these triangles. It may also be said that the impedance *Z* is the geometric sum of the impedances z_1 and z_2. The same is true of three or more impedances in series.

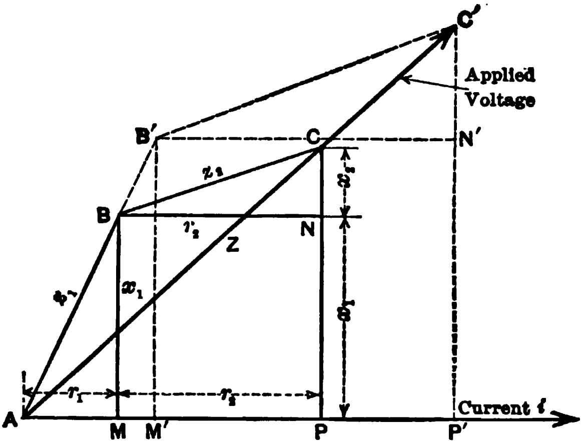


FIG. 121. Electrical relations with two impedances in series.

By changing the scale of the diagram in the ratio of *AC'* to *AC*, the diagram *AB'C'* of component voltages is obtained, shown by dotted lines. *AM'* is the voltage across r_1; *M'P'* is that across r_2. *M'B'* = *P'N'* is the voltage across x_1 and *N'C'* is that across x_2. The voltage across *AB* (Fig. 120) is represented by the vector *AB'*; the voltage across *BC* by *B'C'*. It may be said that *AC'* is the geometric sum of *AB'* and *B'C'*. The same is true when three or more impedances are connected in series. The angles of phase displacement between the current and the various voltages may also be measured on the diagram.

If, instead of the current, the terminal voltage *AC'* is given, the diagram is constructed on the basis of a current arbitrarily assumed; then the values obtained are reduced or increased in the ratio of the given voltage to that found by construction.

143. The Three-Voltmeter Method of Measuring Power. — A special case of the diagram shown in Fig. 121 is represented in Fig. 122. The reactance $x_1 = 0$, so that one of the impedances is reduced to a non-inductive resistance r_1; in other respects both diagrams are identical. This case is of some practical importance, as it leads to a method of determining power in an inductive circuit, without the use of a wattmeter. Let *BC* (Fig. 120) be any apparatus, such as a single-phase motor, and let it be necessary to determine the power consumed in it, without using a wattmeter. According to eq. (16), the power depends upon the current, the voltage, and the phase displacement between the two. The current i and voltage across *BC* are measured directly. In order to determine the phase angle between the two, a non-inductive resistance r_1 is connected in series with the apparatus *BC*, and the voltages *AB* and *AC* are also measured. The resistance r_1 being non-inductive, the voltage drop *AB* is in phase with the current, so that the triangle *ABC* (Fig. 122) may be constructed in its true phase relation

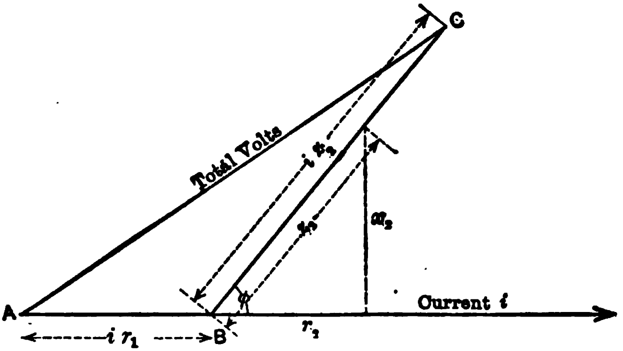


FIG. 122. A special case of the diagram shown in Fig. 121 when the reactance x_1 is zero.

to the current i. This gives the desired phase angle ϕ at B. Projecting *BC* on i gives E Cos ϕ; this value, multiplied by the current, gives the required power. If the value of r_1 is known, it is not even necessary to have an ammeter, since the current can be calculated as the ratio of the voltage to the resistance.

This method is called *the three-voltmeter method*, because the phase angle is determined from three voltmeter readings. It has been used to some extent in former years, but is now applied in exceptional cases only, since indicating wattmeters have come into universal use.

144. EXPERIMENT 6-E. — Voltage and Power Relations with Impedances in Series. — The connections are shown in Fig. 120; a wattmeter should be added, as in Fig. 114. (a) Adjust the desired values of resistances and reactances, and take readings of volts, amperes, and watts. Read total watts, and watts across each impedance separately. Gradually increase the applied voltage by regulating the resistance *R*; take similar readings with each setting of the rheostat. Measure the ohmic resistances with direct current. (b) Repeat the same experiment with different values of resistances and reactances. (c) Determine the power consumed in a given impedance, first using a wattmeter, and then the three-voltmeter method.

Report. Plot the results to total volts as abscissæ. Draw the diagram of voltages $AM'B'N'C'$ (Fig. 121), and check the angles with those calculated from the wattmeter readings. Check the power determined by the three-voltmeter method with that read on the wattmeter, and with the calculated i^2r loss.

145. Impedances in Parallel. — Two impedances in parallel are shown in Fig. 123; it is required to find the total line current i and its phase relation to the alternating voltage E applied between M and N. Consider first the simplest case, when one branch has a pure inductance x only; the other an ohmic resistance r (Fig. 124). The current in the first branch is $i_1 = E/x$, and lags 90 degrees behind E; the current in the second branch is $i_2 = E/r$, and is in phase with E. The total current i is the geometric sum of the two and is represented by the diagonal of the rectangle. Let the *equivalent* impedance of the circuit be defined as one which will give the same current i and the same phase angle ϕ as the given parallel combination, at the same voltage E. Denoting this impedance by z, we have:

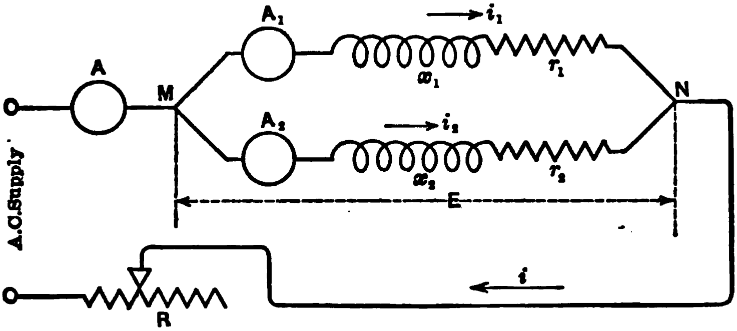


Fig. 123. Two impedances in parallel.

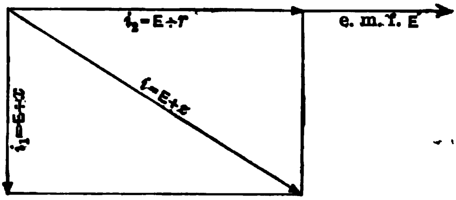


Fig. 124. Vector diagram of currents, with resistance and reactance connected in parallel.

$$(E/z)^2 = (E/r)^2 + (E/x)^2$$

or

$$\frac{1}{z} = \sqrt{\left(\frac{1}{x}\right)^2 + \left(\frac{1}{r}\right)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

This is different from the expression (11), deduced for the case of a resistance and a reactance in series.

When resistance and reactance are present in both parallel branches, the two component currents, i_1 and i_2, are lagging behind the impressed e.m.f., *OA* (Fig. 125), by certain angles ϕ_1 and ϕ_2. The total current $i = OC$ is the geometric sum of the two and lags behind *OA* by the angle ϕ.

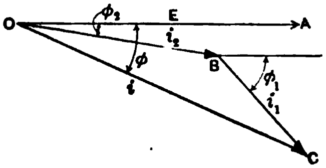


FIG. 125. Component currents and total current in the case of two impedances connected in parallel.

The same relations are shown in greater detail in Fig. 126. The applied voltage between the points *M* and *N* (Fig. 123) is represented by the vector *OE*. The voltages across the resistance and across the reactance in branch 1 are represented by the triangle OP_1E, as in Fig. 116. OP_1 is the drop in the resistance r_1; P_1E is that in the reactance x_1. The current i_1, being in phase with the ohmic drop, is represented by the vector OC_1. The triangle OP_2E gives similar relations for the branch 2. If there are more branches in parallel, a triangle may be constructed for each branch. The apexes P_1, P_2, . . . of all such triangles lie on a semicircle drawn on *OE* as a diameter. This is because the triangles are right triangles, and have *OE* for hypotenuse. The total current i in the line is the geometric sum of i_1 and i_2 and is represented by the vector *OC*. If there are more than two branches in parallel, the vectors of the component currents are added together as if they were mechanical forces radiating from the point *O*. These relationships may be checked experimentally with an oscillograph (Vol. II).

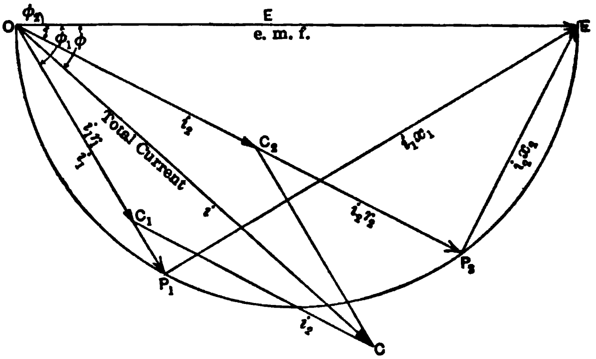


FIG. 126. Vector diagram of currents and voltages, corresponding to the connections in Fig. 123.

146. Admittance, Conductance, and Susceptance. — The reciprocal of a *pure resistance* (without inductance or capacity) is called *conductance*, is denoted by g, and is measured in units written ohm^{-1} or mho. Thus, if the resistance of a wire is $r = 2$ ohms, its conductance is $g = 0.5$ mho (§ 10). When conductors are connected in parallel, their conductances are added arithmetically.

Similarly, the reciprocal of a *pure reactance* (without resistance) is called *susceptance*, is denoted by b and measured in mhos. The reciprocal of an *impedance* is called *admittance*, is denoted by y and measured in mhos. These concepts are useful in the study of circuits containing impedances in parallel.

Thus, in the simplest case of pure resistance and pure reactance (Fig. 124) we have a conductance g in parallel with a susceptance b. The currents through the two are $i_1 = Eb$ and $i_2 = Eg$, respectively. The total current is $i = Ey$, and eq. (17) becomes

$$y = \sqrt{b^2 + g^2} \quad \text{. . . . .} \quad (18)$$

which is analogous to eq. (11)

In the more general case, shown in Figs. 125 and 126, the series connection of a resistance r_s and a reactance x_s in any one of the branches, is first replaced by its *equivalent* parallel combination of a resistance r_p and a reactance x_p, where the subscripts s and p stand for "series" and "parallel," respectively. With the same applied voltage, this equivalent combination must allow the same total current at the same resultant phase angle to flow, as does the given series combination of r_s and x_s. The resistance r_p must therefore carry the in-phase component of the total current E/z in that branch. Hence,

$$E/r_p = (E/z) \operatorname{Cos} \phi.$$

But, according to eq. (10), $\operatorname{Cos} \phi = r_s/z$. Hence

$$r_p = z^2/r_s \quad \text{. . . . .} \quad (19)$$

By analogy we find $$x_p = z^2/x_s \quad \text{. . . . .} \quad (20)$$

The pure resistance r_p may now be replaced by the corresponding conductance, and we get $g = r_p^{-1} = r_s/z^2$ . . . . (21)
Similarly, the reactance x_p may be replaced by the susceptance b, where

$$b = x_p^{-1} = x_s/z^2 \quad \text{. . . . .} \quad (22)$$

Thus, each branch is reduced to the case shown in Fig. 124. All the conductances can be added to form the equivalent conductance

$$g_{eq} = \Sigma g \quad \text{. . . . .} \quad (23)$$

and all the susceptances to form the equivalent susceptance

$$b_{eq} = \Sigma b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

The process may also be reversed, and eqs. (21) and (22) used to determine the values of the equivalent r_s and x_s. These values will give the impedance equivalent of the whole combination of the given impedances in parallel.

Equivalent admittances and impedances. Any arbitrary network of impedances in series, in parallel, or in any other more complicated combination, may be replaced by a single equivalent impedance. This impedance, z_{eq}, may be thought of as a plain series combination of a single resistance r_{eq} in series with a reactance x_{eq}, these values being such that, with a given terminal voltage, the same current and the same power factor would exist in the equivalent combination as in the actual network. Or else, the given network of impedances may be replaced by a single admittance, y_{eq}, consisting of a conductance g_{eq} in parallel with a susceptance b_{eq}, satisfying the above conditions. Since z_{eq} and y_{eq} are equivalent to the same network, they are equivalent to each other, so that eqs. (21) and (22) are satisfied for both, and moreover

$$z_{eq} = 1/y_{eq} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

In practical computations of alternating-current problems it is often convenient to use such equivalent combinations of the series or parallel type, so as to be able to add conductances or susceptances in parallel, and resistances or reactances in series. The method is applicable to any number of impedances and makes it possible to dispense with graphical constructions, such as are shown in Figs. 121 and 126. For an application of this method see § 149 below.

147. EXPERIMENT 6-F. — Study of Impedances in Parallel. — The purpose of the experiment is to illustrate the relations described in §§ 145 and 146. The connections are shown in Fig. 127; they are identical with those in Fig. 123, except that one ammeter is used for all branches, being connected through a polyphase board (see § 58). A wattmeter is provided for determining phase relations. One branch is assumed to contain a pure reactance x and the other a pure resistance r. This represents a typical case, although the student is expected to investigate also the more general case shown in Fig. 123. The resistance r_1, shown by dotted lines, represents the effect of the core loss in the reactive coil (§ 149). See § 134 regarding some precautions and difficulties. (a) Begin with the simplest case, illustrated in Fig. 124, when one branch contains inductance only, and the other resistance only. Measure the three currents, volts, and watts. Gradually increase the voltage and take similar readings. Finally measure the resistance r with direct current. (b) Repeat the experiment with other values of resistance and reactance. (c) Take the more general case of r and x in both branches, as in Fig. 123. Read amperes, watts, terminal volts, and the component voltages in each branch. (d) Illustrate experimentally the existence of equivalent series and parallel combinations explained in § 146. Show that the total current and the power factor obtained in one of the readings under (a) can be also obtained by means of a series combination of a resistance and a choke coil. Show that two impedances in parallel may be replaced by one. Should difficulty be experienced in adjusting the required values empirically, by trials, an estimate according to eqs. (21) and (22) may be helpful.

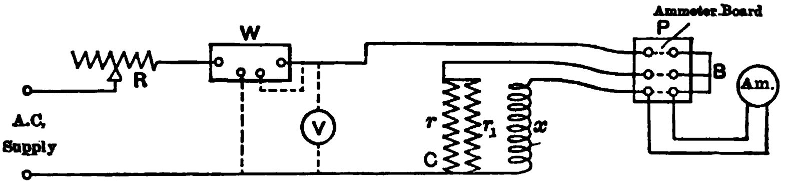


FIG. 127. A diagram of connections for studying two impedances in parallel.

Report. (1) Show that the three currents form a right triangle when a pure resistance is connected in parallel with a pure inductance; check the phase angle ϕ with that determined by the wattmeter. Also show that the total power corresponds to the i^2r loss in the resistance, and that no power is lost in the reactance.

(2) Draw one or more vector diagrams, as in Fig. 125, and show that the relations obtained from the volt and ampere readings check with those calculated from the wattmeter readings.

(3) For at least one set of readings construct a complete diagram, as in Fig. 126.

(4) Apply the theory given in § 146 to the readings obtained under (d).

148. EXPERIMENT 6-G. — Motors and Lamps Connected in Parallel to the Same A.C. Supply. — This experiment is a practical illustration of the case discussed in §§ 145 to 147. The load in most power plants consists partly of incandescent lamps, which are practically a non-inductive load, and of induction motors operating at a comparatively low power factor. The resultant load in the power house is the sum of the power taken by the lamps and of that taken by the motors;

the resultant power factor lies somewhere between the average power factor of the motors and 100 per cent.

In studying the electrical relations between an inductive and a non-inductive load it is *not necessary* to have the experiment arranged on a large scale, since the relations are the same at 5 amperes as they are at 5000 amperes. A laboratory experiment may be considered as representing the actual conditions when performed, say, on a 0.5 kw. motor and some 10 to 15 incandescent lamps connected in parallel with it. By multiplying the results, say by 1000, we shall get the relations which take place in a city having about 500 kw. in motors and from 10,000 to 15,000 incandescent lamps connected to the electric distributing system.

In actual service, the load and the power factor vary in innumerable combinations. To make the problem more definite select two extreme cases: (1) All the lamps are turned on; motor load increases from zero to maximum. (2) All the motors are running at full load; the lamps are gradually turned on. The relations taking place in intermediate cases can be readily understood from the results of these two extreme conditions.

Connect the lamps and the motor in parallel to the power supply, as in Fig. 127, and arrange switches and instruments to read watts and amperes supplied to the lamps, to the motor, and to both. The voltage must be kept as nearly constant as possible during the whole experiment. Take two comprehensive sets of readings, under the conditions described in (1) and (2) above.

Report. To total watts as abscissæ plot kw. lamp load, kw. motor load, lamp current, motor current, total current, resultant power factor, and motor power factor; also calculate and plot on the same curve sheet the reactive amperes in quadrature with the voltage, and the energy component of the current in phase with the voltage. Two separate sets of curves should be plotted: one for a constant motor load, the other for a constant lamp load. The diagram, Fig. 125, is simplified in this case to that in Fig. 128 because the load in one of the branches is non-inductive. Check a few points on the above curves by means of this diagram. For instance, take the observed component currents, find from the diagram the total current and the resultant power factor, and compare them with the values directly observed.

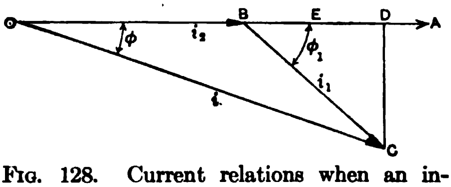


Fig. 128. Current relations when an inductive load and a non-inductive load are connected in parallel (specific case of Fig. 125).

Or else, take the observed total current and one of its components, and determine from the diagram the other component current.

149. The Effective Resistance and Reactance of a Coil with an Iron Core. — When hysteresis and eddy currents in a core (§§ 209 and 210) are of considerable magnitude, it is convenient to express their effect upon the circuit by a suitable change in the ohmic resistance of the coil. Let, for example, the power input into such a coil be 400 watts at a current of 10 amperes and at a terminal voltage of 50. Let the resistance of the winding measured with direct current be 3 ohms. Then the copper loss alone is 300 watts, and the core loss is 400 − 300 = 100 watts.

The effective resistance of a coil or circuit is defined as the total power converted into heat (copper loss, eddy current loss, core loss, skin effect, etc.) divided by the square of the effective value of the current. Accordingly, the effective resistance of the coil under consideration is 4 ohms. This value depends upon the magnitude of the core loss and consequently varies with the current and with the frequency.

The reactance computed by using the effective resistance differs from that computed by using the true ohmic resistance; the former value is known as the effective reactance. Thus, in the preceding example the impedance of the coil is 5 ohms and the two reactances are 3 ohms and 4 ohms respectively. It will be seen that the effective resistance is greater and the effective reactance is smaller than the corresponding true values.

Sometimes an additional power loss is caused by eddy currents induced in surrounding metal objects by the magnetic flux of the circuit. With high frequencies the flux within the conductor causes a non-uniform distribution of current throughout the cross-section of the conductor (skin effect). In all such cases the so-called effective resistance of the coil is taken as the actual watt input divided by the square of the current. The effective impedance is found by dividing the terminal volts by the current. From these two quantities the effective reactance is computed by taking the square root of the difference of the two squares (Fig. 115).

In some cases it is preferable to express the additional losses in a coil by means of a fictitious resistance shunted across the coil, as is shown by dotted lines in Fig. 127. This resistance, or leakage conductance, is of such a value as to cause a power loss equal to that in the core. Thus, in the foregoing numerical example, the whole coil may be replaced by an equivalent parallel combination of a conductance of 0.16 mho and a susceptance of 0.12 mho; see eqs. (21) and (22) in § 146. Of the whole conductance of 0.16 mho, the part equal to 0.12 mho accounts for the loss in the ohmic resistance of the actual coil. The remaining 0.04 mho represents a shunted path, in which the Joulean heat is equal to the core loss. As a check, we have $50^2 \times 0.12 = 300$ watts, and $50^2 \times 0.04 = 100$ watts.

The parallel combination of a 0.12 mho conductance and a 0.12 mho susceptance may be converted into a series combination of a 4.167 ohm resistance and a 4.167 ohm reactance. This new coil is shunted by a resistance of $1/0.04 = 25$ ohms, to account for its core loss. An in-phase current of 2 amperes flows through this shunted resistance, while the current through the coil itself consists of an energy component of $50 \times 0.12 = 6$ amperes and of an equal reactive component. The total current is $[(6 + 2)^2 + 6^2]^{1/2} = 10$ amperes, which checks with the given value.

Resistance and reactance at high frequencies are treated in the chapter on High-Frequency Measurements in Vol. II.

150. EXPERIMENT 6-H. — Effective Resistance and Reactance. — The purpose of the experiment is to determine the effect of core loss and of eddy currents in adjacent metal objects upon the true resistance and reactance of a coil, as discussed in the preceding section. To make the conditions more striking use a solid core or one consisting of thick laminations or wires. The connections are the same as in Fig. 114, except that the resistance r is not necessary. Be sure that the constants of the measuring instruments are known within the range of frequencies used in the following tests. (1) Run the source of alternating current at the highest possible frequency, and send as large an alternating current through the coil as it can stand without damage; read volts, amperes, and watts. Reduce the current in steps, keeping the frequency constant, and take similar readings at each step. (2) Repeat the preceding run at a few lower values of frequency. (3) Keep the current constant, and take readings at different frequencies. (4) Repeat several characteristic settings of the preceding three runs with metal masses placed near the coil in such positions that eddy currents are induced in them by the magnetic flux. (5) Measure the resistance of the coil with direct current.

Report. (a) Plot the readings against the variable quantities as abscissæ, and draw smooth curves. (b) From these curves compute the values of the effective resistance by dividing the power input by the square of the current. (c) Compute the values of impedance by dividing volts by amperes. (d) Knowing the impedance and the resistance, compute and plot the corresponding values of reactance.

(e) For different values of current and frequency, plot the effective values of resistance, reactance, and impedance as affected by the proximity of metal masses, and compare with the values of true ohmic resistance and true reactance. (f) Explain theoretically the general character of the variation of these quantities. (g) Show by one or two numerical examples, taken from the curves, how the effect of the core loss and of the adjacent metal bodies may be taken into account by a conductance shunted around the coil.

MUTUAL INDUCTION

151. The reactance of a coil, such as the coil C_1 shown in Fig. 129, is reduced by the proximity of another coil, C_2, if the circuit of the latter contains no external source of e.m.f., such as an alternator. The reason for this reduction in the primary reactance is that the flux excited by the first coil induces a current in the secondary coil, and this current opposes the action of the primary current. As a result, the total m.m.f., the flux, and consequently the counter-e.m.f. induced in the first coil, are reduced.

The effect of the secondary coil depends, among other factors, upon its distance from the primary coil and on the character of the circuit on which it is closed (resistance r_2 and reactance x_2). The action is greatly intensified if both coils are mounted on the same iron core, I. This interaction of two coils, without direct metallic connection, is called *mutual induction.*

In some practical cases, mutual induction is highly desirable; thus the action of a transformer (Chapter XVI) is based entirely upon it. The so-called " coupled circuits " in radio work are another example of application of mutual inductance. In other cases mutual induction has a harmful effect; for example, when a power-transmission line parallels a telephone line, currents are induced in the latter which cause noises in the telephone receivers and make an understanding of the speech difficult or impossible (inductive interference).

The electrical relations between two circuits subjected to a mutual induction are usually too complicated to be expressed by practical formulæ or diagrams. All that is desired here is to give the student at least a qualitative understanding of the physical phenomenon, and the influence of the factors which enter into it (Fig. 130). This is explained in the following experiment.

152. EXPERIMENT 6-I. — Effect of Mutual Induction on the Reactance of a Circuit. — The phenomenon described in the preceding paragraph may be studied experimentally with the apparatus shown in Fig. 129. The coil C_1 under test is denoted the "Primary Coil." It is connected to an A.C. supply, with an ammeter A_1, a voltmeter V, and a wattmeter W in the circuit, as usual. Another coil, C_2, marked "Secondary Coil," is closed on a circuit consisting of a resistance r_2, reactance x_2 and an ammeter A_2. The secondary coil may be placed at any desired distance from the primary coil. In order to increase the inductive action between the two coils, they may both be put on a common iron core I. To investigate the influence of the position of the secondary coil, short-circuit it upon itself through the ammeter A_2, and bring it as close as possible to the primary coil (position "0"). Adjust the primary current by the resistance R to the maximum safe limit; read volts, amperes, and watts. Gradually move C_2 away, keeping the primary amperes constant. Take readings until the secondary coil is removed so far that its influence on the primary circuit is hardly noticeable. Finally open its circuit and again read the primary values.

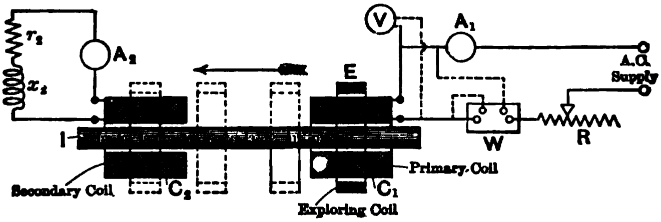


FIG. 129. Effect of mutual induction on the electrical relations in a circuit.

As the coils are separated, the flux between them varies. The values of the flux can be ascertained by an exploring coil E connected to a separate voltmeter. The coil E is moved from place to place, as shown by the dotted lines, and the variations in the flux are measured by the variations in the induced e.m.f. The voltmeter current is so small that the presence of the exploring coil does not appreciably affect the electrical relations between the primary and secondary coils. Repeat the same test without the iron core.

Next investigate the influence of the character of the secondary circuit upon the mutual inductance. Place the secondary coil at a short distance from the primary coil, and take several readings of volts, amperes, and watts, varying the values of x_2 and r_2. Keep either the primary volts or primary amperes constant, in order to have a basis for a comparison of the results.

Report. Plot the results to positions of the secondary coil as abscissæ (Fig. 130). Plot the effect of varying secondary inductance and resistance; use as abscissæ the quantity which was varied. Give a physical explanation of the observed relations and construct a few vector diagrams.

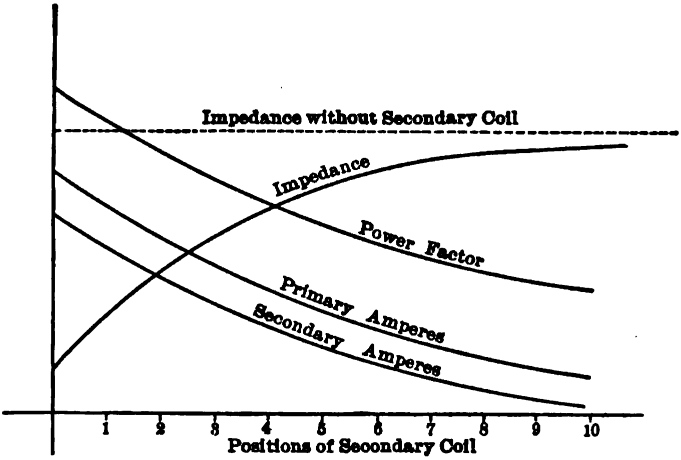


FIG. 130. Curves showing the effect of mutual induction, with the connections as in Fig. 129.

153. Coefficient of Mutual Inductance. — By analogy with the definition of the coefficient of self-induction in § 131, the coefficient of mutual induction, M, is defined by either of the following equations:

$$e_2 = -M\frac{di_1}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

or

$$e_1 = -M\frac{di_2}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

In these equations the subscripts 1 and 2 refer to two electric circuits linked by a common magnetic flux (Fig. 129). It is shown in works on theoretical electricity and magnetism that the coefficient M is the same in both equations. In other words, when the current in circuit 1 varies at a rate of 1 ampere per second, an e.m.f. of M volts is induced in circuit 2, and *vice versa*. The coefficient of mutual induction M is measured in henrys, as is the coefficient of self induction L.

The coefficient of self induction, L, is always positive. The coefficient of mutual induction, M, may be either positive or negative. Let, for example, M equal + 8 milli-henrys between a stationary and a movable coil, in a certain position of the latter in which M is a maximum. Now, if the movable coil be gradually turned about its axis, the value of M would decrease, become zero, and then become negative.

When the coil has been turned by 180 degrees the coefficient of mutual induction is − 8 milli-henrys.

Let the current in circuit 1 vary according to the sine law, eq. (2) in § 133. Then the instantaneous secondary induced voltage is

$$e_2 = -2\pi f M I_m \text{Cos}\, 2\pi f t,$$

that is, the secondary induced voltage lags behind the primary current by 90 electrical degrees. Using the effective values, we get

$$E_2 = x_m I_1 \quad \ldots\ldots\ldots\ldots \quad (28)$$

where

$$x_m = 2\pi f M \quad \ldots\ldots\ldots\ldots \quad (29)$$

The quantity x_m is called the *mutual reactance* of the two circuits and is measured in ohms. Equations (28) and (29) correspond to eqs. (4) and (5) in § 133. For a definition of *mutual impedance*, in application to telephone and telegraph circuits, see the Standards of the American Institute of Electrical Engineers.

The mutual inductance or reactance of two circuits may be measured by sending a pure alternating current I_1 through one of them and measuring the voltage E_2 induced in the second one, on open circuit. The ratio of the two, according to eq. (28), will give the mutual reactance x_m. The frequency being known, M can then be computed from eq. (29). See also § 155.

154. EXPERIMENT 6-J. — Determination of the Mutual Inductance of two Circuits. — Provide two coils without iron cores and place them in such a relative position that an alternating magnetic flux produced by one induces an appreciable voltage in the other. (1) Send a large alternating current through coil #1; read this current and the voltage induced in coil #2. Reduce the current in steps and read the secondary volts at each step, keeping the frequency constant. (2) Repeat the readings at one or two other frequencies. (3) Keeping the primary current constant, vary the position of the secondary coil in a systematic manner and read the secondary volts in each position. Investigate separately the influence of the distance between the centers of the coils and of the angle between their axes. Find the positions of maximum, minimum, and zero M. Note positions for which M is positive and those for which it is negative. (4) For two or three positions of the coils send an alternating current through #1 and measure the voltage induced in #2; then send a current through #2 and measure the voltage in #1. The two currents need not be equal, but the frequency must be the same in both cases. (5) Try the effect of an iron core in each of the two coils in succession, and then in both simultaneously.

Report. (a) Compute the coefficient of mutual induction for the positions tested. (b) Give theoretical reasons for the observed positions of maximum and minimum M. (c) From the readings under (4) show that the two voltages are proportional to the currents and that this result proves the statement following eq. (27). (d) Give the results of the test with the iron cores and explain them theoretically.

155. Measurement of Inductance with Wheatstone Bridge.—Small inductances, such as are used in telephone and telegraph work, and in scientific research, are sometimes measured by means of a Wheatstone bridge, with interrupted or alternating currents. Some of these methods, which are particularly convenient for the measurement of self and mutual induction and of capacitance, will be found in Vol. II.

CHAPTER VII

THE MAGNETIC CIRCUIT

156. The fundamental electromagnetic relations may be conveniently studied on the apparatus shown in Fig. 131. It consists of several sets of U-shaped pieces *m*, *m′*, of steel or iron, and of magnetizing or exciting coils *pp*, connected to a source of direct-current supply. *K* is a reversing switch by means of which the iron cores may be magnetized in either direction. When *K* is closed, a current flows through the coils *pp* and produces a magnetic flux, as shown by the dotted lines. This flux may be varied by the following means:

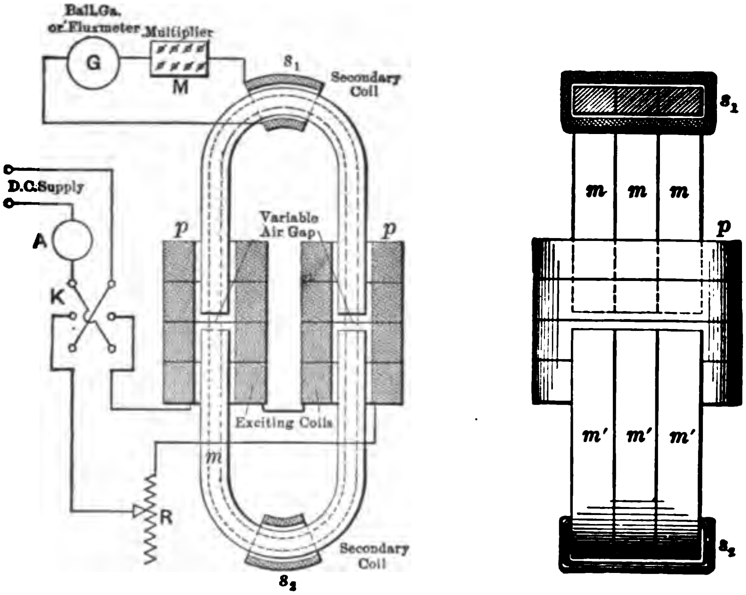


Fig. 131. An apparatus for studying the fundamental relations in a magnetic circuit.

(a) By regulating the exciting current with the rheostat *R*.

(b) By changing the number of coils in the circuit.

(c) By increasing or decreasing the air-gap between the upper and the lower cores.

(d) By changing the number of sections *m*, *m′*.

(e) By replacing the cores by others of different magnetic material. The exciting current is read on the ammeter A. The magnetic flux in the core is measured by means of the exploring coils s_1 and s_2, connected to a ballistic galvanometer (§ 158) or to a fluxmeter (§ 161).

157. Ampere-Turns and Magnetic Flux. — The magnetic flux produced in the cores m, m', depends on the magnitude of the exciting current and on the number of turns in the exciting coils. The flux remains constant as long as the product "current times turns" remains constant. For example, a current of 5 amperes flowing through a coil of 20 turns produces the same magnetizing effect as 20 amperes through 5 turns, or 1 ampere through 100 turns. The *magnetomotive force*, usually abbreviated to m.m.f., is the same in each case and is equal to 100 *ampere-turns*. The reason for this is as follows: Imagine a current of 1 ampere flowing through 1 turn of wire, and take 100 such elements. When they are combined in series, there will be 1 ampere flowing through 100 turns. When combined in parallel, there will be 100 amperes flowing through 1 large turn. The m.m.f. of the coil, being its external magnetizing effect, does not depend on the manner of connection of the turns, whether series or parallel, and is completely determined by the number of elements producing the action. Therefore it depends on the number of ampere-turns only.

The magnetic state in a medium has a definite direction at each point, and may be considered as some kind of tension or attraction along certain paths. These imaginary directions are called "magnetic lines" or "lines of force." They are shown by dotted lines in Fig. 131. The familiar experiment in which iron filings arrange themselves around a magnet in definite patterns illustrates the directions of the lines of force. Apart from their directions these lines are purely imaginary, but it is convenient to measure the strength of a magnetic field by assigning a *numerical* value to each line of force. With this assumption, lines of force are considered to be closer to one another in places where magnetization is more intense, and vice versa. This leads to the concept of *density of lines of force*, or *flux density*, which plays an important part in the theory of magnetism.

The total number of lines of force within an area is called the magnetic flux. The flux within a coil, such as s_1 (Fig. 131), is usually measured in terms of the total or integrated electromotive force which can be induced by it. Experiment shows that when the magnetic flux linking with a coil increases or decreases, an e.m.f. is induced in the coil proportional to the rate of variation of the flux with the time. This is the familiar Faraday's law of induction. The rational unit of magnetic flux in the volt-ampere-ohm system is therefore based on the following definition: *When a magnetic flux varies uniformly at the rate of one unit per second, the e.m.f. induced in a coil which surrounds it is one volt per turn.* Such a unit of flux is called the *weber*, or the *volt-second;* it is too large for practical purposes. The unit actually used, and called a *line of force*, is based on a similar definition, except that the C. G. S. unit of electromotive force is taken instead of the volt. This e.m.f. unit is only 10^{-8} times as large as the volt; this coefficient therefore enters into all formulæ which contain an induced e.m.f. in volts, in terms of the rate of change of flux linkages expressed in lines of force.

To make the above definition clearer, let a certain magnetic flux be produced in the cores m, m' by exciting the coils p, p with the necessary current, and let it be required to express this flux numerically. Let the secondary or exploring coil s_1 consist of 100 turns of wire and be connected to a very sensitive voltmeter or galvanometer. As long as the flux remains constant, the voltmeter shows no deflection. Now let the exciting current be reduced to zero by means of the rheostat R, at such a steady rate that the voltmeter shows a constant deflection of 0.05 volt. Suppose that 30 seconds of time elapse before the current is reduced to zero and the flux disappears. From these data the original flux is figured out as follows: The e.m.f. induced in each turn of the secondary coil is $0.05 \times 10^8/100 = 50{,}000$ C. G. S. units. According to the above definition of flux, this result shows that the flux was decreasing at the rate of 50,000 lines of force per second. As it took 30 seconds for the flux to be reduced to zero, the original flux was $50{,}000 \times 30 = 1{,}500{,}000$ lines of force.

The unit of flux, or one line of force, as defined above, is called *the maxwell,* so that fluxes are expressed in maxwells. While the weber, or the unit of flux defined on the basis of one volt per second, is too large, the maxwell is too small a unit for many practical purposes. A compromise is reached by using the following intermediate units:

$$1 \text{ kilo-maxwell (or kilo-line)} = 1000 \text{ maxwells} \left(= \frac{1}{100{,}000} \text{ weber}\right).$$

$$1 \text{ mega-maxwell (or mega-line)} = 1{,}000{,}000 \text{ maxwells} \left(= \frac{1}{100} \text{ weber}\right).$$

It would be almost impossible to measure a magnetic flux accurately by gradually reducing it to zero, so as to get a constant voltmeter deflection as described above. The ballistic method described below, though based on the same principle, is much more convenient and is largely used in practice.

158. The Ballistic Galvanometer. — A ballistic galvanometer is a measuring instrument whose deflections are proportional to very brief electric discharges through it. An electric discharge through such a galvanometer may be made proportional to a sudden change in a magnetic flux. For this reason the ballistic galvanometer is widely used for measuring magnetic fluxes. Like most galvanometers used in physics, a ballistic galvanometer consists of a coil of fine wire suspended in the field of a permanent magnet. When a current passes through the coil, the latter is deflected according to the strength of the current. The torsion of the suspension wires, or spiral springs, returns the coil to zero when no current flows. The construction is similar to that shown in Fig. 34, except that the coil is suspended by means of straight vertical wires which also provide the restoring force. The coil is made wider, that is, its vertical parts are farther from the axis of rotation, to increase the moment of inertia.

This greater moment of inertia is the only essential difference between the ballistic galvanometer and the usual type of galvanometer used for steady deflections. A greater inertia is necessary in order to increase the time of natural swing of the moving system, and thus to prevent an appreciable movement of the coil before the total electric discharge has passed through it. Only under this condition does the instrument give deflections which are proportional to total discharges, and are independent of their duration or of the instantaneous values of the current. If, for example, the movable part is made so heavy that its period of swing is 10 seconds, the coil will not move appreciably before the discharge is over, with any discharge that lasts, say, less than 0.1 second.

For measuring a magnetic flux (Fig. 131), the ballistic galvanometer is connected to the secondary coil s_1, — if necessary, through a resistance M, to reduce deflections. As long as the flux remains constant, the galvanometer reads zero, but if the flux is suddenly changed, a transient electric current is sent through the galvanometer circuit, and the coil is thrown by a definite amount. It then continues to swing to and fro, in a manner determined by its inertia, the restoring force of the suspension wires, and the dissipative forces of induced currents and friction. The first deflection, multiplied by a constant of the instrument, is a measure of the change in flux. Let, for example, the galvanometer constant be 50,000 maxwells per 1 cm. scale deflection. Then, should the deflection be 15 cm., the change in flux is $50{,}000 \times 15 = 750{,}000$ maxwells, or 750 kilo-lines.

A change in flux is brought about by opening or closing the switch K (Fig. 131), by reversing the current with the same switch, or by changing the setting of the rheostat R.

The theory and calibration of the ballistic galvanometer are explained in §§ 159 and 160.

159. Theory of the Ballistic Galvanometer. — The general features of the instrument are described in the preceding section, and it will now be shown that deflections of the instrument are proportional to variations in the flux through the secondary coil. It will be proved; (a) that an electrical discharge through the galvanometer is proportional to the corresponding change in flux linking with the test coil; (b) that the total impulse communicated to the movable coil of the galvanometer is proportional to the total quantity of electricity that has passed through it; and (c) that the deflection of the galvanometer coil is proportional to the impulse communicated to it. From these three facts it will follow that deflections are proportional to variations in flux.

(a) *An electric discharge through a ballistic galvanometer is proportional to the change in flux.* Let Φ be an instantaneous value, in webers, of a variable magnetic flux linked with an exploring coil, for example the flux within the test coil s_1 in Fig. 131. The instantaneous e.m.f., e, induced in the coil by a variation of this flux, according to § 157, is

$$e = -n\, d\Phi/dt \quad \dots\dots\dots\dots \quad (1)$$

where n is the number of turns in the coil. The minus sign is necessary because of the usual convention as to the positive direction of the e.m.f. Namely, when the flux increases, so that its derivative is positive, the induced e.m.f. tends to produce a current which would reduce the flux. Hence, if the magnetizing current of the flux itself be considered positive, the induced e.m.f. and the current which it would induce are negative.

Let r be the total resistance of the galvanometer circuit and L its inductance. According to eq. (6) in § 138, the instantaneous current i in the galvanometer is determined by the relation

$$e = ir + L\, di/dt.$$

Substituting the value of e from eq. (1) above, gives

$$-n\, d\Phi/dt = ir + L\, di/dt$$

or

$$-n\, d\Phi = ir\, dt + L\, di \quad \dots\dots\dots \quad (2)$$

Let Φ_1 and Φ_2 be the initial and the final values of the flux during the short interval of time t of the change. Integrating the preceding equation over the time t, we obtain

$$n\,(\Phi_1 - \Phi_2) = r\int_0^t idt + Li\Big|_0^t \quad \dots\dots\dots \quad (3)$$

The integral on the right-hand side represents the total quantity of electricity, in coulombs or ampere-seconds, which is discharged through the galvanometer; let this quantity of electricity be denoted by Q. The second term on the right-hand side of eq. (3) is equal to zero, because the galvanometer current is zero both at the beginning and at the end of the discharge period. Thus we have

$$n\,(\Phi_1 - \Phi_2) = rQ \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

This proves the above statement that an electrical discharge through the galvanometer is proportional to the change in flux.

(b) *The total impulse communicated to the movable coil is proportional to the total quantity of electricity* Q. The instantaneous mechanical torque between the movable coil and the magnetic flux of the galvanometer itself, due to its permanent magnet, is proportional to the instantaneous current i and to the flux density in the air-gap. The latter density being constant, the torque impulse, J, may be written in the form

$$J = C\int_0^t idt = CQ \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

where C is a coefficient of proportionality which depends upon the construction data of the instrument. The coil begins its motion as a result of this impulse.

(c) *The deflection is proportional to the impulse communicated to the movable coil.* The galvanometer coil moves against the following forces:

(1) The torsion of the suspension wire or springs.

(2) The currents induced in the coil and in its aluminum frame.

(3) The air resistance.

Let α be the final deflection with a given discharge of Q coulombs, and let x be an instantaneous intermediate deflection. The values of α and x may be in degrees, or in length units on the scale. The torque due to torsion is proportional to the angle of torsion. Hence, the total resisting impulse of the suspension wire may be written as

$$J_1 = C_1'\int_0^\alpha xdt, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

where C_1' is a coefficient of proportionality. As in a pendulum, so in the case of the galvanometer coil, if the current impulse is over before the coil has begun to move, the motion of the coil is harmonic, and with small angles the time of one swing is independent of its amplitude. For a double α, all intermediate angles x are doubled, while the time elements dt remain the same. Hence, the value of the integral in eq.

(6) is doubled. Thus, generally, J_1 may be said to be proportional to α, or

$$J_1 = C_1\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6a)$$

where C_1 is another coefficient of proportionality

The currents induced in the circuit of the coil and in its aluminum frame, during the motion of the coil, are proportional to the instantaneous values of the angular velocity dx/dt. Their impulse is

$$J_2 = C_2\int_0^\alpha (dx/dt)\, dt = C_2\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where C_2 is a coefficient of proportionality.

The air resistance, or windage, may also be assumed to be proportional to the instantaneous angular velocity, so that its impulse is

$$J_3 = C_3\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Now, equating the communicated impulse, eq. (5), to the sum of the resisting impulses, we obtain

$$CQ = (C_1 + C_2 + C_3)\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

This proves that the final deflection, α, is proportional to Q.

(d) *The final formula.* Denoting the "ballistic" constant of the galvanometer by b, we have from eq. (9)

$$Q = b\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Consequently, eq. (4) becomes

$$n\,(\Phi_1 - \Phi_2) = br\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Let, for example, the galvanometer constant be $b = 15 \times 10^{-6}$ coulombs (ampere-seconds) per cm. of scale deflection. Let the resistance of the galvanometer circuit, r, be equal to 2000 ohms, and the number of turns in the exploring coil be $n = 100$. Let a certain flux be established through the test coil, and then the exciting circuit suddenly opened, causing a galvanometer deflection $\alpha = 30$ cm. The final $\Phi_2 = 0$, and we have, according to eq. (11),

$\Phi_1 = 15 \times 10^{-6} \times 2000 \times 30/100 = 9 \times 10^{-3}$ weber $= 900{,}000$ maxwells.

160. Calibration of a Ballistic Galvanometer. — A ballistic galvanometer is usually calibrated with a standard mutual inductance, although it may also be calibrated with a standard condenser or with a standard inductance coil.

(a) The secondary winding of a *standard mutual inductance* M of a known fixed value (§ 153) is left permanently in the galvanometer circuit, in series with the secondary coil s_1 (Fig. 131) used for measuring the unknown flux. The primary coil of the mutual inductance is connected separately to a source of direct current, and provisions are made for quickly reversing this current. A galvanometer deflection may thus be produced by reversing the current either in the magnetizing coils *pp* of the magnetic circuit under test, or in the primary winding of the mutual inductance. In the first case the instantaneous induced secondary voltage in the galvanometer circuit is expressed by eq. (1); in the second case it is $e = -M\, di_1/dt$, where i_1 is the instantaneous primary current in the mutual inductance. A comparison of these two expressions shows that the product Mi_1 takes the place of the product $n\Phi$ in the theory given in § 159, and in the final formula (11).

With the circuit as arranged, the galvanometer resistance r and its constant b are the same in both cases. Hence from eq. (11) we get

$$(n\Phi)/(MI) = \alpha_1/\alpha_2 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (12)$$

In this expression I is the initial or final value of the current i_1 in the primary coil of the mutual inductance, and α_1 and α_2 are the two galvanometer deflections. From this formula the galvanometer constant Φ/α_1, that is, the flux per cm. of scale deflection may be computed. If M is in henrys and I is in amperes, Φ is in webers (§ 157).

(b) Let a *condenser* of a known capacitance, say C microfarads (Vol. II), be charged at a known voltage e, and then discharged through the ballistic galvanometer; let the deflection be α. According to the definition of capacitance, the charge $Q = Ce \cdot 10^{-6}$ coulombs, so that eq. (10) gives $Ce \cdot 10^{-6} = b\alpha$. In this equation all the quantities except b, are known, so that the latter can be determined.

(c) *A standard inductance coil* is usually a long straight solenoid without iron core, with a short concentric exploring coil placed about its central part. The flux density B in the middle part of the coil can be computed as is explained in § 165. When the cross-section A of the coil is known, the flux $\Phi = BA$ becomes known, and formula (11) may be used for the determination of the constant b. The standard inductance coil is excited with a known primary current I, and the exploring coil is connected in the galvanometer circuit. The primary circuit is then opened or the current is reversed, and the deflection is noted. In the first case the change in flux is equal Φ, in the second case $2\,\Phi$. This is a modification of the mutual inductance method described under (a) above.

161. Grassot Fluxmeter. — The disadvantages of the ballistic galvanometer, described in the preceding sections, are:

(1) It is necessary to bring about a change in the flux in order to measure it.

(2) The change must be sudden, in order that the electric discharge be completed before the galvanometer coil begins to move.

(3) The indications are only instantaneous, since the moving coil immediately begins to return to zero.

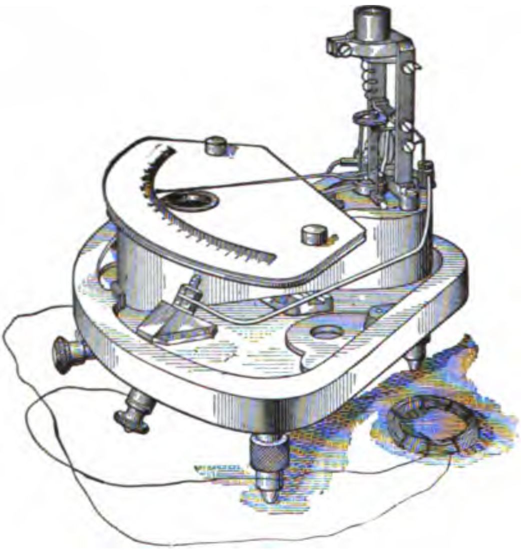


Fig. 132. The Grassot fluxmeter.

These disadvantages are eliminated in the Grassot fluxmeter, shown in Figs. 132 and 133. Its construction is essentially that of a ballistic galvanometer, except that the moving coil is suspended from a single cocoon fiber of negligible torsional stiffness. In an ordinary ballistic galvanometer the coil is returned to zero under the influence of the torsion of the suspension wire, while in the fluxmeter the coil remains at rest in any position, and creeps back but very slowly. The upper end of the fiber is attached to the flat spiral spring *R* to minimize the effect of shocks. The stiff wire frame *E*, fastened to the coil *B*, allows of a central attachment of the very thin silver strips *s* and *s′*, through which the current is led to and from the coil *B*. Because of the soft-iron core *A*, placed between the pole-pieces *NS* of a permanent magnet, the field in the air-gap is of a high and uniform intensity. An arrestment, controlled by a milled button in front of the instrument, serves to lock the coil when not in use; at the same time it brings the pointer back to zero.

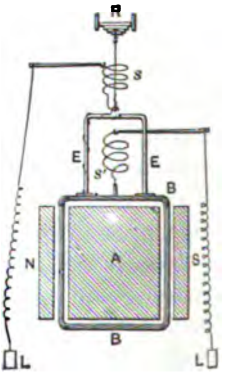


Fig. 133. Arrangement of parts in the Grassot fluxmeter.

The instrument is connected in the same way as the ballistic galvanometer shown in Fig. 131. When the switch *K* is closed or opened, the coil of the fluxmeter is set in motion and comes to rest in a new position. The deflection is determined solely by the magnitude of the flux. Whether the flux is increased suddenly or gradually, the deflection is precisely the same. When the flux is varied, the needle of the instrument follows the variations. When the exciting circuit is opened, the coil returns once more to its zero position. The fluxmeter may be calibrated directly in maxwells or in any other flux units. It permits the measurement of magnetic fluxes with the same facility with which currents and voltages are read on indicating ammeters and voltmeters. In practice, the handling, the setting, and the adjustment of a fluxmeter are somewhat more troublesome and delicate, and for this reason the ordinary ballistic galvanometer continues to be largely used.

The theory of the fluxmeter may be understood with reference to that of the ballistic galvanometer (§ 159). The current impulse, as expressed by eq. (5), is the same. The impulse of the dissipative forces, eqs. (7) and (8), is also the same. But the restoring force of torsion is lacking, the moving coil is no more an oscillating system, and eqs. (6) and (6*a*) are to be omitted. The current impulse is simply dissipated in Joulean heat and in windage, and then the coil stops. In eq. (9) the quantity $C_1 = 0$, but the deflection α still remains proportional to the electrical discharge Q. Because eq. (6*a*) is omitted, the limitation under which it has been deduced is also unnecessary, and the current may flow through the coil while it is moving.

162. Saturation in Iron. — A magnetic flux in air and in other non-magnetic substances is strictly proportional to the number of ampere-turns which produce it. In iron and steel the flux, while intensified a great many times, is proportional to the exciting m.m.f. at low values only. As the ampere-turns are further increased, the flux increases less and less rapidly; see the upper curve in Fig. 134. This peculiar property of iron is expressed by saying that the iron becomes gradually *saturated* or that its magnetic resistance or " reluctance " increases with the flux-density. The following example may make this clearer.

Let the iron cores in the apparatus shown in Fig. 131 be removed, and a current of 10 amperes sent through the exciting coils. Let the fluxmeter register a small total flux of 2 kilo-lines in the air near the center plane of the coils *pp*. With an exciting current of 20 amperes it will register 4 kilo-lines. Now let the cores be put in place and the coils excited again with 10 amperes. A much larger flux will be found, say 800 kilo-lines. But at 20 amperes the flux will probably be only 1200 instead of 1600 kilo-lines, showing that the iron is approaching the saturation limit. The influence of saturation is shown in Fig. 134 in that the curves bend gradually toward the axis of abscissæ.

For a given percentage of increase in excitation, the increase in flux becomes less and less.

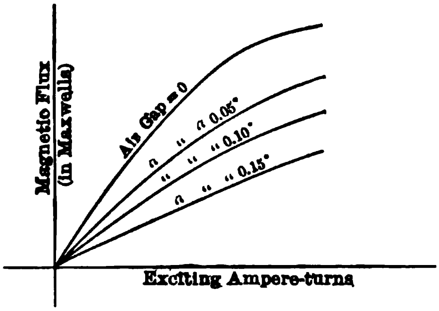


Fig. 134. Influence of small air-gaps in an iron circuit upon the magnetic flux.

The curves in Fig. 134 also illustrate clearly the difference in the magnetic properties of air and iron. The addition of a very small air-gap — a few thousandths of an inch — appreciably reduces the flux produced by a given number of ampere-turns. This is expressed by saying that the *permeability* of air is much lower than that of iron.

When obtaining such a saturation curve experimentally, care should be taken to increase the exciting current continuously, without allowing it to decrease even momentarily, if a smooth curve is desired. Otherwise the effect of hysteresis will make itself noticeable (§ 209).

163. EXPERIMENT 7-A. — Fundamental Relationship between Magnetic Flux and Magnetomotive Force. — The purpose of the experiment is to illustrate the fundamental relationship between a magnetic flux and its exciting ampere-turns. The connections are shown in Fig. 131. The constant of the ballistic galvanometer or of the fluxmeter is supposed to be known; otherwise the instrument is calibrated as described in § 160. Before beginning the experiment, thoroughly demagnetize the cores from the residual flux which they may contain after a previous experiment. To do this, excite the cores to a considerable degree and then gradually reduce the current to zero by the rheostat *R*, at the same time continually reversing the current with the switch *K*. Instead of reversing the current an alternating current may be used. Eliminate the air-gap as thoroughly as possible by bringing the cores tightly together, and excite the circuit with a small current, keeping the galvanometer circuit open.

(1) Close the galvanometer circuit, and with its coil at rest, read the exciting current. Then suddenly reverse switch *K* and observe the galvanometer deflection. It is preferable in practice to reverse the exciting current, instead of opening the circuit, on account of the indefinite residual magnetism which remains when the circuit is opened. By reversing the current the flux is changed by a definite amount $\Phi - (-\Phi) = 2\,\Phi$. Again open the galvanometer circuit, increase the

exciting current, and produce a second discharge through the galvanometer by reversing the switch K. Proceed in this way as far as the exciting coils will stand the increasing current, being careful not to allow the exciting current to decrease even momentarily. To keep galvanometer deflections within reasonable limits, either the resistance M is varied, or the number of turns in the coil s_1. Note each time the value of this resistance and the number of turns in the secondary coil, since the galvanometer constant depends upon these two quantities.

(2) Again carefully demagnetize the cores and repeat the same experiment with a small air-gap, say 0.010 inch. Such an air-gap may be easily obtained by interposing a thin piece of fiber or paper between the cores; these materials, being non-magnetic, have the same effect as air. Take a magnetization curve as before. Make similar tests with larger air-gaps.

(3) Substitute cores made of other kinds of magnetic materials, say wrought iron, steel, and cast iron, and repeat the experiment.

(4) Check the statement made in § 157, namely that the magnetomotive force depends on the number of ampere-turns, and not on the number of turns or on the current separately.

If a fluxmeter is used instead of a ballistic galvanometer, it is not necessary to reverse the exciting current. After the cores have been demagnetized, connect the fluxmeter to the secondary coil s_1, and increase the magnetizing current in steps. Read the exciting current and the fluxmeter deflection at each step.

Report. (1) Plot curves as in Fig. 134. (2) Explain the characteristic shape of these curves. (3) Plot curves of ampere-turns necessary for the air-gaps alone, by subtracting from the total ampere-turns the excitation required for the iron parts of the circuit. (4) From these latter curves show that the air-gap ampere-turns are proportional to the length of the gap and to the flux density.

164. Influence of the Length and Cross-Section of a Magnetic Circuit on its Reluctance. — Let the magnetic circuit, shown in Fig. 131, be entirely closed; that is to say, let it consist of a solid piece of iron without an air-gap. If the arrangement of the exciting winding is such that the magnetic leakage (§ 168) is negligible, then experience shows that the number of ampere-turns necessary to produce a given flux is proportional to the length of the magnetic circuit. This length is the average length of the lines of force, as indicated by the dotted lines. This is analogous to a similar relation in the electrical circuit, namely, that the e.m.f. necessary to produce a certain current in a cylindrical conductor is proportional to its length. The exciting ampere-turns, or the magnetomotive force, is analogous to the electromotive force, and the magnetic flux corresponds to the electric current. The magnetic circuit may thus be said to possess a certain resistance to the passage of lines of force. To distinguish this magnetic resistance from the electrical resistance, the former is called the "reluctance" of a magnetic path. Thus we may say that the reluctance is proportional to the length of a uniform magnetic circuit.

With a non-magnetic material the reluctance is inversely proportional to the cross-section of the path of the flux. This is again analogous to an electrical resistance, which is inversely proportional to the cross-section of the conductor. For steel and iron this relationship is strictly true only at moderate flux densities at which the saturation curve is a straight line (Fig. 134). At higher flux densities, due to the peculiar phenomenon of saturation described in § 162, the reluctance is a function of the flux density (§ 165) and increases with it. When the cross-section is increased, keeping the total flux constant, the flux density is reduced and the reluctance is reduced more than the increase in the cross-section would indicate. Conversely, when the number of lines of force per square cm., or the magnetic density, increases, the required number of ampere-turns increases more rapidly than the flux. This is equivalent to saying that the permeability (magnetic conductivity) of iron decreases as the flux density increases.

Because of this variable permeability, the concept of reluctance is not often used in application to steel and iron. The properties of the magnetic circuit are in this case better expressed in the form of a saturation curve for the whole circuit (Fig. 134), or as a relationship between the flux density and the magnetizing force (Fig. 135).

For an air-gap the concept of reluctance is very convenient, the permeability being constant. By analogy with the familiar expression for the electrical resistance of a cylindrical conductor, we have for the reluctance of a short air-gap, in which the lines of force are nearly parallel to one another:

$$R = l/(\mu A) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Here l is the length of the gap in the direction of the lines of force, A is its cross-section, and μ is the permeability of the air. The latter may or may not be equal to unity, depending upon the units used for the magnetomotive force and flux. For further details in regard to this question see the table in § 165 and also the author's "Magnetic Circuit," Chapter I.

165. Flux Density and Ampere-Turns per Unit Length (B-H Curve). — The magnetic properties of a given piece of steel or iron may be conveniently represented in the form of a curve, such as is shown in

Fig. 135. This curve gives the relationship between the magnetomotive force and the corresponding flux for a sample one inch (or one cm.) long and one square inch (or one sq. cm.) in cross-section. For such a sample the magnetomotive force becomes equal to the *magnetizing force,* or magnetic intensity, H, which by definition is *the number of ampere-turns per unit length of the material.* The flux for such a sample is identical with the flux density B, which by definition is equal to *the flux per unit cross-section of the material.* The curve shown in Fig. 135 is known as the saturation, magnetization, or B-H curve. Another term used for flux density is *induction;* it is somewhat misleading because of the use of the same word in connection with induced e.m.f.

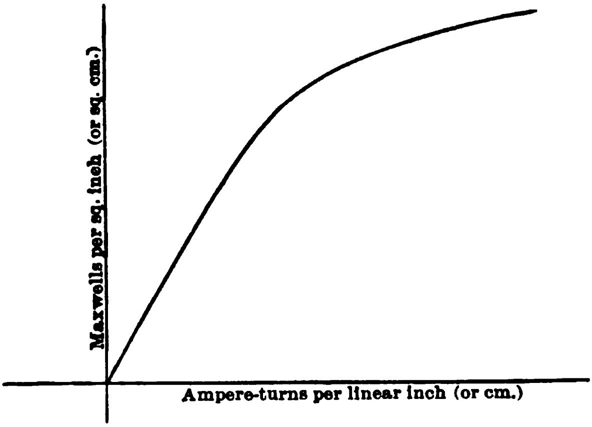


Fig. 135. Magnetization curve of a sample of iron.

As a numerical example, let the total number of turns in the exciting coils be 320, let the measured flux be 160,000 maxwells, and the length of the magnetic circuit be 100 cm. Then, with an exciting current of 10 amperes, the magnetizing force is $H = 320 \times 10/100 = 32$ ampere-turns per linear cm. Let the cross-section of the iron be 10 sq. cm., then the flux density is $B = 16$ kilolines per sq. cm. The value $H = 32$ characterizes the material at the density $B = 16$ kilolines per sq. cm., without regard to the length or the cross-section of the whole circuit. The curve shown in Fig. 135 is plotted from the upper curve in Fig. 134, by computing the corresponding values of B and H.

With the aid of such a B-H curve the number of ampere-turns for a uniform magnetic circuit of any dimensions, and made of the same kind of magnetic material, may be readily computed, when the total flux is given. It is only necessary to find the flux density B, to obtain the corresponding value of H and to multiply H by the length of the magnetic circuit.

For the air, the theoretical relationship between B and H is a straight line

$$B_a = \mu_{aa} H_a \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (14)$$

where μ_{aa} is the absolute permeability of the air for the particular units in which B_a and H_a are expressed. The symbols B and H are provided with the subscript a to indicate the air. The numerical value of μ_{aa} depends upon the units used, and is not always equal to unity as is sometimes erroneously assumed (§ 178). The following table gives the values of μ_{aa} for the principal units in practical use.

Flux Density B_a	Magnetizing Force H_a	Absolute Permeability of air μ_{aa}
Maxwells per sq. inch.........	Amp.-turns per inch.........	3.193
Maxwells per sq. cm...........	Amp.-turns per cm...........	1.257
Maxwells per sq. cm...........	Gilberts per cm..............	1.000

It will be seen from the table that with the units for H_a and B_a in common use μ_{aa} is different from unity. In order to make $\mu_{aa} = 1$, a special unit of m.m.f. has been introduced, called *the gilbert.* Theoretically

$$1 \text{ gilbert} = 10/(4\pi) \text{ amp.-turn} = 0.796 \text{ amp.-turn}.$$

In this connection it may be noted that the unit *gauss* stands primarily for the flux density of "one maxwell per square centimeter," and is simply an abbreviation for this expression. The name gauss is also sometimes applied to the expression "one gilbert per centimeter length." It will be seen from the third line in the above table that numerically the two expressions for B_a and H_a are equal, because $\mu_{aa} = 1$, but from a physical point of view they have different dimensions, since μ_{aa} is *not* a numeric. Hence, the name gauss can be properly applied only to one of them. See also § 180.

As an example of application of eq. (14), let the magnetic circuit shown in Fig. 131 have two air-gaps of 1 mm. each. With a flux density $B_a = 16$ kilolines per sq. cm., the corresponding $H_a = B_a/\mu_{aa} = 16{,}000/1.257 = 12{,}730$ ampere-turns per cm. Therefore, the magnetomotive force required for the two gaps is $2 \times 0.1 \times 12{,}730 = 2546$ ampere-turns.

The foregoing method of computation of the ampere-turns required for a flux in the air is applicable whenever the flux density is fairly constant, so that H_a may be directly multiplied by l. Such is the case of a uniformly wound torus ring, and of the middle portion of a very long straight solenoid where the lines of force are parallel to one another. In the latter case let n_1 be the number of turns per unit length, and i the exciting current. Then $H_a = n_1 i$, and B_a may be found from eq. (14). If the cross-section of the solenoid is A, the total flux is $\Phi = B_a A$. This relationship is referred to in § 160*c*.

The total number of ampere-turns required for a composite magnetic circuit, consisting of iron parts and air-gaps in series, is obtained by adding together the magnetomotive forces required for the separate parts. Thus, in the foregoing example $10 \times 320 = 3200$ ampere-turns are necessary for the iron parts and 2546 ampere-turns for the air-gaps. Hence, the whole required excitation or magnetomotive force is $3200 + 2546 = 5746$ ampere-turns. Since the number of turns in the exciting coils is 320, the required magnetizing current is $5746/320 = 17.95$ amperes.

166. EXPERIMENT 7-B. — Influence of the Length and Cross-Section of a Magnetic Circuit on its Reluctance. — The arrangement of apparatus and the method of measuring the flux are the same as in § 163. (a) Take one section of the cores, *m*, *m′* (Fig. 131), and press the ends together as accurately as possible, so as to reduce the air-gap to a minimum. Take a magnetization curve, as in Fig. 134. (b) Take the cores apart and fasten between them two straight pieces of iron, of the same cross-section and same quality of material as the rest, so as to increase the length of the magnetic circuit, and take a similar curve. More ampere-turns are necessary to produce the same flux, because of the greater length of the magnetic circuit. (c) Take out the straight pieces, bring the U-shaped pieces together as before, and add more sections *m*, *m′* in parallel, so as to increase the cross-section of the magnetic circuit. Again take a magnetization curve. It will be found that the same flux is obtained with a considerably smaller number of ampere-turns. (d) Repeat the same experiment with cores made of a different magnetic material. (e) Before leaving the laboratory, measure all the lengths and cross-sections of the samples.

Report. (1) Plot all the magnetization curves for which data were taken in the laboratory. (2) Show that the number of ampere-turns necessary to produce a given flux is proportional to the length of the magnetic circuit. (3) Show that with a given number of ampere-turns, the total flux is proportional to the cross-section of the circuit, in other words, that the reluctance at a given flux density is inversely proportional to the cross-section. (4) Show that with a given length and cross-section of the magnetic circuit, the reluctance increases when the flux density increases.

167. EXPERIMENT 7-C. — Influence of a Small Air-Gap Upon the Reluctance of a Magnetic Circuit. — The arrangement of the apparatus and the method of measuring the flux are the same as in the two preceding experiments. (a) Take magnetization curves indicated under (a) and (c) in experiment 7-B, eliminating the air-gap as much as possible. This part of the experiment may be omitted if data are available from the preceding experiment. (b) Take similar curves with several small values of air-gap, by interposing pieces of paper, fiber, or some such material, of known thickness. The length of the air-gap must be very small as compared with its transverse dimensions, in order to minimize the leakage, which is treated later. Measure the cross-section of the air-gap.

Report. Plot the magnetization curves obtained in the experiment, and subtract the ampere-turns for the iron from the total ampere-turns. The difference will give the ampere-turns for the air-gap, and these must be proportional to the flux, for each size of air-gap. Compute the corresponding values of B_a and H_a, and check formula (14).

168. Magnetic Leakage. — When two or more paths in parallel are offered to an electric current, it is divided into parts inversely proportional to the resistances of the paths. In a similar way, a magnetic flux is divided when two paths are offered to it, as in Fig. 136. The apparatus shown there is the same as in Fig. 131, except that the exciting coils *pp* are placed unsymmetrically. The whole flux goes through the lower core because the exciting coils are placed on it, but only part of this flux goes through the upper core. The rest finds its way back directly through the air, as shown by dotted lines. This latter portion is called the *leakage flux*, because in most practical cases it is not utilized for the purpose for which the magnetic circuit is produced. Thus, in an electric generator or motor part of the flux — instead of going through the armature of the machine — is shunted around it, and passes directly from pole to pole through the air. The lower core in Fig. 136 corresponds to the field frame of a machine, the upper one to its armature.

Magnetic leakage makes it necessary to produce a larger flux than is actually utilized. This means more exciting ampere-turns, and consequently more copper, and more expenditure of energy for excitation. Magnetic leakage is particularly objectionable at high saturation in iron, since an increase in flux means a disproportionately larger increase in the number of exciting ampere-turns. Let, for example, the leakage in a certain case amount to 20 per cent of the useful flux (this is not an uncommon figure in practice). The increase in the exciting ampere-turns necessary for producing these 20 per cent additional lines of force may be 50 per cent or more, as compared to the ideal case of no leakage.

For this reason it is of importance to know how to measure magnetic leakage, and to learn the principal factors upon which it depends, so as to minimize it. In the apparatus shown in Fig. 136, the leakage flux, or the difference between the total and the useful fluxes, may be measured with a ballistic galvanometer or a fluxmeter as before, by taking discharges first through the coil s_2 (total flux), then through s_1 (useful flux). It is more difficult to study accurately the actual distribution of the leakage flux in space, because this requires measuring the flux density in its magnitude and direction from point to point. At the same time, a study of the map of magnetic leakage is of considerable practical importance, because it enables the designer to modify the form and the dimensions of the frame of a machine so as to reduce the leakage to a minimum.

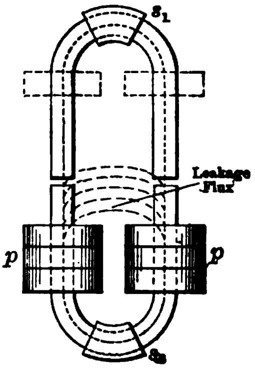


Fig. 136. Arrangement of coils for studying magnetic leakage with the apparatus illustrated in Fig. 131.

A preliminary study of the distribution of a leakage flux in air may be made with a small magnetic needle freely suspended from a silk thread. The direction of the needle in different places of the stray field gives an idea as to the directions of the leakage lines of force. After this, the flux may be investigated either with a ballistic galvanometer, a fluxmeter, or a bismuth spiral (§ 202). When a ballistic galvanometer is used, a small exploring coil, connected to it, is brought into the place in which it is desired to measure the leakage. The coil is held on a pivot by a wound spring locked by a catch. With the galvanometer coil at rest, the catch is released and the test coil quickly turns by 180 degrees. The flux cut during this movement produces in the galvanometer a discharge proportional to the flux. If turning a test coil is impracticable, for instance because of a narrow space, the coil is made flat, and after having been brought into the desired place, is quickly withdrawn out of the field. In some other cases the exciting current must be broken or reversed in order to produce the desired galvanometer deflection.

169. Factors on which Magnetic Leakage Depends. — Generally speaking, the magnitude of a leakage flux depends on the relative values of the reluctances of the useful path and of the stray paths. Any factor which decreases the reluctance of a stray path or increases the reluctance of the useful path tends to increase the leakage. Thus, for example, making the U-shaped cores (Fig. 136) narrower, that is, bringing the two legs of each core closer together, will increase the leakage, because the path for the leakage flux through the air becomes shorter. Increasing the air-gap between the two U's also increases the leakage flux (Fig. 137), because the reluctance of the useful path is thereby increased. For the same reason the percentage of leakage increases with the increase in the saturation. Moving the exciting coils lower increases the cross-section of the leakage flux (in the vertical plane perpendicular to the paper), and consequently increases the leakage flux itself.

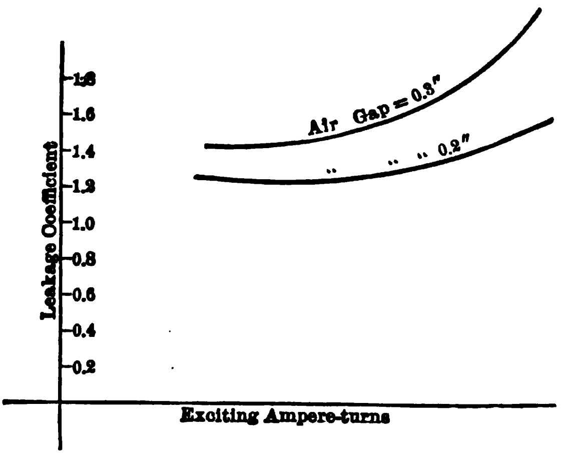


Fig. 137. Increase in percentage of leakage with increased saturation in iron.

An important case of increase in leakage is that due to some counteracting ampere-turns (shown by dotted lines on the upper core). This corresponds in practice to armature currents which counteract the excitation produced by the field coils. The phenomenon is known in generators and motors as *armature reaction* (§§ 358 to 363). That armature reaction increases magnetic leakage in certain cases may be understood from the following example. Let it be required to maintain a certain flux of Φ lines of force in the upper core (Fig. 136), whether or not a current flows through the upper coils shown by dotted lines. Suppose that each section of the coils has 40 turns, and that at no load the necessary flux Φ is produced with 10 amperes exciting current, or $10 \times 6 \times 40 = 2400$ ampere-turns. Let the leakage flux at no load be 20 per cent of Φ. Now let an opposing current of 15 amperes be sent through the upper coils, creating an armature reaction of $15 \times 2 \times 40 = 1200$ ampere-turns. Without the leakage, the lower exciting coils would have to furnish 2400 + 1200 = 3600 ampere-turns, in order to give the same flux as before. With the leakage, an increase of 1200 a.t. is not sufficient, as shown by the following simple calculation.

First assume the reluctance of the iron paths to be negligible as compared to that of the two air-gaps. Then the m.m.f. across the leakage paths is proportional to the excitation on the lower cores. When this excitation is increased from 2400 to 3600 ampere-turns, the leakage flux is increased from $0.2\,\Phi$ to $0.3\,\Phi$. The lower core must now carry a flux of $1.3\,\Phi$ instead of $1.2\,\Phi$, and the leakage constitutes 30 per cent instead of 20 per cent of the useful flux Φ. In reality the reluctance of the lower cores is not negligible, so that more than 3600 ampere-turns must be placed on it in order to leave 3600 between the lower pole tips. Thus, on account of the armature reaction, the magnetic leakage is increased, and an additional m.m.f. is required on the exciting poles which is greater than the corresponding armature ampere-turns. This shows the importance of keeping the magnetic leakage as low as possible.

The magnitude of the leakage flux, with the same excitation, depends upon the reluctance of the leakage paths, and is therefore affected by the shape of the pole-pieces, and particularly by that of the pole tips. This matter is of extreme importance in the design of the magnetic circuit of electrical machinery. In Fig. 136 the amount of the leakage flux may be increased by adding pieces of iron on the inner side of the pole tips so as to reduce the reluctance of the air paths.

The ratio of the total flux to the useful flux is called the *leakage factor* or *leakage coefficient*. For example, if the total flux is 2.4 megalines, and the useful flux is 2.0 megalines, the leakage coefficient is 1.2. The value of the leakage factor gives a good idea of the quality of a magnetic circuit or of the skill with which it has been designed. In Fig. 137 the influence of the saturation in iron and of the length of the air-gap is shown by plotting the corresponding values of leakage coefficient as ordinates.

170. EXPERIMENT 7-D. — A Study of Magnetic Leakage. — The purpose of the experiment is to illustrate the relations explained in the two preceding articles. The apparatus used is shown in Figs. 131 and 136. Magnetic leakage depends upon several factors, and the influence of each factor should be investigated separately. The total flux is measured by discharges through the coil s_2 (Fig. 136) and the useful flux by those through the coil s_1.

(a) *Influence of the air-gap and saturation.* Set the exciting coils *pp* as shown in Fig. 136, and place the upper core as close to the lower core as possible (no air-gap). Take an accurate curve of fluxes in s_1 and s_2 with various values of the exciting current (Fig. 134). Read the fluxes as in Experiment 7-A (§ 163). Make similar runs with two or three different values of air-gap.

(b) *Influence of the position of the magnetizing coils.* Select a value of air-gap, place the exciting coils symmetrically with respect to the two cores, as in Fig. 131, and measure the fluxes in both cores. Then gradually move the coils lower, as in Fig. 136, and take similar readings. Repeat the same experiment with different values of air-gap and of the exciting current.

(c) *Influence of the shape of pole-pieces.* Put pieces of iron on the inner side of the ends of the lower core, so as to assist the leakage through the air. Do the same with the pole tips of the upper core. Investigate the influence of the shape of these extra pole-pieces with different degrees of saturation in the iron and with different lengths of the air-gap.

(d) *Effect of the armature reaction.* Excite the coils on the upper core to oppose the lower coils, as shown in Fig. 136. Determine the value of the flux in s_2 when the flux in s_1 has a given value, with and without the opposing coils. Reverse the current in the upper coils, so as to make them assist the lower exciting coils; observe the change in the leakage. Make similar tests with different positions of the coils, different degrees of saturation in the iron, and various lengths of air-gap.

Report. Tabulate or plot the results in such a form as to bring out clearly the influence of each factor. Make use of the leakage coefficient in order to reduce the results of the experiment to a more general form (Fig. 137).

171. EXPERIMENT 7-E. — Maps of Stray Flux. — This experiment supplements the preceding one, the purpose being to determine the actual paths of leakage lines of force. The method of measurement is described in § 168. Establish certain magnetic conditions, as in Fig. 136, and investigate as closely as possible the direction and the magnitude of the stray flux density at various places. Change the conditions, as indicated in the preceding experiment, and find the effect upon the distribution of the stray flux. Preferably select the same conditions for which values of the leakage coefficient have been determined in the preceding experiment.

Report. Plot the principal directions of the stray flux in several vertical and horizontal planes; mark the magnetic densities per square inch or per square cm. Make one or two free-hand sketches, showing actual lines of force, their distribution, and relative density. State whether the densities at all points increase proportionately with the increase in the exciting current. Discuss the influence of the factors enumerated in § 170, and show how they should theoretically affect the distribution and the density of the stray flux.

Note. For a further study of magnetic flux and leakage in direct-current machines see Chapter XV.

LIFTING AND TRACTIVE ELECTROMAGNETS

172. Electromagnets are used in practice for various duties, particularly for exerting mechanical pull, and for lifting weights. They are especially convenient for intermittent work, where a short stroke and quick action are required. They are used for releasing crane and elevator brakes, for operating switches and motor starters, for supporting weights carried by a crane (Fig. 138), etc. Electromagnets are also widely used as relays for closing all kinds of auxiliary or operating circuits.

Electromagnets used for heavy mechanical duty may be subdivided into two classes: *lifting magnets* and *tractive magnets.* The first kind (Fig. 138) is employed for supporting weights, the second kind (Figs. 139, 140, and 141) for performing mechanical work by the movement of the plunger. The characteristics of each class will be considered separately.

173. Lifting Electromagnets. — Electromagnets of this type, Fig. 138, are used in shops and in rolling mills, where large pieces of steel are regularly carried by cranes. An electromagnet is suspended from the crane in place of the usual hook, and is lowered to the piece of steel to be carried. When the electromagnet is energized by closing the circuit of its exciting coil, the magnetic object to be lifted is attracted to it and may be safely carried by the crane. To release the piece, it is sufficient to open the exciting circuit of the electromagnet. The advantage of this method over the ordinary hook is that the work is performed more quickly, as no time is lost in fastening the hook, and in unhooking the piece at the end of the travel. The lifting electromagnet shown in Fig. 138 consists of a steel casting with a ring-shaped hole in it for the exciting coil. Lines of force pass through the inside core, then through the object to be lifted, and the path is completed through the outside part of the electromagnet frame. The armature and the hook, shown in the sketch by dotted lines, are not used in regular operation; they are shown merely to indicate the method by which the electromagnet may be tested for tractive effort.

Lifting electromagnets are made in different shapes, according to the purpose for which they are intended to be used. Thus, in rolling mills, where large sheets are carried, they are made with several short, projecting poles, each provided with an exciting coil. This is done in order to support the sheet in several places simultaneously. For carrying pig-iron of irregular form, electromagnets are provided with long, projecting poles, which may be sunk into a heap of pig-iron; the pieces stick to the poles on all sides. To make the action still more effective, the poles, or fingers, may be made movable in the vertical direction, so that each pole is sure to come in contact with the heap.

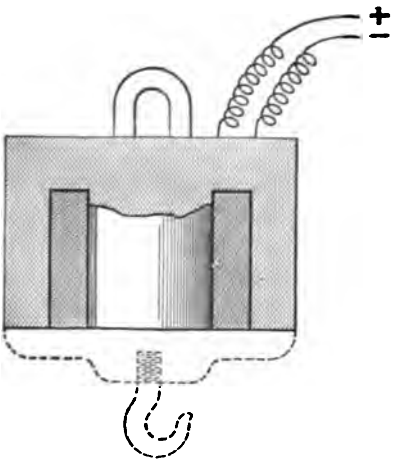


FIG. 138. A lifting electromagnet.

The lifting power of an electromagnet depends not only on its construction and on the exciting current, but also on the form of the piece to be lifted, and on the quality of iron in it. This is because the magnetic flux is closed through the piece to be lifted; hence the reluctance of this piece determines the value of the flux. To make the conditions definite, when testing such an electromagnet, an armature should be used with the smallest possible reluctance; such an armature is shown by dotted lines in Fig. 138. Under such conditions the electromagnet develops its maximum lifting power. If, however, the electromagnet is intended for a definite service, such as for lifting certain kinds of plates in a rolling mill, the test may be performed with such plates.

174. EXPERIMENT 7-F. — Test of a Lifting Electromagnet. — Suspend the electromagnet (Fig. 138) from a secure place, and provide an iron armature with a hook, as shown by dotted lines; weigh the armature before using it. Place the armature under the electromagnet, and gradually excite the coil until the armature is supported by the magnetic attraction. Note this critical value of the current. Place some weight on the hook, and increase the exciting current until it becomes again sufficient to support the armature. Proceed in this way until the saturation limit is reached, or until the safe limit of heating is reached. In performing this test, have under the armature a suitable support, preferably on springs, to limit the motion and to reduce the noise when the armature is released and strikes the support. Repeat the same test, varying the individual factors, *viz.*: (a) use a different armature; (b) interpose an air-gap, by placing a piece of paper, fiber, etc., between the magnet and armature; (c) lift irregular objects. Before leaving the laboratory, measure the dimensions of the magnet and determine the number of turns in the exciting coil. In the absence of a regular lifting magnet, the apparatus shown in Fig. 131 may be used for the experiment, and the influence of various factors investigated.

Report. To exciting current as abscissæ plot curves of maximum supported weight. From formula (8) or (9) in § 183 compute the corresponding values of magnetic flux density in the air-gap, and plot them on the same curve sheet. Show by a few examples that this density checks within reasonable limits with that computed from the exciting ampere-turns and the dimensions of the magnet. Use a standard *B-H* curve (§ 165) such as is found in various textbooks and handbooks. Indicate the influence of the various factors on the lifting force of the electromagnet.

175. Tractive Electromagnets. — Three common types of tractive electromagnets are shown in Figs. 139 to 141; the difference in their characteristics may be judged from the curves of pull shown in Fig. 142. The electromagnet shown in Fig. 139 consists of a coil surrounding a plunger; the curve of pull shows that as the plunger is sucked into the coil, the pull first increases and then again decreases. The addition of an iron frame on the outside (Fig. 140) increases the pull at the end of the stroke, as shown by the curve marked "Iron Clad." A further increase in pull is obtained by adding an inside core *C* (Fig. 141). The plunger is tapered and the core is countersunk, in order to increase the pull by reducing the reluctance of the air-gap. In many cases, however, a flat-faced core and stop are sufficient. Because an iron-clad electromagnet with a core gives the greatest pull, it does not follow that this type should be used in all cases. The selection of one or another type depends upon the character of the work for which the magnet is intended. Knowing the requirements in a particular case, the best type may be selected with reference to the curves of pull.

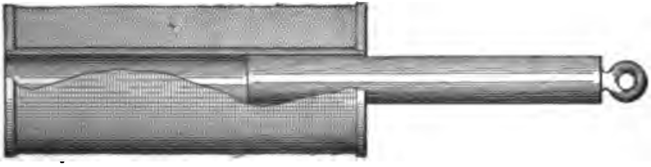


FIG. 139. A coil-and-plunger type tractive electromagnet.

Alternating-current electromagnets are built along similar lines, except that the magnetic circuit usually has to be laminated to reduce eddy currents (§ 210). If the plunger consists of iron strips, a square cross-section is more convenient. The pull, being proportional to the square of the flux density, varies from zero to a maximum and back twice during each cycle. This causes the plunger to vibrate, and if it touches a stop, produces chattering. This is sometimes remedied by the use of a *shading coil,* or solid metal sleeve at the end of the plunger (Fig. 314). Secondary currents induced in this sleeve are considerably out of phase with the primary current, so that the resultant m.m.f. in the air-gap is never equal to zero. Consequently, the variations in the pull are not as great as without the shading coil.

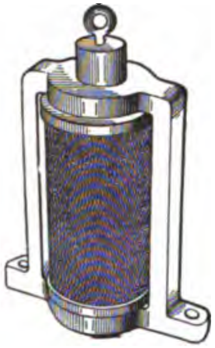


FIG. 140. An iron-clad tractive electromagnet.

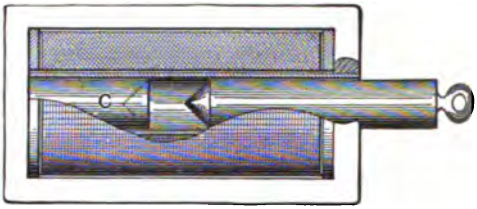


FIG. 141. An iron-clad tractive electromagnet, provided with a core *C* for increasing the pull at the end of the stroke.

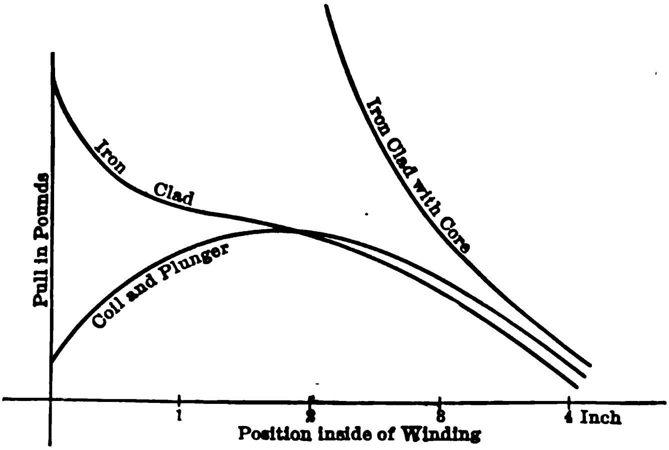


FIG. 142. Curves of comparative pull of the electromagnets shown in Figs 139 to 141.

A convenient apparatus for testing tractive electromagnets is shown in Fig. 143. The coil C of the electromagnet under test is connected to a source of direct- or alternating-current supply, through the switch S, regulating rheostat R, and the ammeter A. The plunger P is suspended from the beam of a balance. The balance has fulcrums at F_1 and F_2, and two sliding weights, w_1 and w_2. With this construction a large pull is counter-balanced with small weights, so that the apparatus is sensitive and convenient to handle. The curves shown in Fig. 142 were taken by means of such a device.

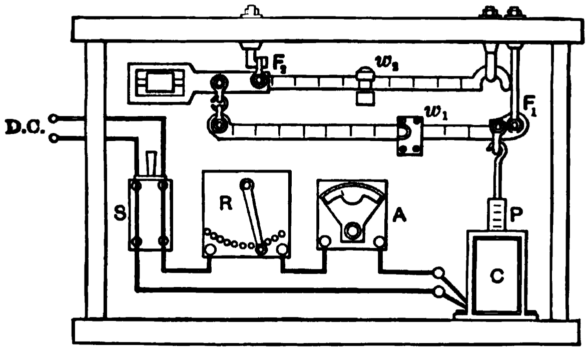


Fig. 143. An apparatus for testing tractive effort of electromagnets.

176. EXPERIMENT 7-G. — Test of a D.C. Tractive Electromagnet. — The purpose of the experiment is to determine curves of pull (Fig. 142) of different forms of tractive electromagnets. An apparatus, similar to that in Fig. 143, is convenient for the purpose. (a) First test an electromagnet of the type shown in Fig. 139; take curves of pull with a certain position of the plunger, through a wide range of exciting currents. (b) Repeat the same test with different positions of the plunger. (c) Use the same coil and provide it with an iron core, as in Fig. 140. Perform a similar test throughout the same range of currents and with the same positions of the plunger. (d) Provide a stop C (Fig. 141) and again test the magnet for pull.

Report. Plot curves showing variations in the pull with the position of the plunger; also curves showing variations in the pull with the exciting current. Give a theoretical justification for the general shape of the curves.

176a. EXPERIMENT 7-H. — Test of an A.C. Tractive Electromagnet. — Investigate the performance of an alternating-current electromagnet, if possible of the same dimensions as the one tested in the preceding experiment. Use a solid and then a laminated plunger and core. The general arrangement and sequence of tests is the same as in Experiment 7-G. Use a wattmeter to find the power factor and the power consumption. Compare the performance with and without a shading coil, and investigate the effect of the dimensions and position of this coil.

CHAPTER VIII

PERMEABILITY AND HYSTERESIS LOOP

177. Steel and iron intended for use in the magnetic circuit of electrical machinery and apparatus, is usually tested for its magnetic properties. There are two standard tests of this kind, *viz.*, one for permeability and one for core loss. The permeability test is considered in this chapter and the core-loss test in the next chapter.

178. Magnetization or Saturation Curve. — The properties of a sample of magnetic material are conveniently represented by means of a magnetization or *B-H* curve, such as is shown in Fig. 135 in § 165. From such a curve the ampere-turns necessary for producing a desired flux density can be computed, and one can also judge as to whether or not the shipment from which a sample was taken comes up to the standard established for the particular application.

By definition, the ratio of B to H is called the *absolute permeability* of the material; we shall denote it by μ_a. Thus we have

$$\mu_a = B/H \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Since both B and H may be expressed in different units the value of the absolute permeability is not definite unless the units are stated. For example, let a piece of steel require $H = 25$ ampere-turns per cm. at a flux density of $B = 15$ kilolines per sq. cm. According to the foregoing definition, $\mu_a = 15/25 = 0.6$. A flux density of 15 kilo-gausses is identically the same as one of $15{,}000 \times 2.54^2 = 96{,}800$ maxwells per square inch. The corresponding H in the English measure is $25 \times 2.54 = 63.3$ ampere-turns per inch. The new value of μ_a is $86{,}800/63.3 = 1525$.

To avoid this ambiguity, the *relative permeability*, μ_r, is often used; it is defined as the ratio of the flux density produced in the sample under test to that in the air, with the same magnetizing force H. In other words,

$$\mu_r = B_{iron}/B_{air} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The kind of units used for B is immaterial, as long as it is the same for both the iron and the air. In the foregoing example the flux density in the air for $H = 25$ is $1.257 \times 25 = 31.4$ maxwells per sq. cm. (§ 165). Hence, $\mu_r = 15{,}000/31.4 = 477$. This number means that at the flux density of 15 kilo-gausses the sample of steel under test is magnetically 477 times " better " than the air.

It is not always required to figure out the values of permeability from a *B-H* curve. Often the *B-H* curve itself is the desired characteristic of a sample. The *B-H* curve is also called the *saturation curve* or the *magnetization curve,* and the three names are used interchangeably. The measuring devices described below are intended for obtaining the *B-H* relationship and are called permeameters.

179. Hysteresis Loop. — The magnetization curve shown in Fig. 135 is also shown in Fig. 144 as the curve *OAM.* It is assumed that at *O* the sample is completely demagnetized. The saturation curve *OAM* is therefore called the *virgin curve.* Let this curve be obtained experimentally, beginning at point *O.* If now, after point *M* has been

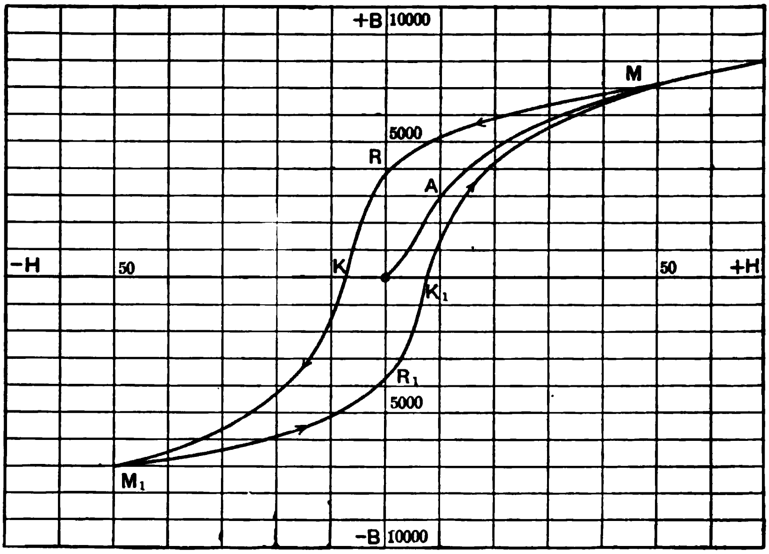


FIG. 144. Magnetization curve and hysteresis loop of a sample of iron.

reached, the excitation is reduced again, the flux density does not decrease as rapidly as it increased, but varies according to the curve *MR.* When the exciting circuit is opened, the iron still possesses a *residual* magnetic density *OR.* When the exciting current is reversed this density is gradually decreased. The value *OK* of the magnetizing force which reduces the residual magnetism to zero is called the *coercive force* (or *retentivity*). When the exciting current is further increased (in the negative direction), the magnetic flux in the iron is reversed, and varies according to the part KM_1 of the curve. Decreasing the current again and reversing it causes the densities to vary according to the branch M_1K_1M of the curve.

This peculiar phenomenon, *viz.*, that two different flux densities are possible with the same value of exciting current, — is called *hysteresis*. It is attributed to some kind of molecular friction in iron; the root of the word signifies "to lag behind." The curve MKM_1K_1 is called the hysteresis loop. In accurate permeability tests, it is advisable to take both a virgin curve and one or more hysteresis loops.

When a hysteresis cycle is performed for the first time on a piece of virgin iron the loop is not closed, but the final point M lies somewhat above the initial point M. After the cycle has been repeated for a number of times the loop becomes closed and the iron is said to be in a *cyclic* state. The locus of points, such as M and M_1, for different final values of H in the cyclic state, is called the *normal* or *commutation curve*, and may be slightly different from the true virgin curve.

A large value of residual magnetism OR is objectionable in cases where the flux is supposed to follow closely any variations in exciting current, as for example in the field poles of a generator or motor. On the other hand, large residual magnetism is desirable in steel intended for permanent magnets, such as are used in certain types of measuring instruments, in telephone receivers, and in magneto generators. For a further discussion of hysteresis see Chapter IX. The testing of permanent magnets for residual magnetism and for coercive force is described in §§ 204 to 207.

180. Magnetic Flux in Air. — In some of the tests described below, it is required to compute the ampere-turns necessary to produce a certain flux density in air, instead of iron. This can be done with sufficient accuracy only when the lines of force are parallel to each other, as for example in a small air-gap between two large pieces of iron, or near the center of a long solenoid. Theory and experience show that under these conditions *one ampere-turn per cm. of length of path in the air produces a flux density* $B_a = 1.25$ *lines of force* (maxwells) *per sq. cm.* (theoretically $4\pi/10$ lines per sq. cm., which value more accurately is equal to 1.257). The magnetic density in air is denoted by B_a to distinguish it from that in iron, which is designated by B. Thus, to produce a flux density of B_a lines per sq. cm. (gausses) in an air-gap l cm. long (in the direction of lines of force) the required number of ampere-turns is

$$ni = B_al/1.25 = 0.8\,B_al \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

If l is in inches, and B_a in lines per sq. inch, the formula becomes

$$ni = 0.796\,(B_a/2.54^2) \times (2.54l) = 0.313\,B_al \quad . \quad . \quad . \qquad (4)$$

Here in place of 0.8 a more accurate value 0.796 = $10/(4\pi)$ is used. Let it be necessary, for example, to produce a flux of 300,000 lines in an air-gap 0.3 cm. long, and of a cross-section of 50 sq. cm. The flux density B_a = 300,000/50 = 6000 maxwells per sq. cm. If the air-gap were 1 cm. long, the necessary number of ampere-turns would be 0.8 × 6000 = 4800. As the air-gap is only 0.3 cm. long, the actual number is 0.3 × 4800 = 1440 ampere-turns. See also § 165.

181. Types of Testing Devices. — The methods and the devices for experimentally obtaining the *B-H* curve and a hysteresis loop (Fig. 144) of a given specimen of steel or iron are described below under the following three headings: (1) Tractional method, (2) Ballistic method, (3) Steady deflection methods. The *B-H* curve can be also determined with alternating current, by the wattmeter method, when measuring the core loss (§ 224).

With the tractional method the magnetic flux density is measured in terms of the mechanical attraction between two parts of a magnetic circuit. This is one of the oldest methods and is gradually giving way to the more accurate and convenient ballistic method. In the latter method the flux within the specimen is suddenly reversed by reversing the current in the magnetizing winding (§ 158). A transient current is thereby induced in a secondary winding and the total electric discharge is read on a ballistic galvanometer. From this deflection the change in flux may be computed. Steady deflection methods are not much used, except for testing permanent magnets. In one of these methods the piece of steel under test is used as part of the magnetic system of a moving-coil galvanometer, and the deflection is a measure of the flux in the specimen. In another arrangement the flux in the specimen is made equal to that in a standard sample, and the magnetomotive forces in the two are directly compared. The property of bismuth to increase its electrical resistance in a magnetic field has also been utilized. When steel laminations are to be used with A.C. excitation exclusively, as for example in a transformer core, it is sometimes preferred to determine their magnetization curve under the same conditions (§ 224), that is, with alternating current.

The selection of this or that piece of apparatus or method of taking a *B-H* curve depends upon the purpose of the test, for example, a scientific investigation, a special industrial test, routine commercial tests, etc. The desired rapidity of individual determinations, the required accuracy of the results, and the form of available samples also influence the choice of this or that equipment and procedure.

Samples of solid iron are usually tested in the form of straight rods of circular cross-section and of convenient length. Sometimes it is desired to test a specimen in ring form, so as to avoid an air-gap; in exceptional cases it may be necessary to test a casting or a forging in bulk (§ 192). Laminations are tested whenever possible in the form of rectangular strips assembled into packages. Sometimes it is required to test them in ring form, or in the form of more complicated stampings of the same shape in which they are used in the assembly of a piece of electrical apparatus. Devices and methods are available for all such cases, and the selection of the best device for the purpose on hand is a matter of judgment essentially based upon first-hand experience with various devices and methods of testing.

182. Procedure in Testing. — The following remarks are taken almost verbatim from the circular No. 17 of the *Bureau of Standards.* While these instructions have been written specifically for the ballistic method, they will be found useful with any other method of determination of permeability and hysteresis loop. Some of the recommended precautions are necessary only in extremely accurate testing, and in each particular practical case one has to decide which of them may be omitted without detriment to the results.

The induction (flux density) which a bar of iron or steel will assume under a given magnetizing force depends upon the previous magnetic condition of the specimen and upon the rate of change from one magnetic state to another. It is modified by the presence of mechanical vibration and depends to some extent on temperature. It is therefore desirable to state the conditions under which the test is made.

All the ordinary tests on magnetic material except ageing should be made at a constant temperature of 25° C. This is in view of the fact that all the magnetic quantities, including the normal induction, the hysteresis loop, and the losses due to hysteresis and eddy currents, depend upon the temperature of the specimen. The relation is not a simple one and is not the same for all materials. In view of these facts it seems desirable to make all such measurements at a fixed temperature.

It is important that there be no mechanical vibration of the specimen during the test. Such vibrations tend to give an induction greater than normal for increasing magnetizing forces, and too small values for decreasing forces. Hence, the test specimen should always be protected from mechanical vibrations in ballistic measurements.

The results found for rolled sheets usually depend upon whether the material is magnetized parallel to the direction of rolling or at right angles to this direction. When not otherwise specified and the dimensions of sheets submitted permit it, the test pieces should be so cut that the flux traverses half of them parallel to the direction of rolling, and half normal thereto.

The measurement of flux density requires a knowledge of the cross-sectional area of the specimen. For rods and bars the cross-section is determined from the dimensions. In sheet metal, however, it should not be determined by direct measurement, but from values of mass, length, and density. The density of each specimen should be determined experimentally, as experience shows that the assumption of any specified value introduces an uncertainty in the result which is greater than the inaccuracy of the magnetic measurements.

Normal Induction. — If a bar of thoroughly demagnetized iron is subjected to a magnetizing force, it experiences a certain induction (flux density). This induction will be greater if the magnetizing force is applied suddenly than for a slower growth of magnetizing current. If the magnetizing force is repeatedly applied and removed, the values of the induction obtained differ somewhat. If the magnetizing force is reversed, a change of induction approximately twice the preceding values is obtained. For the first few reversals the change of induction is not constant, but becomes so after a large number of reversals. One-half this constant value of the change in induction on reversal of the magnetizing force is the normal induction, and the locus of such points is the curve of normal induction.

The magnetic properties of a piece of iron or steel may be considered as defined by the curves of normal induction and hysteresis. Before determining the normal induction data, it is necessary that the specimen be freed from its previous magnetization. This is accomplished by subjecting it to a cyclic magnetizing force of one period per second, which is gradually reduced from an initial value, and carries the induction (flux density) well beyond the point of maximum permeability to a final value somewhat lower than the lowest induction to be studied.

After thorough demagnetization, the lowest magnetizing force to be used is applied and reversed many times, until the iron is brought to a cyclic magnetic state. The induction is then measured and the next higher value of the magnetizing force applied in the same manner. This process is repeated until the required number of points is determined. This is a somewhat laborious operation, but has been found necessary in order to obtain reliable results.

Hysteresis Loop. — Before determining the hysteresis loop, the iron is demagnetized as above, and the magnetizing force is applied and increased until the iron is brought up to the maximum induction for which the loop is required. This magnetizing force is repeatedly reversed until the iron is in a normal condition. The magnetizing force is now reduced from its maximum value to a lower one, and the change in magnetic induction corresponding to the change in force is noted.

After determining this pair of values, the maximum magnetizing force is again applied and the iron once more brought back to a normal magnetic condition. It is not necessary, however, to repeat the process of demagnetization. Another point is then determined in the same manner as the first. Points corresponding to negative values of the magnetizing force are obtained by simultaneously reversing and reducing the magnetizing force. Before each determination of a point on the loop the iron is brought back to its normal condition.

This method of measuring the magnetic constants differs somewhat from the old " step by step " method, which is still employed in many of the modern commercial permeameters. It has the advantage of making the measurement under conditions more nearly approximating those of commercial practice, and is practically free from the effects of magnetic viscosity. Further, it is possible to get more consistent results by this method than by the older one, as the effects of imperfect initial demagnetization are not so serious. The numerical data obtained by these two methods are not identical, and in publishing results of work of the highest precision it is desirable to specify the method of measurement

TRACTIONAL METHOD

183. Thompson Permeameter. — One of the simplest pieces of apparatus for testing iron and steel, the so-called *Thompson permeameter*, is shown in Fig. 145. It is based on the measurement of the mechanical force necessary to separate two parts of a magnetic circuit. The sample *T* to be tested is made in the form of a rod; it is placed within an iron yoke *y*, of negligible magnetic resistance (reluctance). The closed iron circuit thus formed is magnetized by the coil *C*. Then the sample *T* is pulled out of the yoke by turning the handwheel *H*; the pull is read on the spring balance *B*.

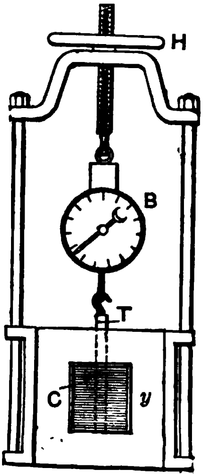


FIG. 145. Thompson permeameter.

The apparatus is intended primarily for approximate relative measurements; the better the sample (the higher its permeability) the more force is required to pull it out of the yoke, with the same magnetizing current. The device has been used for such approximate tests by some electric manufacturing companies. Whenever a new lot of steel is received, samples made out of it are tested in the permeameter. The force necessary to pull a specimen out of the yoke must be at least equal to that previously found for the lowest acceptable grade of material; otherwise the entire consignment is rejected. A knowledge of the *absolute* value of the permeability is not necessary in this case. The number of turns on the field winding of a machine cannot be changed to suit the permeability of each lot, but is designed for a certain average grade of iron. The right number of ampere-turns in each individual machine is obtained by regulating its field rheostat, so as to get the desired voltage.

The permeameter may also be used for the determination of the actual magnetization curve (Fig. 144) of a sample, by comparing the mechanical force necessary to pull out the sample under test, to that required with a standard sample whose magnetization curve has been previously determined by some other method. The comparison is made on the basis of the fact that the mechanical force of attraction between two parts of a magnetic circuit is proportional to the square of the magnetic density at the separating surface and to the area of this surface (Maxwell's law). This law may be roughly explained as follows: The attraction at the separating surface may be thought of as due to two equal and opposite magnetic poles. When the magnetic density increases n times, the strength of each pole also increases n times. The force of attraction is proportional to the product of the pole strengths; therefore it increases n^2 times. For a more rigorous proof of this law and of eq. (6) below see, among other works, the author's "Magnetic Circuit," the last chapter.

Thus, with a given magnetizing current, if F_1 is the spring balance reading with the sample under test, and F_2 that with the standard sample, we have

$$\left(\frac{B_1}{B_2}\right)^2 = \frac{F_1}{F_2} \quad \text{. . . . .} \quad (5)$$

If the magnetic density B_2 is known from the magnetization curve of the standard sample, B_1 may be calculated, and the magnetization curve plotted for the sample under test.

The magnetization curve of a sample can also be obtained with the Thompson permeameter even without having a standard sample. Let B be the flux density in the sample, in kilolines per sq. cm.; let B_a be the density in the air after the sample has been withdrawn; let A be the cross-section of the sample in sq. cm., and F the mechanical pull in kilograms. The theoretical formula for the total mechanical force at the separating surface is

$$F = AB^2/24.7 \quad \text{. . . . .} \quad (6)$$

When the sample is pulled out, not all the flux disappears, since B_a kilolines of force per sq. cm. still remain. Thus, only $(B - B_a)$ kilo-lines per sq. cm. vanish, and the indication of the spring balance corresponds to this number. The preceding equation in our case becomes

$$F = A(B - B_a)^2/24.7 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

Solving it for B, we get

$$B = 4.97\sqrt{F/A} + B_a \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (8)$$

In this expression B_a may be computed from eq. (3) in § 180, if the number of exciting ampere-turns ni and the length l of the coil, that is, the height of the window in the iron yoke, are known. It must be remembered, however, that B_a in formula (3) is in gausses, while in eq. (8) it must be in kilo-gausses.

If B and B_a are expressed in kilo-maxwells per sq. inch, A is in inches, and F in pounds, the preceding formula becomes

$$B = 8.5\sqrt{F/A} + B_a \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

In this case B_a must be computed from eq. (4) and the result divided by 1000, to convert it into kilolines.

The Thompson permeameter is adapted for making quick comparative tests rather than for taking exact magnetization curves. A spring balance is not reliable enough to allow of accurate measurements, and moreover the results are vitiated by the two air-gaps between the yoke and the sample. To reduce the error due to this cause the sample must fit nicely into the hole in the upper part of the yoke, and the two surfaces which are pulled apart must be carefully machined.

The *Ewing Magnetic Balance* is another device based on the same principle. It has a beam with a sliding weight instead of a spring balance. The specimen rests on the yoke with its cylindrical surface, instead of touching the yoke with an end, as in the Thompson permeameter. This feature does away with the surfacing of one end of the sample, and makes it unnecessary to have a hook on the other end; moreover, a better contact is obtained. The scale is calibrated directly in flux densities by means of a standard sample, previously tested by some other method.

184. EXPERIMENT 8-A. — Magnetization Curves with Thompson Permeameter.— The purpose of the experiment is to obtain, for a given sample, a B-H curve and a hysteresis loop, such as are shown in Fig. 144. It is desired that the student test samples of forged, rolled, and cast steel, wrought iron, and cast iron, to see the difference in the magnetic properties of these materials. For the general procedure in testing see § 182, although this particular permeameter is not an accurate device and many refinements recommended there are hardly necessary. (a) To demagnetize the sample, place it in the apparatus and magnetize it well above the maximum density to be used in the test. A regulating rheostat and a reversing switch should be provided in the circuit of the exciting winding (Fig. 131). By operating the double-throw switch, reverse the magnetism of the sample continuously and at the same time gradually reduce the exciting current by means of the regulating rheostat. A frequency of about one reversal per second is recommended. (b) First take a virgin curve OAM; read the exciting current and the pull necessary to tear the sample from the yoke. Carry the magnetizing current as high as the coil will permit. Then take a complete hysteresis loop. (c) Repeat the same test with other samples. Before leaving the laboratory, measure the cross-section of the samples tested, and the axial length of the coil; also ascertain the number of turns in the coil. Each point should be obtained as an average of several readings, because of the inaccuracies in each observation. The sample and the yoke must be nicely surfaced and the spring balance must exert an axial pull. Another precaution, important in all permeability tests, is that once the current has been raised to a certain value, it should not be reduced again, until that part of the magnetization curve is completed. Otherwise, confusing hysteresis effects will be introduced.

Report. Plot the spring-balance readings to exciting amperes as abscissæ. This should be done before making any calculations, in order to obtain smooth curves and to eliminate possible errors of observation. For one of the samples change the curve into a magnetization curve, such as is shown in Fig. 144, by using formula (8) or (9). If the sample has been previously tested by another method, check the results. With this curve as a standard, plot a similar curve for another sample, using formula (5).

185. Du Bois Magnetic Balance. — This is also a tractional device (Figs. 146 and 147), more sensitive and accurate than either of the two described in § 183. The sample T under test is clamped between two heavy pole-pieces PP and magnetized by the coil C. Clamps are provided for accommodating either a round or square sample, or a bundle of laminations. In Fig. 146 two specimens are shown lying on the table. The magnetic circuit is closed through the iron yoke YY; the yoke is supported at O by knife-edges, like the beam of a balance. The shorter arm of the yoke is made of the same weight as the longer one, by weighting it with lead; the yoke is in balance when the coil C is not energized and the weights W_1 and W_2 are on zero.

When the sample is magnetized, the yoke tips over to the left, because the attraction in the left-hand air-gap acts on a longer lever. To balance the yoke, the weights are moved to the right. When the balance is obtained, the flux density in the sample is read directly on the scale, in maxwells per sq. cm. The movement of the yoke is limited by adjustable stops, *SS* (Fig. 146). To simplify the balancing, one of the stops and the yoke may be provided with platinum tips, connected to a local circuit consisting of a dry cell and a bell or a galvanometer so that a signal is given when the yoke touches the stop.

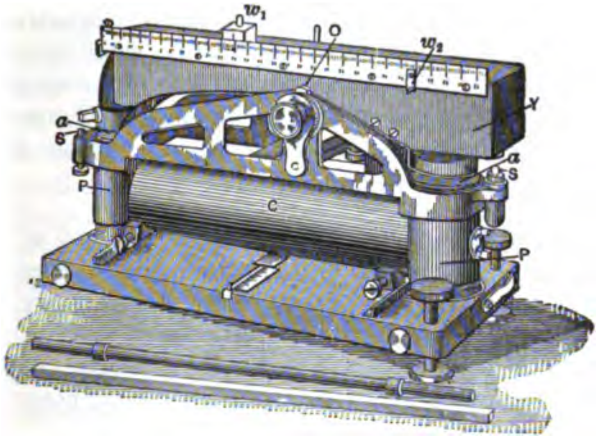


Fig. 146. Du Bois magnetic balance.

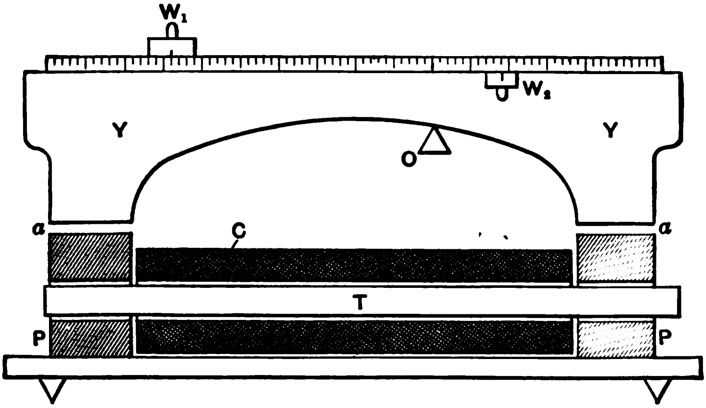


Fig. 147. Schematic representation of the Du Bois magnetic balance.

A certain flux in the sample and in the pole-pieces gives a definite force of attraction at the air-gaps *aa*. Therefore the scale of the instrument may be calibrated directly in fluxes. But as all samples are of the same cross-section, a definite density corresponds to each flux. Therefore the scale is calibrated directly in magnetic densities. The calibration is done by means of a specimen whose magnetization curve has been determined by the ballistic method described below. A good feature of the instrument is that the reluctance of the air-gaps is large as compared to that of the sample. Therefore small inaccuracies in the contacts between the sample and the pole-pieces are of no consequence.

186. EXPERIMENT 8-B. — Magnetization Curves of Iron with Du Bois Balance. — The experiment is performed in the same order as Experiment 8-A (§ 184). Before leaving the laboratory, weigh the sliders, measure the length and the cross-section of the air-gaps and their horizontal distance from the knife-edge supports. Make a copy of the scale; measure the cross-section of the samples; note the number of turns in the magnetizing coil.

Report. (1) Plot curves of flux density to ampere-turns per cm. (or inch) as abscissæ (Fig. 144). If possible, use the same sheet of cross-section paper for all the samples, so as to have a direct comparison for the quality of the materials. (2) Show on a numerical example how to use these curves for determining the number of ampere-turns required to produce a given flux in a magnetic circuit. Take a circuit like the one shown in Fig. 131. The dimensions of the cores and the length of the air-gap are supposed to be given. (3) Compute the actual pull on the yoke of the balance (§ 183) and compare the calculated magnetic torque with that of the weights.

BALLISTIC METHOD

187. The Development of the Ballistic Method. — When a magnetization curve is taken, two quantities are measured, the exciting ampere-turns and the flux produced by them. The chief difficulty lies in measuring the flux. It can be measured more accurately by a discharge through a ballistic galvanometer than by the tractive method described in §§ 183 and 185. The use of the ballistic galvanometer for measuring fluxes is described in §§ 158 to 161. While this is the most accurate method of measuring fluxes, it is only recently that devices have been perfected which make the method convenient for technical purposes. It may not be amiss to give here a brief history of the development of this method.

The original arrangement (Fig. 148) required samples made in ring form, each provided separately with an exciting and a secondary winding. This, of course, was a serious objection from a practical point of view. Hopkinson introduced the "divided-bar" apparatus (Fig. 149) in which the samples *TT* are made in the form of rods; these are much easier to prepare than rings. Permanent coils are used; the magnetic circuit is closed through a heavy yoke *YY*. An objection to this method is that because of the imperfect contact between the yoke and the sample, the exciting current is greater than the real quality of the sample requires.

Ewing remedied this defect by eliminating the reluctance of the contacts and the yoke (Fig. 150). Two measurements are taken with different useful lengths of the magnetic circuit, and from these data the disturbing reluctance of the joints is eliminated. An objection to this method is that it requires two identical samples which are sometimes difficult to obtain. Picou, on the other hand, eliminated the influence of the imperfect contacts (Fig. 151) by introducing additional windings which compensate for the reluctance of the air-gaps. This compensation has been further developed in the Fahy permeameter (Fig. 156) which permits of a rapid and convenient testing of a large number of similar samples with a considerable degree of accuracy.

The Burrows permeameter (Fig. 155) offers a still greater refinement in the compensation for imperfect magnetic joints and is used in standardization work or where a great accuracy is required. Drysdale's device (Fig. 153) has been developed to meet a need for approximate magnetic testing of large castings or forgings, without the preparation of special test samples.

Another objection to the ballistic method has been found in the ballistic galvanometer itself; in its original form, with a telescope and with undamped oscillations, it was not a convenient device for practical work. Ballistic galvanometers of sufficient sensitiveness have been more recently developed, provided with a pointer and a scale like ordinary voltmeters and ammeters. They are perfectly damped electromagnetically, so that readings may be taken in quick succession. The fluxmeter (§ 161), where it can be used, makes it possible to obtain continuous indications of fluxes instead of instantaneous deflections of a ballistic galvanometer.

The above-mentioned devices, based on the ballistic method, will now be described more in detail.

188. Ring Method. — A sample of iron or steel to be tested is turned in the form of a ring *I* (Fig. 148) of a comparatively small radial width. If sheet steel is to be tested, separate rings are punched, assembled together, and turned down on the sides. The ring is uniformly wound with a known number of turns of wire, connected to the terminals *P* (primary). A secondary or exploring coil *E* is wound on the first coil and connected to a ballistic galvanometer. The core is excited from a source of D.C. supply, through a regulating rheostat *R* and a double-throw switch *D*. An ammeter is connected in the primary circuit. This is essentially the same arrangement as that shown in Fig. 131 and described in §§ 156 and 157, except that the magnetic circuit consists of iron only, and no inaccuracy is introduced because of an air-gap.

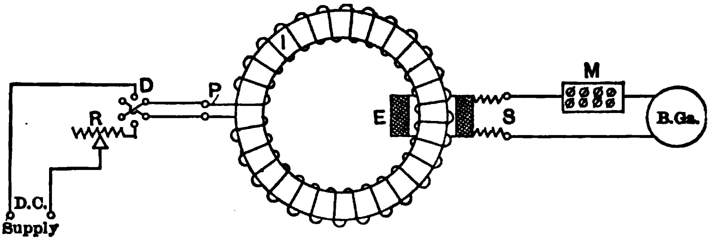


FIG. 148. Testing permeability of a sample in ring form.

The first step is to demagnetize the sample thoroughly by exciting it to as high saturation as possible, and then gradually reducing the excitation to zero, at the same time continually reversing the current. After this, the sample is magnetized again in steps, and the corresponding deflections of the galvanometer are noted. This gives the virgin curve (Fig. 144); a complete hysteresis loop may also be taken in a similar way, if desired. The curves shown in Fig. 144 may then be plotted if the dimensions of the core, the number of turns on both windings, and the constant of the ballistic galvanometer are known. For the exact procedure in testing see § 182.

In some cases it is possible to have specimens in the form of very long rods. Experience shows that a rod whose length is at least 500 times its diameter may be considered magnetically as infinitely long. This means that the same number of ampere-turns *per unit length of its middle part* are required in order to produce a certain flux, whether the rod is straight or made into a closed ring without air-gap. This is not true for a shorter rod because of the leakage and of the demagnetizing action of the ends. With a very long rod the reluctance of the path of the lines of force back through the air is practically zero, because of the very large cross-section of this path. For this reason the required number of ampere-turns for the middle part is nearly the same as it would be if the rod were bent into a ring.

189. Divided-Bar Method. — An objection to the practical use of the ring method is that the preparation of samples is rather expensive and slow. Hopkinson found a way out of this difficulty by making samples in the form of two straight bars, *TT* (Fig. 149). The magnetic circuit is closed through a heavy iron yoke *YY* of negligible reluctance, so that the reluctance of the circuit is practically equal to that of the samples and of the joints. The left-hand bar is securely clamped to the yoke by the screw *n*. The right-hand bar is provided with a handle *H*, by means of which it may be pulled out of the yoke. The exciting or primary coil *C* is wound in two sections and is connected to the terminals *P*. The exploring coil *E* is inserted between the two sections and is connected to the terminals *S*. The electrical connections are the same as in Fig. 148.

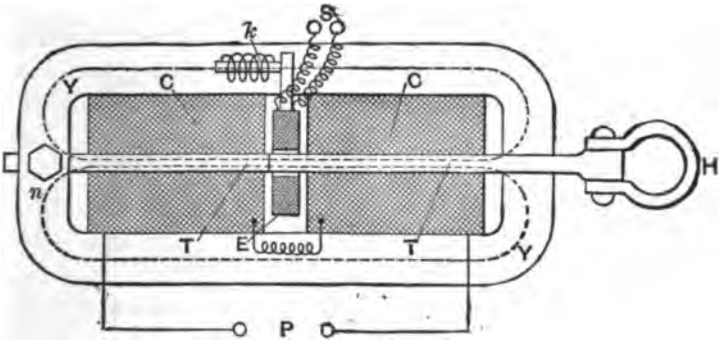


FIG. 149. Hopkinson divided-bar method of testing iron.

To measure the flux produced in the samples by the coils *CC*, the right-hand sample is suddenly withdrawn from the yoke. This releases the coil *E*, and the spring *k* instantly snaps it out of the magnetic circuit. The change in flux embraced by this coil produces a discharge in the ballistic galvanometer, as in the ring method. In calculating the value of *H*, the length of the magnetic circuit is taken as that between the inside faces of the yoke, because of the high permeability of the rest of the circuit. There is also a slight leakage error due to the flux through the coil that does not pass through the bar, but this may be predetermined by a test with a non-magnetic bar.

Instead of using two samples and pulling out one of them, one long sample may be used, and the ballistic discharge obtained by changing or reversing the current in *CC*, as in the ring method. The apparatus is fairly accurate for some practical purposes, but great care must be exercised to have good magnetic contacts between the yoke and the sample.

190. Ewing Double-Bar Method. — To eliminate the influence of the contacts and the reluctance of the yoke, Ewing proposed the apparatus shown in Fig. 150. The magnetic circuit is formed by two identical test bars *AA′*, *BB′*, and two heavy yokes *YY′* made of wrought iron. Each bar is surrounded by a magnetizing coil, as in Fig. 160. A secondary coil, connected to a ballistic galvanometer, is wound on one of the exciting coils. The coils are mounted in the wooden case *C*; the terminals *PP′* are connected to a source of D.C. supply, the terminals *SS′* to a ballistic galvanometer. The number of turns in the secondary coil may be varied at will by the switch shown on top of the box. At the same time a resistance is automatically inserted into the galvanometer circuit, equal to that of the sections of the coil that are cut out. In this way the total resistance of the galvanometer circuit is kept constant, and its deflections are proportional to the number of secondary turns; see eq. (11) in § 159. The reason for varying the number of turns is to have the galvanometer deflections within the best range of the instrument with widely different values of flux.

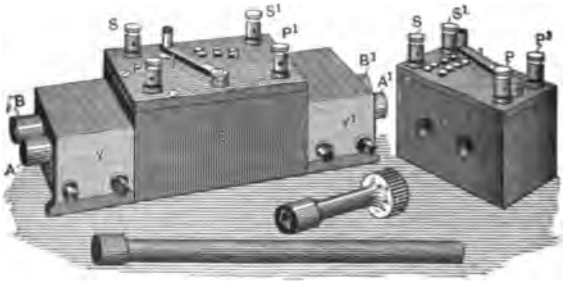


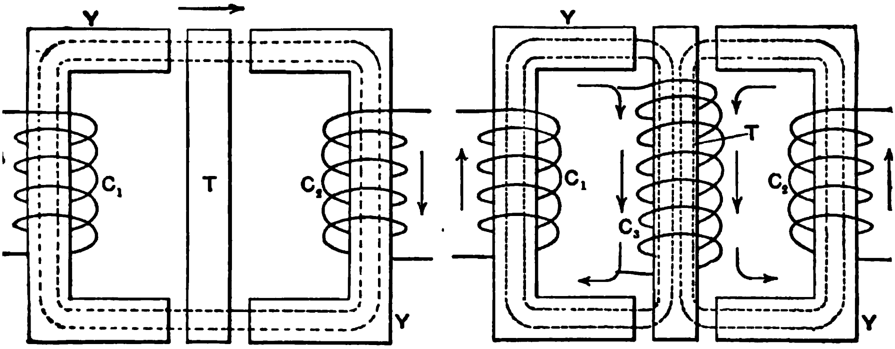
FIG. 150. Ewing double-bar method of testing iron.

The test is performed similarly to that with a ring sample. After a magnetization curve has been obtained, one of the yokes and the box *C* are removed by loosening the screws. A shorter box *D* is slipped in place, and the magnetic circuit completed again, the yokes being brought closer together. A magnetization curve is taken as before. It differs from that previously obtained because the circuit is shorter and the number of magnetizing turns in the box *D* is smaller. The reluctance of the yokes and that of the contacts is practically the same in both cases, and may be eliminated by comparing the two curves. Let, for example, 850 ampere-turns be necessary in order to produce a certain flux with the box *C*, 10 inches long. Let 450 ampere-turns give the same flux with the box *D*, 5 inches long. This shows that 400 ampere-turns were used in the first case for $5 \times 2 = 10$ inches of the length of the samples, so that the true ampere-turns per inch of the sample are $400/10 = 40$. If the shorter box were not used, the reluctance of the contacts and the yokes would have to be neglected;

the number of ampere-turns per inch of the sample would have to be taken as 850/20 = 42.5, which is about 6 per cent too high. The readings with the shorter box enable one to separate the 50 ampere-turns lost in overcoming the reluctance of the contacts and the yokes.

The details of the apparatus shown in Fig. 150 are due to Mr. J. W. Esterline. In particular he deserves credit for the boxes, for the method of clamping, and for the scheme of using variable secondary turns with constant resistance.

191. Picou Permeameter. — In this apparatus the disturbing influence of the magnetic contacts is compensated by additional magnetizing windings. The principle of compensation is shown in Figs. 151 and 152. The sample T under test is of rectangular cross-section, and may consist of a bunch of steel laminations. It is clamped within the magnetizing coil C_3 between two U-shaped yokes YY, provided with compensating windings C_1, C_2. The winding C_3 supplies just enough ampere-turns to maintain the flux in the test sample. The coils C_1 and C_2 provide the magnetomotive force necessary to overcome the reluctance of the yokes and the air-gaps. A secondary coil is wound on each of the three magnetizing coils, and may be connected at will to a ballistic galvanometer.



Figs. 151 and 152. Principle of compensation for the reluctance of air-gaps in the Picou permeameter.

The first step in testing a sample is to compensate for the reluctance of the yokes and the air-gaps. For this purpose the circuit of the main coil C_3 is left open; the coils C_1 and C_2 are energized so as to produce a flux through the yokes only, as shown by the dotted lines in Fig. 151. Each coil supplies the ampere-turns necessary for overcoming the reluctance of one yoke and of two air-gaps. The desired value of the flux is established by trial discharges through a ballistic galvanometer. The current in one of the coils is then reversed, so that the flux is made to pass through the sample under test, as shown by the dotted lines in Fig. 152. Each of the coils, C_1 and C_2, has now to overcome the reluctance of its yoke, of two air-gaps, and in addition the reluctance of one longitudinal half of the sample under test. The flux is naturally reduced; to bring it to its former value, the middle coil C_3 is excited so as to assist the two other coils.

When the former value of the flux has been re-established in the yokes (as recognized by ballistic deflections), it is evident that the two outside coils again supply the ampere-turns necessary for the yokes and air-gaps, while the middle coil carries the flux through the sample under test. Knowing the number of ampere-turns per unit length of the coil C_3 and taking a ballistic discharge through the secondary coil which surrounds it, a point on the magnetization curve is obtained. The same test is repeated with other values of flux to obtain a complete set of data for a *B-H* curve.

192. Drysdale Permeameter. — This instrument, shown in Figs. 153 and 154, has been used for testing frame castings of electric generators and motors, without preparing special samples. The magnetizing and the exploring coils (Fig. 153) are wound on a plug (Fig. 154); the plug is inserted into a hole drilled at any desired place in the casting to be tested. The magnetic circuit is excited as shown by dotted lines, and a discharge taken through a ballistic galvanometer, or a fluxmeter.

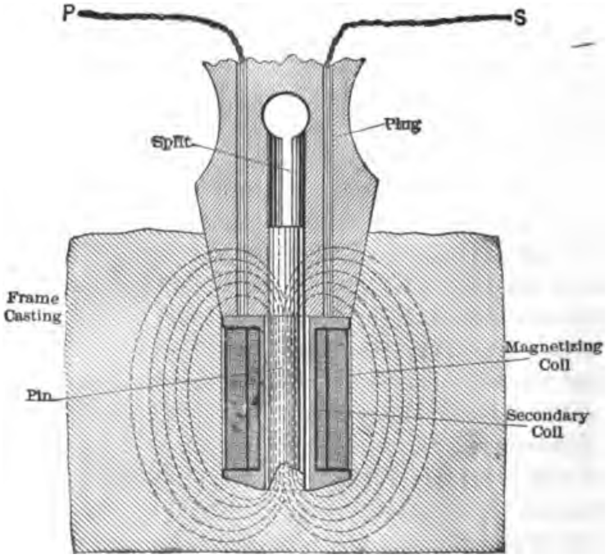


FIG. 153. Cross-section of the Drysdale permeameter, showing the plug in the casting under test.

The hole in the casting has the form shown in Fig. 153. The pin left in the center constitutes the sample under test. A special hollow drill is used for making such holes, so that they will exactly fit the testing plug. It is well to test a casting in several places, in order to get its average permeability. The instrument is calibrated empirically by means of a sample of known magnetic properties.

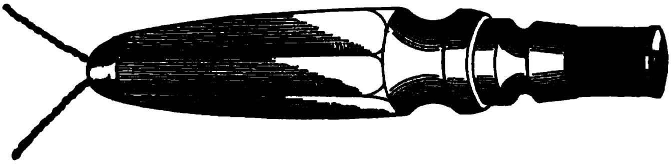


Fig. 154. The exploring plug of the Drysdale permeameter.

193. Burrows Permeameter. — The magnetic circuit and the windings of the Burrows permeameter are shown in Fig. 155. This permeameter is a modification of the ring method (Fig. 148) and at the present writing is considered by the Bureau of Standards and by the American Society for Testing Materials as the standard device. The magnetic circuit consists of two soft-iron yokes, B and C, joined by the rod under test and by an auxiliary rod. The two rods should be of the same size and material. The magnetic leakage over the middle portion of the test bar is reduced to zero by proper distribution of the magnetomotive force. As a result, the true magnetizing force H may be accurately computed from the exciting current, knowing the concentration of the magnetizing winding over the middle part of the bar. The magnetizing winding over the test bar is denoted by T, that of the auxiliary bar by A. The four coils marked J are for the purpose of providing an adjustable magnetomotive force for the joints and the yokes, so as to reduce the leakage to zero. The secondary or test coils, which may be connected at will to the ballistic galvanometer, are denoted by the corresponding small letters t, a, and j.

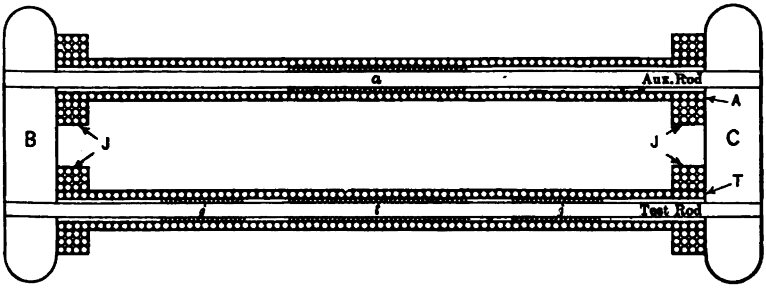


Fig. 155. The principle of the Burrows permeameter.

The adjustment and compensation described below consist in distributing the magnetomotive forces *A*, *T*, and *J* in such a proportion that (a) the fluxes in the middle parts of the two bars are equal, and (b) the flux over a considerable portion of the test bar remains constant. The purpose of this adjustment is to cut down as much as possible "free magnetomotive forces," that is, magnetomotive forces produced at one point of the magnetic circuit and used up at some other point. Such free magnetomotive forces, or differences of magnetic potential, cause leakage fluxes through the surrounding air and therefore impair the accuracy of measurement. The magnetomotive force must be consumed as much as possible in the reluctance of the iron path bit by bit, where produced, leaving no free ampere-turns. If the permeability of the two bars and the contact reluctances were exactly the same, the magnetomotive force of each exciting winding (*A* or *T*) with the same number of ampere-turns, would be used up completely upon the corresponding half of the magnetic circuit. Because of unavoidable differences in the material and clamping such is not the case, and one of the magnetomotive forces may partly exert its influence upon the other half of the circuit, thus shunting part of the flux through the air. Therefore the exciting current in the auxiliary bar is changed until the flux in the middle portions of the two bars is the same. This is done by connecting the two secondary coils, *a* and *t*, in opposition and discharging the secondary current through a ballistic galvanometer when the primary current is reversed. With the proper adjustment of the two primary currents the galvanometer deflection should be zero.

Because of the reluctance of the joints and the yokes, a free magnetomotive force, and consequently a leakage flux, would exist along the test bar if it were not for the compensating coils *JJ*. The current in these coils must be so adjusted that the main winding *T* furnishes the magnetomotive force only for the bar itself, while the ampere-turns in *JJ* are just sufficient for maintaining the flux through the joints and the yokes. In this case the magnetomotive force in *T* is consumed bit by bit where produced and there is no leakage flux, so that the flux along the bar remains constant, as in a ring sample. The secondary coils *jj* have only 50 turns each, while *t* has 100 turns. The two *j* coils are therefore connected in series with the ballistic galvanometer and in opposition with *t*. The current in *JJ* is then adjusted until there is no galvanometer deflection when the primary currents are reversed.

The induction or *B-H* curve is taken by a step-by-step process beginning at the point of lowest magnetization. The sample is previously demagnetized, and at each step the magnetic circuit is compensated as described above. The flux in the sample is measured by reversing all the magnetizing currents simultaneously and noting the deflection on a calibrated ballistic galvanometer. The values of H are determined from the number of turns and the exciting current in T.

The permeameter is capable of a considerable degree of precision, and various auxiliary switches and rheostats have been developed by the *Bureau of Standards* and the makers of the device in order to simplify its manipulation. Detailed instructions accompany the permeameter and should be closely followed in actual work. Publications of the Bureau of Standards on the subject are also available.

194. Fahy Permeameter. — This device is also of the compensated ballistic type, like the Burrows and Picou permeameters described above. While not as accurate as the Burrows arrangement, it represents a distinct advance over older permeameters and gives readings which are amply consistent and accurate for practical requirements. The magnetic and electric circuits are shown in Fig. 156. An H-shaped magnetic core YJY' forms two symmetrical magnetic circuits, each of which includes the cross-bar J in common. The remaining portions of one of these circuits embrace the core arms Y, Y and the air path between such arms; while the other includes the core arms Y', Y' and a similar air path. The cross-bar J is divided midway between its ends into two sections separated by a small air-gap, which

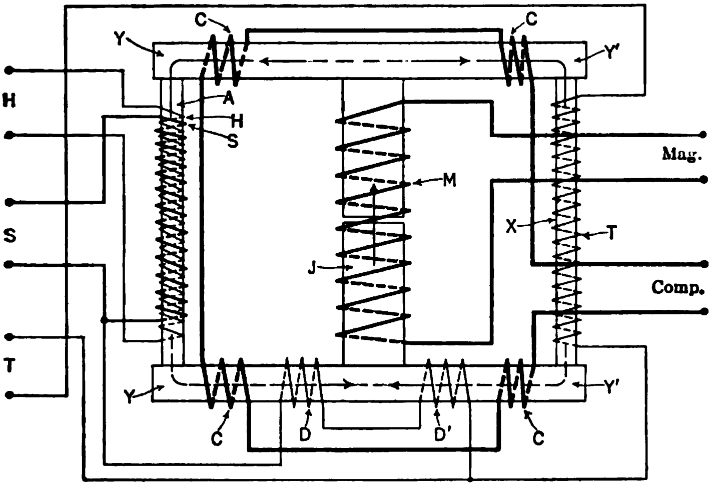


FIG. 156. The Fahy Permeameter.

may be closed to a greater or lesser extent by a screw piston of magnetic material.

The ends of the transverse core portions, forming a part of each of the two parallel magnetic circuits, are adapted for bridging by specimens of magnetic material. A standard specimen, *A*, the magnetic characteristics of which are known, bridges the core arms *Y*, and a test specimen, *X*, the magnetic characteristics of which are to be determined, bridges the core arms *Y′*. Clamps are provided to hold the specimens tightly in place. The ends of the core arms *Y* and *Y′* are slotted to permit the insertion of bushings which vary in form according to the shape of the material bridging the core arms. The adjustable air-gap in the cross-bar *J* is designed to increase the reluctance of the core to suit requirements. When the gap is open to its fullest extent, this high reluctance is of material assistance in reducing the time constants of the magnetic circuits, when the magnetizing current is broken in the act of reversing the current. This air-gap may be so adjusted as to reduce to a negligible amount the magnetic field due to the residual properties of the core itself, in the region occupied by the test specimen. Under this latter condition direct measurements of residual induction are simplified.

The magnetomotive force is applied in two sections — one over the cross-bar *J* by means of the current in the magnetizing coil *M*, and the other distributed in equal parts over the core arms *YY′* by means of the current in the compensating coils *C*. The currents in these two sections are independently adjustable. When current is flowing in both the magnetizing and the compensating coils, one magnetic circuit has the magnetomotive force due to the magnetizing coil augmented by the current in the compensating coils, while the other magnetic circuit has its magnetomotive force decreased by the same amount.

The secondary or test coils indicated by *D*, *D′*, *S*, and *T* are each of the same number of turns and placed as shown in the figure; *D* and *D′* on opposite arms of the same transverse core portion, and *S* and *T* uniformly wound on coil forms through which the standard and test specimens *A* and *X* respectively may be inserted. A test coil, *H*, of a relatively large number of turns is wound uniformly on the same form which carries the test coil *S*. The test coils *S* and *T* are of the same mean cross-sectional area. The magnetizing and compensating coils are connected to the binding posts marked "Mag" and "Comp" respectively. The test coils, *S*, *D*, *D′*, and *T* are permanently connected in series, taps being brought out to binding posts marked *S* and *T*. This series connection is carried out in such a manner that upon reversal of the current in the magnetizing coil *M*, the integrated e.m.f.

induced in the coils S and D' acts in the same direction and in opposition to the integrated e.m.f. simultaneously induced in the coils D and T. Connection to the adjacent binding posts of the pairs marked S and T enables observation of the differential effect of the coils D and D' to be made. The permeameter is initially so adjusted that this differential effect is zero when no specimens are in the apparatus.

The test coil H is connected to the binding posts marked H. This coil is used to determine the magnetic potential difference existing between its ends when it encircles air only, that is, when no specimen is inserted within it. The average magnetic potential drop per unit length of the winding H is equal to the average magnetizing force. The product of " area times turns " of the coil is determined by measurements on the coil in a uniform field such as is produced by a long solenoid. This value is corrected empirically to allow for the difference between the length of the coil and the free length of the test specimen between contacts at the opposite pole faces.

The current in the compensating coils CC is adjusted to such a value that the magnetic leakage flux in parallel with the bar A is equal to that in parallel with the bar X. These leakage fluxes are in the air and are due to the m.m.f.'s between the ends YY, $Y'Y'$ of the core arms respectively. If the leakage fluxes are equal, then the m.m.f.'s which produce them are also equal, because of the symmetry of the two halves of the apparatus. Thus, under these conditions the two bars under test are subjected to the same m.m.f. and the fluxes within them may be directly compared ballistically. The leakage flux to the left is equal to the difference of those treading the coils D and S respectively. The leakage flux to the right is equal to the difference of those linked with the coils D' and T. The coils on each side are connected in opposition, and the two groups are in series. Thus, when the current in the magnetizing winding M is reversed, the deflection of the ballistic galvanometer is a measure of the difference of the two leakage fluxes, and when the deflection is reduced to zero by adjusting the current in CC, the two leakage fluxes are equal.

The flux through the bar A is measured by reversing the current in M and measuring the ballistic discharge through coil S. Similarly, the flux in X is determined by discharging the induced current in coil T through the ballistic galvanometer. The B-H curve of the bar A is supposed to be known, so that the magnetization curve of the test sample may be immediately plotted. The compensation for equal leakage must be made for each point on the saturation curve.

Absolute measurements of flux density are made without the use of a standard bar, which must be removed from the permeameter. Compensation for unequal magnetomotive force drops in the two magnetic circuits should be made as before. The measurement of the magnetizing force and induction is made while reversing simultaneously the currents in *M* and *C*. The deflection of the galvanometer (when it is connected to *H*), gives directly the magnetizing force acting on the specimen *X* (§§ 165 and 180). The coil *H* which is wound uniformly along the length of the coil *S*, may be used to measure the difference in magnetic potential and hence the mean magnetizing force for this region. This is also the mean magnetizing force along the path of the coil *T* within which the test specimen lies. The flux within the specimen *X* is read when the galvanometer is connected to *T*, as before.

Special switches have been designed for a convenient manipulation of the several circuits, and the instructions which accompany the device must be closely followed. For quick approximate results, especially when the two bars are of nearly the same magnetic quality, the compensation may be omitted and the coils *CC* not used at all.

A simpler permeameter has also been designed, based on the same principle, but without compensating coils. It is not as accurate as the device with compensating coils, but is manipulated more readily by a less skilled operator and enables a large number of readings to be taken within a shorter time. This "Simplex" type of permeameter has been successfully used in commercial magnetic testing, and also in studies on the correlation of magnetic and mechanical properties of steels as dependent upon their heat treatment. Drills, reamers, high-speed cutting tools, etc. may be thus readily tested magnetically, by comparison with standard samples. Any considerable departure in permeability or in residual magnetism usually indicates structural irregularities or stresses which may be corrected by a subsequent heat treatment, without destroying a valuable tool.

195. EXPERIMENT 8-C. — Magnetization Curves of Iron by the Ballistic Method. — The experiment comprises the use of any of the instruments described in §§ 187 to 194. The sample is thoroughly demagnetized (§ 182) and a virgin curve *OAM* (Fig. 144) taken step by step. The test should be performed on samples of steel, soft wrought iron, and cast iron. If the constant of the ballistic galvanometer is not known, the instrument must be calibrated, as explained in § 160. Aside from learning the method itself, and the operation of a permeameter, it is expected that the student will make clear to himself: (a) the details of its construction, (b) the sources of error, and (c) the limits of accuracy.

Report. (1) Plot the magnetization curves of the samples, using ampere-turns per cm. as abscissæ and flux densities in kilo-lines per square cm. as ordinates. Also plot values of absolute and relative permeability (§ 178) to values of B as abscissæ. (2) Illustrate on a numerical example the use of these curves for determining the number of ampere-turns in a given case,— for instance, to produce a given flux in a magnetic circuit like the one shown in Fig. 131. The dimensions of the cores and the length of the air-gap are supposed to be given. (3) Discuss the advantages and disadvantages of the apparatus used, sources of error, limits of accuracy, etc.

196. EXPERIMENT 8-D. — Hysteresis Loop of Iron by the Ballistic Method. — The meaning of the hysteresis loop is explained in § 179, and the steps in the experimental procedure in § 182. Any of the devices described in §§ 187 to 194 may be used. See also the instructions for the preceding experiment. For each sample tested obtain data for at least two different hysteresis loops corresponding to different values of maximum H.

Report. (1) Plot the hysteresis loops of the samples tested, using ampere-turns per cm. as abscissæ and kilo-lines per sq. cm. as ordinates. Mark in particular the values of coercive force and of residual flux density and compare the materials tested with respect to these points. (2) Compute the hysteresis loss per cycle, in joules, by integrating one of the curves, as explained in § 225. (3) Discuss the advantages and disadvantages of the apparatus used, sources of error, limits of accuracy, etc.

STEADY DEFLECTION METHODS

197. Koepsel Permeameter. — In its principle the device is a movable-coil milli-voltmeter (Figs. 157 and 158), in which the permanent magnet is replaced by an electromagnet; the sample P under test is a part of this electromagnet. The instrument consists of two heavy pole-pieces J, J (Fig. 157) in which is fastened the rod P to be tested; S is the magnetizing coil. The flux produced in the sample and in the pole-pieces is measured by means of a movable coil s, similar to those used in direct-current ammeters and voltmeters (Fig. 34). The coil is supplied with current from a small auxiliary dry battery; deflections are indicated by a pointer on the

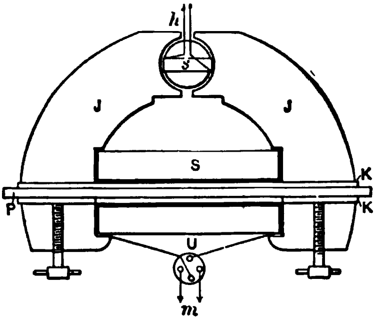


Fig. 157. A cross-section of the Koepsel permeameter.

dial (Fig. 159). The scale is calibrated directly in magnetic densities B per sq. cm.

U in Figs. 157 to 159 is a double-throw switch for reversing the current in the magnetizing coil S; W_m in Fig. 159 is a rheostat for regulating the current in the same coil. W_h is a rheostat for regulating the current in the moving coil; it has three handles, for coarse, medium, and fine adjustment. S_1 and S_2 in Fig. 158 are compensating coils connected in series with S and opposing it magnetically. These coils are adjusted so that the instrument shows zero when the sample is taken out.

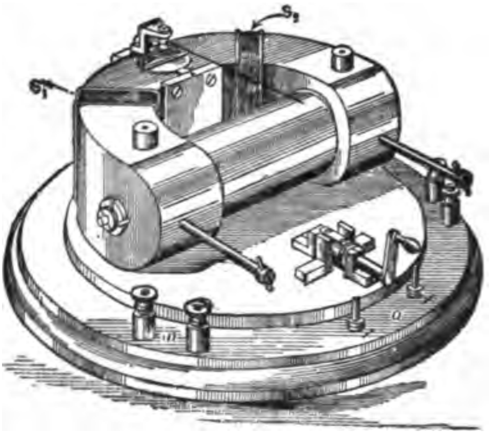


FIG. 158. The Koepsel permeameter, with the cover off.

The apparatus is intended for taking complete magnetization curves, similar to one shown in Fig. 144. The values of B are read off directly on the scale (Fig. 159) when the current i in the moving coil has been adjusted to a value given on the instrument.

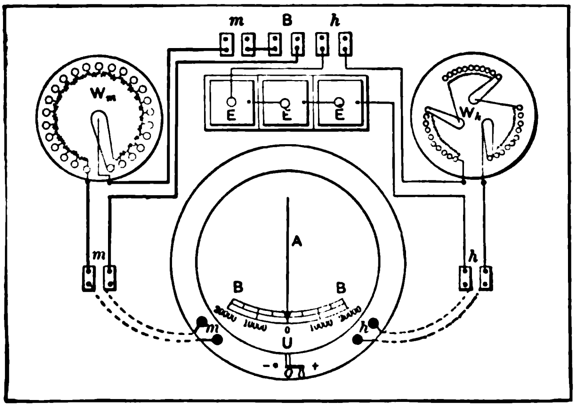


FIG. 159. The electrical connections in the Koepsel permeameter.

This current $i = k \div q$, where k is an instrument constant, and q is the cross-section (in sq. cm.) of the sample under test. The smaller this cross-section the smaller the flux, and the larger must be the current in the moving coil in order to produce the same deflection.

The exciting ampere-turns are obtained by multiplying the current I in the coil S by its number of turns per unit length.

Care must be taken to protect the apparatus from external magnetic fields; strong magnets or large pieces of iron should not be allowed near it when readings are taken. The sample itself must have no ends protruding out of the yoke, as they produce a stray field. The apparatus is sensitive even to terrestrial magnetism; in order to eliminate this error the device must be placed so that the line marked on the dial is in the direction "north-south." When the instrument is in this position and both currents are "on," the pointer must remain on zero when there is no test sample in the apparatus.

In this permeameter, as well as in some others, there is a source of error which is difficult to eliminate, namely, imperfect contacts between the yoke and the rod under test. As a result, it takes more ampere-turns to produce a certain flux than it would without these unavoidable air-gaps, and the apparent permeability comes out lower than it is in reality. An air-gap only 1/1000 of an inch long is equivalent in its magnetic resistance to nearly ½ inch of a good iron rod; therefore, when accurate results are required, the experimentally observed B-H curve must be properly corrected. Such correction curves are usually supplied with the Koepsel apparatus.

198. EXPERIMENT 8-E. — Magnetization Curves of Iron with Koepsel Permeameter. — Test a steel specimen, a cast-iron specimen, and a bunch of iron laminations. Special clamps are provided with the instrument for accommodating round and rectangular samples. Set the instrument in the N-S direction; see that the needle shows no deflection, without the sample, and with both currents on. Insert a sample and demagnetize it (§ 182); use the commutator shown in front in Fig. 158 for reversing the current. Take the curves shown in Fig. 144. Correct the obtained data in accordance with the standardization certificate of the instrument. Investigate the sensitiveness of the apparatus to stray magnetic fields, to terrestrial magnetism, and to large iron masses in its proximity.

Report. Similar to that in § 195.

199. Esterline Permeameter. — This device is essentially a small direct-current generator; the sample bar to be tested completes the magnetic circuit of the machine. The working principle of the device may be understood with reference to Fig. 157, if the moving coil s be replaced by an ordinary direct-current armature (§ 226) driven by a motor. The magnetizing coil S, the sample P and the pole-pieces J, J are essentially the same as in the Koepsel permeameter. The flux in the sample, instead of being measured by a deflection of the moving coil *s*, is measured by the voltage induced in the revolving armature. The magnetic density is computed from the formula

$$\text{density } B = \text{const.} \times \text{volts/speed}.$$

This follows from the fact that the voltage induced in the revolving armature is proportional to the flux and to the speed of rotation. Compensating coils are used to correct for the reluctance of the air-gap and of the parts of the circuit other than the bar under test.

The exciting ampere-turns are varied by changing the number of turns in the magnetizing coil, or by regulating the current in the coil by a suitable resistance, as in the Koepsel permeameter. The Esterline apparatus can be substantially built, is not affected by terrestrial field, and is suitable for ordinary commercial work. It is claimed that an inexperienced operator, with hardly any knowledge of electricity and magnetism, can obtain accurate results after very little practice. The device is described in detail in the *Proceedings of the American Society for Testing Materials*, VI, 1906.

200. Ewing Permeability Bridge. — An ingenious method of comparing specimens of steel and iron to a standard sample was devised by Prof. Ewing (Fig. 160). *S* is a standard rod whose magnetization curve is known; *T* is the sample under test. Both are surrounded by magnetizing coils and are clamped in heavy iron yokes *P P*. The magnetic circuit is closed as shown by the arrows. The standard sample is excited with a certain number of ampere-turns; the number of ampere turns on the sample under test is varied until the fluxes in both samples become equal. This condition obtains when the pivoted magnetic needle *m* returns to zero. When the fluxes in the two samples are different, some lines of force must find their path through the yokes *H H* and the air-gap between them, deflecting the needle. When the fluxes become equal, no stray flux passes through *H H*, and the needle returns to zero under the influence of the permanent magnet *g*.

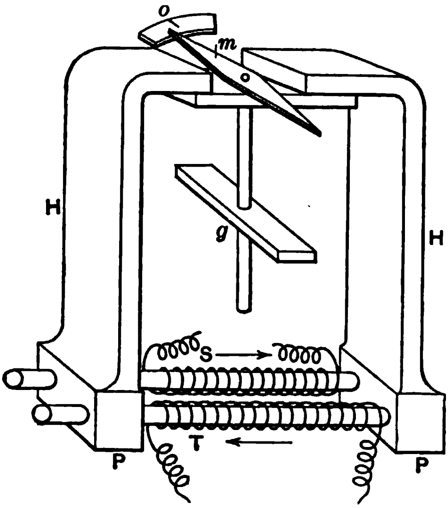


Fig. 160. The Ewing magnetic bridge.

In the apparatus, as it is actually made, the number of turns around the standard sample *S* is fixed; that around the specimen *T* may be varied by a radial arm and contacts, somewhat as in Fig. 150. A double row of contacts is provided, so that when a section of the coil is cut out of the circuit, an equivalent resistance is inserted in its place. This is done in order to maintain a constant current. The two magnetizing coils are connected in series, so that the ratio of active turns gives directly the ratio of ampere-turns. With this arrangement one ammeter is used instead of two.

The magnetization curve of the sample under test is plotted by changing the abscissæ of the magnetization curve of the standard sample. Let, for example, the standard sample require 80 ampere-turns per inch at a density of 100 kilo-maxwells per sq. inch. Send through the magnetizing coils a current, such as to produce 80 ampere-turns per inch in the standard sample; adjust the number of turns of the specimen under test, until the needle *m* returns to zero. Let the number of turns in the standard coil be 100, that in the other coil 150. This shows that in order to obtain a density of 100 kilo-maxwells in the sample under test, $80 \times 150/100 = 120$ ampere-turns per inch are required. A calibration curve is usually furnished with the instrument to correct for unavoidable air-gaps.

The apparatus is called a "bridge," because of its resemblance to the Wheatstone bridge; the two horns *H H* and the magnetic needle correspond to the galvanometer circuit of the bridge (§ 16).

201. EXPERIMENT 8-F. — Magnetization Curves of Iron with Ewing Permeability Bridge. — The apparatus is described in § 200. The conduct of the experiment and the requirements for the report are the same as in § 195.

/ **202. Bismuth Spiral.** — The metal bismuth has the peculiar property of increasing its electrical resistance appreciably when placed in a magnetic field. Per cent increase in resistance is nearly proportional to the magnetic density of the field. The curve shown in Fig. 161 gives an idea of this increase for an average specimen of bismuth wire. This property of bismuth has been utilized to some extent for measuring magnetic densities. Fine bismuth wire is made into a flat spiral wound non-inductively, the wire being doubled back on itself. The winding is held rigidly between two thin pieces of mica. Such a spiral is placed in a magnetic field whose flux density it is desired to measure. The ends of the spiral are connected to a Wheatstone bridge (§ 16). From the increase in the electrical resistance of the bismuth wire, the flux density is determined by using the curve shown in Fig. 161. Bismuth spirals used for measuring flux density in air-gaps of electrical machines, stray fluxes, etc., are provided with handles and with suitable terminals. For testing permeability of iron with a bismuth spiral, an apparatus similar to that shown in Fig. 149 has been used. The exploring coil *E* is omitted, and the spiral is inserted between the ends of the two

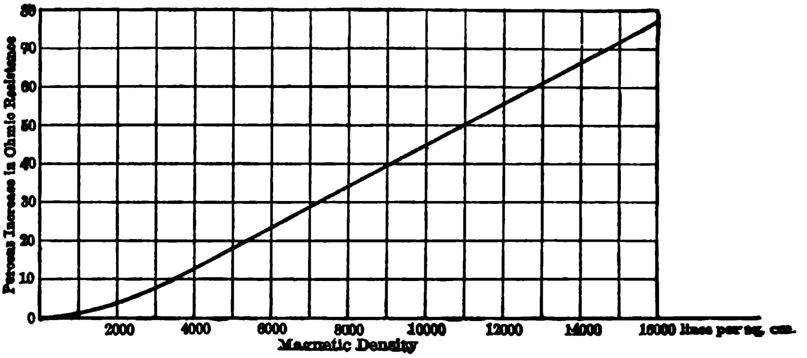


FIG. 161. The resistance of a bismuth spiral as a function of magnetic flux density.

samples *T*, *T*. Bismuth has an appreciable resistance-temperature coefficient, so that an observed change in resistance may in part be due to this cause. A carefully computed correction for temperature must be applied when using this method.

203. EXPERIMENT 8-G. — Magnetization Curves of Iron with Bismuth Spiral. — The method is described in § 202. The general conduct of the experiment and the requirements for the report are the same as in § 195.

TESTING OF PERMANENT MAGNETS

204. Permanent magnets are used in a number of electrical devices, such as magneto-generators, d'Arsonval-type measuring instruments (§ 42), watt-hour meters (Fig. 93), and telephone receivers (§ 591). The magnetic flux of a permanent magnet is due to its residual magnetism (§ 179), and before using a magnet in a piece of apparatus it is often necessary to measure its residual flux, or at least to see that it is not below a certain required value. The magnitude of the residual flux *OR* (Fig. 144) in itself is not always a sufficient indication of its "permanency." It is also necessary to determine the width of the hysteresis loop, in other words the value *OK* of the coercive force. The greater the coercive force, the more constant will the strength of the magnet remain under various adverse conditions of service, such as vibration, temperature changes, external magnetic fields, etc.

In the regular manufacture of permanent magnets, sample bars are taken from each lot of magnet steel, as received from the steel manufacturer, and several bars are heat-treated or " quenched " at each of four different temperatures near 800° C. After the bars have been carefully heat-treated, hysteresis curves are taken on each sample to determine that the coercive and residual values for the steel are above permissible minimum values. After being magnetized and heat-treated, all magnets are carefully aged, being demagnetized by a definite percentage in the course of the ageing process. After ageing, the magnets for indicating instruments are not tested, but are brought through the processes of manufacture in such a way as to require a small amount of demagnetizing to give the proper strength after complete assembly in the meter.

When it is necessary to protect a permanent magnet against losing a considerable portion of its original magnetism, the following precaution may be used. After the magnet has been properly energized, it is lifted through the exciting coils, a U-shaped keeper coming up with it. A flexible keeper is then laid over the exterior of the magnet, remaining there until the magnet has been placed on the magneto frame, or in whatever piece of apparatus it is designed for.

The method of testing depends on the purpose for which a magnet is to be used. Narrow-gap magnets used in watt-hour meters are simply " weighted " as described in § 205. Wide-gap magnets used in ammeters and voltmeters are usually tested in a d'Arsonval-type device described in § 206. Magnets for magneto generators may be tested in the same kind of device, or in a hand-operated magneto described in § 207.

205. Testing of Narrow-gap Magnets. — A magnet intended to be used in a watt-hour meter is usually tested with an aluminum disk. The latter is mounted in a vertical plane on a horizontal shaft delicately pivoted and balanced. The magnet is mounted in the same position with respect to the disk as it would occupy in a meter, so that its lines of force thread through the disk. The disk receives a sudden impulse from a weight arm moving through a definite angle. The disk then begins to move, and is finally stopped by the eddy currents induced in it by the magnet under test. The angle of deflection of the disk is a measure of the strength of the magnet. The stronger the magnet the smaller the angle of deflection.

The test is a purely relative one, and gives an indication of the strength of the magnet under test as compared to a standard magnet, the residual flux of which has been determined ballistically (§ 158), by withdrawing a test coil from one of its legs. A watt-hour meter usually has two permanent magnets. After having been "weighted" as described above, these magnets are paired to give the various strengths required by meters of different torques. The strength of a pair of magnets after assembling is determined by placing them on a special test meter having a definite torque, and noting the angle of rotation of the test meter when a like test meter with standard magnets has completed one revolution.

206. Testing of U-shaped Permanent Magnets. — (*a*) *With a d' Arsonval Meter.* — The principle is shown in Fig. 162. *A* is a direct-current D'Arsonval-type instrument (§ 42) from which the permanent magnet has been removed and replaced by a pair of soft-iron poles which project through the case of the instrument. A small current of known and fixed value from a cell *C* is put through the movable coil of the instrument. *B* is a direct-current milli-ammeter on which the value of this current may be read and kept constant by adjusting the rheostat R_1. If desired, the winding of the movable coil and the current through it may be so adjusted that when a permanent magnet under test is placed across the soft-iron pole-pieces, the scale of the instrument *A* will indicate the flux directly in maxwells, or the density in gausses. The principle is similar to that used in the Koepsel permeameter (§ 197).

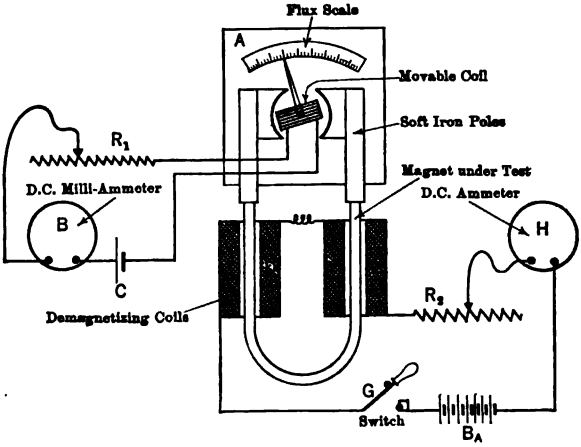


FIG. 162. A movable-coil device for testing permanent magnets.

Surrounding the magnet are two coils *DD*, wound and connected in such a way as to weaken the magnet. *H* is a direct-current ammeter which measures the demagnetizing current through these coils. G is a switch and R_2 is an adjustable rheostat for regulating the demagnetizing current. The scale of the instrument H can be so drawn as to read directly in demagnetizing ampere-turns, or in ampere-turns per unit length of the magnet. B_A is a source of d.c. current.

The procedure in using this instrument is as follows: After the current through the instrument B has been adjusted to the desired constant value, for which it is calibrated, the magnet under test is placed upon the pole-pieces, the switch G being open. Due to the reaction between the magnetic flux of the magnet and the current in the movable coil, the latter is deflected and the instrument reads the residual magnetism of the magnet.

When this has been done, and with the resistance R_2 at a maximum, the switch G is closed. The resistance R_2 is then gradually reduced until the instrument A reads zero, showing that the coercive force of the magnet is exactly balanced by the demagnetizing effect of the coils DD. While in this condition, the indication of the instrument H gives the current necessary to demagnetize the magnet, or directly the coercive force of the magnet.

(*b*) *With a Magnet-Testing Generator.* — A modification of the preceding method, sometimes used for testing large permanent magnets for magnetos, consists in replacing the movable coil (Fig. 162) by an ordinary direct-current armature with a commutator and brushes (§ 226). The principle is similar to that used in the Esterline permeameter (§ 199). The armature is connected to a low-reading voltmeter and is driven by hand through a suitable gearing. The driving mechanism includes a centrifugal clutch which allows the armature to slip at or above a critical speed. Thus, as long as the handle is rotated above the operating speed of the clutch, the armature is driven at a fixed speed. Under these conditions the indication of the voltmeter is proportional to the flux of the magnet under test, and the scale may be calibrated directly in flux units.

The generator used for this test is similar to the familiar " megger " employed in insulation measurements (Vol. II). An electromagnetic tachometer driven at a constant speed can also be used. It is only necessary to replace the regular permanent magnet of the instrument by the magnet under test, and to provide suitable pole-shoes made of soft iron.

If the number of armature turns and the induced electromotive force are known, the useful flux through the armature, at a given speed, may be computed from the usual formula for a direct-current machine, eq. (1) in § 227. Or else, in addition to the usual winding on the armature, a coil of a known number of turns is so wound that it can be made to embrace the whole magnetic flux through the armature. To carry out the calibration by means of this additional coil, a permanent magnet is slipped upon the pole-pieces to provide the necessary magnetic flux, and the search-coil terminals are connected to either a ballistic galvanometer (§ 158) or a fluxmeter (§ 161) by flexible wires. The armature is now turned through 180°, starting and finishing through the search-coil. By comparing the throw on the ballistic galvanometer with that produced by reversing the current in a standard solenoid provided with a secondary winding (§ 160), the total flux through the search-coil, and therefore through the armature, may be readily calculated. It must be remembered, however, that with either method it is the useful flux of the magnet that is determined, and not the total flux. The coercive force is measured in the same manner as in the arrangement shown in Fig. 162.

207. EXPERIMENT 8-H. — Testing of Permanent Magnets. — The principal methods in use are described in § § 204 to 206. The student should make clear to himself the following points: (a) The heat-treatment and magnetization of steel bars; (b) The effect of vibration, high and low temperatures, external fields, etc. upon the residual flux; (c) Different methods of testing, their advantages, disadvantages, and accuracy.

CHAPTER IX

MEASUREMENT OF CORE LOSS

208. A LAMINATED steel core, used in the armature of an electrical machine or in a transformer, is subjected to an alternating magnetization. Each particle of the core is regularly magnetized in one and then in the opposite direction many times a second, according to the frequency of the supply or the speed of the machine. Under such conditions the core may become appreciably heated; this heating is objectionable for two reasons:

(1) The temperature may reach a limit dangerous to the insulation of the windings.

(2) The efficiency of the machine is lowered, because of the energy thus converted into heat.

This heating of a steel core, or the corresponding loss of energy, is due to two distinct and independent causes; *viz.*, (1) molecular friction, or hysteresis; (2) induced eddy currents.

209. Physical Nature of Hysteresis. — The phenomenon of hysteresis is described in § 179. A definite amount of energy is necessary to subject a unit mass of iron to one complete cycle of magnetization (Fig. 144). This energy depends upon the limits of magnetic flux density at points M and M_1, and upon the chemical and physical properties of the sample, its heat treatment, etc. It is proved in § 225 that this amount of energy is proportional to the area of the hysteresis loop. Therefore, the hysteresis loss per cycle increases with the flux density and is greater for kinds of steel which possess greater residual magnetism and higher coercive force.

The energy for such a cyclic magnetization is furnished electrically when a stationary iron core is magnetized by a stationary coil, and it is furnished mechanically when a piece of steel or iron is being rotated in a stationary magnetic field. In either case the energy input is converted into heat in the material undergoing cyclic magnetization. Thermodynamically speaking, the phenomenon is " irreversible," that is, it is impossible to transform this heat back into electrical or mechanical energy by performing the cycle in the opposite direction. It appears that the molecules of iron do not follow the gradual increase in the magnetizing force continuously, but in small steps; and every time a new configuration of equilibrium or orientation of molecules takes place, they oscillate about their new stable positions, and dissipate some energy in these vibrations.

The power P_h lost in hysteresis in a mass M of magnetic material may be represented by the following empirical formula:

$$P_h = \eta f M B^n \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where f is the number of complete cycles of magnetization per second, B is the maximum flux density reached during each cycle, and η is a physical constant which characterizes a given material. The exponent n usually has a value of between 1.5 and 1.8, and in some computations is assumed to be equal to 1.6. Another empirical formula which sometimes can be made to represent experimental results more closely is:

$$P_h = fM(\eta_1 B + \eta_2 B^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This formula has the advantage of two empirical coefficients, η_1 and η_2, and has no fractional exponent.

In stationary machinery, for example in a transformer, the energy necessary for overcoming hysteresis loss is supplied electrically in the form of an additional component of the magnetizing current. In a generator, the hysteresis causes an opposing torque between the armature and the field, and the extra energy for overcoming this torque is supplied by the prime mover. In a motor the loss is supplied from the line, and it reduces the useful torque available on the shaft.

It is of interest to note that the counter-torque caused by hysteresis is independent of the speed of the machine. Let this torque be T and the speed of the machine N r.p.m. The power necessary for overcoming this torque is proportional to the product " torque times speed," so that

$$P_h = K'NT \text{ watts,}$$

where K' is a numerical constant which depends on the units used. Equating this expression to eq. (1), we have

$$K'NT = \eta M f B^n.$$

The number of cycles of magnetization f is proportional to the speed N of the machine, that is, $f = K''N$, where K'' is another constant. Substituting and dividing both sides of the equation by $K'N$, we finally get

$$T = K\eta M B^n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where K is another constant. Equation (3) shows that the hysteresis torque does not depend on the speed of the machine, but only on the flux density and on the quality and quantity of magnetic material.

210. Physical Nature of Eddy Currents. — Steel and iron not only possess high permeability for magnetic flux but also have a comparatively good conductivity for electric currents. Therefore, when a magnetic flux in iron varies with the time, currents are induced in it as in any other conductor subjected to an alternating flux. These currents are called eddy currents, or Foucault currents, after a French physicist by that name. According to Lenz's Law, these currents tend to oppose the changes in the magnetic flux, and therefore form closed paths linking with the flux in planes perpendicular to it. By laminating the core in the proper direction, the paths of eddy currents are restricted and the resistance of their paths is increased. In order to prevent these currents more effectively from flowing from one sheet to another, the laminations are sometimes insulated from each other by japan, varnish, tissue paper, etc. However, for many practical purposes the iron oxide, formed on the surface of laminations during the process of annealing, constitutes a sufficient insulation against eddy currents.

The power loss due to eddy currents may be represented theoretically by the formula

$$P_e = \epsilon M(tfB)^2 \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where ϵ is a constant which depends upon the electrical resistivity of the iron, its temperature, the distribution of the flux, the wave-form of the flux, and the units used. M is the mass of the laminations, t the thickness of each; B is the maximum flux density during a cycle, and f is the frequency, or the number of cycles of magnetization per second.

By a reasoning similar to that used in the preceding section, it can be proved that in a revolving machine the resisting torque due to eddy currents is proportional to the speed of rotation. Since the hysteresis torque is the same at all speeds, this difference in the character of the two losses is utilized in practice for separating the hysteresis loss from that due to eddy currents (§ 223).

211. Methods of Measuring Hysteresis and Eddy Currents. — The core loss due to hysteresis and eddy currents lowers the efficiency and increases the temperature rise in machinery. It is important, therefore, to know how to measure this loss on samples of steel intended for use in the magnetic circuit of electrical apparatus. Results of core-loss tests are usually plotted in the form of curves, such as are shown in Fig. 163. The curves give the total loss which comprises that due to both hysteresis and eddy currents. In most practical cases it is not required to separate the two; but should this be necessary, it can be done as is explained in § 223.

There are two practical methods of determining core loss, *viz.*, (1) the wattmeter method and (2) the mechanical torque method. With the wattmeter method, steel laminations under test are assembled into packages so as to form the core of a choke coil which is energized with an alternating current. The power input into the coil, read on a wattmeter, less the copper loss, is equal to the core loss. With the mechanical-torque method, the laminations under test and an external magnetic field are rotated relatively to each other, as if the core were that of the armature of an electrical machine. The torque due to the core loss is measured by means of a torsion spring or counter-weight.

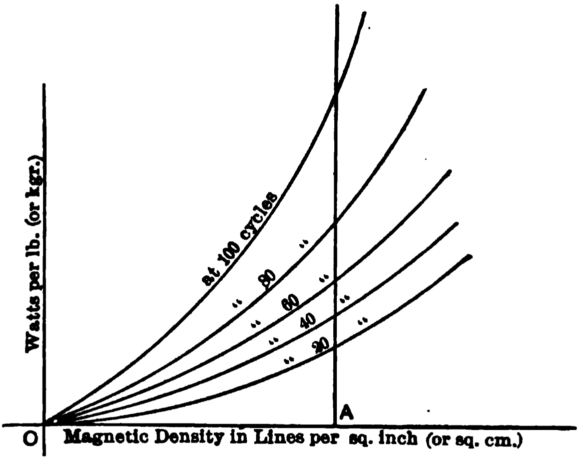


FIG. 163. Core loss at different magnetic densities and at different frequencies of magnetization.

The torque method should give more accurate results for sheet steel intended for armature cores of generators and motors, because they are subjected to a "rotating" hysteresis. The wattmeter method is more suitable for transformer iron subjected to alternating magnetization. In practice, however, no such strict distinction is made between the two methods, because in most cases it is sufficient to know that a new lot of steel laminations has a core loss which is not above a certain limit. It makes little difference in which way this limit is ascertained, provided that the same method is used in all cases. The wattmeter method has important practical advantages and is being used more and more, in preference to the mechanical-torque method.

A third possible method consists in integrating the area of a hysteresis loop of the sample (Fig. 144). It can be shown theoretically (see § 225) that the area of this loop is proportional to the hysteresis loss per cubic unit of steel, per cycle of magnetization. This method is seldom used because it is rather lengthy; besides, it gives the hysteresis loss only, and not the total core loss.

THE MECHANICAL-TORQUE METHOD

212. Ewing's Magnetic Tester. — This is the simplest and the oldest device (Fig. 164) based on the principle of measuring hysteresis loss by its mechanical torque. A few strips of sheet iron under test are clamped in the carrier *C* which is made to revolve by turning the handle *H*. The carrier turns between the poles of a permanent magnet which is suspended on a knife-edge. The hysteresis in the specimen causes the magnet to be deflected, and the deflection is observed by means of a pointer on the scale *S*.

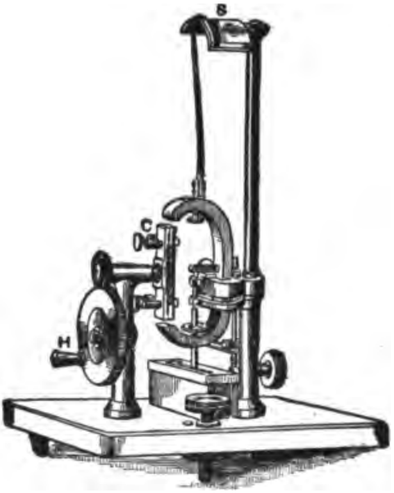


Fig. 164. The Ewing hysteresis tester.

The same test is repeated with a standard sample whose hysteresis loss has been previously determined by some other method. The ratio of the deflections gives the ratio of the values of hysteresis loss in the two samples.

The deflections of the pointer are practically independent of the speed of rotation; it is shown in § 209, eq. (3), that hysteresis torque is independent of the frequency of magnetization. When the speed of rotation is quite high, the influence of eddy currents becomes noticeable, and the deflection is somewhat increased. The loss determined with this tester refers to a certain flux density only, and no provision is made in the apparatus for varying the density. The density within the samples is practically independent of the quality of iron tested, the value of the flux being determined by the reluctance of the two air-gaps. The instrument is adapted only for rough relative tests.

213. EXPERIMENT 9-A. — Hysteresis Loss in Iron with the Ewing Magnetic Tester. — Test a few samples of sheet steel and iron as explained in the preceding article. Investigate the influence of the speed of rotation. Make a sketch of the mechanical details of the device. Test one of the samples with laminations thoroughly insulated from each other, and then without insulation; see if the influence of eddy currents makes itself perceptible in the second case. Form an opinion in regard to the accuracy of the instrument and its applicability for practical work.

214. Blondel's Hysteresimeter. — This device (Figs. 165 and 166) is based on the same principle as the above-described Ewing magnetic tester, but is mechanically superior to it. It has a U-shaped permanent magnet *MM* which can be rotated about a vertical axis by means of the handle *H*. The sample sheets under test are made in the form of a ring *R* and are fastened on the support *S*. This support, with its pivoted vertical shaft *P*, tends to revolve, but is restrained by an opposing spiral spring shown in the figure. When the magnet *M* revolves, the support with the sample turns until the hysteresis torque just balances the torsion of the spring. The deflection is practically independent of the speed of rotation, as in the Ewing apparatus described above, and is directly proportional to the hysteresis constant η of the sample (§ 209). A standard sample, whose hysteresis constant has been determined by another method, is used with this instrument. The ratio of the deflections obtained with the standard sample and with the sample under test, is equal to the ratio of their hysteresis constants.

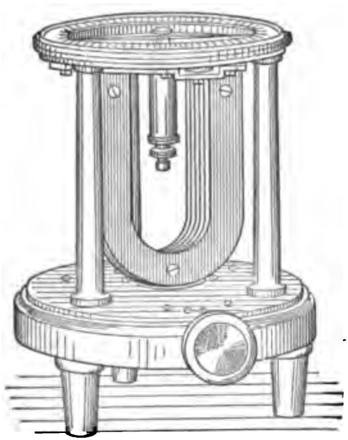


FIG. 165. The Blondel hysteresimeter.

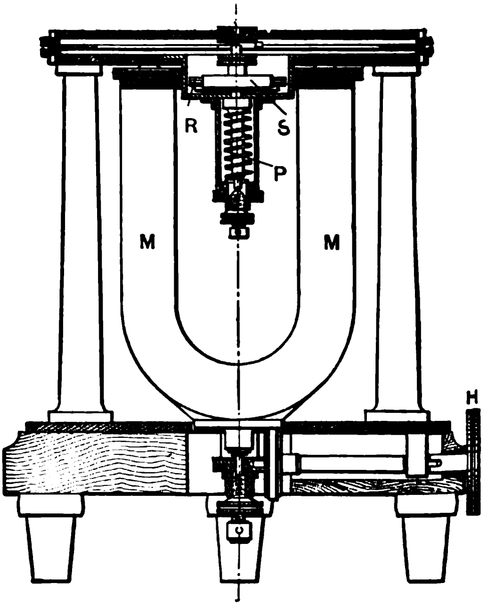


FIG. 166. A cross-section of the Blondel hysteresimeter.

The magnet *M* is selected of such a strength as to give a magnetic density *B* of about 10,000 lines per sq. cm. within the sample. The reluctance of the air-gap is so large, as compared to that of the sample itself, that differences in permeability of different samples hardly affect the above value. Thus all samples are compared at a standard density of 10,000 maxwells per sq. cm. To insert a sample into the apparatus, first remove the glass which covers the scale, then take off the pointer and finally the support *S*. The sample is then clamped to the support and the parts returned to their respective places. In testing samples with this hysteresimeter it is always necessary to take readings with rotation both ways; the average of the two indications is the true zero position. This eliminates the error due to an uncertain previous magnetization of the sample. It is advisable to rotate the handle at a speed of 2 to 3 revolutions per sec.

215. EXPERIMENT 9-B. — Hysteresis Loss with the Blondel Hysteresimeter. — See the directions for the preceding experiment.

216. Holden-Esterline Core-Loss Meter. — The two above-described hysteresis testers are very simple and convenient to use, but have the following disadvantages:

(a) Since they are rotated by hand, the frequency is so low that samples are tested essentially for hysteresis loss only, while usually the total core loss is of importance.

(b) Hysteresis loss is determined at one density only.

(c) Readings are merely relative, so that a standard sample is required.

These objections are remedied in the device shown in Fig. 167. In this instrument the magnetizing field is revolved by a motor at any required frequency. A six-pole electromagnet is used in place of a permanent magnet, so that it is possible to test samples within a wide range of magnetic densities, and the instrument can be made to read the core loss directly in watts. The exciting field, with its six inwardly pro-

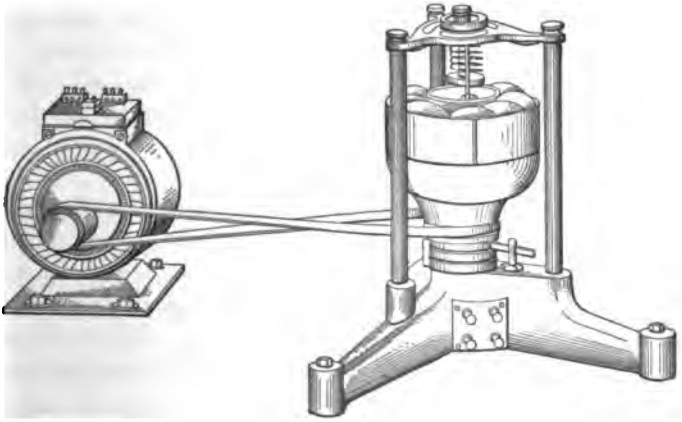


Fig. 167. The Holden-Esterline core-loss meter.

jecting poles, is made of stamped laminations and mounted on a hub which can be rotated by means of a belted motor. The poles of this field are wound with exciting coils. For alternating-current work the frame is left stationary, the coils being connected to a two-phase or three-phase circuit so as to produce a rotating magnetic field (§ 472).

The sample to be tested consists of concentric rings mounted on a brass spider whose shaft has a conical pivot below and rests on a jeweled bearing. The upper end of the shaft is guided by a bearing in the dial plate. When the poles are rotated, or a rotating field is produced by A.C. currents, the eddy currents and hysteresis in the sample tend to cause it to rotate also. This rotation is prevented by the coiled spring attached to the spider at one end and to the milled head on the dial plate at the other end. The pointer attached to the milled head is set at zero when there is no twist in the spring. The angle of torsion by which it is necessary to turn the core back to zero, when the field is rotating, is a measure of the loss.

Instead of calibrating the dial in torque units, such as ft.-lbs., or gram-centimeters, it can be calibrated directly in watts loss at a certain standard frequency, for instance, at 60 cycles per sec. At lower frequencies the readings must be reduced in proportion, and at higher frequencies increased. The air-gap is of sufficient length to make the flux density in the specimen practically proportional to the exciting current. Moreover, the permeability of the sample at the usual densities has very little effect on the flux in the instrument, so that the dial can be calibrated once for all. The apparatus is well adapted for commercial work; it is strong mechanically, and not much skill or electrical knowledge is required to operate it. A somewhat similar device has also been constructed in which the punchings are made to revolve and the field is stationary.

217. EXPERIMENT 9-C. — Testing Iron with the Holden-Esterline Core-Loss Meter. — For a description of the device see the preceding section. Test a few samples over a wide range of densities and frequencies, driving the field with a motor. Take a few readings with a revolving field produced by polyphase currents. Study the mechanical details of the apparatus. If possible, check the calibration of the dynamometer spring. See also § 213.

Report. Plot curves of core loss in watts per unit weight of laminations at various densities and frequencies (Fig. 163). Separate the hysteresis loss from eddy currents, as is explained in § 223. Show the difference in the values of the loss with the field produced by direct current and by polyphase currents; explain the discrepancy.

THE WATTMETER METHOD

218. In the above-described core-loss testing devices, the sample is subjected to the so-called "rotating hysteresis" of the same general character as in the armature of a generator or a motor. In a transformer core the iron is subjected to "alternating hysteresis," and the power loss may be considerably smaller.

A package of laminations to be tested under the latter conditions is placed within a magnetizing coil excited with an alternating current of the desired frequency and magnitude, and the core loss is read on a properly connected wattmeter. The general arrangement of the magnetic and electric circuits described below is due to Dr. J. Epstein and has been used for a number of years by the *German Association of Electrical Engineers*. Later it was adopted with some modifications by the *American Society for Testing Materials*, and another modification has been developed for accurate work by M. G. Lloyd of the Bureau of Standards.

Where a complete knowledge of the core loss in a particular sample is required, curves of such loss are taken at different values of flux density and frequency. Where only a check measurement is desired on a shipment, it is sufficient to determine the loss at one or two standard flux densities and at a standard frequency. Such a value is sometimes known as the *loss value* (Verlustziffer). In Europe the loss value is usually given in watts per kilogram of steel laminations at 50 cycles per second, and at the flux density of 10 or 15 kilo-lines per square centimeter. In this country the loss value is sometimes given for the same flux densities, for 60 cycles, and per pound of laminations.

219. The Epstein Apparatus. — This device, in the form recommended by the *American Society for Testing Materials*, is shown in Fig. 168. The arrangement of the magnetizing coils and of the test specimens with butt joints is shown in Fig. 169*a*. The joints are held tight by the clamps indicated in Fig. 168. The specifications for the samples are as follows: "The magnetic circuit shall consist of 10 kg. (22 pounds) of the test material, cut with a sharp shear into strips 50 cm. (19 11/16 inches) long and 3 cm. (1 3/16 inches) wide, half parallel and half at right angles to the direction of rolling, made up into four equal bundles, two containing material parallel and two containing material at right angles to the direction of rolling, and finally built into the four sides of a square with butt joints and opposite sides consisting of material cut in the same manner. No insulation other than the natural scale of the material (except in the case of scale-free material) shall be used between laminations, but the corner joints shall be separated by tough paper 0.01 cm. (0.004 inch) thick."

The coil specifications are as follows: The inside cross-section of the spool on which a coil is wound is 4 by 4 cm.; the thickness of the wall not over 0.3 cm., the winding length 42 cm. The primary winding of each of the four coils to consist of 150 turns of copper wire, the total resistance per coil to be between 0.3 and 0.5 ohm. The secondary winding on each coil to consist of 150 turns of copper wire, and the resistance to be not over 1 ohm.

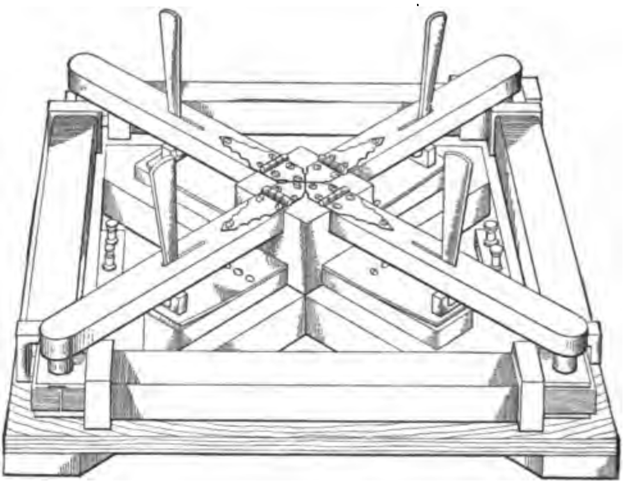


FIG. 168. The Epstein apparatus for measuring core loss.

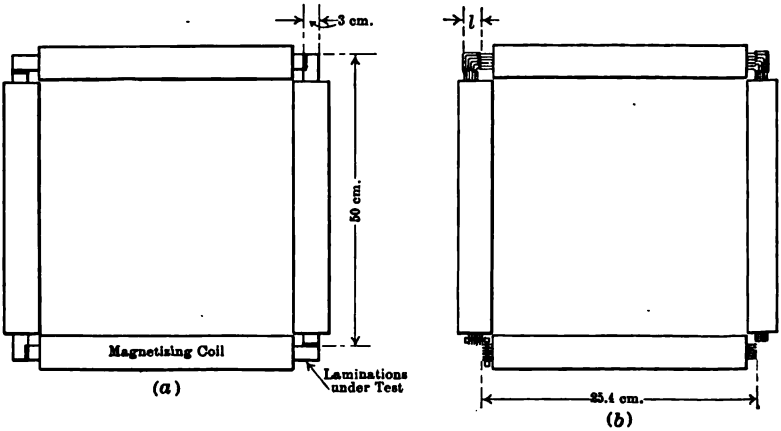


FIG. 169. The arrangement of coils and test pieces for core loss; (a) with butt joints, (b) with corner pieces.

The arrangement shown in Fig. 169*b* is used by the *Bureau of Standards.* It requires less than 2 kilograms of iron for a sample, as against the 10 kilograms required for the standard Epstein device. The sheets are placed on edge and are connected at the ends by corner pieces of the same material or of a material of known core loss constants. This arrangement does away with the air-gaps unavoidable with butt joints. The specifications are as follows: " The material to be tested is usually submitted in the form of 10 sheets 30 cm. (12 inches) square. This material is cut into strips 25.4 by 5 cm. (10 by 2 inches). The final cutting to size is done at the Bureau. These are assembled into four bundles, in each of which adjacent strips are separated by strips of pressboard of equal width and thickness, but 2 cm. shorter. Each bundle is wrapped with gummed paper and is inserted in a solenoid, and the four bundles are then arranged in a square so that the plan view shows the edges of the strips. The solenoids are wound upon fiber forms, which are 22.7 cm. long, and have inside dimensions 5 by 1 cm. At the corners of the square, short pieces of test material are bent at right angles and interleaved between the strips of adjacent bundles, as shown in the figure. There are as many of these corner pieces as there are test pieces, and they are graduated in length so as to give a uniform lap of about 2 mm. A clamp is tightened over these laps so as to give a good magnetic joint.

Each solenoid has two windings. The secondary consists of 121 turns of No. 14 double-cotton-covered wire. Over the secondary are wound 242 turns of No. 14 double-cotton-covered wire. The four solenoids are connected in series, making a total of 968 magnetizing turns and 484 turns in the secondary."

The *electrical connections* for use with either Fig. 169*a* or Fig. 169*b* are shown in Fig. 170. Ordinary indicating instruments are used in commercial testing, while the *Bureau of Standards*, in its scientific work, has used an astatic electro-dynamometer-type instrument with a mirror and scale. The current winding of the wattmeter is in the primary circuit, while its potential winding is connected across the secondary terminals of the core-loss tester. When so connected, the wattmeter measures the core loss in the iron samples plus the I^2R loss in the whole secondary circuit, consisting of the potential windings of the testing device and the wattmeter, and a multiplier resistance. When the *secondary* resistance R is known and the induced voltage E_2 has been read on the voltmeter, the secondary copper loss E_2^2/R is computed and subtracted from the wattmeter reading, which then gives the true iron loss. The corresponding flux density is computed from formula (6) given below. With the potential winding of the wattmeter connected in the secondary circuit, an error will arise when there is some flux threading the core and linked with the primary but not with the secondary (leakage flux).

This is avoided by winding the secondary under the primary and making the two co-extensive in length.

After a number of values of core loss have been read on the wattmeter, at different values of induced voltage and frequency, curves of core loss may be plotted (Fig. 163). If it is desired to separate the hysteresis loss from the eddy currents, readings are preferably taken at 60 and at 30 cycles, or at some other two frequencies in the ratio of two to one, because computations are thus simplified (§ 223).

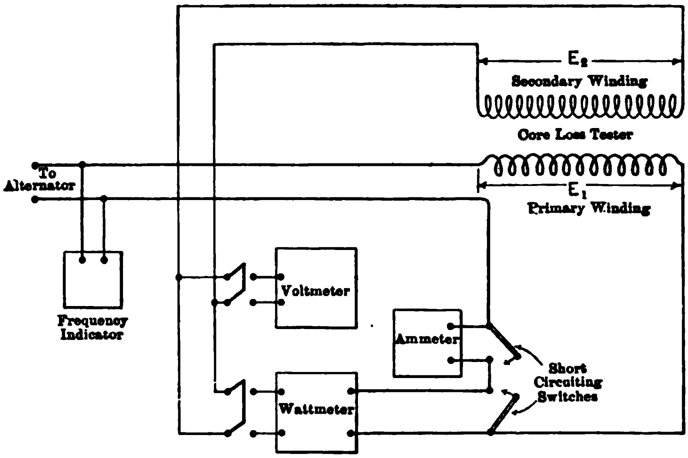


FIG. 170. Electrical connections for measuring core loss.

The hysteresis loss depends upon the amplitude of the exciting current and is independent of its wave-form, unless the current has more than one maximum for each alternation (saddle-form wave). The eddy current loss depends upon the wave-form of the primary current. In all core-loss tests it is therefore essential either to have an alternator which gives a nearly sinusoidal voltage, or to test samples with a voltage of the same wave-form as that on which the corresponding lot of laminations is to be used (see however § 222). The applied voltage should be adjusted either by means of the field rheostat of the alternator or by using an auto-transformer (§ 395) with numerous taps. As little resistance as possible should be used in the main circuit, so as to keep the power factor of the load as low as possible. The reason for this lies in the nature of the armature reaction of the alternator (§ 452). With a purely inductive load the armature reaction merely reduces the main flux, still keeping it of nearly the same shape as at no load. On the other hand, the energy component of the armature current distorts the flux and shifts it to the trailing pole tip. The result is that the induced e.m.f., and consequently the current wave, are distorted.

With the growing use of high frequencies for various purposes, there is some need for determining core loss at such frequencies. A dynamometer-type wattmeter and voltmeter are no longer suitable on account of their inductance. One method that has given satisfactory results at high frequencies consists in measuring the core loss with a calorimeter, while the induced secondary voltage is determined with an electrostatic voltmeter. A tungsten arc oscillator can be made to furnish the required high-frequency current.

220. Theory of the Wattmeter Method. — First assume the iron core to be perfect, without hysteresis or eddy currents, and let the secondary circuit (Fig. 170) be open. Let a sinusoidal voltage be applied to the primary winding. The flux ϕ excited in the core (Fig. 171) is then in phase with the primary magnetizing current I_0, being excited by it. The instantaneous secondary induced voltage is

$$e_2 = -N_2 d\phi/dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Here N_2 is the number of turns in the potential winding, t is time in seconds, and ϕ is measured in webers (§ 157). This voltage is in a lagging time quadrature with the flux ϕ and with the primary current, because $d\phi/dt$ varies according to the cosine law if ϕ itself varies according to the sine law. Therefore, the wattmeter, connected as shown in Fig. 170, would read zero. The vector E_0 represents that part of the primary applied voltage which is equal and opposite to the counter-e.m.f. induced by the flux ϕ in the primary circuit. The ratio of E_2 to E_0 is equal to that of the numbers of turns in the two windings. Adding to E_0 the voltage drop I_0r parallel to I_0, where r is the resistance of the primary winding, the total applied primary voltage E_1' is obtained.

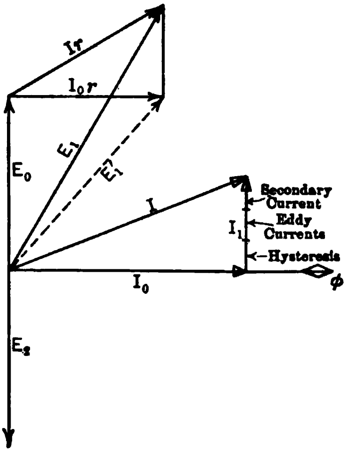


FIG. 171. The vectorial relationship of the electric quantities in Fig. 170.

The presence of hysteresis and eddy currents makes it necessary for the magnetizing current to have an energy component in phase with E_0 in order to furnish the necessary power for overcoming these losses. Strictly speaking, the presence of hysteresis also makes the primary current non-sinusoidal, but approximately it may be replaced by an "equivalent" sine-wave and a vector diagram may be used as before.

The secondary current which flows through the potential winding of the wattmeter is practically in phase with the induced voltage E_2, because of the high resistance and small inductance of the circuit. According to the theory of the transformer (§ 387), the corresponding component of the primary current must also be in phase with E_0. The total energy component of the primary current is I_1, and the total primary current is I. The primary terminal voltage E_1 is equal to E_0 plus the primary voltage drop Ir.

The wattmeter connected as in Fig. 170 indicates the average power due to the component I_1 of the current, times the induced voltage E_2. If the number of primary turns is the same as in the secondary, this reading gives the true power input into the core and into the secondary circuit. Subtracting the secondary loss E_2^2/R, the result gives the true core loss. If the number of the secondary turns is different from that in the primary winding, the wattmeter reading must be reduced or increased in the proper ratio, to find the true power input.

In the early forms of Epstein apparatus there was no secondary winding, and the potential winding of the wattmeter (as well as the voltmeter) used to be connected directly across the primary terminals. In this case the wattmeter reading includes the I^2r loss in the primary winding, and moreover, the voltmeter reading must be corrected for the primary voltage drop in order to obtain the true induced electromotive force E_0 from which the flux density is computed. For these reasons, a secondary winding has been added in the later types of apparatus.

The relationship between the voltmeter reading E_2 (*effective volts*) and the corresponding *maximum* flux density B in the iron, in kilo-lines per sq. cm., is as follows:

$$E_2 = 4.44 f\, N_2 A B 10^{-5} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

where f is the frequency in cycles per second, N_2 is the total number of secondary turns, and A is the net cross-section of the core, in sq. cm. The latter is usually computed from the weight of the laminations, their length, and the known specific gravity of iron. A specific gravity of 7.5 may be assumed for high-resistance steel, that is, for all steels having a resistance of over 2 ohms per metergram, and 7.7 for low-resistance steel. The flux in the air spaces between the laminations is neglected.

Eq. (6) may be proved as follows: The instantaneous flux density in the core is $B \operatorname{Sin} 2\pi ft$, so that the instantaneous flux (in webers) is

$$\phi = AB \operatorname{Sin} 2\pi ft 10^{-5} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

substituting this value of ϕ in eq. (5) we find

$$e_2 = -2\pi f N_2 A B \operatorname{Cos} 2\pi f t 10^{-8} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Hence, the amplitude of the secondary voltage is $2\pi f N_2 A B 10^{-8}$. Dividing by $\sqrt{2}$ to change from the amplitude to the effective value, eq. (6) is obtained.

In the apparatus built according to the specifications of the *American Society for Testing Materials* (Fig. 169*a*) the flux density is practically uniform throughout, and therefore the wattmeter reading of loss is divided by the total weight of laminations. In the apparatus designed by the *Bureau of Standards* (Fig. 169*b*) a small correction is necessary for the overlap of the corner pieces where the flux density and the loss are lower. Assume the corner pieces to be made of the same material and to be of the same thickness as the samples under test. Let M be the mass of the test pieces and m that of the corner pieces. The wattmeter reading is divided not by $(M + m)$ but by $(M + m)(1 - 1.4c)$, where $1.4c$ is an empirical correction for the lower flux density in the overlaps. The factor c is equal to the mass of the corner pieces which lap, expressed in terms of the mass of the test pieces. In other words

$$c = (m/M) - (l/25.4) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where l is the dimension shown in Fig. 169*b*, in cm., and 25.4 is the length of the test pieces, in cm. When the corner pieces are of different material the correction is somewhat more involved. See the Circular of the *Bureau of Standards*, No. 17, 1916, p. 30.

221. EXPERIMENT 9-D. — Determination of Iron Loss by the Wattmeter Method. — The method and the apparatus are described in §§ 218 to 220. The diagram of connections is shown in Fig. 170. (1) Weigh the samples, measure their length, and compute the coefficient of proportionality between voltmeter readings and flux densities, according to formula (6). (2) Begin readings at a standard frequency and at the highest feasible flux density. Read watts, volts, amperes, and the frequency. Be sure that the resistance of the secondary circuit is known for each set of readings. Keep the voltmeter circuit open when reading watts. Reduce the voltage in steps and take readings at each step, keeping the frequency constant. Between the steps reduce the current gradually so as to demagnetize the material properly. Whenever the magnetizing circuit has been broken, it must be closed through a considerable resistance which is continuously reduced to zero, in order to prevent a large first surge and consequent high magnetization which would require subsequent demagnetization. (3) Repeat the preceding test at several other frequencies, above and below standard, including the frequency exactly equal to one-half standard. (4) Make a comparative test of iron loss with and without insulation between the laminations, taking care that in one set of readings bright metal surfaces touch each other. This test is preferably made at a high frequency at which the effect of eddy currents is more noticeable. (5) Before leaving the laboratory, record all the data concerning the apparatus that are needed in computations.

Report. (1) Correct the wattmeter readings for the secondary copper loss and plot values of total core loss against flux densities as abscissæ, for each frequency investigated. It will save time to plot these data for the whole weight of laminations as tested. (2) Separate hysteresis from eddy currents by the straight-line method indicated in § 223, and compare one or two results with those obtained by the two-point method. (3) Compute the exponent according to which the hysteresis loss varies with the flux density. (4) Write down the final numerical formula from which the loss per kilogram of material may be computed at any flux density and frequency within the range investigated experimentally. (5) Give numerical data to illustrate the effect of imperfect insulation upon the increase in eddy-current loss. (6) Plot curves of magnetizing volt-amperes per kilogram of iron (§ 224) for different frequencies and show that for a given flux density the values are proportional to the frequency. (7) For one or two flux densities compute the actual instantaneous ampere-turns per cm., corresponding to maximum B (§ 224), and compare with an available B-H curve for similar grade of steel.

222. Iron-Loss Voltmeter. — This meter is for use in connection with a wattmeter for determining the iron loss in distributing and power transformers on the basis of sine-wave voltage and normal frequency, when the testing is done on a circuit of any wave-shape and approximate frequency. The iron loss in a transformer varies with the wave-shape and frequency. Guarantees are, therefore, based on sine-wave and a normal frequency; but the standard conditions are difficult to obtain in practice. However, for any other wave-shape and frequency there is some equivalent voltage that will produce the same iron loss as would the sine-wave of normal frequency at normal voltage. The iron-loss voltmeter enables the tester to adjust to this equivalent voltage for the circuit at hand.

The iron-loss voltmeter consists of a wattmeter movement in series with the winding on an iron core (Fig. 172). With this connection the meter element measures the loss in the iron core. The scale is calibrated on a circuit of sine-wave voltage and is marked in " volts." The reading therefore indicates the sine-wave voltage of normal frequency equivalent to the actual voltage, frequency, and wave-shape of the circuit to which the instrument is connected. If the voltage or frequency of the circuit on which a transformer test is being made is adjusted until the iron-loss voltmeter indicates the rated sine-wave voltage of the transformer being tested, the iron losses in the transformer will be the same as they would be on a circuit of that voltage and sine-wave shape. There is also a watt scale marked on the meter which indicates the total watts consumed in the meter. This is for the correction of wattmeter readings taken when the voltmeter is in circuit.

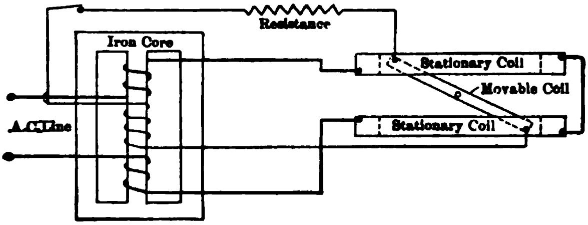


FIG. 172. The iron-loss voltmeter.

In application the iron-loss voltmeter is connected across the terminals of the transformer under test in the same manner as an ordinary voltmeter. A wattmeter is also connected in the circuit in such a way as to measure the total input of both the transformer and the iron-loss voltmeter. The voltage of the circuit is then adjusted by any convenient means until the voltmeter reads the normal voltage of the transformer. The total power input is read on the wattmeter and the watts input of the instrument is read on the watt scale of the iron-loss voltmeter, the difference being the normal iron loss of the transformer. The standard iron-loss voltmeter is usually calibrated for 60 cycles and for 125 and 250-volt circuits.

223. Analysis of Loss Curves. — At a given flux density, the hysteresis loss is proportional to the frequency, while the eddy-current loss is proportional to the square of the frequency (§§ 209 and 210). Thus, the total core loss P per unit weight of laminations may be represented by the formula

$$P = Hf + Ff^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (10)$$

where f is the frequency, and H and F are functions of the flux density B. The letter H stands for hysteresis and F for Foucault or eddy currents. Dividing both sides of eq. (10) by f, we get

$$P/f = H + Ff \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (11)$$

This relationship is utilized by the separation of hysteresis from eddy currents in two ways, according to whether the total loss curves (Fig. 163) are available at two frequencies only, or at three or more frequencies.

(a) *The two-point method.* Let values of total loss P be available at a certain flux density, at two frequencies, f_1 and f_2. Writing eq. (11) for these two frequencies, we have

$$\left.\begin{aligned} P_1/f_1 &= H + Ff_1 \\ P_2/f_2 &= H + Ff_2 \end{aligned}\right\} \quad \cdots\cdots\cdots \quad (12)$$

Solving these as simultaneous equations for H and F, we get

$$F = \quad [(P_1/f_1) - (P_2/f_2)]/(f_1 - f_2) \quad \cdots\cdots \quad (13)$$

$$H = [f_1(P_2/f_2) - f_2(P_1/f_1)]/(f_1 - f_2) \quad \cdots\cdots \quad (14)$$

Knowing F and H, the individual losses $P_h = Hf$ and $P_e = Ff^2$ may be computed at any frequency.

The computations are particularly simple when the values of the losses are required only at a standard frequency f_1, and when the value of the total loss is known at this frequency and also at half the frequency. Substituting in the preceding equations $f_2 = \frac{1}{2}f_1$, we get after reduction

$$\text{hysteresis loss } P_h = Hf_1 = 4P_2 - P_1 \quad \cdots\cdots \quad (15)$$

$$\text{eddy-current loss } P_e = Ff_1^2 = 2P_1 - 4P_2 \quad \cdots \quad (16)$$

(b) *The straight-line method.* Let OA in Fig. 163 represent the flux density at which the losses are to be separated, and let the total loss be known for at least three frequencies. The preceding method is then inconvenient, since the values of H and F determined from each pair of data would be somewhat different on account of unavoidable inaccuracies in the experimental data and in the computations. In such a case it is both simpler and more accurate to draw graphically the *most probable straight line* which covers all the data. Equation (11) represents a straight line between f and P/f, and the corresponding points may be easily plotted on a sheet of cross-section paper. A straightedge is placed over these points and a straight line is drawn which passes as nearly as possible through these points. The intercept on the axis of ordinates gives the value of H, and the remainder of the ordinate at any frequency represents the value of Ff for that frequency. After this, the values of $P_h = Hf$ and $P_e = Ff^2$ may be readily computed.

The exponent according to which the hysteresis loss varies with the flux density is determined in the following way: Let the exponent of B in the formula (1), § 209, be unknown. The formula may be written in the form

$$P_h = aB^n \quad \cdots\cdots\cdots\cdots \quad (17)$$

where a is a constant with which we are not at present concerned. Taking logarithms of both sides of this equation, we get

$$\log P_h = \log a + n \log B \quad . \; . \; . \; . \; . \; . \; . \quad (18)$$

This is the equation of a straight line between $\log P_h$ and $\log B$; the unknown exponent n is numerically equal to the trigonometric tangent which this line makes with the axis of abscissæ (Fig. 173). Thus, in order to determine n at a certain frequency, the eddy-current loss must first be separated from the total loss, as is explained above. The rest is then replotted to a logarithmic scale, as in Fig. 173. The points usually lie on a nearly straight line; the slope of this line gives the exponent n.

It is convenient to plot this curve on a sheet of logarithmic paper. In this case the abscissæ and the ordinates are obtained directly without any calculation. Do not attempt to mark the origin; with the logarithmic scale it is at minus infinity. All that is needed is the slope n of the curve; this slope is independent of the position of the origin.

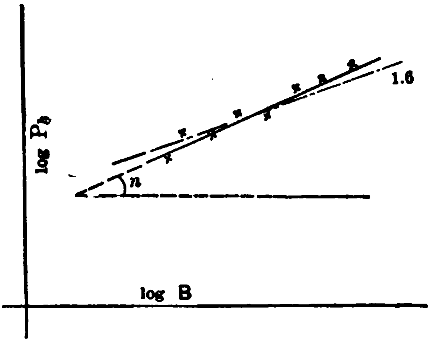


FIG. 173. Determination of the exponent according to which hysteresis loss varies with the magnetic density.

In some cases it may be desired to accept the usual exponent $n = 1.6$ as being correct, and it is merely required to find the value of a in eq. (17); in other words, the constant η in formula (1). In such a case a line having the slope of 1.6 is drawn as closely as possible through the experimental points. The coefficient a is then calculated for this line from eq. (18). In a similar manner one may check the fact that the eddy-current loss increases as the square of the magnetic density.

The total core loss, P, also increases as some power of the flux density; the exponent has a value between 1.6 and 2. This value can be found by plotting the total loss curve to a logarithmic scale, as is explained for the hysteresis loss above.

In some investigations it is inconvenient to use a fractional exponent in the expression for the hysteresis loss or for the total core loss. Moreover, for some grades of iron these exponents are not constant, but are higher at higher flux densities than at lower. It is sometimes more convenient to represent either the total loss or the hysteresis loss (as the case may be) at a constant frequency by the expression

$$P = cB + gB^2 \quad \text{. . . . .} \quad (19)$$

where c and g are certain constants. In this formula the two terms do not represent the hysteresis loss and the eddy-current loss respectively, but the total loss is represented empirically by a parabolic expression. Dividing both sides of this equation by B, we get

$$P/B = c + gB \quad \text{. . . . .} \quad (20)$$

The values of c and g may be determined by either method described above for the computation of H and F.

224. Magnetizing Current from Wattmeter Test. — When performing a core-loss test with the Epstein apparatus (§ 219) it is possible at the same time to determine the magnetization curve of the sample (OAM in Fig. 144). For this purpose it is only necessary to compute the magnetizing or reactive component I_0 of the total primary current I (Fig. 171). The latter is read on the ammeter shown in Fig. 170. Let the wattmeter reading be P watts and let the induced primary voltage be E_0. The value of E_0 is computed from the voltmeter reading E_2, by multiplying it by the ratio of the numbers of turns, N_1/N_2. Thus, the component of the primary current in phase with E_0 is

$$I_1 = P/E_1 \quad \text{. . . . .} \quad (21)$$

and the magnetizing or reactive component of the primary current is

$$I_0 = \sqrt{I^2 - I_1^2} \quad \text{. . . . .} \quad (22)$$

In some cases I_0 differs but a little from the total I, so that the correction for I_1 may not be necessary.

The amplitude of the magnetizing force, or the ampere-turns per unit length of iron, is

$$H = N_1 I_0 \sqrt{2}/l \quad \text{. . . . .} \quad (23)$$

where l is the total length of the magnetic circuit. In this formula the amplitude of the current, $I_0\sqrt{2}$, is used because the corresponding value of B from eq. (6) also refers to the maximum flux density. In this way the data for the B-H curve of the laminations may be computed. Care must be taken to make the butt joints as perfect as possible, since even a small air-gap will appreciably increase the magnetizing current. No such precaution is needed in the arrangement with corner pieces shown in Fig. 169*b*.

The excitation data per kilogram of material of an A.C. core, at a constant frequency, may also be given as reactive volt-amperes plotted or tabulated against values of flux density B. This may be shown as follows: By analogy with eq. (6), the voltage induced in the primary winding is

$$E_0 = 4.44 f N_1 A B \, 10^{-8} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (24)$$

Multiplying both sides of this equation by the magnetizing current I_0, dividing by the volume Al of the laminations, and denoting the result by Q, we obtain

$$Q = \frac{\textit{magnetizing volt-amperes}}{\textit{volume}} = \frac{E_0 I_0}{Al} = \pi f \left[\frac{N_1 I_0 \sqrt{2}}{l}\right] B \, 10^{-8}. \quad (25)$$

According to eq. (23), the expression in the brackets on the right-hand side represents the value of magnetizing force H. This magnetizing force is a function of B, so that the whole right-hand side of the equation is a function of B, say $F(B)$, and we have

$$Q = f\,F(B) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (26)$$

This equation shows that it is possible to plot an *experimental* curve which will give magnetizing volt-amperes per pound or kilogram of material at a standard frequency, as a function of magnetic density B. Furthermore, eq. (26) shows that for a given B the values of Q are proportional to the frequency, so that it is sufficient to plot the curve for one frequency only.

This method of representing the magnetizing current is very convenient in designing transformers, because the current can be computed immediately from the dimensions of the core. The dimensions of the core may also be determined from the condition that the magnetizing current should not exceed a certain per cent of the full-load current.

An experimental curve represented by eq. (26) may be readily obtained from a wattmeter test on a bunch of laminations with the Epstein apparatus, because from the results of such a test the primary induced voltage E_0, the magnetizing current I_0 and the flux density B can be computed. All that is necessary then is to plot the values of $Q = (E_0 I_0 / \mathit{Volume})$ against B as abscissæ.

225. Hysteresis Loss by Integrating B-H Curve. — The hysteresis loss, in joules or watt-seconds per cycle, per cubic unit of iron, is proportional to the area of the hysteresis loop (Fig. 144). The magnitude of the loss depends on the maximum magnetic density reached during the cycle, corresponding to the points M and M_1.

The proof of this proposition is as follows: When a piece of well-laminated iron is undergoing a cycle of magnetization, e.m.f.'s are induced in the exciting winding by the varying magnetic flux. The instantaneous energy supplied by the exciting circuit, or returned to it, is equal to $eidt$, where i and e are instantaneous values of the exciting current and of that part of the applied voltage which is equal and opposite to the induced voltage; dt is an infinitesimal element of time. The total net electric energy, in joules, supplied to the exciting coil by the source of a.c. power, during one complete cycle of duration T, is

$$W = \int_0^T iedt. \quad \ldots \ldots \ldots \quad (27)$$

According to the fundamental law of induction, eq. (5), we have

$$e = n\,d\phi/dt = nA\,dB/dt \quad \ldots \ldots \quad (28)$$

where n is the number of exciting turns, ϕ is the instantaneous flux, B the corresponding flux density (in webers per sq. cm. or per sq. inch), and A is the cross-section of the iron core. Substituting this value of e in eq. (27) and dividing both sides of the equation by the volume Al of the laminations, we obtain

$$W/(Al) = \int_{\bigcirc}(ni/l)\,dB = \int_{\bigcirc} HdB \quad \ldots \ldots \quad (29)$$

The circle as a subscript to the integral signifies that the integration is to be extended over a closed figure. Thus, eq. (29) represents the area of the complete hysteresis loop, and the above stated proposition is thereby proved. If the sample undergoes f cycles of magnetization per second, the hysteresis loss in watts per cubic unit is proportional to f × (area of hyst. loop). Hence

Hyst. loss in watts = Const. × f × loop area × volume. .(30)

This method is seldom used in practice for determining the hysteresis loss, because:

(1) It is tedious, since it requires a complete magnetization curve to be taken and integrated for each point on the hysteresis loss curve.

(2) In ballistic determinations of hysteresis, the iron is undergoing the magnetizing process slowly, while in actual machines it is magnetized and demagnetized many times per second. There are indications that the hysteresis loss is somewhat different in the two cases.

(3) The eddy-current loss is not taken into account, while in most practical cases the engineer is interested in the total core loss.

Nevertheless the method is of interest from a theoretical point of view, and it is recommended that the student integrate at least one hysteresis loop in connection with the experiments in Chapter VIII. The scale to which the loop is to be plotted has to be carefully considered in order to obtain the result in the desired units.

CHAPTER X

DIRECT-CURRENT GENERATOR

226. The essential parts of a direct-current generator are shown in Fig. 174. The machine consists of a revolving part, or armature, in which e.m.f.'s are induced (generated), and a stationary magnetic field, necessary for inducing these e.m.f.'s. In the particular case shown, the field has four poles. The armature is driven by a prime mover whose mechanical energy is transformed into electrical energy and delivered to the line through the commutator and the brushes.

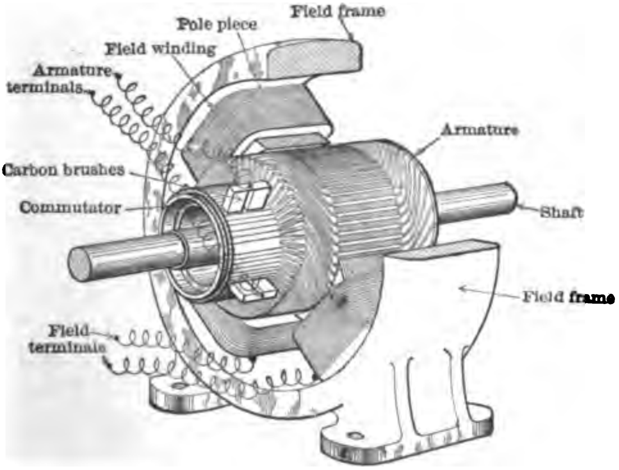


Fig. 174. The principal parts of a direct-current generator or motor.

The armature core is made of laminated iron with slots on its periphery, and is keyed to the shaft. A winding consisting of copper bars or coils is placed in the slots. The coils are properly interconnected so as to assist each other in their electrical action, and are also connected to the so-called commutator shown to the left. For details see the chapter on "Armature Windings" in Vol. II. The commutator consists of copper segments mounted on a sleeve and insulated from each other by sheets of mica, or some plastic insulating compound. Two or more sets of stationary carbon brushes make contact with the commutator segments and conduct direct current to the line.

The stationary frame is made of magnetic material, either cast iron or steel. The pole-pieces are of steel and may be cast with the frame, cast into the frame, or bolted to it. The poles are provided with coils through which a direct current is sent, necessary for the excitation of the magnetic field. In practice, the exciting current is usually generated by the machine itself, but for the first experiment the generator is excited from a separate source.

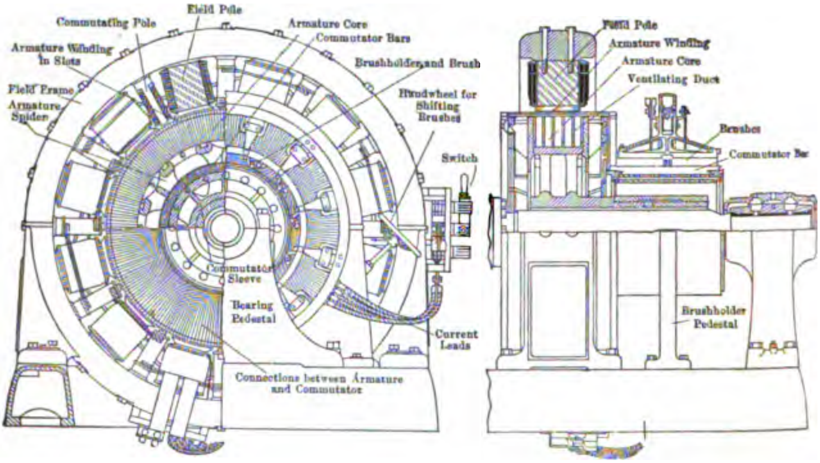


FIG. 175. A multipolar direct-current machine.

A direct-current machine is shown more in detail in Fig. 175. The view to the left is a plane perpendicular to the shaft. The upper left-hand quadrant is a cross-section through one of the field poles and through the armature and commutator. In the lower left-hand corner the radial connections are shown between the armature winding and the commutator. The brush rigging and the leads are shown on the right-hand side of the same view. The upper part of the sketch to the right is a longitudinal cross-section of the machine, in the vertical plane passing through the center line of the shaft. The lower part is a side view of the machine. The armature laminations are shown assembled in five stacks with ventilating spaces between. The radial lines shown in the armature core, in the upper left-hand quadrant of the other sketch, represent the ribs of a spacing plate which forms a ventilating duct.

In addition to the main or field poles, the so-called commutating poles, or interpoles, are shown midway between the main poles. The purpose of the interpoles is to provide a strong magnetic field over those armature coils in which the current is undergoing commutation under the brushes. This field helps the reversal of the current and makes the commutation practically sparkless. For further details regarding interpoles see § 268.

NO-LOAD CHARACTERISTICS

227. Separate Excitation.—It is advisable to begin the study of the direct-current generator with the simplest possible case, namely, that of the machine excited from a separate source and running at no-load (Fig. 176). A voltmeter is connected across the armature terminals to measure the induced voltage. The field winding is connected to a source of direct-current supply, in series with a regulating rheostat and an ammeter. A double-throw switch will permit reversing the direction of the exciting current, and therefore the polarity of the induced e.m.f., without changing the direction of rotation of the machine.

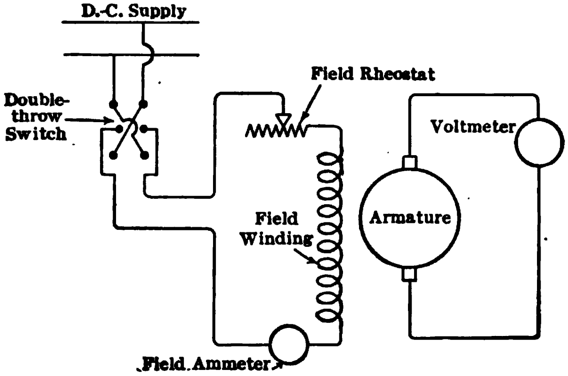


FIG. 176. A separately excited generator.

It is proved in the theory of direct-current machinery that the voltage induced in the armature at no-load is proportional to the following three factors: the magnetic flux per pole; the speed of the machine; and the number of armature conductors in series. The exact formula is

$$e = (p/p') \, (r.p.m/60) \, C\Phi \times 10^{-2} \quad . \; . \; . \; . \; . \; . \quad (1)$$

where e is the induced e.m.f. in volts, p is the number of poles, p' is the number of armature circuits in parallel, C is the total number of armature conductors, and Φ is the flux per pole, in megalines.

Let us assume first that the speed of the machine is kept constant, and that the flux Φ is varied by regulating the exciting current with the field rheostat. Let us assume also that the magnetic circuit of the machine has been thoroughly demagnetized so that it has no appreciable residual magnetism (§ 179). Then the relationship between the field current and the flux is found to be like the curve *Odb* in Fig. 177. According-ing to the foregoing formula, the induced voltage is proportional to the flux, so that while the curve is plotted between the field current and the induced voltage, it also gives a relationship between the exciting current and the flux which it produces. This curve is known as *the no-load saturation curve* of the machine, and its curvature on top is caused by the increasing magnetic saturation in the iron parts (§ 162).

If the residual magnetism has not been completely removed, some voltage is induced by it even when the field circuit is open. For this reason the saturation curve may begin somewhat above the origin, and the initial voltage *Of* depends upon the amount of residual magnetism present. If the exciting current is now gradually increased, a voltage curve *fgb* is obtained, which lies somewhat above *Odb*. When the magnetizing current is again reduced from its maximum value *Oa*, the voltage curve *bhf* lies still higher in its middle part, forming a hysteresis loop (§ 179). In practice, the difference between these curves on their upper "working part" is unimportant, but the student should see clearly the reason for discrepancies at low values of the exciting current.

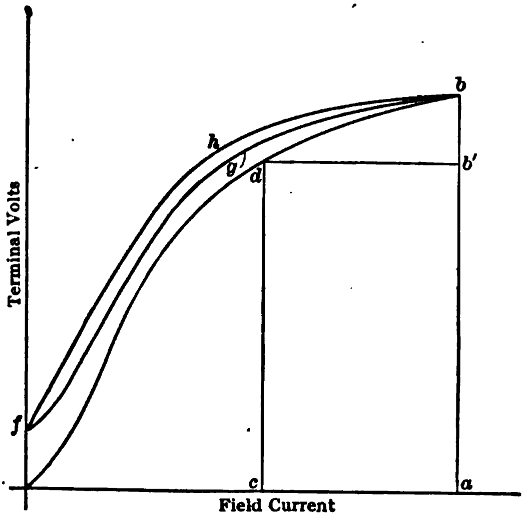


FIG. 177. No-load saturation curve of a direct-current generator.

Assume now that the exciting current, and consequently the flux, is kept constant, and that the speed of the machine is varied from zero to the highest safe limit. According to the fundamental law of induction, the induced voltage is proportional to the rate at which the armature conductors cut the flux. Therefore, with constant excitation, the relationship between the induced voltage and the speed of rotation is a straight line through the origin (Fig. 178). Three such straight lines are shown, each corresponding to a different value of the exciting current. At any given speed, it will be noted that increasing the field current from 20 to 30 amperes does not raise the voltage as much as increasing the current from 10 to 20 amperes. This is due to the saturation in the magnetic circuit of the machine.

It is not so easy to investigate in the laboratory the effect of the number of armature conductors, C, upon the induced e.m.f. Manufacturers of electrical machinery often use this relationship in changing a generator from one rated voltage to another. For example, a standard 100-kw., 220-volt generator may have 400 armature conductors of a certain size, placed in 100 armature slots. Should a 110-volt generator of the same speed and output be desired, the same frame and armature may be used, but the required number of conductors would be 200, and each conductor would be of about double the cross-section, thus occupying practically the same space in the slot as two conductors of the standard machine. The current density and the I^2R loss in the two machines will be nearly the same.

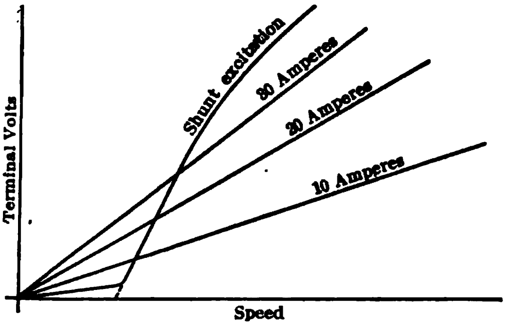


Fig. 178. Induced voltage at no-load, at different values of exciting current and at a variable speed.

228. Brush Shift. — Another factor which affects the induced e.m.f. is the position of the brushes with respect to the field poles. Part of the armature and field of a direct-current machine is shown in Fig. 179.

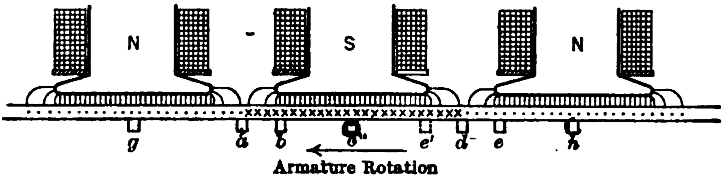


Fig. 179. The effect of brush shift upon the induced e.m.f.

The direction of the e.mf.'s induced in the individual conductors is shown by the usual dots and crosses. At no load, and with the brushes midway between the poles (that is, in the position *ad*), the action of all the conductors under one pole is cumulative, and the induced voltage is a maximum. The brushes are said to be placed on *the geometric neutral.*

Let the brushes be shifted into position *be*, and let *e'* be a position (*not* an actual brush) such that $de = de'$. Then the voltages induced in the groups of conductors *de* and *de'* neutralize each other, and the active group of conductors is reduced to *be'*. The e.m.f.'s induced in the belt *ee'* are comparatively small, because the conductors are subjected only to a rather small leakage flux of the main poles. For this reason the total induced voltage is not appreciably affected with small angles of brush shift. When, however, the brushes are shifted beyond the edges of the poles, the induced voltage is rapidly reduced and becomes zero when the brushes are opposite the centers of the poles, *i.e.*, in the position *ch*.

Sparking may occur under the brushes when they are shifted into such a position that the corresponding armature conductors are in a strong magnetic field. This sparking is due to the electromotive force and the resulting currents, induced in the coils short-circuited by the brushes through adjacent commutator segments (Fig. 241). When the contact with a commutator segment is broken, the inductance of the coil causes a spark to jump across the widening gap between the edge of the brush and the segment. For details see Chap. XV.

In recording the qualty of commutation it is convenient to use some such comparative terms as perfect commutation, slight spark, pin spark, spark in spots, hot spark, vicious sparking, near flash, etc., or other terms descriptive of the observed phenomena.

When plotting results of tests or computations relating to induced e.m.f., commutation, etc., it is convenient to use the so-called *electrical degrees* for abscissæ. The distance between the centers of two adjacent pole-pieces, or between two sets of brushes of opposite polarity, is always taken as 180 electrical degrees. Curves so plotted are independent of the actual number of poles of the machine.

With a given polarity of the field poles, the polarity of the brushes is reversed when the armature is driven in the opposite direction. This follows from the fundamental law of induction. The polarity is also reversed when the exciting current is made to flow in the opposite direction, because the flux polarity of each pole becomes reversed.

229. EXPERIMENT 10-A.—No-Load Characteristics of a Separately Excited Direct-Current Generator.—The purpose of this experiment is to study the performance of a separately excited direct-current generator at no-load, and to illustrate its theory, as explained in the preceding articles. The four parts of the experiment are practically independent of one another and may be performed during separate laboratory periods. Connect the machine for separate excitation (Fig. 175) and arrange to drive it by an adjustable-speed motor. For the first two parts of the experiment the brushes should be set on the geometric neutral. Use a data sheet similar to the one shown below.

	Field Amps.	Volts	Speed	Remarks
Instr. No........				
Constant........				

(1) *No-load saturation curve.* With the generator running at its rated speed, bring the field current up to as high a value as is possible, by cutting out the rheostat resistance. Record this maximum field current and the corresponding voltage and speed. Throughout this run the speed should be kept as nearly constant as possible, and small deviations recorded, in order to correct the observed curve accordingly. Reduce the field current slightly by inserting a small part of the external resistance in series, and again read the voltage and the speed. Be careful not to change the residual magnetism by reducing the field current too much and then raising it again. Every time this is done the saturation curve will appear somewhat irregular. Proceed in this way for several values of the field current, from maximum to zero. This will give data for the descending branch *bhf* (Fig. 177) of the saturation curve. To obtain the corresponding data for the ascending branch *fgb*, begin with the field circuit open and read the residual voltage. Then introduce all the field resistance into the circuit, close the field switch and read the current, the voltage, and the speed, as before. Increase the field current in steps, recording the same data at each step, and use as nearly as possible the same values of field currents as in the descending branch. The warning about overrunning the values of the field current applies here also. If desired, the field poles may be demagnetized (§ 182, under "normal induction") and the virgin saturation curve *Odb* taken.

(2) *Speed characteristic.* The purpose of this run is to obtain the relationship shown in Fig. 178. Bring the speed of the machine up to the highest possible value and excite the field with the highest possible current. Record the current, the voltage, and the speed. Now reduce

the speed in small steps to zero, taking similar readings at each step. Repeat this run with two or three lower values of field current, always beginning with the maximum speed, and keeping the field current constant.

(3) *Brush shift.* Provide a template by means of which the position of the brushes may be determined either in degrees or in terms of commutator segments, with reference to the geometric neutral of the machine. With the generator running at a constant speed, shift the brushes backward and forward to satisfy yourself that the voltage is a maximum when they are in the neutral position. Shift the brushes in steps between the neutral and the quadrature position and at each step record the angle of shift and the voltage; also note the quality of commutation, that is, the amount of sparking, if any. The speed and the field current should be kept constant, or small deviations recorded. The same data sheet may be used by adding another column for the angle of brush shift.

Before leaving the laboratory, note the number of poles of the machine and count the total number of commutator segments.

(4) *Direction of induced e.m.f.* Prove by experiment that reversing either the field current or the direction of rotation reverses the polarity of the machine, while reversing both leaves the polarity unchanged. In recording the results of this experiment the direction of rotation is considered clockwise or counter-clockwise, looking from the commutator end of the machine. Make a rough sketch of the generator and denote the armature and field terminals by letters or numerals. Record the field current as entering the winding, say through terminal *C* and leaving through terminal *D*. Similarly indicate the direction of the induced e.m.f. by marking the plus and minus armature terminals.

Report. (a) Using the data obtained in part (1) of the experiment, plot the descending and ascending saturation curves. Explain why different values of voltage were obtained for the same field current in these two curves. (b) Plot the results of run (2) as in Fig. 178, and explain why the curves are straight lines. Find the value of the induced voltage for normal speed on each of these curves and check with the saturation curves obtained in part (1). (c) Plot the results obtained in part (3), using the values of induced voltage as ordinates and the angles of brush shift (in electrical degrees) as abscissæ. Explain why the curve has only a slight slope for small angles of shift, and why the slope becomes considerable as the angle becomes larger. (d) Tabulate the results of part (4) giving the polarity of the field and armature terminals, and the direction of rotation. Show from these results that the polarity of the brushes is reversed when either the field current or the direction of rotation is reversed.

230. Self-Excitation. — In the introductory treatment, the generator has been assumed to be excited from a separate source. In practice the machine is usually made to excite itself; in other words, the field current is generated in the armature of the machine. One of the usual connections between the field winding and the armature of a self-excited direct-current generator is shown in Fig. 180. The field winding consists of many turns of comparatively small wire, and is connected across the brushes of the machine in series with a regulating rheostat, so that the exciting circuit is in parallel with the external circuit. Such a generator is said to be *shunt-wound*, or *shunt-excited*, the field winding being " shunted " across the armature terminals.

The no-load saturation curve (Fig. 177) which represents the relationship between the field current and the induced voltage at a constant speed, is practically the same whether the machine is self-excited, or separately excited. This is because the exciting current usually amounts to but a few per cent of the rated armature current, and hence its voltage drop in the armature and its armature reaction are very small. Similar curves for other than normal speeds are of interest in studying the initial self-excitation of the shunt-wound generator (Fig. 181). The ordinates of these curves, for any given field current, are proportional to the speed, since the e.m.f. generated in the armature winding is proportional to the rate of cutting the magnetic flux.

With the field connections shown in Fig. 180, the self-excitation is made possible because of the *residual magnetism* in the iron parts, which, after being magnetized, retain a small part of their magnetism (§ 179). Let the machine be running somewhere near its rated speed, and let both the load circuit and the field circuit be open. The residual magnetism induces in the armature a small voltage, and this voltage

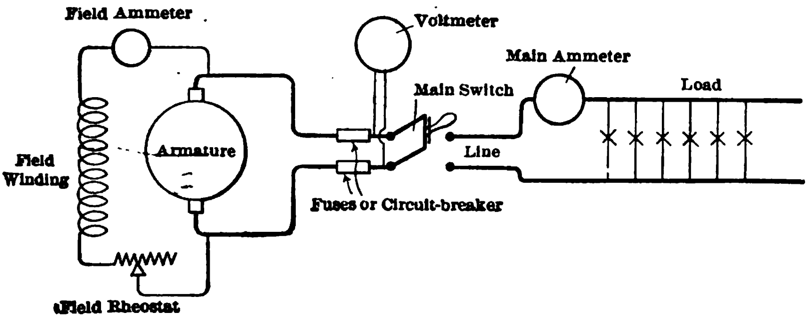


Fig. 180. Diagram of connections of a shunt-wound generator and load.

is available at the brushes. Now, if the field circuit be closed, a small current will flow through the field winding. With the proper polarity, this current strengthens the original flux, so that a higher voltage is induced in the armature. This higher voltage causes a larger current to flow in the field winding, which current in turn increases the flux, etc., until the flux, the induced e.m.f., and the exciting current reach their full stable values.

231. Limits and Direction of Self-Excitation. — Let the straight line *Of* (Fig. 181) represent the values of the voltage necessary to maintain various values of the exciting current in the field circuit. In other words, if the resistance of the field circuit (including the rheostat) is R, and an exciting current i is plotted as abscissa, the corresponding ordinate is equal to Ri. The line *Of* will be referred to as the *resistance line.* The limiting voltage of self-excitation is determined by the point b of intersection of Of and the saturation curve Og. At any point to the left of b, say p, the induced e.m.f., sp, is higher than the voltage sq required for the ohmic drop in the field winding. Hence the excess voltage qp will increase the field current. At b there is no excess voltage to increase the field current further, and the voltage will not build up beyond this point.

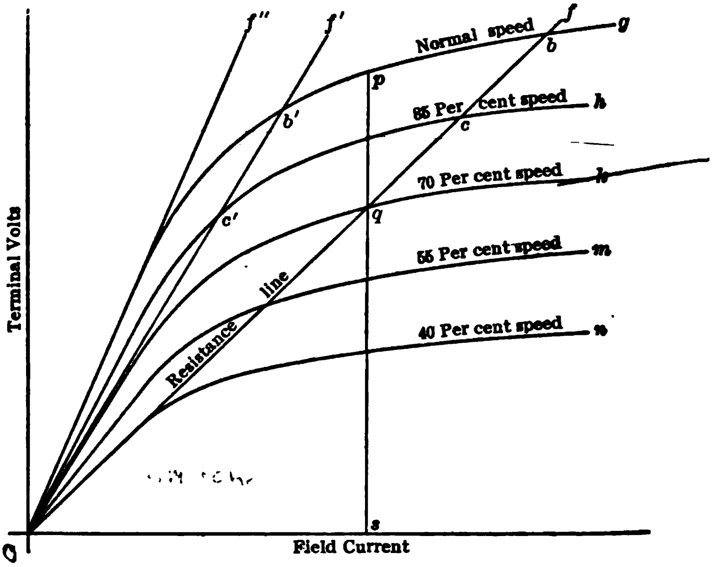


Fig. 181. No-load saturation curves of a shunt-wound generator at different speeds.

Now let the resistance of the field circuit be increased so that the resistance line becomes *Of'*. The machine will build up its voltage only to *b'* which is less than *b*. Finally, let the resistance of the field circuit be so increased that the resistance line assumes the position *Of''*. In this case there can be no decided building up since the straight line is practically tangent to the saturation curve, and the current produced in the field circuit by the residual magnetism is not sufficiently large to produce any decided increase in the flux. In reality, the saturation curve does not start from the origin, because of the residual magnetism, while *Of''* does pass through the origin. Consequently, *Of''* must cut the saturation curve at a point slightly above that represented by the residual magnetism and raise the voltage a little. Practically, however, there is a definite limit to the field resistance, above which there is no appreciable building up of induced voltage.

So far, we have considered the process of self-excitation at the rated speed. At a lower speed, for example that represented by the curve *Oh*, the same resistance lines *Of* and *Of'* intersect the saturation curve at lower voltages corresponding to points *c* and *c'*. The resistance line *Of'* is almost tangent to *Oh*, so that at this speed the machine will lose its voltage when the field resistance is slightly increased beyond that corresponding to *Of'*. We thus see that at a lower speed the generator has a lower critical value of field resistance. There is a speed, sometimes called the critical speed, below which no appreciable building up of voltage will occur even with all the external resistance cut out of the field circuit. The resistance line of the field winding itself is in this case practically tangent to the saturation curve at that speed (curve *On* in Fig. 181).

With separate excitation (§ 227), the polarity of the induced voltage is reversed with the reversal of either the field current or the direction of rotation, but remains the same when both are reversed simultaneously. Now, with shunt excitation, let either the field connections or the direction of rotation be reversed. The induced voltage due to the residual magnetism will force a current through the field winding in the direction in which this current will tend to destroy the original residual magnetism, and the generator will not build up. When both the field connections and the direction of rotation are reversed the machine will build up as at first.

232. EXPERIMENT 10-B. — No-Load Characteristics of a Self-excited Shunt-wound Generator. — The purpose of this experiment is to determine the relation between the exciting current, the speed, and the terminal voltage at no-load, and to investigate the ability of the machine to build up its voltage. Connect up the generator as shown in Fig. 180, except that no load is necessary, and arrange to drive it by an adjustable-speed motor. If the voltage does not build up, try reversing the field connections. If the field still fails to build up, excite it from a separate source (Fig. 176), so as to restore the residual magnetism in the right direction. Use a data sheet similar to the one given for the preceding experiment.

(1) *Saturation Curves.* Take the data necessary for plotting the descending branch of the no-load saturation curve at normal speed, as explained in the experiment on separate excitation (§ 229). Take similar data at two other speeds, for example at 85 and at 70 per cent of the normal.

(2) *Maximum Field Resistance for Building Up.* Drive the generator at its rated speed and find the rheostat setting at which the field just begins decidedly to build up. Read the field current, the final voltage, and determine the time required for the voltage to reach its final value. Repeat for one or two other rheostat settings. Measure the total resistance of the field circuit, at least approximately. Repeat for one or two other rheostat settings.

(3) *Minimum Speed for Building Up.* (a) With the external field resistance cut out, gradually increase the speed in small steps until the voltage just begins to build up definitely. Record this speed, the final voltage, and the field current; measure the total resistance in the field circuit. (b) Increase the speed in small steps and take similar readings at each step. (c) To determine the effect of external field resistance on the minimum speed of building up, repeat the preceding runs with one or two different values of external field resistance.

(4) *Effect of the Direction of Field Current and Rotation on Building Up of Voltage.* Before beginning this experiment it is convenient to connect a reversing switch in the field circuit. Provide means for driving the generator in both directions. Run the machine in the direction in which it excites itself. If it fails to build up the voltage with either direction of rotation and with either position of the field switch, restore the residual magnetism by exciting the fields from an external source, and see that the voltage builds up. For recording the results see Exp. 10-A, part (4). Investigate the possibility of self-excitation (a) with reversed field current; (b) with reversed direction of rotation, and (c) with both the field current and the direction of rotation simultaneously reversed. Before each test make sure that the residual magnetism is still of the original polarity.

(5) Observe the effect of brush shift upon the induced voltage, by the method described in Exp. 10-A, part (3); but keep the setting of the field rheostat constant instead of maintaining a constant field cur-

rent. Read the voltage, the field current, the speed, and the position of the brushes.

Report. (a) From the data obtained in part (1) plot the saturation curves shown in Fig. 181, correcting the voltages in direct proportion for small variations from the assumed constant speed. Show from the curves that for a given flux the induced e.m.f. is directly proportional to the speed (§ 227). Compute the saturation factor and the percentage saturation of the machine, according to the Standards of the American Institute of Electrical Engineers. From each plotted curve compute the maximum resistance of the field circuit at which the machine will excite itself. (b) Compare the critical values of the field resistance measured in part (2) with the theoretical values computed from the saturation curves. Explain possible reasons for discrepancies, if any. (c) From the saturation curves calculate by proportion the minimum speed at which the machine will excite itself with the field rheostat cut out, and compare with the data obtained under (3a). Check the values of the final voltages and field currents obtained in (3b) and (3c) with the computed values, as shown in Fig. 181. (d) Show by means of curves or tables the time which it took the machine to excite itself to the final voltage under different conditions. (e) Tabulate the results of part (4) of the experiment, giving the field and armature connections and the direction of rotation. Explain in detail the conditions under which the machine did not build up. (f) Plot the results of part (5), using the angles of brush shift in electrical degrees as abscissæ and values of voltage and field current as ordinates. Explain (a) why the field current varies with the brush shift and (b) why the induced voltage varies much more rapidly with the same brush shift than in the case of a separately excited machine.

LOAD CHARACTERISTICS OF SHUNT-WOUND GENERATOR

233. Voltage Drop and Regulation. — After the student has become familiar with the characteristics of a shunt-wound generator at no load, the next step is to obtain its electrical characteristics when it is loaded. Referring to Fig. 180, this means that the main switch is now assumed to be closed and the machine furnishing electric power to the load. One of the most important requirements in the practical operation of a generator supplying current for light and power is that the terminal voltage must be nearly constant and independent of the load. This practically means that each customer should get the same quantity of light from his lamps, and his motors should run at the same speed, whether other customers are using current at the same time or not.

This condition of constant voltage cannot be strictly fulfilled with a shunt-wound machine, unless the field current is being continuously adjusted by an automatic regulator (§ 245). Without regulation the voltage drops considerably as the load increases. This is due to the following three causes:

(a) Part of the induced voltage is consumed in the resistance of the armature winding and in the resistance and contact e.m.f. of the brushes. The *ir* drop in the winding is proportional to the armature current, that is, approximately proportional to the load of the machine. The loss of voltage under the brushes is more of the nature of a counter-e.m.f., follows a more complicated law, and depends upon the grade and quality of the brushes, the current density, the pressure of the contact, etc. (§§ 375 to 378).

(b) The armature winding with the currents flowing in it becomes a source of magnetomotive force which is combined with the ampere-turns of the field winding (§ 358) and modifies the flux produced by the same exciting current at no load. Usually the effect is such as to reduce the useful flux, and the voltage induced in the armature is correspondingly lower.

(c) As the terminal voltage of the machine decreases, on account of the above two causes, the current in the shunt winding also decreases in the same proportion. This again weakens the field and reduces the voltage still further.

Curves which show the influence of the load on the voltage of a machine are generally called its *performance characteristics* (Fig. 182). Such a characteristic curve may represent the terminal voltage of the machine as a function of the load current, at a constant field current. Another curve of this type may show the required variations in the field current in order to keep the voltage constant when the load varies, etc. For a generalized representation of such curves by means of a surface, see § 237.

The load resistance in Fig. 180 may be conveniently taken as the independent variable. At no load it is infinitely great, and then gradually decreases. Its limiting value is zero, which it reaches on short circuit. As this resistance decreases from infinity to zero, the terminal voltage in Fig. 182, plotted against the load current as abscissæ, varies according to the curve $VWKS$, from V to S. As long as the load resistance is comparatively high, any decrease in this resistance causes the load current to increase, and results in a comparatively small additional drop in voltage. This corresponds to the working part, VW, of the curve. As the load resistance is further decreased, the corresponding increase in the load current causes such an armature reaction and voltage drop in the armature that the terminal voltage is decreased very appreciably. This also reduces the field current and causes a further voltage drop. Finally point *K* is reached; beyond this the load current cannot be increased. When the load resistance is reduced below the value corresponding to this point, the electrical and the magnetic conditions in the machine become such that the load current is reduced instead of being increased. This corresponds to the part *KS* of the curve. A more detailed explanation of this part is given at the end of § 237.

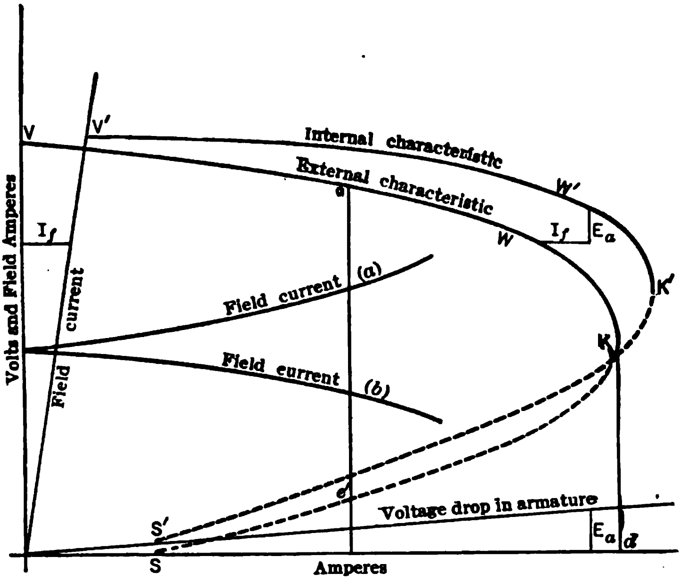


Fig. 182. Voltage and field-current characteristics of a shunt-wound generator.

The maximum value of the load current, corresponding to point *K*, depends upon the saturation of the magnetic circuit, and consequently on the value of the field resistance. The higher the resistance the lower the induced voltage and the sooner the maximum current is reached. With a comparatively small machine, having relatively large armature reaction, this maximum current may be readily obtained on test, at least with high values of field resistance, without causing excessive heating or sparking.

The curve *VWKS* in Fig. 182 represents the external characteristic of the machine. A similar curve, *V′W′K′S′*, represents the relation between the induced voltage and the armature current, and is called the internal characteristic of the machine. It may be obtained from the first by adding the voltage drop in the armature, marked E_a, to the terminal volts, which gives the induced voltage; and adding the field current, marked I_f, to the load current to obtain the armature current.

When a shunt-wound generator is running at a variable load, the operator usually regulates the field rheostat from time to time, trying to keep the voltage constant. In order to see what voltage regulation may be expected under such conditions, two characteristics are usually taken when testing a machine, to correspond to the following two extreme cases: (1) The machine is left *entirely without attention*, so that the voltage fluctuates freely with the load (§ 235). (2) The switchboard attendant (or an automatic device) *regulates the field continuously*, thus keeping the voltage absolutely constant (§ 236). In actual practice any intermediate condition may exist: the attendant does not watch the voltmeter constantly, but corrects the voltage from time to time.

These two extreme conditions are represented in Fig. 182 by means of the curves of field current, (*a*) and (*b*). The (*b*) curve corresponds to the first case, namely, that in which the field current is allowed to decrease as the load increases. The (*a*) curve shows the increase in excitation necessary to keep the terminal voltage of the machine constant, irrespective of the load.

The relative variations in the field current, necessary to keep the terminal voltage constant, depend on the degree of saturation of the magnetic circuit, and are less with highly saturated iron parts. This is because a much greater number of field ampere-turns is required for high saturation, so that the same number of armature ampere-turns constitutes a smaller percentage of the total magnetomotive force, and its influence is correspondingly less. The magnetic circuit of the machine is then said to be stable and less affected by the armature reaction. A large air-gap also requires an increased excitation and therefore gives a more stable magnetic field, less affected by the armature reaction.

According to the Standards of the American Institute of Electrical Engineers, the regulation of a self-excited machine is defined as per cent increase in voltage when a full-load is thrown off the machine, the speed and the setting of the field rheostat remaining unchanged. To illustrate, let the rated voltage of a shunt-wound generator be 100 volts. Let the machine be loaded to its full-load current and the field rheostat so adjusted that the terminal voltage is 100 volts; then let the main switch be opened. When the prime mover comes back to the normal speed, let the terminal voltage be 109 volts; according to the above definition, the regulation of the machine is 9 per cent.

234. Components of Voltage Drop. — The curve of terminal volts, *VWK*, shown in Fig. 182, refers to the case of a constant setting of the field rheostat, so that all the three causes of the voltage drop in the armature (§ 233) are present. These three factors are shown separately in Fig. 183.

The voltage drop *a* is calculated by multiplying the armature current by the resistance of the armature, and adding to it the voltage drop due to the brush contact. The drop *c*, due to the decrease in the field current, can be found from the no-load saturation curve (Fig. 177). If the values of the field current at no load and at the load under consideration are known from Fig. 182, the corresponding decrease in voltage, *bb'* (Fig. 177), may be read directly from the curve. The remainder of the total observed drop (*b*, Fig. 183) is caused by the armature reaction. It is sometimes stated that the total voltage drop in a shunt-wound generator is caused by *two* factors: the voltage drop in the armature and the armature reaction. This is true when the machine is separately excited, or when the exciting current is kept constant. In actual operation the exciting current itself usually varies with the terminal voltage, so that in analyzing the voltage drop of a machine *left to itself* (without regulation), this third cause must also be considered.

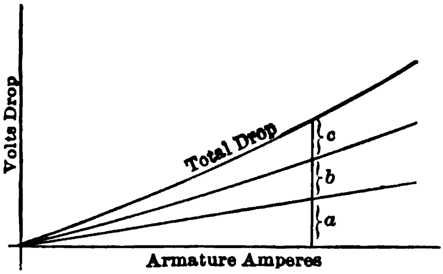


FIG. 183. The components of voltage drop in a shunt-wound generator.

When a special study of the armature reaction is made, the field current can be kept constant from no load to full load. In this case, the voltage drop is due to *two* factors only; *viz.*, *a* and *b*. Subtracting the former, we readily find the magnitude of the armature reaction, with a given load and excitation. Armature reaction may also be studied on the machine with the armature stationary; see Chapter XV.

235. EXPERIMENT 10-C. — Voltage Characteristics of a Shunt-Wound Generator. — The purpose of the experiment is to obtain performance curves, such as *VWK* in Fig. 182, and to separate the voltage drop into its components (Fig. 183). This corresponds to the first extreme condition of operation mentioned in § 233. The machine is connected as in Fig. 180, and is driven at its rated speed by a motor. The readings may be recorded on a data sheet similar to that shown below.

	Normal Voltage at ____ Load.			
	Field Amps.	Load Amps.	Volts.	Speed.
Inst. No.........				
Constant........				

Bring the machine up to its rated speed, and load it on suitable resistances. Adjust these resistances and the field rheostat, so as to get the rated voltage of the machine at a load exceeding its rated capacity by about 25 per cent. Read load amperes, terminal volts, and field current; the speed should be kept as nearly constant as possible throughout the whole test and small deviations noted. Reduce the load in steps to zero leaving the field rheostat in the same position. Read the volts, the amperes, and the field current at each step.

Repeat the same test with a few other settings of the field rheostat, for example such that the full rated voltage is obtained at 100 or 75 per cent of full load. It is also desirable to take a curve corresponding to such a position of the field rheostat that the machine gives its full rated voltage at a small load, say about 25 per cent of full load, or even at no load. In each case, after having properly set the rheostat, begin the run at about 25 per cent overload in load current and gradually reduce it to zero.

Having finished the test, measure the voltage drop in the armature, with and without the brushes. Use the drop-of-potential method (§ 12). When measuring the voltage drop with the brushes in the circuit, use currents from very large to very small values in order to find out the influence of the current density. The no-load saturation curve (Fig. 177) is supposed to be known from Experiment 10-B; otherwise it should be taken before leaving the laboratory.

Report. Plot curves such as are shown in Fig. 182. Add a curve of output in kilowatts; the ordinates are calculated as a product of load amperes by terminal volts, divided by 1000. Compute per cent regulation of the machine at full load, as explained at the end of § 233. For one of the curves, separate the voltage drop into its three components, as in Fig. 183. Using the data obtained with the stationary armature, subtract the voltage drop due the armature resistance from the total drop across the brushes, for each of the currents used. The differences obtained are the drops due to the brush contact and the brush resistance. Plot these values as ordinates against the values of current as abscissæ. If the curve does not pass through the origin, give the reason. Suggest an experiment to prove that the principal voltage drop occurs at the contact surfaces and not in the body of the brush.

236. EXPERIMENT 10-D. — The Field Characteristic or Compounding Curve of a Shunt-Wound Generator. — This experiment corresponds to the second extreme condition mentioned in § 233, *viz.*, when the field rheostat is adjusted for each load, so as to keep the terminal voltage constant. The purpose of this experiment is to determine the variations in the field current necessary for maintaining a constant terminal voltage; see field curve (*a*) in Fig. 182.

The connections are shown in Fig. 180, and the same data sheet may be used as in Exp. 10-C. Throughout the experiment, the speed of the machine must be kept constant and small deviations noted. Bring up the load and the field current so as to have a full rated voltage at a reasonable overload current, say 25 per cent. Read the terminal voltage, the field current, and the load current. Reduce the load in small steps, keeping the terminal voltage constant, and at each step take similar readings. To investigate the influence of saturation of the magnetic circuit, repeat the preceding test at as high a terminal voltage as it is possible to obtain at the overload used, and also at a voltage considerably below normal.

Report. Plot the field current curves as in Fig. 182. Determine the percentage of variation in the field current between no load and full load. Show that the higher the saturation of the magnetic circuit the better is the voltage regulation of the machine. Explain in detail why the field current must be increased with the load to give a constant terminal voltage. Explain why, for a given increase in the armature current, this increase in the field current must be greater at a heavy load than at a light load.

237. Load Saturation Curves. — These curves (Fig. 184) are obtained by plotting terminal volts as ordinates against the corresponding values of field current as abscissæ. Each curve corresponds to a constant value of load current. As a special case, when the load current is zero, the no-load saturation curve is obtained, the same as the upper curve in Fig. 181. A set of such load saturation curves contains all the data needed for the plotting of several other characteristics, and for this reason is valuable in the study of the machine.

To obtain an external characteristic (Fig. 182) from the load saturation curves of the machine, draw *OL*, the field resistance line (§ 231) for normal excitation, and find its points of intersection *c*,*c''*, etc., with each load characteristic. From these ordinates, the external characteristic, that is, terminal volts *vs.* load current, may be plotted on another curve sheet. The upper points of intersection, such as *c*, will give the principal part of the curve, and the lower points, such as *c'''*, the dotted part (Fig. 182).

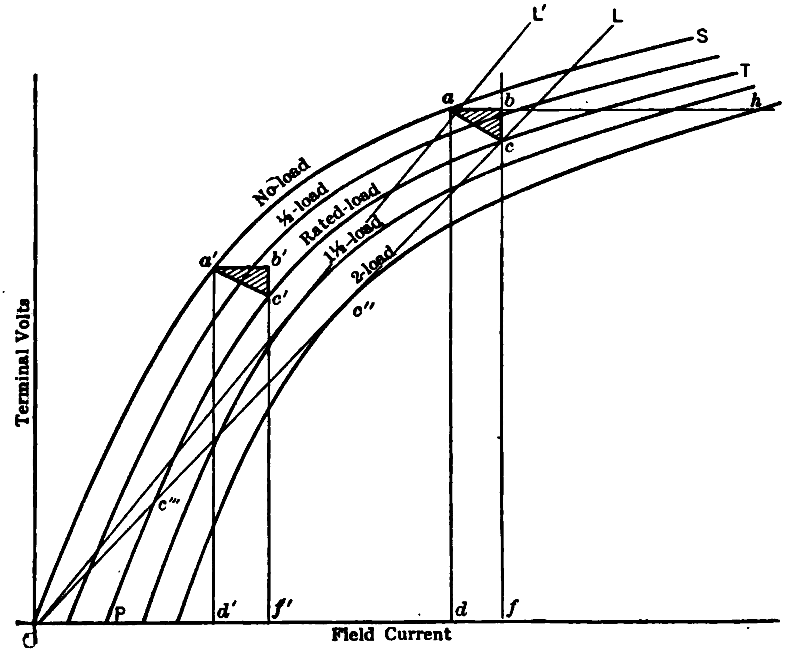


FIG. 184. Load characteristics of a shunt-wound generator.

To obtain a voltage characteristic of the machine at a constant field current, or when separately excited, draw a vertical line, such as *ad*, corresponding to the chosen value of excitation. The intersections of this line with the various load characteristics will give the data for the desired curve. Similarly, to find a field-current characteristic to give a constant terminal voltage (Fig. 182), draw a horizontal line such as *ah*, for the desired constant voltage, and find its points of intersection with the load characteristics.

A more complete insight into the performance of a shunt-wound generator at a constant speed may be obtained by means of a *characteristic surface.* Let values of the load current be plotted along the X-axis, the field current along the Y-axis, and terminal voltages along the Z-axis. Such a surface will give the value of the terminal voltage for any desired values of field and load currents. Intersections of this surface by planes parallel to the XZ-plane will give voltage-regulation curves at a constant field current. Intersections by planes parallel to the YZ-plane will give load-saturation curves. Intersections by planes parallel to the XY-plane will give the excitation characteristics. Let a field resistance line (§ 231) be drawn in the YZ-plane and a plane imagined through this line, perpendicular to the YZ-plane. The intersection of this "resistance plane" with the original surface is a curve which gives the voltage regulation at that particular setting of the field rheostat.

Another surface may be made to represent the relationship between the induced voltage, the armature current, and the field current. These two surfaces may be called the external and the internal characteristic surfaces of the machine, respectively. It is only because we know of no convenient means of constructing such surfaces in practical work that we replace them by the various sets of curves (Figs. 182 and 184) in each of which some quantity or condition is assumed to remain unchanged.

For a special study of a machine, such a characteristic surface may be realized as follows: Paste a piece of cross-section paper on a board, and mark along the edges of the paper some suitable scales to comprise the limits of the expected values of the load current and of the field current. Drill small holes on the principal intersections of the two rulings, everywhere on the sheet where an experimentally determined point may be expected. When performing load runs on the machine, adjust the values of the load current and of the field current to correspond to these holes, and read the values of the terminal voltage. Use pieces of wire of proper length to represent these voltages to a certain scale, and insert them in the corresponding holes. The upper ends of these wires will then lie on the characteristic surface of the machine. With a sufficiently large number of points, the eye readily visualizes the surface; or else the whole space between the wires may be filled with plaster of Paris or some such plastic material smoothed over to agree with the tops of the wires.

Theory of Load Saturation Curves. In order to see the relationship between the no-load saturation curve OS (Fig. 184) and the full-load saturation curve, PT, consider two corresponding ordinates, such as da and *fc*. The no-load voltage *da* requires field ampere-turns equal to *Od*. With a certain current *I* in the armature, an additional number of ampere-turns, $ab = df$ is required on the field poles to induce the same voltage, because the armature reaction has to be compensated for. Thus, the same voltage, $bf = da$, is induced in the loaded armature at the field excitation equal to *Of*. Deducing from *fb* the internal voltage drop, $Ir = bc$, in the armature and the brushes, we obtain the terminal voltage *fc*. Thus, for every point *a* on the no-load saturation curve there is a corresponding point *c* on the load saturation curve. This is shown in the figure by drawing another triangle, $a'b'c'$, between the same curves.

The existence of the dotted part of the external characteristic, shown in Fig. 182, may be demonstrated in Fig. 184 by drawing a field resistance line *OL* (§ 231). This straight line intersects *PT* at two points, *c* and c''', and a stable operation of the machine is possible at both points. Thus, for given values of armature current, field resistance, and speed, two values of terminal voltage and of field current are possible. These two values of terminal voltage correspond in Fig. 182 to points *c* and c' respectively.

The field resistance line *OL* is tangent at point c'' to the load saturation curve drawn for twice the rated load current, and this is the upper limit of load current possible with that particular field resistance. Point c'' corresponds to the ordinate *dK* in Fig. 182. With a higher field resistance (line OL') the maximum load current is only 1.5 times the rated current.

238. EXPERIMENT 10-E. — Load Saturation Curves. — The purpose of this experiment is to obtain load saturation curves of the machine and from them to plot the other characteristic curves, as explained in the preceding section. The wiring, the general conduct of the experiment, and the data sheet are the same as in the experiments 10-C and 10-D.

With the generator running at its rated speed, build up the voltage to the maximum safe value attainable, and load the machine to as high a current as is feasible. Adjusting the load resistance at each step, so as to make the load current the same each time, vary the field current in small steps from this maximum to the minimum value at which the load current can be maintained. For each step record the field current, terminal volts, and small unavoidable variations in speed. This will give the lower curve in Fig. 184. Make similar runs with several smaller values of armature current, so as to get a complete set of curves shown in Fig. 184. Six to eight curves are necessary (besides the no-load characteristic) to obtain satisfactory cross-curves asked for in the report.

With the armature stationary and the field winding disconnected, determine the voltage drop across the brushes for each of the load currents used in the preceding test, remembering that the armature current is larger than the load current by the amount of field current. Find the rheostat setting that gives the normal voltage at normal speed and full rated load, and measure the total resistance of the field circuit at this setting. If possible, determine the maximum values of load current (point *K* in Fig. 182) for several rheostat settings, and measure the total field resistance in each case.

Report. Plot all the saturation curves on one curve sheet, as in Fig. 184. From these, by drawing the proper construction lines, determine and plot the following curves: (a) External characteristics (Fig. 182), showing both the upper and lower portions as far as the data will permit; these curves should be plotted for three or more values of field resistance, to show the effect of saturation on the maximum load current. (b) Armature and brush voltage drop, from the test data. (c) Internal characteristics. (d) External and internal characteristics at a constant field current. (e) Field characteristics (field current *vs.* load current), each curve for a constant terminal voltage.

SERIES-WOUND GENERATOR

239. Series Excitation. — When the field winding of an electrical machine is connected in series with the armature (Fig. 185), the machine is said to be series-wound or series-connected. The electrical charac-

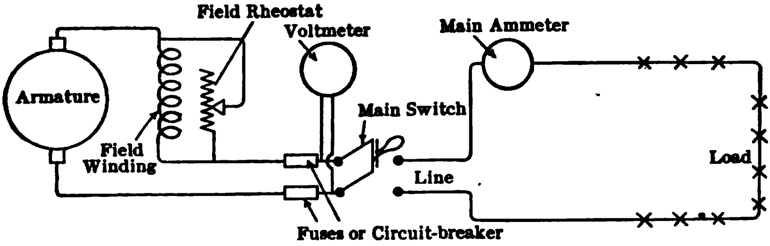


FIG. 185. A series-wound generator and its load.

teristics of such a generator are shown in Fig. 186. While the series-wound generator is not much used in practice, a knowledge of its performance is useful when studying the compound generator and the series motor treated in the following chapters. Because the field winding is connected in series with the load, the generator cannot build up its voltage unless the external circuit is closed. To obtain a no-load characteristic, *i.e.*, a relationship between the field current and the induced voltage, it is necessary to use separate excitation (Fig. 176).

In the external characteristic of the series generator (Fig. 186), which shows the relationship between the terminal voltage and the load current at a constant speed, the load current is equal to the field current, and the general shape must therefore resemble that of the saturation curve. It will lie below the no-load saturation curve by an amount equal to the sum of the voltage drops in the armature and field windings and that due to the armature reaction (§§ 233 and 234). After a certain maximum value of voltage has been reached, the curve drops quite rapidly as the external resistance is further reduced toward zero. This is due to the combined effect of armature reaction, voltage drop in the armature and field, and the increased saturation of the magnetic circuit.

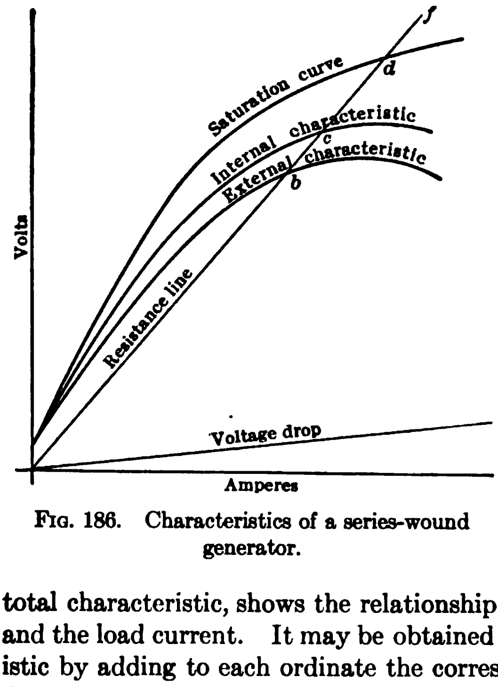


FIG. 186. Characteristics of a series-wound generator.

The internal characteristic, sometimes called the total characteristic, shows the relationship between the induced voltage and the load current. It may be obtained from the external characteristic by adding to each ordinate the corresponding value of the voltage drop in the armature and field windings. The difference between any ordinate of this curve and the corresponding ordinate of the no-load saturation curve is a measure of the voltage drop due to the armature reaction at that particular load current.

The influence of the direction of rotation and of the polarity of the field connections upon the building up of the series-wound generator is similar to that in the shunt machine (§ 231). A reversal of the residual magnetism will not affect the building up but will merely reverse the final polarity of the generator. Since each point on the external characteristic gives the terminal voltage and the load current, the ratio of the two is equal to the corresponding total resistance of the load circuit. To find the limit of the building up of the voltage and the current for a given load resistance, draw a straight line *Of* such that the ratio of its ordinates to its abscissæ equals the given resistance. The intersection *b* of this straight line with the external characteristic will give the limit of building up. When this load resistance becomes so great that the line *Of* becomes practically tangent to the external characteristic, the series-wound generator will not build up its voltage. This is similar to the case of the shunt-wound generator with a high resistance in the field circuit (§ 231), and the corresponding load resistance is called the critical resistance of the series-wound generator.

The voltage regulation in a series-wound generator is accomplished either by shunting the field winding by an adjustable rheostat (Fig. 185), or by cutting out a part of the field turns. Both methods may also be used simultaneously.

240. EXPERIMENT 10-F. — Characteristics of a Series-Wound Generator. — The purpose of this experiment is to obtain the characteristics (Fig. 186) of a direct-current generator excited by a field winding connected in series with its load (Fig. 185). An adjustable-speed motor should be used for driving the generator, and means provided for reversing the direction of rotation.

(1) *No-load Saturation Curve* (§ 229). Set the brushes on the geometric neutral. Disconnect the field winding and arrange to excite it from a separate source (Fig. 176). Drive the generator at its rated speed, record the terminal voltage induced with different values of field current, and note any unavoidable deviations in speed. Start at a maximum value of exciting current and reduce the excitation gradually so as to obtain a smooth curve. An ascending curve may also be taken if desired.

(2) *External Characteristics.* Replace the series connection, load the machine on an external resistance and keep the speed constant throughout the following test. A data sheet similar to that in Exp. 10-C may be used. Beginning with the highest safe value of load amperes, record the terminal voltage, the load current, and any unavoidable deviation in speed. Increase the load resistance in steps, and at each step take similar readings. Make similar runs at one or two different speeds.

(3) *Shunting the Field.* Connect an ammeter in series with the field winding and shunt both by a resistance such as to reduce the field current, for example, to 80 per cent of the load current. Take external characteristics as in (2) above. Make similar runs with the field current reduced to 70, 60 etc., per cent of the load current.

(4) *Building Up of Voltage.* Investigate the effect of the direction of rotation, field connections, and the polarity of the residual magnet-

ism on the building up and the final polarity of the generator. This can be done as in Exp. 10-C, except that the load circuit must be kept closed.

Before leaving the laboratory, measure the voltage drop in the armature and field windings with the machine stationary. Use several values of current so as to plot a curve between volts and amperes.

Report. (1) Using currents as abscissæ and voltages as ordinates, plot the no-load saturation curve and all the external characteristics (Fig. 186) taken in the laboratory. (2) Plot the voltage-drop curve for the field and armature. (3) From one or more external characteristics, obtain the corresponding internal characteristics by adding the internal voltage drop in the machine. (4) Determine the voltage drop due to the armature reaction, at different values of load current and field saturation, and plot the results so as to bring out their meaning. (5) Explain the comparative shapes of the internal characteristics taken at different speeds. (6) For one or two characteristic curves, calculate the value of the critical resistance. Is this resistance a function of the speed of the machine? (7) Explain the comparative shapes of the external characteristics with the field winding shunted. Show that by this means the machine can be made to deliver a constant current with a varying terminal voltage and load resistance. (8) Describe and explain the results on the building up of the generator.

CHAPTER XI

DIRECT-CURRENT GENERATOR *(Continued)*

241. Compound Winding. — The preceding study of the shunt-wound generator shows that when the field rheostat is left unchanged the terminal voltage decreases appreciably as the load current increases. For many purposes the terminal voltage must be kept at least approximately constant, and this can be accomplished in three ways:

(1) The field rheostat is adjusted from time to time by hand.

(2) The field current is adjusted continuously by an automatic regulator (§§ 245 to 247).

(3) A second field winding is provided (Figs. 187 to 189) through which the load current (or the armature current) of the machine is made to flow. Thus, as the load of the machine increases, the number of exciting ampere-turns is automatically made larger, and the voltage is correspondingly raised (Fig. 190).

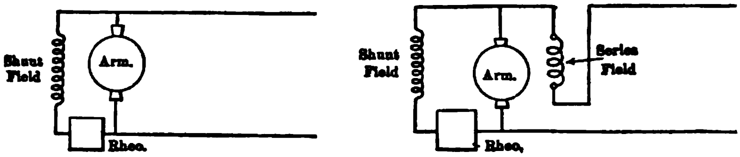


FIG. 187. Shunt-wound generator to the left, compound-wound generator to the right.

When a generator, in addition to a shunt field winding, is provided with a series winding, it is said to be *compound-wound* (Fig. 187). The shunt winding in this case furnishes the excitation necessary to produce the rated voltage at no load. The series winding raises the voltage approximately by the amount required to compensate for the armature reaction and for the voltage drop in the series circuit of the machine itself. In the most general case this includes the voltage drop in the armature, the brushes, the series winding, the interpole winding and the compensating winding, if present. If the machine is situated at a considerable distance from the place where the voltage must be maintained constant, the number of turns in the series winding may be increased so as to cover the line drop as well. In this case the voltage of the machine is greater at full load than at no load, and the generator is said to be *over-compound*. When the number of series ampere-turns is such that the terminal voltage at full load is practically equal to that at no load the generator is said to be *flat-compound.*

In a compound-wound machine the shunt winding may be connected either across the armature alone (Fig. 188), or across the armature and the series winding (Fig. 189). In the first case it is called the *short shunt* and in the second case the *long shunt.* Since the voltage drop across the series winding hardly amounts to one per cent of the terminal voltage, the electrical characteristics of the two machines are approximately the same. One or the other type of connections is used, according to the local conditions.

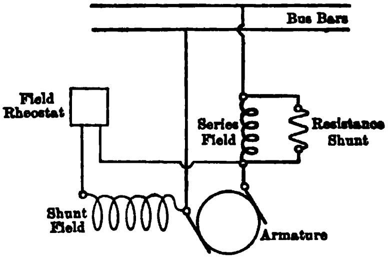


Fig. 188. Diagram of connections in a compound-wound generator, with the shunt-field winding connected across the armature (short shunt).

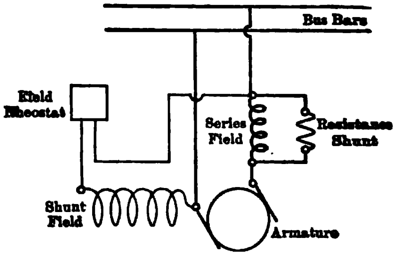


Fig. 189. Diagram of connections in a compound-wound generator, with the shunt-field winding connected across the terminals of the machine (long shunt).

242. Deviations from Constant Voltage. — When the series winding has been so adjusted that the voltage at full load is the same as at no load (curve $fc'e'$ in Fig. 190), the voltage at the intermediate loads is somewhat higher than would be required by an ideal flat-compounding. The ideal straight line is marked $fb'e'$. To keep the voltage absolutely constant, small adjustments of the shunt-field rheostat must be made by hand, unless an automatic voltage regulator is provided in addition to the series winding. The existence of this "hump" on the voltage curve may be explained as follows: Let the required number of field ampere-turns at no load be 8000; these must be provided by the shunt winding (see table below). Let the total required number of exciting ampere-turns at full load for flat compounding be 11,000. Of this number the shunt winding furnishes 8000 and the series winding 3000. At half load the machine is not as highly saturated as at full load, so that the additional number of turns is less than 1500 and may be only 1000. Thus the total excitation actually required at half load for flat compounding is 9000. But in the machine the series-field winding furnishes 1500 ampere-turns at half load, so that the total excitation at this load is at least 9500. In reality it is more, because the current in the shunt-field winding is increased by the higher armature voltage. The resulting excitation is therefore higher than that required to produce the normal voltage, and the " hump " in the curve results.

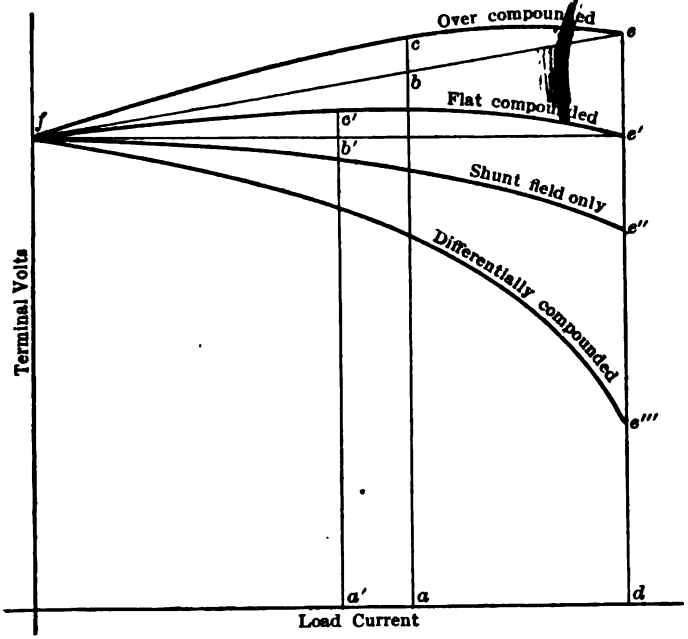


FIG. 190. Effect of series-field winding on voltage characteristics of a generator.

Per cent load	Excitation required	Excitation actually obtained
0	8,000	8,000
50	9,000	over 9,500
100	11,000	11,000

It is difficult for a manufacturer to design a machine with just the right number of series turns for a desired percentage of voltage rise. The permeability of iron may be somewhat different from that assumed in the calculations, and the armature reaction cannot be predetermined with very great accuracy. Moreover, different users may desire the same type of machine with different degrees of over-compounding. Therefore generators are usually provided with a number of series turns sufficient for a reasonably high degree of over-compounding. The *series winding is shunted* or weakened, as desired, by resistance strips or grids (Figs. 188 and 189), so that only a part of the total current flows through the series field. By varying the resistance of the shunt, a desired characteristic may be obtained without changing the design of the machine.

If the terminals of the series-field winding be reversed, the machine is said to be *differentially compounded.* In this case the series excitation opposes the shunt-field excitation, and increases the voltage drop for any given load. Such machines are used for special purposes only, for example, when it is necessary to limit the load current to a safe maximum value as the load resistance is decreased. The characteristics of the machine approach those of a constant-current generator rather than those of a constant-voltage generator.

In view of the deviation of the external characteristic (Fig. 190) of a compound-wound generator from the straight line, it is convenient to characterize this departure as the ratio (or percentage) of the maximum departure from the straight line to the straight-line voltage at the same point. The straight line is drawn between the points of no-load and full-load voltage on the actual curve. For the upper curve the departure ratio is bc/ab; for the flat-compound curve the departure ratio is $b'c'/a'b'$. In an ideal machine, without saturation, the voltage curve becomes a straight line and the percentage departure becomes zero.

243. Number of Turns in the Series Winding. — The necessary number of turns in the series winding for flat compounding may be determined by successive trials, or by the following experiment. Let the machine under test be a 110-volt, 200-ampere generator, and let it give its rated voltage at no load with a field current of 5.5 amperes. With the series winding disconnected, and the field rheostat setting unchanged, the voltage will drop as the machine is loaded. Suppose further that the field current must be increased to 6.1 amperes to maintain the rated terminal voltage at full load. If the total number of turns in the shunt winding is 2400 per pole, then $(6.1 - 5.5) \times 2400 = 1440$ ampere-turns, represents the additional excitation to be furnished by the series winding. At full load the current is 200 amperes; consequently there must be about 8 turns in the series winding $(200 \times 8 = 1600)$. This is somewhat larger than is necessary, but an allowance must be made to compensate for the ohmic drop in the series winding itself, and the surplus may be reduced by a resistance shunt around the series turns.

The number of turns required for a certain degree of over-compounding may be determined in a similar manner. With the same data as before, let it be required to provide a sufficient number of turns in the series winding to increase the full-load voltage to 116 volts. To determine experimentally the required number of series turns, load the machine, using the shunt field only, until it gives 116 volts at 200 amperes. Let the necessary field current be 6.8 amperes, or the total number of exciting ampere-turns per pole 6.8 × 2400 = 16,320. With the series winding in the circuit, the shunt current at full load will be 5.5 × 116/110 = 5.8 amperes, and the corresponding number of ampere-turns will be 5.8 × 2400 = 13,920. Thus the series winding will have to supply 2400 ampere-turns, which means 12 turns at 200 amperes.

If the number of turns in the shunt winding is not known it can be determined experimentally. Excite the machine separately by an auxiliary winding of a known number of ampere-turns and measure the terminal voltage at no load. Determine the current in the shunt-field winding necessary to induce the same voltage at the same speed. Let it take 72 turns of the auxiliary winding at a current of 100 amperes to produce 50 volts at the terminals, and let it take 3 amperes to induce the same voltage by using the shunt winding. The voltage in the two cases being the same, the flux per pole is also the same, and consequently the numbers of exciting ampere-turns must be equal. Thus the unknown number of turns in the shunt winding is 100 × 72/3 = 2400.

The number of turns in the shunt-field winding may also be determined ballistically (§ 158) by providing one of the poles with an auxiliary exciting winding of a known number of turns and with a tertiary winding connected to a ballistic galvanometer. The armature is kept stationary and the shunt-field winding on the pole under test is disconnected from the rest of the pole windings. The pole under test is then excited once by means of the regular field coil and then by means of the auxiliary winding. The two currents are so adjusted that the deflections of the ballistic galvanometer upon a reversal of the current are the same in the two cases. This means that the flux, and consequently the number of exciting ampere-turns, is also the same. Thus we have the condition $n_1 i_1 = n_2 i_2$, from which the number of turns in the shunt-field winding may be computed.

244. EXPERIMENT 11-A. — Exercise in Compounding a Direct-Current Generator. — The purpose of this experiment is to investigate the action of the compound winding explained in §§ 241 to 243. The machine used for this experiment should be a shunt-wound generator, and the student himself is expected to predetermine and provide a suitable series winding for different amounts of voltage rise.

(1) Take an external characteristic of the machine as a shunt-wound generator, without the series winding (§ 235). Set the field rheostat so as to get the rated voltage at full-load current. Keeping the rheostat in this position, increase the load in steps until the machine is considerably overloaded. At each step record the field current, terminal volts, and the load current. This will give the data necessary for calculating the voltage regulation of the machine without the series winding.

(2) To obtain the number of turns that must be supplied by the series winding for flat compounding, measure the field current necessary to produce the normal voltage at no load and subtract this current from that required to produce the normal voltage at full load. Determine the number of turns in the shunt winding by first taking a few points on the no-load saturation curve, using the shunt winding alone. Then place a known number of turns on the field poles and determine the values of current in this auxiliary winding necessary to produce the same voltages as before. In this last run, the shunt winding must be kept open. From these data compute the required number of turns in the series winding to make the generator flat compound (§ 243), and actually place a slightly greater number of turns on the machine. The number of turns in the shunt winding can also be measured ballistically, as is explained in the preceding section.

(3) Adjust a shunt around the series winding so as to have the same voltage at full load as at no load, and take data necessary for plotting an external characteristic of this compound-wound generator at normal speed and normal full-load voltage.

(4) Determine the number of series turns necessary to over compound the machine, say by 10 per cent, and check the results experimentally. Shunt the new series winding by a variable resistance and adjust it so as to make the machine flat compound as before.

(5) Reverse the connections to the series-field winding and take the data necessary for plotting an external load characteristic, beginning with the rated voltage at no load (differential compound winding).

(6) Disconnect the shunt-field winding and take the necessary data for plotting a curve of the voltage produced by the series field alone (series-wound generator, § 240).

Report. (1) Plot the external characteristics of the machine without the series winding, and for all the cases where it was used. (2) Plot the saturation curves obtained by using the shunt winding alone and the auxiliary turns alone. Explain how the number of turns in the shunt winding was obtained from these two curves and give the numerical results. (3) Explain how the number of turns in the series winding was predetermined to make the generator flat compound, and over-compound. (4) Give the results of applying the resistance shunt to the series winding. (5) Calculate the regulation of the machine as a shunt, flat-compound, over-compound and differentially-compound generator.

AUTOMATIC VOLTAGE REGULATORS

245. Tirrill Regulator. — The compound winding described above does not give a perfect voltage regulation, nor does it correct for speed fluctuations of the prime mover. Moreover, the machine must be made somewhat larger and heavier, in order to accommodate the series winding. Various attempts have been made to do away with the series winding and to regulate the voltage of the machine automatically — for example, by solenoids acting on the shunt-field rheostat. An objection to such regulators is that they are rather sluggish, and do not follow variations in voltage quickly enough.

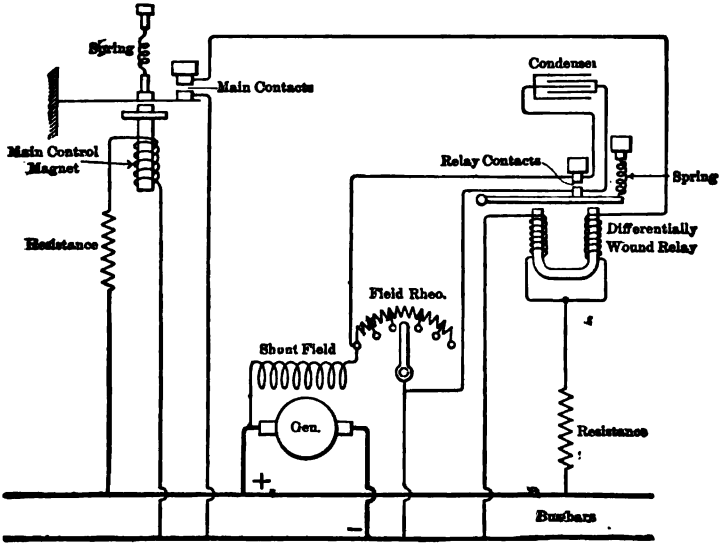


Fig. 191. Diagram of connections of a Tirrill voltage regulator for use with a direct-current generator.

Mr. Tirrill conceived the idea of accomplishing automatic voltage regulation by periodically short-circuiting the field rheostat, or a portion of it (Fig. 191). This is done by light vibrating contacts with practically no inertia. The relative lengths of time during which the rheostat is short-circuited or active, determine the average value of the field current and consequently the voltage of the machine. The leads from the field rheostat are short-circuited between the silver tips marked " relay contacts; " a condenser is placed across the contacts to reduce sparking. The closing and opening of these contacts is done by the differentially wound electromagnet, or relay, controlled by the " main contacts." The main contacts are closed and opened by the control electromagnet connected across the main bus-bars.

The action of the regulator is as follows: Let the main contacts be open; then only the left-hand side of the differential electromagnet is energized; it opens the relay contacts and introduces the field rheostat into the circuit. The terminal voltage of the machine immediately drops below normal. This reduces the current in the control electromagnet, and the spring above it overpowers the attraction of a small iron armature and closes the main contacts. Now a current flows through the right-hand branch of the differential relay and destroys its previous magnetization. The spring above it closes the relay contacts and short-circuits the field rheostat. The line voltage rises slightly above normal, the main contacts are again opened, and the operations of the two electromagnets are repeated in the same order. In reality, both contacts are continually vibrating; the periods during which the relay contacts are closed adjust themselves automatically, so as to maintain a constant voltage at the terminals of the machine.

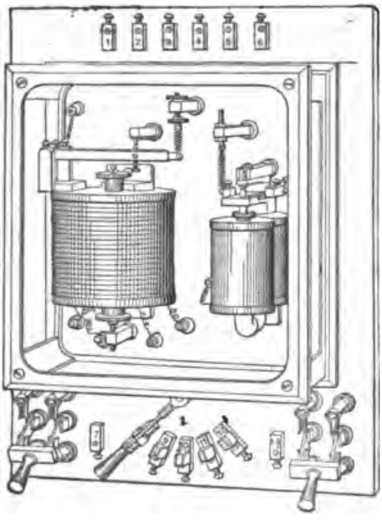


FIG. 192. Tirrill voltage regulator for direct-current generators.

An effect similar to over-compounding is produced by providing an additional differential series winding on the main control electromagnet. The voltage then rises as the load increases. It is not necessary for the whole main current to flow through this winding, but only for a small current proportional to it. To accomplish this, a low-resistance shunt is inserted in the main circuit; the series winding of the electromagnet is connected across the terminals of this shunt, as in a direct-current ammeter (Fig. 38). The series winding opposes the action of the shunt winding; therefore, with a heavy load on the line the bus-bar voltage must be higher before the control magnet can overcome the tension of its spring. In other words, the spring keeps the main contacts closed for a comparatively longer time.

Fig. 192 shows the external view of a Tirrill regulator; the main contacts are seen to the left and the differential electromagnet to the right. The radial switch below is for connecting the regulator to the field of any of the machines working in parallel. The double-throw switches are for reversing the polarity of the contacts so as to wear them equally.

246. EXPERIMENT 11-B. — Study of the Tirrill D.C. Regulator. —The regulator described in the preceding section is connected as shown in Fig. 191, and its performance studied under various conditions met with in practice. Each factor which affects the performance of the regulator should be studied separately, keeping all other factors constant. A very explicit and detailed book of instructions, furnished by the makers of the device, gives all the necessary directions for adjustments and operation. Vary the current output of the machine, the terminal voltage, the setting of the rheostats, the adjustment of the springs, etc. Vary the load gradually, and then suddenly. Observe the promptness of the action of the regulator under the extreme conditions. Devise some means for finding out if there is any flicker in the lamps connected to the terminals of a machine controlled by a Tirrill regulator. If an oscillograph is available (Vol. II) take actual curves of the terminal voltage under a few extreme conditions.

Report the results systematically, giving a theoretical explanation of the principal features of the observed performance.

247. Counter-e.m.f. Voltage Regulator. — From the above description of the Tirrill regulator it will be seen that the main contacts are closed or opened only when the line voltage is slightly below or above normal. This sometimes causes an objectionable flicker, noticeable in lamps. For this reason another regulator has been developed in which a small counter-e.m.f. motor is connected in series with the shunt-field of the generator, and the current through the latter is regulated by changes made automatically in the counter-e.m.f. of the motor. The motor field is energized from the bus-bars through a resistance. This device is also provided with a main control magnet and contacts corresponding to the main contacts of the Tirrill regulator. These contacts are connected across the motor field so that they short-circuit it when closed.

The counter-e.m.f. motor is equipped with an eddy-current brake consisting of a low-resistance metal disk which rotates in a strong magnetic field produced by one or more magnets energized by the generator field current. This brake prevents the motor from over-speeding and gives it greater uniformity of rotation.

The generator field rheostat is so adjusted that without the regulator it holds the voltage somewhat above normal. Consequently, the control contacts are originally open, putting full field on the motor. The counter-e.m.f. induced in the motor opposes the generator field current and at once lowers the generator voltage. When the voltage is lowered sufficiently, the main contacts close, short-circuiting the motor field and thus lowering the counter-e.m.f. This again causes the generator voltage to rise slightly, and the same cycle is repeated. The contact action is so rapid that an average field is held on the counter-e.m.f. motor, and its action varies from time to time with changes in the load and in the speed of the main generator.

Sometimes an additional winding is provided over the main winding on the control magnet, to prevent hunting and to insure a speedy contact action. The leads to this "speed" winding, when used, should be connected to give the most rapid regulation. This anti-hunting winding opposes the main winding when the contacts are open, thus causing them to close rapidly again, preventing the building up of the counter-e.m.f.

A detailed instruction book and an exact diagram of connections are furnished by the makers of this device. For an experimental study of it, see the preceding experiment on the Tirrill regulator.

OPERATION OF GENERATORS IN PARALLEL

248. It is common practice in power houses to have more than one generator connected to the same bus-bars and operated simultaneously (Fig. 193). This gives a more flexible arrangement than one large machine of a size sufficient to supply the heaviest load of the day. The chief advantages of having more than one machine are:

(1) The number of machines actually used during certain hours can be made to correspond more closely to the demand; this permits the operation of the machines near the point of their maximum efficiency.

(2) An accident to one machine does not shut down the whole station.

In large power houses it is necessary to use several machines, simply because the total output is considerably above the capacity of the largest generator built.

With the system of distribution of electrical energy used at present (constant-potential system), machines which operate simultaneously are connected *in parallel*, so that each runs at full voltage and supplies a part of the current demand.

Certain conditions must be fulfilled in order that two or more machines may supply power to the same bus-bars without interfering with each other, and that each may take a proportionate share of the total load. When but one machine is connected to the line, the power generated can only flow into the load. When, however, two generators are connected to the same line, the current from the first machine, instead of flowing to the line, may also flow into the second machine, driving it as a motor. In this case, the second machine not only does not help the first one, but constitutes an additional useless load for it. Even if this is prevented, and both machines are working as generators sending power into the line, it does not necessarily follow that each machine is taking its proper share of the load. The conditions necessary for satisfactory parallel operation of two or more direct-current machines are discussed in the following articles. Shunt-wound machines are considered first, and then compound-wound machines.

249. Shunt-Wound Generators in Parallel. — Two identical shunt-wound machines driven by identical prime movers and with the centrifugal governors properly set, usually run satisfactorily in parallel, and automatically divide the load equally, for there is no reason why one machine should carry a larger share of the load than the other. Should one machine tend to take more load, its voltage would drop, and the other machine with its higher voltage would automatically share this load.

A diagram of connections for two shunt-wound generators in parallel is shown in Fig. 193. Let machine No. 1 be supplying the whole load at a certain time, and then let the load increase to such a value that it becomes necessary to add the other machine. Generator No. 2 is first brought up to speed and excited so as to give the same terminal voltage as No. 1 (and the right polarity, of course); then its main switch is closed. The first machine still continues to supply the total load, unless the excitation of the second machine is somewhat *increased* and that of the first machine *reduced*. By regulating the field rheostats of both machines the load can be divided in any desired proportion. One voltmeter only is used with any number of machines in parallel. Each machine is provided with voltmeter terminals, such as p_1 and p_2; a plug is inserted in one of the receptacles, and thus connects the voltmeter to the desired machine.

If it is desired to disconnect machine No. 1 and to transfer the whole load to the other machine, this should *not* be done by simply opening the main switch of the first machine. This would cause an arc at the switch, the line voltage would suddenly drop, and the second machine might be damaged by the extra load suddenly thrown upon it. This may not be noticeable on small laboratory generators, but is of consequence with large machines. Therefore, before the main switch of one of the machines is opened, its field excitation must be *lowered* and that of the other machine or machines *increased*, until the machines which are to continue in operation carry practically the whole load. Then the main switch of the first machine can be opened without disturbing the established electrical relations. In some cases the throttle on the prime mover also has to be adjusted in transferring the load from one machine to the other.

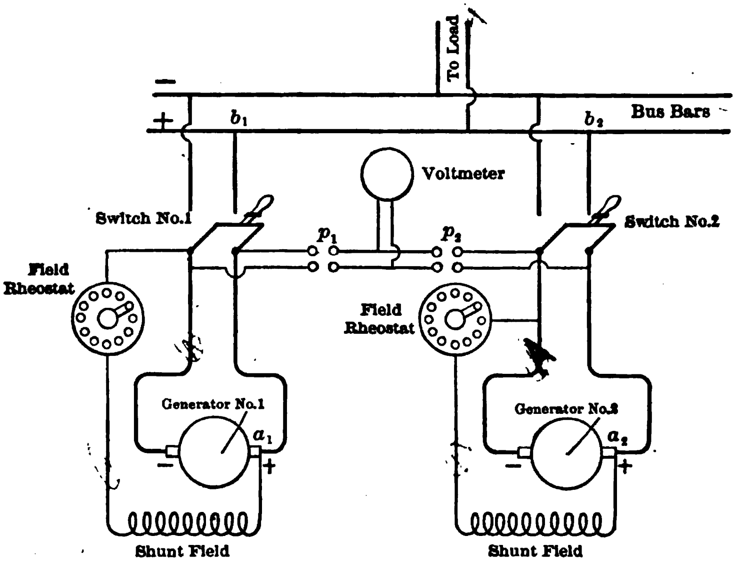


FIG. 193. Two shunt-wound generators in parallel.

By reducing the field excitation of the first generator below the no-load point, its armature current is reversed, and the machine begins to run as *a shunt motor*, driving its prime mover. This, of course, must be avoided in practice, but should it accidentally occur, no particular harm would result, as the machine continues to run in the same direction. In some installations, reverse-current relays are provided in connection with circuit-breakers to open the circuit of the machine in case the current becomes reversed.

If the two machines have different voltage characteristics (Fig. 194), that is to say, if the curve of voltage drop in one differs from that in the other, it may easily occur that while the machines divide the current properly at a certain load, the machine which has the smaller internal drop becomes relatively overloaded on heavier loads, and vice versa. Thus, in Fig. 194 the load currents *ab* and *ab′* are equal to each other, but the load current *cd* is considerably higher than *cd′*. At this latter load the machine whose characteristic is shown to the left will carry less than its proportionate share of the load. Under such circumstances it may be necessary to connect some resistance in the armature circuit of the better machine, until it gives the same per cent drop as the other. While it is not always possible to make the two external characteristics identical by this means, the two machines can usually be made to work satisfactorily under all load conditions unless the saturation of their magnetic circuits and the percentages of armature reaction differ too greatly.

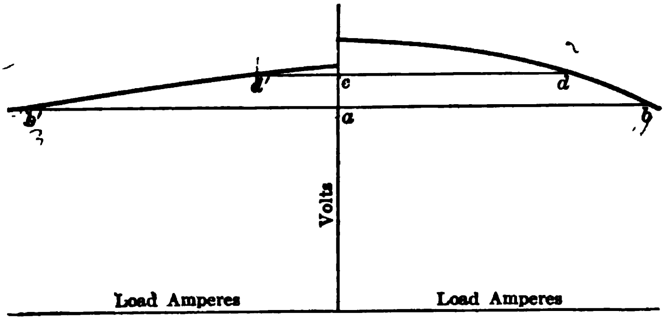


Fig. 194. Division of load between two generators of different characteristics.

250. EXPERIMENT 11-C. — Operation of Shunt-Wound Generators in Parallel. — The student is given two shunt-wound generators of different types with different characteristics. The machines are to be connected in parallel, as shown in Fig. 193, and loaded on a rheostat. An ammeter should be used to measure the total load and one between each machine and the bus-bars. Ammeters should also be provided in the field circuits.

At first take a moderate load and practice transferring it from one machine to the other by regulating the field rheostats. Start and stop one of the machines while the other is running, without disturbing the load. Weaken the field of one of the machines until the current in it is reversed and it is driven as a motor by the other. Try to run one or both machines in parallel with the laboratory direct-current supply.

After this, investigate whether the machines divide the current properly at all loads. Take a load nearly equal to the total capacity of the two machines, and adjust the field rheostats so that each machine takes its share of the load. Read the load current and voltage; also the field current, the load current, and the speed of each machine. Gradually reduce the load and at each step read the same quantities. If the machines do not divide the load even approximately in the correct proportion, introduce some resistance into the armature circuit of the machine that has the tendency to take a larger share of the load. Adjust the resistance to the best value, and then take a complete curve from full load to no load, showing the division of current between the two machines.

251. Compound-Wound Generators in Parallel. — Two compound-wound generators connected in parallel to the same bus-bars, are shown in Fig. 195. The general scheme of connections is the same as in Fig. 193, except for the addition of the series windings and the equalizer connection explained below. If necessary, the series windings are provided with resistance shunts to insure approximately the same degree of compounding in each machine (Figs. 188 and 189). It may seem at first that the addition of series windings should in no way affect a satisfactory operation of the machines, and that all that has been said in regard to shunt-wound machines is directly applicable to compound-wound generators in parallel. Experience shows, however, that two compound-wound generators in parallel do not operate in a stable manner unless an *equalizing* connection, $a_1E_1E_2a_2$, is established between the brushes a_1 and a_2, adjacent to the series windings.

The reason for this is as follows: Let the two machines be identical and let the switch E_1 be open at first, so that there is no equalizing connection; let machine No. 2 for some reason have a slightly higher induced e.m.f. This machine will then take a slightly larger share of the load than No. 1. Therefore, a larger current will flow through the series winding of No. 2 than through that of No. 1. The e.m.f. of No. 2 will thereby be *still more increased*, and the share of the load of the other machine decreased. This will continue until machine No 2 carries the whole load, while the other machine runs at no-load. Unless the machines are provided with reverse-current circuit-breakers, generator No. 2 will reverse the current in No. 1 and will drive it as a motor. Such an incident would do no harm to a shunt-wound machine, but with a compound-wound machine the reversed current flowing through the series field weakens the total field, and the machine begins to run faster and faster, driving its prime mover. This causes the other machine to become heavily overloaded, until finally one of the circuit-breakers disconnects the two machines.

In a machine provided with interpoles, the interpole winding must be considered as part of the armature and not as part of the series field.

The equalizer connection must therefore be made between the main series winding and the interpole winding, and not directly to the brush terminal of the machine. Otherwise the equalizing current will flow not only through the series coils but also through the interpole coils, thus making the commutating field either too strong or too weak for the actual armature current.

252. Action of the Equalizer. — An equalizer connection between the points such as a_1 and a_2 (Fig. 195) remedies the foregoing unstable operation. Its action is as follows: Let, as before, the two machines be identical, but suppose that for some reason the induced e.m.f. of machine No. 2 is higher, and therefore the current supplied by it is larger, than that of machine No. 1. This tends to make the ohmic drop between a_2 and b_2 larger than that between a_1 and b_1. But with an equalizing cable of negligible resistance between a_1 and a_2, this is impossible, and thus part of the current of machine No. 2 flows to the positive bus-bar through the equalizer and the series winding of machine No. 1. Therefore, the field of machine No. 2 is relatively weakened and that of the weaker machine, No. 1, is strengthened by the excess of the current of the other; machine No. 1 is thus helped to keep up its voltage. In short, *the equalizing connection prevents the currents in the series fields of two or more identical machines from differing widely from each other, however different their armature currents may be.*

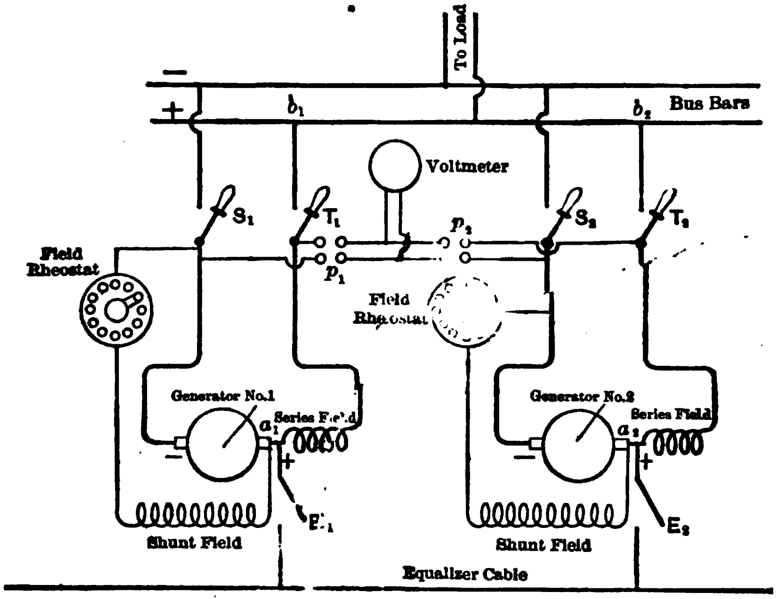


FIG. 195. Two compound-wound generators in parallel.

Therefore, when connected by an equalizing bus-bar or cable, two or more identical machines cannot have widely different voltages, nor can their load be disproportionately distributed. Even if one machine should pump power back into the other, the series field current cannot become reversed. The equalizer is mainly useful on the rising part of the external characteristic (Fig. 190). Beyond the hump in the curve, if one machine tends to take a larger load its voltage will drop and the other machine will be made to take more load also. The operation is thus seen to be similar to that of two shunt-wound machines in parallel.

A difficulty arises when the machines have *different characteristics*, as is explained in the case of the shunt-wound generators. The two conditions necessary for satisfactory parallel operation of two compound-wound machines of different types are as follows:

(1) Both machines must have similar external characteristics when running separately. If such is not the case, the machine which has a higher degree of compounding will take a larger share of the total output as the load on the two increases, because its induced e.m.f. will be higher than that of the other machine. The equalizing connection cannot remedy this fault.

(2) When each machine is delivering its proper share of load, no current should flow through the equalizer. This means that the resistances of a_1b_1 and a_2b_2 must be inversely as the rated currents of the respective machines. As a general rule, the resistances a_1b_1 and a_2b_2 do not satisfy this requirement and it becomes necessary to introduce additional resistance in one of the circuits. Then the machines will divide the current properly at all loads.

253. Directions for Connecting in Parallel. — Let machine No. 1 be supplying the load alone, and let it to be required to connect generator No. 2 in parallel with it. The main switches S_2 and T_2 are open, as well as the equalizer switch E_2; the equalizer switch E_1 being considered closed. As a first step, machine No. 2 is brought up to speed and excited so as to give a voltage slightly below that across the bus-bars. Then the switches E_2 and T_2 are closed, causing part of the main current of machine No. 1 to flow through the series field of machine No. 2, raising its voltage. Then, after the voltage of machine No. 2 has been properly adjusted to that of the bus-bars, the last switch, S_2, is closed and the excitation brought up so as to distribute the load properly between the machines. In disconnecting one of the machines the switches should be opened in the reverse order. The fundamental rule to be followed either in connecting or in disconnecting a machine is to do it with as little electrical disturbance as possible. This means

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that the load must be transferred from a machine before it is disconnected from the line, and its voltage must be made as nearly equal to the bus voltage as possible before it is thrown in again.

With large machines it is desirable to have three separate switches, as described above. With medium size machines the two switches connecting a machine to the bus-bars can be united into one double-pole switch, though in this case the machines cannot be thrown in parallel as smoothly, because the main circuit is closed before the field is properly adjusted. The double-pole connecting switches can be used to better advantage if a second equalizer connection is established between the other extremities of the series-field coil, so that merely closing the equalizer switch will connect the two series windings in parallel. In this case an equalizer switch is placed in the second equalizer connection and switches E_1 and E_2 are omitted. In inexpensive isolated plants, three-pole switches are sometimes used, combining all the three switches into one; this is done when a momentary rush of current is not considered harmful.

254. EXPERIMENT 11-D. — Operation of Compound-Wound Generators in Parallel. — The student is given two direct-current generators of somewhat different characteristics. The machines may be provided with series windings beforehand, or the student himself may be expected to put them on. As is explained in §§ 248 to 253, three conditions are necessary in order that the machines may operate satisfactorily in parallel:

(1) An equalizing connection must be provided.

(2) Both machines must have similar external characteristics when running alone.

(3) The voltage drop in a_1b_1 must be equal to that in a_2b_2.

First try to run the machines without any of these conditions being fulfilled and see what happens. Connect an ammeter in the equalizer circuit, introduce the necessary modifications, one by one, and observe the improvement in operation. Record the observed data throughout the experiment, so as to be able to describe the performance intelligently in the report. Work on the set until a satisfactory operation has been obtained at all loads, and take data for the curves showing the division of the load between the two machines.

The performance is considerably influenced by the resistance in the equalizer circuit. The lower this resistance the better is the equalization of the series exciting currents and therefore the more satisfactory the operation of the machines. Take several readings at a constant load but with different resistances in the equalizing circuit, so as to observe the influence of this factor. Make a few runs to ascertain how the same machines divide the load at different speeds, or at values of terminal voltage different from those used in the first run.

Report. (1) Plot the external characteristics of the two machines and explain the necessary adjustments for the best operation in parallel. (2) Describe what happened when you tried to operate the set without the equalizing connection. (3) Explain the extent of the unsatisfactory operation when the resistances of a_1b_1 and a_2b_2 were not inversely proportional to the rated currents of the machines. (4) Plot a curve showing the division of the current between the two machines at various loads under the best conditions you were able to obtain. (5) Report your findings regarding the effect of the resistance in the equalizer connection upon the operation in parallel. (6) Explain the operation at a speed or at a voltage different from the rated values.

THE THREE-WIRE SYSTEM

255. There are cases in which it is of advantage to distribute direct-current energy by three wires instead of two (Figs. 196 to 199). The two outside wires are the main wires; the middle conductor carries the current corresponding to the unbalanced part of the load. The three-wire system is used in two principal cases:

(1) On large lighting and power circuits on which the economy in copper thus obtained warrants the complication.

(2) In some industrial establishments where adjustable-speed motors are used and a speed range of two-to-one is obtained by running the motors at full voltage and at half voltage (§ 275).

The present tendency is away from the three-wire system because of the gradual change to alternating currents for lighting and power. A speed range as high as three-to-one and more can be obtained with modern commutating-pole motors (§ 268) by field control on an ordinary two-wire system. Nevertheless, there are a great many three-wire direct-current installations in actual operation, and it is essential that the student become familiar with their fundamental electrical relations.

In a three-wire lighting circuit the lamps are connected between the middle wire and either of the outside wires (Fig. 199). The voltage across the outside mains is usually about 220 volts, and the copper economy (not counting the middle wire) corresponds to this pressure; at the same time standard 110-volt lamps are used. The weight of copper with the same power output and with the same percentage of line loss is inversely proportional to the square of the line voltage. The cross-section of the middle conductor need not be more than 25 per cent of that of either of the outside wires, so that the addition of a third wire on a 220-volt system still leaves a considerable margin of economy, as compared to an ordinary 110-volt two-wire distribution.

256. Methods of Dividing the Voltage. — The two outside wires in the three-wire system are connected to the terminals of the main generator (Figs. 196 to 199); special devices are used for dividing the voltage so as to obtain a point for connecting the middle conductor. The methods in common use are:

(1) Balance coils (Fig. 196).
(2) Motor-generator or balancer set (Figs. 197 and 199).
(3) Storage battery (Fig. 198).

The original method of obtaining a three-wire system was to use two direct-current machines in series and to connect the outside terminals to the main conductors, while the common terminal was connected to the neutral wire. This method is hardly ever used, except perhaps in the case of high-voltage direct-current generators for traction purposes.

(1) The balance coils (Fig. 196) are ordinary reactance coils or autotransformers (§ 395) provided with taps B_1, B_2 at the center. The generator G has four slip-rings S, S, connected to four equidistant points of the armature winding, as in a two-phase rotary converter. The balance coils form a star-connected quarter-phase system, and the middle wire c is connected to the neutral point of this polyphase system. This neutral point being symmetrically situated with respect to the outside wires a and b on the direct-current side, its potential is always midway between the potentials of these wires, and thus the voltage is divided into two equal parts. The energy consumption in the balance coils is small, their inductance being large as compared to their resistance.

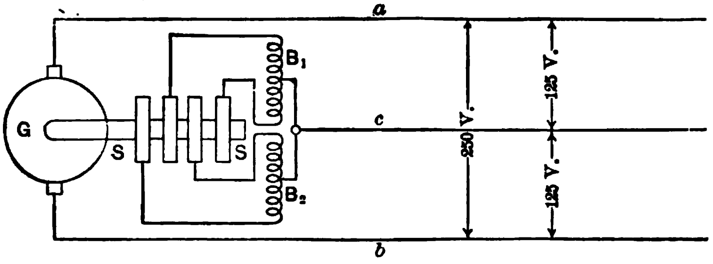


Fig. 196. Dividing line voltage in two by means of balancing coils.

Instead of two coils and four slip-rings, three slip-rings and three coils are sometimes used in a three-phase Y-connection. The middle conductor is connected to the neutral point of Y. In order to reduce the number of slip-rings the balance coils are sometimes made to revolve with the armature of the generator. In this case only one slip-ring is necessary for connecting the neutral wire to the mid-points of the balance coils. This method of dividing the voltage is particularly well adapted for use with rotary converters, because they are already provided with slip-rings. Even special balance coils may be often dispensed with, because it is usually possible to obtain a neutral point on the main transformers. For a description of rotary converters and polyphase systems see Vol. II.

(2) The balancer set (Figs. 197 and 199) consists of two identical shunt-wound machines, B_1 and B_2, of comparatively small current capacity. The armatures are directly connected mechanically and are connected in series electrically, between the two outside wires. The middle wire is connected at *c*, between the two armatures. The two machines of the balancer set being identical, excited to the same voltage, and run at the same speed, the voltage is divided at *c* into two equal parts. When the load becomes unbalanced, one of the machines runs as a generator and the other as a motor; the voltages still remain approximately the same, the induced e.m.f.'s being approximately equal. The details of the distribution of currents are explained in the next article.

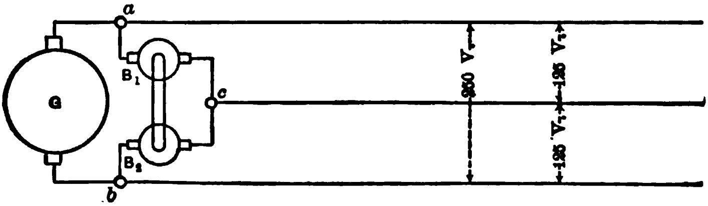


FIG. 197. Dividing line voltage in two by means of a balancer set.

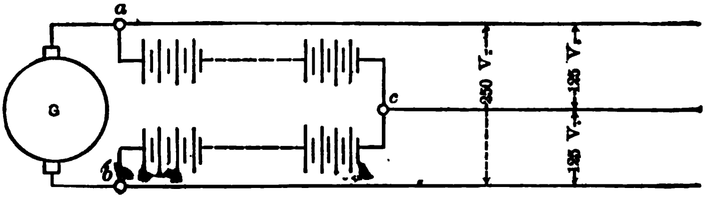


FIG. 198. Dividing line voltage in two by means of a storage battery.

(3) A storage battery may also be used for dividing the voltage (Fig. 198), but the comparatively high cost and depreciation of storage cells prevent their extensive use for this service.

An advantage of balance coils over a motor-generator set is that the former are less expensive and require but slight attention. On the other hand, a three-wire generator with slip-rings is more expensive than an ordinary two-wire machine. Here, as in most practical problems, preference is given to one or the other system according to the local conditions in each particular case.

257. Effect of Unbalancing. — The distribution of currents in a three-wire system with an unbalanced load is shown in Fig. 199, the total voltage of 220 being divided in two by a motor-generator set. It is assumed that the load is 500 amperes on one side and 400 amperes on the other. The difference, equal to 100 amperes, returns through the middle wire to the balancer set. The balancer set, not being connected to a prime mover, cannot generate additional electric energy; it merely equalizes or transfers power from the less loaded side to the more heavily loaded side. If the machines were ideal, 100 amperes would be divided into 50 and 50 in each machine, flowing in the opposite directions, the lower machine running as a motor, the upper as a generator. The main generator would supply the average of the two currents, *i.e.*, 450 amperes at 220 volts. With this distribution of currents, Kirchhoff's first law is fulfilled at points *a* and *b*, namely $500 = 450 + 50$ and $400 = 450 - 50$.

In reality, the current distribution is somewhat different, because of the losses and the voltage variations of the balancer set. A certain no-load current x flows through the set even when the load is perfectly balanced; this current is necessary for overcoming the iron loss and friction in the two machines. Moreover, the main generator has to supply some current for the shunt fields of both machines. Superimposing this current x upon the useful currents mentioned above, we get the distribution of currents shown in Fig. 199. The current in the balancer machine working as a generator is reduced and that in the motor increased by the amount x. The condition $\Sigma i = 0$ at the points *a* and *b*, is again fulfilled.

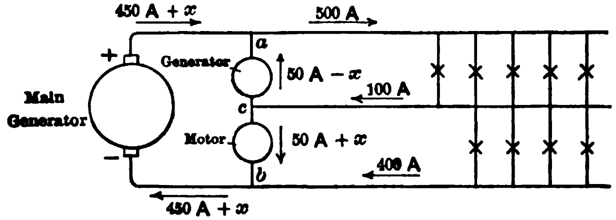


Fig. 199. Distribution of currents in a three-wire system with an unbalanced load.

With an unbalancing of load currents, the voltages between the middle wire and each of the two outside conductors become somewhat different. With a properly designed balancer set, the difference should not be more than a very few per cent; moreover, it may be corrected to some extent by regulating the field rheostats of the balancer machines. An investigation of the current relations in the balance coils (Fig. 196) leads to similar results.

258. Efficiency of a Balancer Set. — Referring to Fig. 199, the efficiency of the balancer set is equal to the ratio of the output of the machine working as a generator, to the input into the machine working as a motor; the excitation losses of both machines should be added to the input. This is according to the standard definition of efficiency of any motor-generator set. It will be easily seen, however, that in the case of a balancer set the efficiency, according to this definition, becomes negative when the load is nearly balanced, because then both machines operate as motors, each overcoming part of the total losses. As unbalancing increases, the efficiency gradually rises to zero and then becomes positive. There is no physical contradiction in this result, provided that the actual conditions of operation of the set are clearly kept in mind.

There is another way of looking at the efficiency of a balancer set, namely from the standpoint of the three-wire system as a whole. The losses in the balancer set increase the output of the main generator above the useful demand for power; the ratio of this demand to the output of the main generator may be called the efficiency of the three-wire system, with reference to the balancer set.

To illustrate the above definitions of efficiency, assume that the no-load current of the balancer set, x, is 10 amperes (Fig. 199), and that the field circuit of each machine consumes 2 amperes, both fields being connected in parallel across the outside conductors.

The efficiency of the motor-generator set itself, according to the first definition, is

$$\frac{(50 - 10)\,110}{(50 + 10)\,110 + (2 + 2)\,220} = 70.6 \text{ per cent.}$$

The efficiency of the three-wire system, according to the second definition, is

$$\frac{500 \times 110 + 400 \times 110}{(450 + 10 + 2 + 2)\,220} = 97 \text{ per cent.}$$

Each expression has a definite physical meaning, and either is used, according to the conditions of the problem. If a greater accuracy is required, the copper loss in the armatures of the balancer set, and the resulting unbalancing of the voltages, must be taken into account in the above expressions for efficiency.

259. EXPERIMENT 11-E. — Study of a Three-Wire System. — Wire up the generator and the balancer set as in Fig. 197, and provide variable resistances to be used as a load. Load one side of the line until the balancer set runs as nearly as possible at its full rated capacity. Note the input and the output of the two machines of the balancer set, the voltages on the loaded and unloaded sides of the line, the speed of the set, the currents in the three line wires, and the field currents of the balancer machines. Then gradually load the other side of the line, keeping the load on the first side and the total voltage constant. As the unbalancing decreases the balancer load decreases, though the total generator load increases. Take readings as indicated above with different degrees of unbalancing, until the load on both sides is the same, and the balancer runs idle. Increasing the load further will merely reverse the conditions in the balancer set: the machine which has been running as a motor will run as a generator, and vice versa.

If a generator with slip-rings is available, or a rotary converter, a three-wire system with balance coils (Fig. 196) may be tested. It is not absolutely necessary to have three or four slip-rings; if only two slip-rings are available, the neutral point may be obtained with one balance coil. In fact, it is desired that the student investigate the effect of using only one balance coil instead of two. With a three-phase rotary converter, the three balance coils are connected in Y, and the middle wire connected to the neutral point. The same load experiment should be performed with balance coils as with the motor-generator set. Note that a balance coil consumes a magnetizing current, which is an alternating current, and cannot be detected by a moving-coil ammeter (§ 42).

Report. Draw a diagram of actual connections. To amperes in the main generator as abscissæ plot the currents in the two outside wires and in the neutral, the voltages, the generator output, and the currents in the balancer armatures and fields; also the speed of the set. Show that the current readings satisfy Kirchhoff's first law at points *a*, *b*, and *c*. Figure out the efficiency of the balancer set and of the three-wire system, as in § 258. Explain theoretically the observed departures in speed and voltage of the balancer set from the no-load values. Give similar results for the test in which balance coils were used.

AUTOMOBILE LIGHTING GENERATOR

260. The electric generator of a gasoline automobile is always connected in parallel with a storage battery, Fig. 200. The main purpose of the generator is to keep the battery charged, although at times the generator also furnishes current to the lamps, the horn, the ignition coil, etc. An ordinary shunt-wound machine is not suitable for the purpose, because the generator is usually geared or connected by a chain to the main engine, and thus has to run at widely different speeds. If it were adjusted to furnish the proper charging current at a certain moderate speed, it would overcharge the battery at higher speeds, and also overload itself. Of the various arrangements tried for limiting the generator current and voltage at high speeds, the third-brush excitation shown in Fig. 200 has proved to be the simplest and the most reliable.

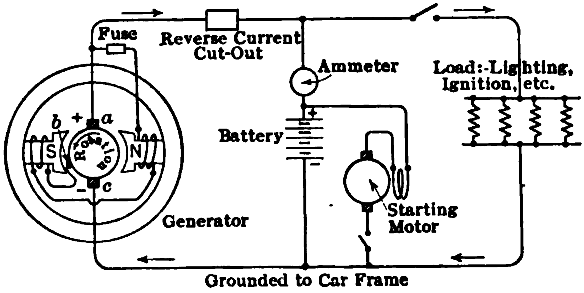


FIG. 200. The electric circuit of a gasoline automobile, showing a third-brush generator.

The third brush bears upon the commutator somewhere between the main brushes, and the shunt-field winding is connected between this brush and one of the terminals of the machine. The operation of such a generator at no load is not much different from that of an ordinary shunt-wound machine, in that the induced voltage is proportional to the speed. When, however, the generator is connected in parallel with a storage battery, the generator current affects the field and the induced voltage of the machine in such a way as to produce current-speed characteristics shown in Fig. 201. Each curve refers to a particular position of the third brush. Beginning with a certain low speed, the generator current increases and reaches a maximum, and when the speed is further increased the current drops again, never exceeding the desired safe limit. At low generator speeds, when its e.m.f. is below that of the battery, the two are automatically separated by the reverse-current cut-out shown in Fig. 200. This prevents the battery from discharging into the generator, and at these speeds the load is supplied by the battery alone.

The automatic regulation of current in this machine is due partly to the armature reaction, and partly to the currents induced in the armature coils short-circuited by the third brush. Each of these factors will now be considered separately.

261. Armature Reaction and the Effect of Short-Circuited Coils.— The difference between the flux density distribution in the air-gap of a direct-current generator at no load and with the armature loaded is shown in Fig. 237. The whole flux is shifted in the direction of the rotation of the armature. Thus, with the connections shown in Fig. 200, as the armature current increases, the air-gap flux between the brushes *a* and *b* decreases. Therefore the voltage induced in the armature between *a* and *b* is reduced, and consequently the exciting current also decreases in the same proportion, still further reducing the flux and the field current. By making the armature ampere-turns relatively large and the air-gap small, the flux is made unstable and easily shifted by the armature current. In this manner any appreciable increase in the armature current is effectively checked.

The armature coils short-circuited by the third brush are in a comparatively strong field, so that large currents are induced in them. By actually determining the direction of these currents it will be found that their m.m.f. opposes that of the field coils between *a* and *b*. Thus, these currents further reduce the flux between *a* and *b*, and the field current. The magnitude of these currents is nearly proportional to the speed of rotation, and it is for this reason that the current output of the machine decreases at high speeds (Fig. 201). The magnitude of the induced currents at a given speed depends somewhat on the brush contact resistance, and the currents may be increased by using a third brush *b* made of soft graphite.

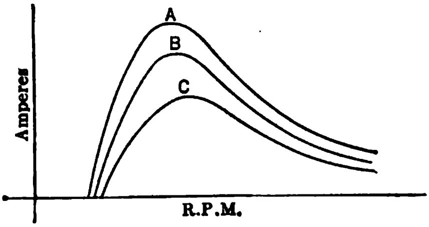


FIG. 201. Current characteristics of a third-brush generator.

262. Operating Characteristics of a Third-Brush Generator.— The three curves shown in Fig. 201 refer to different settings of the third brush. By shifting it in the direction of the rotation of the armature, the field current is increased and the curve marked *A* is obtained. The generator becomes connected to the battery at a lower speed, the charging current reaches a higher absolute maximum, and it reaches it at a lower speed. Such a setting would be used in a car for city service at comparatively low speeds, and with frequent starting and use of the lights. For summer touring at high speed when there is a tendency to overcharge the battery, the third brush should be shifted back to obtain a characteristic corresponding to curve *C*.

The maximum generator current depends on the counter-e.m.f. of the battery. When this counter-e.m.f. is high the generator voltage is also high, and so is the voltage across the field circuit. The exciting current is somewhat higher than normal, the main field is comparatively stiff and it takes a stronger armature current to shift it. The machine is thus able to maintain a higher charging rate. On the contrary, when the battery is nearly discharged, the maximum charging current that the generator is capable of delivering is correspondingly lower.

An important precaution to be observed in the operation of the third-brush generator is that the machine must not be run without the battery, unless the fuse shown in the field circuit be removed, or the armature short-circuited. Otherwise the voltage will rise to a value many times that reached in regular operation, and the insulation of the machine may be injured.

On some cars the so-called single unit is used; this acts as a motor for starting the engine, and as a generator afterwards. It is provided with a series-field winding for starting purposes and with the usual shunt-field winding connected to the third brush. In starting, the shunt field assists the series field, thus increasing the torque. In generator operation the action of the series field is differential, still further opposing the tendency of the armature current to rise at high speeds.

In at least one make of third-brush generator a thermostatic device is mounted in the machine housing for a further regulation of its output. The device has four contacts which in turn short-circuit four sections of a resistance in series with the field winding. In cold weather, or at the beginning of a run when the generator is comparatively cool, all of the field resistance is short-circuited, and the generator delivers its full output, charging the battery at a high rate. As soon, however, as the generator begins to heat up, one section of the field resistance after another is introduced into the circuit, and the generator current is materially reduced.

263. EXPERIMENT 11-F. — Characteristics of a Third-Brush Generator for Automobile Service. — Connect the machine to an adjustable-speed motor and provide some arrangement for conveniently regulating the speed within as wide limits as would be met in automobile service. A suitable storage battery must be provided, also a lamp load or a resistance load, and the connections made as in Fig. 200, omitting the starting motor. In addition to the zero-center ammeter in series with the battery, an ammeter should be inserted in the load circuit and one in series with the field winding. A voltmeter should be connected through a multi-point switch to measure at will the voltage at the generator or battery terminals, as well as the field voltage.

(1) Adjust the reverse-current cut-out to close the generator circuit at a voltage slightly above that of the battery. Keep the load switch open, run the generator at various speeds and adjust the position of the third brush for the maximum possible output. (2) Get the data necessary for the upper curve shown in Fig. 201 and also read the terminal voltage, the field voltage, and the field current. (3) Close the load switch and investigate the influence of a considerable resistance load upon the generator characteristics, and upon the charging current. Investigate also the effect of a reasonable amount of resistance in the field circuit. Shift the third brush into two or three other positions and repeat the preceding run at each position. (4) Set the third brush at the point of maximum output and investigate the influence of the battery counter-e.m.f. upon the charging rate. Take a battery in a different state of charge from the first one, or add a nearly discharged cell to act first cumulatively and then against the main battery. Finally run the generator with a very high counter-e.m.f. at its terminals and also with the armature short-circuited. (5) Run the machine carefully on open circuit at a low speed to prove that the voltage would rise to a dangerous value at higher speeds. (6) Try a hard carbon brush and a soft graphite one for the third brush, and also brushes of different width. Investigate the effect of these factors upon the performance characteristics of the generator.

Report. Plot the various characteristics to speed as abscissæ. State briefly the effect of the variable factors observed. Show that the observed changes in the characteristics could be expected theoretically, at least qualitatively.

CHAPTER XII

DIRECT-CURRENT MOTOR

264. A DIRECT-CURRENT machine, such as is shown in Figs. 174 and 175 and described in § 226, is *convertible;* that is to say, it may be operated either as a generator or as a motor. When such a machine is driven by a source of mechanical power (prime mover) it delivers electrical energy and is said to be operating as a *generator*. When electrical energy is supplied to the machine, it is capable of developing mechanical energy available at its shaft and is said to operate as a *motor.*

This convertibility is a direct result of two fundamental laws of electromagnetism; viz.: (1) When an electric conductor, forming a part of a closed circuit, is moved across a magnetic field, a current is induced in the conductor such as to oppose the motion; the m.m.f. of this current tends to strengthen the field in front of the conductor and to weaken the field behind it. (2) When a current is sent through a conductor located in a magnetic field, the conductor tends to move across the lines of force and the m.m.f. of the current strengthens the field behind the conductor and weakens it in front; the e.m.f. induced in the conductor by its motion through the field tends to oppose the passage of the original current, and is known as the back e.m.f. or *counter-e.m.f.* of the motor. In the first case the conductor acts as an element of a generator, in the second as that of a motor.

265. Types of Motors. — Three types of direct-current motors may be distinguished according to the method of field excitation, viz.: the shunt-wound motor (Fig. 202), the series-wound motor (Fig. 203), and the compound-wound motor (Fig. 204). In the shunt-wound motor the field winding is connected directly across the supply circuit, so that the field current of the motor is practically constant at all loads. In the series-wound motor, the field winding is in series with the armature circuit, and as the load increases the field current is also strengthened automatically. In a compound-wound motor (§§ 298 to 302) the series field may be connected either so as to assist the shunt field (cumulatively compounded motor) or to oppose the shunt field (differentially compounded motor).

There is a marked difference in the performance characteristics of these types of motors and each type has a field of application of its own.

The shunt-wound motor is essentially an adjustable-speed motor, while the speed of the series-wound motor increases automatically as the load decreases. A theoretical explanation of these characteristics is given below, in the treatment of these types of motors. Shunt-wound motors are used for machine-tool drive, and for driving various machines in textile and other industries, where a constant or nearly constant speed is required independent of the load. Series-wound motors are used in railway, crane, and hoisting work, where a heavy starting torque is necessary, and where the motors have to be frequently started and stopped. In these cases the load is usually fluctuating, and it is desirable to have a motor which automatically slows down as the load increases, in order not to produce too large fluctuations in power demand.

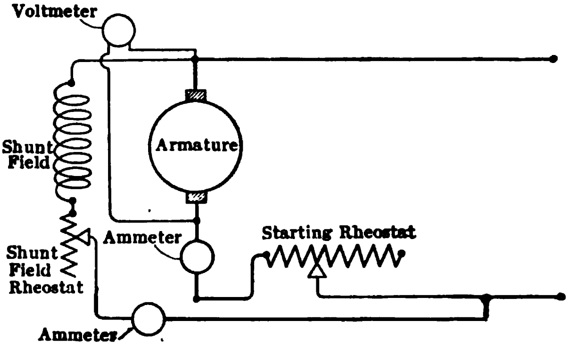


FIG. 202. Diagram of connections of a shunt-wound motor.

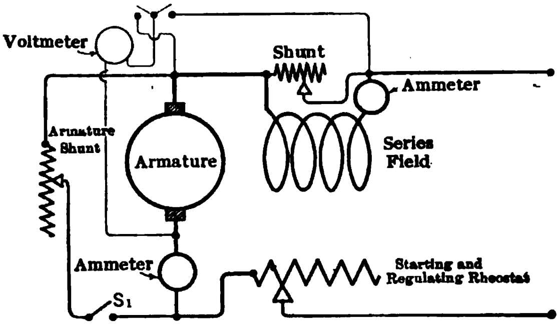


FIG. 203. Diagram of connections of a series-wound motor.

In some special cases a motor is desired, with characteristics intermediate between those of the two above-mentioned types, and for this purpose both series- and shunt-field windings are provided. Such a cumulatively compounded motor possesses a considerable starting torque and slows down with the load, but does not run away when the load is taken off. This type of motor is used with elevators, also for driving planers, punches, etc. In some exceptional cases an absolutely constant speed is required, independent of the load. A differentially compounded motor may be made to satisfy this requirement quite closely, although its speed is somewhat affected by fluctuations in the terminal voltage. A speed regulator (§ 271), will keep the speed of a motor constant even with a fluctuating terminal voltage. By suitably increasing the number of turns in the series winding, a differentially compounded motor can even be made to run faster at full load than at no load, should a particular service require such a characteristic.

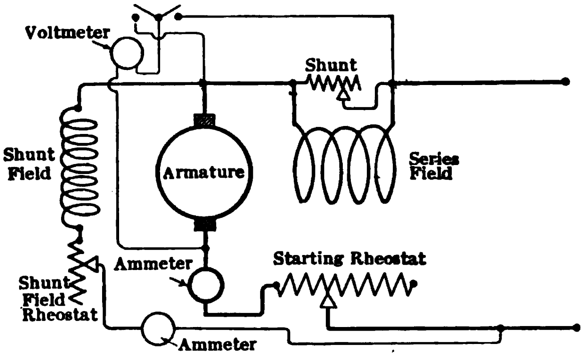


FIG. 204. Diagram of connections of a compound-wound motor.

A direct-current motor usually requires a starting rheostat, and sometimes a field regulating rheostat. In addition to these rheostats, switches may be required for reversing the direction of rotation, for changing connections, etc. Standard auxiliary devices of this kind are described in Chapter XXVI, and the reader is advised to familiarize himself at least with the simplest of these while studying the performance of direct-current motors.

THE SHUNT-WOUND MOTOR

266. The diagram of connections of a shunt-wound or separately excited direct-current motor is shown in Fig. 202. It is an adjustable-speed machine in the sense that the field strength at constant applied voltage remains practically constant and independent of the load, but may be adjusted at will by means of the field rheostat. The interaction between the armature current and the magnetic field of the poles produces a tangential force (electromagnetic torque). By means of the commutator and the brushes, the relative position of the groups of armature conductors remains unchanged in space during the rotation of the armature, so that the action persists indefinitely.

The rotation of the armature in the field causes an e.m.f. to be induced in the armature conductors, as in the D.C. generator (§ 227). According to the principle of action and reaction, the induced e.m.f. opposes the applied voltage and is therefore called the counter-e.m.f. Under normal operation this counter-e.m.f. is but a few per cent lower than the applied voltage, and the small difference between the two allows a current, just sufficient for the required load, to flow through the armature. An electric motor must develop a counter-e.m.f. in order to deliver mechanical power, because otherwise there can be no relative motion between the armature conductors and the magnetic field. On the other hand, no counter-e.m.f. is required for the starting torque, which is due to a static action between the armature conductors and the main poles.

267. Performance Characteristics. — The relationship between the applied voltage E and the counter-e.m.f. e is

$$E = e + IR \quad \ldots\ldots\ldots\ldots \quad (1)$$

where I is the armature current and R is the resistance of the armature circuit including the brushes and the interpole winding, if any. At the rated armature current, the term IR amounts to but a few per cent of E in order to keep the efficiency of the motor sufficiently high. Thus, for many practical computations, the counter-e.m.f., e, is assumed to be essentially equal to the applied voltage, especially when the value of e is not accurately known. Multiplying both sides of the foregoing equation by the armature current I we get

$$EI = eI + I^2R \quad \ldots\ldots\ldots\ldots \quad (2)$$

In this expression, EI is the input into the armature and I^2R is the heat loss in the armature circuit. Hence, according to the law of the conservation of energy, the amount eI must represent the useful output of the machine plus its iron losses and friction. This relationship makes it possible to write down immediately an expression for the electromagnetic torque between the armature and the field. The expression given in § 227 for the induced e.m.f. may be applied directly to the counter-e.m.f., e, that is,

$$e = (p/p')(\text{r.p.m.}/60)\, C\Phi \times 10^{-8} \quad \ldots\ldots \quad (3)$$

Multiplying both sides of this equation by I, we get the following expression for the motor output, including the iron loss and friction:

$$eI = I(p/p')(\text{r.p.m.}/60)\, C\Phi \times 10^{-8} \quad \ldots\ldots \quad (4)$$

Let P be an output expressed in kilowatts, and let it be required to compute the corresponding torque, T, in meter-kilograms. We have then

$$P \times 10^3 = 981 \times 10^5 \times T \times 2\pi \, (\text{r.p.m.}/60) \times 10^{-7} \quad . \; . \quad (5)$$

In this expression, the factor 10^3 converts kilowatts into watts; 981×10^5 converts kilogram-meters into ergs and 10^{-7} converts ergs into watt-seconds (joules). Solving for T, we obtain

$$T = 16.2 \, P/(\text{r.p.m.}/60) = 974 \, P/(\text{r.p.m.}) \quad . \; . \; . \; . \quad (6)$$

If T is desired in foot-pounds, we have

$$T = 974 \times 3.281 \times 2.205 = 7040 \, P/(\text{r.p.m.}) \quad . \; . \; . \quad (7)$$

The use of horse-power units for measuring the input or the output of electrical machinery is being gradually discontinued in favor of the kilowatt. Should P, in the foregoing formulæ, be expressed in horse-power, the following conversion ratios would be used: 1 British horse-power = 746 watts; 1 metric horse-power = 736 watts.

Applying eq. (6) to the output eI in eq. (4), we obtain the following expression for the torque, T, in meter-kilograms:

$$T = 0.0162 \, I(p/p') \, C\Phi \times 10^{-3} \quad . \; . \; . \; . \; . \; . \; . \quad (8)$$

or in foot-pounds

$$T = 0.117 \, I(p/p') \, C\Phi \times 10^{-2} \quad . \; . \; . \; . \; . \; . \; . \quad (9)$$

It will thus be seen that the torque is independent of the speed of the machine and is entirely determined by the current, the flux, and the armature winding. It must be clearly understood that eqs. (5) and (6) do not give the net available torque on the shaft, but the gross electromagnetic torque, which includes that for overcoming the iron loss and the friction and windage losses.

The performance characteristics of a shunt-wound motor at a constant field current (Fig. 205) may be readily understood in the light of the foregoing theory. The field flux remains essentially constant throughout the entire operation range, except for a comparatively small effect of the armature reaction (§ 358) which somewhat reduces the net flux at high values of armature current.

When the terminal voltage E is kept constant, then, according to eq. (1), the counter-e.m.f., e, decreases somewhat as the load current increases. At a constant flux, the speed of the machine decreases in the same proportion as the required counter-e.m.f., that is, but a few per cent between no load and full load, see eq. (3). Because of the armature reaction which reduces the flux, a somewhat higher speed is necessary to induce the same counter-e.m.f. On the other hand, the armature current for a given torque must be larger, so that the counter-e.m.f. need not be as great as it would have to be without the armature reaction. We thus see that the speed curve is shown drooping while the current curve is shown rising faster than the torque. The other characteristic curves shown in the figure require no explanation.

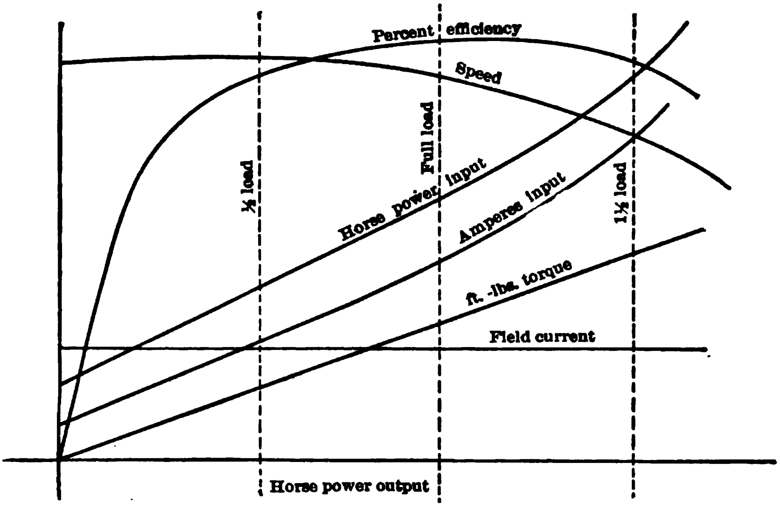


FIG. 205. Performance curves of a shunt-wound motor.

At another value of field current, a new set of performance curves must be obtained or used. The smaller the field current the higher the speed of the machine and the more noticeable the effect of the armature reaction. The machine takes a higher armature current for the same torque, but a given torque corresponds to a greater output. As in the case of the generator (§ 237), a complete picture of the operating characteristics may be obtained by using a three-dimensional surface with the armature current, field current, and speed plotted along the axes of coördinates.

268. Commutating Poles or Interpoles. — A motor intended for heavy duty, frequent overloads, or a wide range of speeds by field control, is usually provided with so-called commutating poles or interpoles (Figs. 206 and 175). The interpoles are placed midway between the main poles and are much narrower than these. The magnetic field produced in the air-gap by these auxiliary poles is concentrated over the conductors undergoing commutation. The induced e.m.f.'s due to these poles are of such magnitude and direction as to assist and to accelerate the reversal of the current in the armature conductors. The e.m.f. of self-induction in the coils under commutation is proportional to the armature current to be reversed, so that the commutating flux must also be nearly proportional to the armature current. For this reason the winding on the commutating poles is connected in series with the armature (Fig. 206), and the magnetic flux density in the interpoles is kept below the knee of the saturation curve.

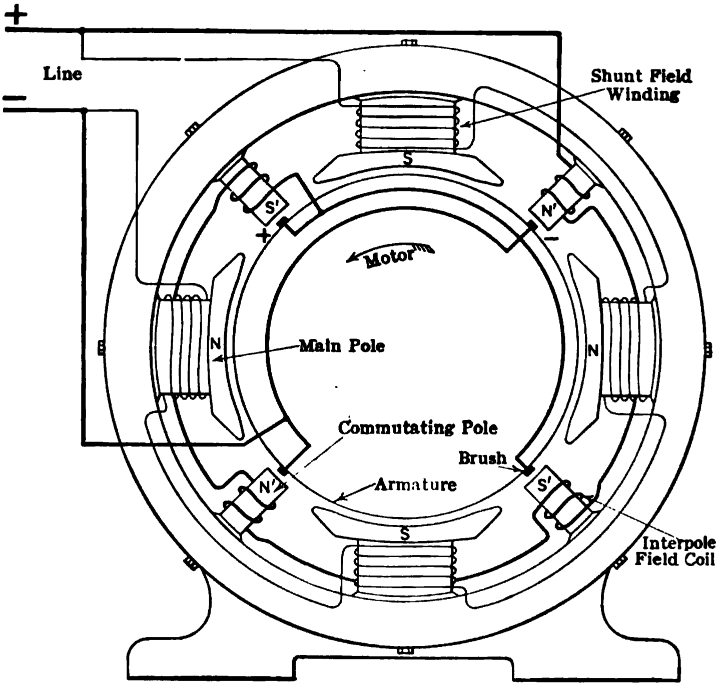


FIG. 206. A four-pole motor with commutating poles (interpoles).

In a motor without commutating poles the main magnetic field is designed to be of sufficient strength and stiffness to make satisfactory commutation possible at the rated speed. With the motor operating in one direction only the brushes may have a permanent shift from the geometric neutral. However, when the motor is intended to run in either direction, the brushes must remain in the neutral position. Some increase in speed may be obtained by weakening the field, but a limit is soon reached at which the armature reaction so distorts the field that serious sparking occurs.

In a motor provided with interpoles the design constants are different, and the magnetic field need not be as stiff. A correct commutating field is maintained independent of the main field, at all values of the armature current, so that the magnetic field can be weakened much more, and a much wider range of speeds can be obtained. Modern shunt-wound motors with commutating poles may have a speed range as high as four-to-one.

The polarity of a commutating pole in a motor is that of the trailing main pole; this corresponds to shifting the brushes against the direction of rotation in a motor without interpoles. In a generator, with the same direction of rotation, the commutating poles are of the opposite polarity.

With very difficult conditions of operation, such as sudden overloads and current reversals met with in some applications, as for example in steel-mill drive, it becomes necessary to remove the effect of the armature reaction more completely. Compensating windings are then provided on the main pole faces to prevent the shifting of the magnetic field. The interpole winding in this case consists of fewer ampere-turns. For further details regarding armature reaction, commutating poles, compensating windings, and commutation see Chapter XV.

269. Speed Control. — An inspection of eq. (3) shows that the speed of the motor may be varied at will by varying one of the following factors: field flux, armature voltage, or the number of armature conductors in series. Of these three methods, the first one, or the so-called field control, is used most, because of the convenience of regulating the comparatively small exciting current. The weaker the field the higher the motor speed with the same applied voltage. The lower speed limit is reached when the field rheostat is short-circuited and the field winding is connected directly across the line. Sometimes the motor cannot be operated at this speed for any length of time without overheating the field winding.

The upper speed limit is determined by several considerations, namely (a) commutation, (b) heating of the armature winding, and (c) centrifugal stresses. At higher speeds, that is with a weakened field, the effect of the armature reaction is more pronounced, and the frequency of commutation is higher. For these two reasons the commutation is less satisfactory at higher speeds. Without commutating poles the limit of satisfactory commutation is usually reached with a comparatively small increase in speed. Motors provided with commutating poles are available with a speed range as high as four-to-one. At any speed within this range the motor is supposed to furnish its rated output without excessive sparking or heating.

In a motor with interpoles (Fig. 206) the brushes are always left at the geometric neutral, and this is especially important when the motor must run in both directions. In a motor without commutating poles, the brushes may be shifted to some extent to obtain the best commutation, or to adjust the speed. The latter possibility is based on the fact that the direct armature reaction is determined by the brush shift (§ 228), so that by shifting the brushes the value of the flux per pole is varied. No considerable speed variation can be obtained by this means, because sparking results as soon as the brushes have been shifted by any appreciable angle from the best position. However, this method is sometimes convenient for adjusting the speed to an exact value, after an approximately correct speed has been obtained with the field rheostat.

270. Special Motors of Adjustable-Speed Type. — Instead of varying the field strength by regulating the field current, the magnetic flux may be varied by changing the reluctance of the magnetic circuit. An adjustable-speed motor of this type is shown in Fig. 207 (Stow motor). The pole-pieces are hollow and have center cores which consist of plungers actuated by a handwheel. As these cores are withdrawn the field flux is decreased without changing the field current. In four-pole motors of this type all the pole-pieces are moved simultaneously by a beveled-gear transmission. The reason why the effect of the armature reaction in this motor is reduced at higher speeds may be seen by a comparison of Fig. 207 with Fig. 208; the latter represents the field of an ordinary motor. In the field of the Stow motor (Fig. 207) the reluctance of the magnetic path of the distorting lines of force, due to the armature current, increases with the effective length of the air-gap. The ratio of the field ampere-turns to the armature distorting m.m.f. is not decreased with increase of speed, as it is in the case of the

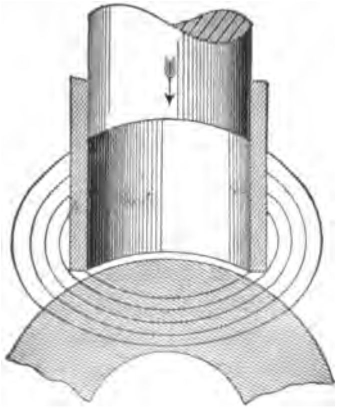


Fig. 207. Pole-piece in a Stow motor; cross-magnetization is reduced by an air column.

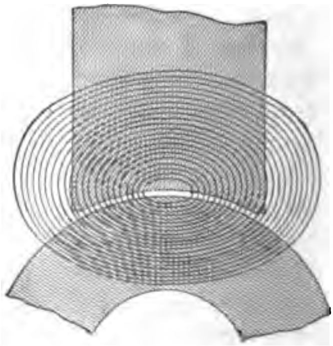


Fig. 208. Pole-piece in an ordinary motor; field distortion is the greatest when the field is the weakest.

ordinary motor where the field flux is varied by decreasing the field current. Therefore, the field distortion does not increase with the speed. In the ordinary motor the field distortion is greater at higher speeds, since the ratio of the field ampere-turns to the armature distorting ampere-turns is smaller. Another advantage of the Stow motor is that the speed adjustment is gradual, and not in steps.

In the Lincoln adjustable-speed motor the armature is made slightly larger at one end than at the other, and can be shifted along the shaft by a handwheel. As the armature is withdrawn from the pole-pieces, the useful magnetic area decreases and the length of the air-gap increases because of the conical shape of the armature. The result is that the useful flux decreases, and the speed of the motor is thereby increased. Since in this motor the useful flux is reduced for speed control by increasing the reluctance of the air-gap, the magnetic distortion is said to be actually less at high speeds than at low speeds. This is the opposite to that which takes place in a motor with field-current control. Motors of this type have been built with a speed range as high as ten-to-one.

271. Centrifugal Speed Regulator. — In some drives it is required to hold the motor speed absolutely constant, independent of fluctuations in the load and in the voltage supply. An automatic centrifugal governor is then mounted on the motor shaft and arranged to control two contacts. One of these contacts is stationary and the other movable, being mounted on the end of a governor arm. The action of the governor arm is controlled by a spring which can be adjusted for any desired speed. The contacts are connected through two collector rings, brushes and brush-holders, to close a shunt circuit across the motor field rheostat. This shunt circuit is thus rapidly opened and closed, resulting in an average field excitation of the correct value to give the required motor speed. There is no danger of breaking the motor field, because the device operates in shunt with the field rheostat.

In a large motor the field current may be too strong for the control contacts. One or more relays are then used; they are mounted separately, and are similar to the Tirrill regulator described in § 245. The motor contacts control the circuits of these relays which in turn close and open the shunted by-pass across the field rheostat.

272. EXPERIMENT 12-A. — No-Load Test of a Shunt-Wound Motor, — Field Current Control. — The purpose of this experiment is to determine the relation between the field current and the speed of a shunt-wound direct-current motor at no load, with a constant applied voltage. The connections are shown in Fig. 202 and the theory is explained in §§ 266 to 271. The data sheet shown in § 281 may be used, the last column being omitted. In performing this and other experiments on motors it is advisable to use a carefully calibrated, reliable speed indicator, such as the vibration tachometer, the centrifugal tachometer, or one of the electromagnetic type. The ordinary revolution-counter takes too much time and gives only the average speed over a comparatively long period.

(1) Set the brushes exactly in the geometric neutral, close the field circuit with the resistance all cut out, so as to have the strongest possible field, and have all the starting rheostat " in," so as to cut down the starting current. Start the motor by closing the armature circuit, and cut out the starting resistance in steps as the armature speeds up. Bring the machine up to the highest safe speed by reducing the field current and let it run for several minutes to warm up the bearings. The voltage at the armature terminals must be kept constant throughout the test. If the supply voltage is fluctuating, connect an adjustable resistance in the armature circuit and always obtain the correct voltage at the armature terminals before measuring the speed. Read the armature current, the field current, the speed, and the armature voltage.

(2) Increase the field current in steps and at each step repeat the preceding readings. Having obtained the speed with the highest field current, reduce the current again to the same minimum value as in the first set of readings and check the speed reading to observe the effect of residual magnetism.

(3) Take a few readings with the brushes shifted from the neutral position by different amounts in each direction. Note the effect upon the speed, and record the degree of sparking, if any (§ 228).

(4) Drive the machine as a generator at a convenient constant speed and take a no-load saturation curve (§§ 229 and 232). See also § 282.

(5) Measure the armature resistance, including the brushes, and determine the resistance of the commutating pole winding, if present.

Report. To the values of field current as abscissæ plot the corresponding speeds; mark the effect of residual magnetism, if any. To the same abscissæ plot the armature current and the power input into the armature, corrected for the I^2R losses. These corrected values represent the watts lost in supplying the iron loss and friction at no-load. To the same abscissæ plot the values of the induced e.m.f. from the no-load saturation run, correcting these readings, where necessary, for speed departures. By means of this saturation curve show that in each run as a motor the product of the speed and the field flux was nearly constant, as it should be, according to eq. (3). Show that the relationship between the field current and the speed (as a motor) could be predicted from the constant-speed saturation curve as generator. Give the results of the test with the brushes shifted.

273. Methods of Controlling Speed by Varying the Armature Voltage. — With a constant field excitation, the speed of a shunt motor may be varied in several ways, the principal ones being:

(1) Separate connections to mains of different voltages.
(2) A source of variable voltage.
(3) Armature rheostatic control.

A variable voltage supply is usually provided in one of the following three ways:

(1) By means of a motor-generator set.
(2) By means of a counter-e.m.f., or booster set.
(3) By means of a three-wire system.

These methods are described below more in detail.

274. The Use of Motor-Generator or Booster for Speed Control. — When the armature voltage of a motor *M'* (Fig. 209) is varied by using a separate motor-generator set, the primary source of supply may be either direct or alternating current, because this source is used only to drive a constant-speed motor *M*, which in turn drives the direct-current generator *G*. A three-phase power line is shown in the diagram. The fields of both the generator *G* and the variable-speed motor *M'* are preferably excited from a separate source, such as a small motor-generator set marked M_1 and G_1, to permit an independent adjustment of the exciting current, and the reversal of the direction of rotation of the main motor *M'*. Instead of the small auxiliary motor-generator, an exciter may be mounted on the shaft of the main set *MG*, or the exciting current may be drawn from an independent source of direct current, such as a storage battery. Such an arrangement, though

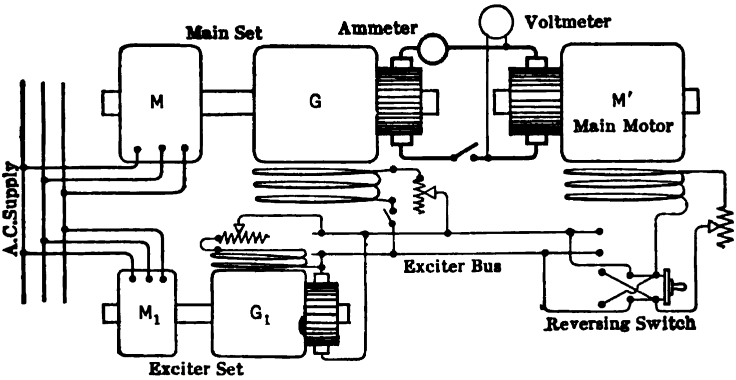


FIG. 209. An adjustable-speed motor driven by a motor-generator set.

quite expensive, has been successfully used for the main drive in steel mills, where the motors are rated at several thousand horse-power. The speed control is almost unlimited in either direction of rotation, and the resultant economies in steel production outweigh the extra investment in the motor-generator sets.

The large motor-generator set is usually provided with a heavy fly-wheel, and the speed of the driving motor M is regulated by a current or power relay. By these means the fluctuations in the power taken from the supply circuit are greatly reduced. When a sudden overload comes on the main motor M', the driving motor M of the motor-generator set slows down and allows the inertia of the fly-wheel to supply a part of the overload. When the load is suddenly thrown off the main motor, the motor-generator set is speeded up, thereby again storing energy in the fly-wheel. The method of driving an adjustable-speed motor from a motor-generator set at a variable voltage is sometimes referred to as the Ward-Leonard system. The use of a fly-wheel on the motor-generator set to reduce fluctuations in the power demand is usually ascribed to Ilgner.

The Series Booster. When the power supply is direct-current, a modification of the Ward-Leonard system is possible, as shown in Fig. 210. In this case the generator armature G is connected in series with the supply and with that of the main motor M'. The generated voltage may be either added to or subtracted from that of the supply, and thus speed variations may be obtained within wide limits.

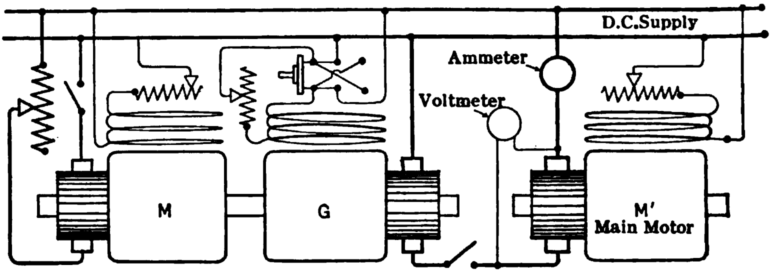


FIG. 210. An adjustable-speed motor driven from a constant voltage supply in series with a booster.

Let, for example, the speed of the main motor at the supply voltage and at no load be 1000 r.p.m. and let the maximum generator e.m.f. be 80 per cent of the supply voltage. Then, with the generator e.m.f. subtracted from the line voltage, the net voltage at the motor terminals is only 20 per cent of the rated voltage, and the speed will be approximately 200 r.p.m., the field current of the motor being assumed to be kept constant. At the other extreme, with the total generator electromotive force added to that of the line voltage, the motor will run at about 1800 r.p.m. Thus, a speed range of nine-to-one is obtained with a motor-generator set of much smaller size than in Fig. 209. If the field control is also used, the total speed range may be considerably increased.

When a generator is used to raise the voltage of the supply it is usually called the booster, and when it lowers the voltage it is called a negative booster. In this particular application it is sometimes called the teaser, and the above-described system is referred to as the teaser system. In spite of its comparatively high cost, it has been used in cases in which an accurate speed adjustment within wide limits is of importance, as for example in printing establishments.

The two above-described methods of varying the armature voltage are used in special cases only. For all ordinary purposes the field control described in § 269 is sufficient and is the simplest and the least expensive solution.

275. The Three-Wire System. — A shunt-wound motor connected to a three-wire system (§ 255) is shown in Fig. 211. The field winding is connected between the outside conductors, and the field current

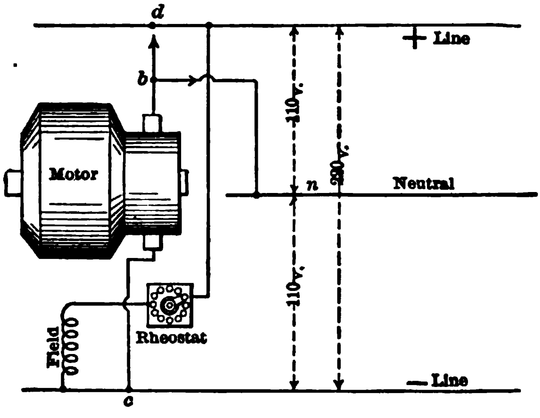


FIG. 211. Motor connected to a three-wire supply.

can be varied within certain limits by the rheostat in its circuit. One armature terminal is connected to the negative line conductor at *c*. The other terminal, *b*, may be connected at will either to the neutral conductor *n*, or to the positive conductor at *d*. To obtain the lowest speed, say 400 r.p.m., the motor field is fully excited, and the armature is connected across 110 volts. By gradually weakening the field, the speed can usually be at least doubled before difficulties in commutation begin; this gives any desired speed between 400 and 800 r.p.m. The armature may then be switched on to the full voltage, 220 volts, and the field again fully excited. This will again give a speed of 800 r.p.m.; by weakening the field as before, the speed can be raised to 1600 r.p.m. This gives at least the same speed range of four-to-one as can be obtained with a commutating-pole motor on a two-wire supply.

A still wider speed range may be had by using the so-called *unsymmetrical* three-wire system. The total line voltage, say 250 volts, may be divided by the middle wire into 160 and 90 volts. This gives three motor speeds in the ratio of 25 : 16 : 9, without the field control. By weakening the field this range can be at least doubled. This system has not been used much, probably on account of additional complication in maintenance.

Instead of a three-wire system, two motors on a two-wire system can be used for driving the same load. By connecting the armatures in series and in parallel (as in an electric car), half speed and full speed can be obtained. It is even unnecessary to have two separate motors; the two armature windings can be put on the same core and made to revolve in the same field. This is called the "two-commutator motor" (Fig. 212). When *a* is connected to *b* and *b′*, and the connections at *c* and *d* are open, the two armature windings are in series and the motor runs at half speed. When *a* is connected to *c*, and *b′* to *d*, the connection between *b* and *b′* being open, the two windings are in parallel and the motor runs at full speed. The field circuit is provided with a regulating rheostat, by means of which the range of speed is further increased and intermediate speeds are made possible

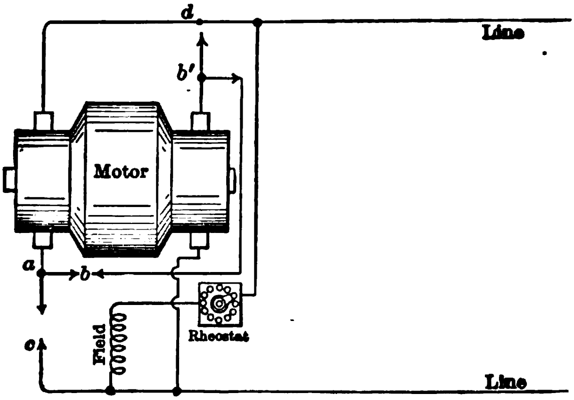


FIG. 212. A two-commutator motor; the armature windings may be connected either in series or in parallel.

276. EXPERIMENT 12-B. — No-Load Test of a Shunt-Wound Motor, — Armature Voltage Control. — The purpose of this experiment is to investigate the limits and the relative advantages of the various methods of speed control described in §§ 273 to 275. This study may be conveniently performed at no load because, with a given field excitation and at a constant applied voltage, the speed of a shunt-wound motor varies but a few per cent between no load and full load. This statement does not hold true for the rheostatic control (§ 277), with which the voltage across the armature is necessarily a function of the armature current.

(1) *Three-Wire System.* If a three-wire system is not available, it can be improvised by one of the means described in § 256. Connect the field circuit across the outside mains and keep the field current constant throughout the test. Run the motor with the armature connected at full voltage and at half voltage (Fig. 211). With an unsymmetrical three-wire system, make a run with the armature across each of the three voltages. Read the field current, armature current, volts, and speed. Perform similar runs with two or three other values of field current.

(2) *Motor-Generator Set.* Connect the motor under test to an available generator forming part of a motor-generator set, as in Fig. 209, and have both machines separately excited from the supply circuit. Set the field excitation of the motor under test at some convenient value and keep it constant throughout the first run. Run the generator at its normal speed *with the field circuit open* and close the switch between the two machines. Start the motor and bring it up to speed by gradually exciting the generator. Vary the generator excitation in steps, and at each step read the amperes and volts at the motor armature terminals, and its speed. Reduce the excitation of the generator again to zero and reverse the direction of the exciting current of the motor. Gradually cut out the field rheostat, causing the motor to start and to run in the opposite direction. Take one or two check readings at the same values of armature voltage as before. Investigate the effect of brush shift in each direction upon the speed and the armature current of the motor. Note the degree of sparking, if any occurs. Repeat the same run with one or two other values of field current in the motor under test.

(3) *The Teaser System.* Connect the generator in series with the laboratory supply, as in Fig. 210. Excite the motor under test to a convenient value and keep the field current constant throughout the run. Bring the motor-generator set to its rated speed, with the generator field circuit open. Start the motor under test and bring it up

to speed by gradually short-circuiting the starting rheostat. Read the armature volts and the ampere input into the motor armature, and measure the speed. Excite the generator field in steps, so as to increase the voltage across the motor armature, and at each step repeat the above readings. Again open the generator field and, after the motor under test has slowed down to its normal speed, take check readings at this speed. Reverse the field connections of the generator and again increase the excitation in steps, taking at each step the same readings as above. Repeat the run with two or three other values of field current in the motor under test.

(4) Measure the resistances of the two armatures.

Report. Describe and draw the various connections used during the experiment, and show by means of curves or tables that the motor speed at no load and with a given field current is nearly proportional to the voltage at the armature terminals. Give a theoretical reason for this fact. Compare the range of speeds obtainable with the various systems tried, and point out the relative complication and expense of each. Give a few examples of industrial applications in which each of these systems might be preferred. Since the armature volt-amperes represent the input necessary for overcoming the iron loss and friction in the motor (§ 267), show from the readings that with a given field current the losses increase with the speed, and that with a given speed they increase with the field excitation. Explain separately the reasons for the increase in hysteresis loss and in eddy currents with the speed and with the field excitation.

277 Speed Control by a Rheostat in the Armature Circuit. — The rheostatic control consists in the use of an adjustable resistor in the armature circuit, similar to the starting rheostat (Fig. 202), but of a larger current-carrying capacity and possibly with a larger number of steps. By varying the resistance of this rheostat, the resistance of the whole armature circuit is varied at will. Consequently, with a constant line voltage and a given torque, the speed of the armature may be regulated as desired.

The rheostatic control is but seldom used in regular operation because it is wasteful of energy, the power loss in the rheostat being nearly proportional to the reduction in speed. For example, to reduce the speed to one-half of its normal value, one-half of the applied voltage must be consumed in the rheostat and the I^2R loss in it will be equal to the power delivered to the armature. With a medium or large-sized motor the rheostat is bulky and expensive, and the heat generated in it is sometimes difficult to dissipate.

Another objection to the rheostatic control is that, with a given setting of the rheostat, the speed of the motor fluctuates within wide limits with the load. This is because the voltage drop in the rheostat, and consequently the voltage at the armature terminals, depends upon the armature current. Thus, a frequent adjustment of the rheostat is necessary in order to keep the speed even approximately constant.

With small motors the so-called *potentiometer method of rheostatic control* (Fig. 213) gives a closer speed regulation, although this method is still more wasteful of energy than the straight rheostatic control. A resistance R is connected across the supply mains, and its value is such that the current through it is several times larger than that through the motor armature. The armature circuit is shunted off at any two desired points of this resistor by means of the connections A and B, one of which may be for larger steps and the other for fine adjustments. The whole arrangement resembles the potentiometer connections shown in Fig. 70, whence its name.

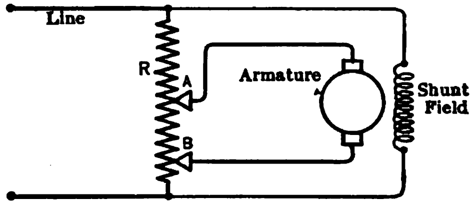


FIG. 213. The potentiometer method of speed regulation.

278. EXPERIMENT 12-C. — Speed Characteristics of the Shunt-Wound Motor with the Armature Rheostat Control. — Connect the motor under test to a generator, a blower, or some such steady load, and provide an adjustable resistor in series with the armature circuit of the motor. This resistor must be able to carry the full rated current of the motor for a considerable time without overheating. Excite the motor field to the highest possible value and start the armature in the usual way by cutting out part of the resistance in the armature circuit. Adjust the load and the armature resistance in such a way as to have the rated current through the motor at about 90 per cent of the line voltage across the armature terminals. Read armature volts and amperes, the field current, the motor speed, and the voltage drop across the regulating resistance. Throw off the load completely and take similar readings. Perform similar runs with the voltage at the armature terminals reduced to 80, 70, 60, etc. per cent of the line voltage and with the same rated armature current. Also take a few runs at a different value of the field current. Make runs with the motor loaded and unloaded when the armature is connected directly across the line, all of the regulating resistance being short-circuited. Make a similar investigation with the regulating rheostat connected across the line, as in Fig. 213.

Use a very small motor for such a test, and keep the current through the resistor at a value equal to several times the motor current.

Report. (1) Use the full-load volts across the armature terminals as abscissæ, and plot the following curves: (a) volts at the armature terminals at full load (45° line); (b) same at no load; (c) volts lost in the regulating resistance at full load and at no load; (d) motor speed at full load and at no load. (2) Devise a method for plotting the results of the test with the potentiometer arrangement, so as to bring out its good and objectionable features. (3) Compare these two methods of speed regulation in regard to convenience, expense, efficiency, and speed fluctuation between no load and full load.

279. The Ampere-Speed Curve of a Shunt-Wound Motor. — A direct measurement of the mechanical output of a motor is avoided in practice whenever possible, because a Prony brake (§ 303) can be used only with comparatively small motors, and a transmission dynamometer (§§ 304 and 305) is expensive and not always available. For these reasons it is often preferred to compute the output from a given or assumed input, and the losses (Chapter XIII). Such computations are simplified and made more accurate if the relationship between the electrical input and the speed is determined experimentally.

This relationship, or the ampere-speed curve, is obtained by connecting the motor under test to a generator, a blower, a centrifugal pump, or some such steady load. The motor is then run within the desired limits of input, voltage, field current, etc., and the corresponding values of speed are carefully measured. Because of a steady load, the speed can be kept constant and can be accurately measured. The electrical input can also be read accurately. The losses within the motor being known from another test, the performance curves (Fig. 205), including the output, may be computed quite accurately, without actually measuring the output.

Let, for example, a set of readings in an ampere-speed test of a small shunt-wound motor be as follows: armature current 15 amperes, speed 880 r.p.m., terminal voltage 220 volts, and the field current 1 ampere. Let the resistance of the armature (including the brush contacts) be 0.50 ohm, and let the iron loss and friction, found from another test, be 200 watts. We then have

Input into the armature	15×220	= 3300	watts
Input into the field	1×220	= 220	"
Total input	$3300 + 220$	= 3520	"
Armature copper loss	$15^2 \times 0.5$	= 113	"
Core loss and friction		= 200	"
Output	$3300 - (113 + 200)$	= 2987	"
Efficiency	$2987/3520$	= 84.8	per cent
Torque, § 267, eq. (6)	$974 \times 2.987/880$	= 3.63	m.-kg.

The foregoing results could not have been so readily obtained without the ampere-speed curve because the speed itself is an important characteristic, and if the exact speed at a given input is known, the proper value of the core loss and friction can be obtained from the no-load loss. Moreover, it would not be possible to compute the useful torque from the output, without knowing the speed. It is true that the full-load speed can be estimated from the no-load speed, by allowing for the armature voltage drop; but the armature reaction and the voltage drop at the brushes make this computation somewhat uncertain.

An ampere-speed test, in place of a brake test, is useful not only in the case of shunt-wound motors, but with all kinds of motors, alternating as well as direct-current.

280. EXPERIMENT 12-D. — Ampere-Speed Curves of a Shunt-Wound Motor. — The purpose of this experiment is to obtain the relationship between the ampere input into the motor armature and its speed, with the different values of field current (Fig 205). This test should preferably be performed on the same motor as Exp. 12-A in order that the performance characteristics may be computed. If the supply voltage is unsteady, provide a regulating rheostat or a booster by means of which the voltage at the motor terminals may be kept constant when taking readings. Use the same data sheet as in § 281, omitting the last column.

(1) Connect the motor under test to a suitable generator, blower, centrifugal pump, or some such steady load. If an electric generator is used it may be convenient to pump the power back into a supply circuit rather than to dissipate it in resistances. Set the brushes exactly in the neutral position. Excite the motor under test with the largest field current possible and bring the motor gradually up to speed on a light load. Increase the load until the armature current in the motor under test reaches its highest safe value. In a motor without interpoles it may be necessary to shift the brushes to a position that will give the best average commutation under all load conditions. Run the set for a sufficient time to make sure that the friction and lubrication are nearly constant. Read armature amperes, field amperes, terminal volts, and motor speed. Take similar readings with the brushes shifted in a few different positions in each direction. Reduce the load in steps to zero, keeping the field current of the motor and the voltage across the armature terminals constant. Take the same readings at each step.

(2) Repeat the same run at several other values of the field current, in each case going to the safe limit of speed, commutation, and heating.

(3) Measure the resistance of the armature winding, including the brushes, and also that of the interpole winding, if present.

Report. Against armature amperes as abscissæ, plot the speed curves at the different values of field excitation. On the same curves indicate the effect of brush shift. For several points on these curves compute the losses, the output, and the efficiency, as in § 279, and plot the performance curves shown in Fig. 205. A separate set of curves must be obtained for each value of field current, and two or three sets may be plotted on the same curve sheet with inks of different colors.

281. EXPERIMENT 12-E. — Brake Test of a Shunt Motor. — The purpose of the experiment is to obtain directly the performance curves shown in Fig. 205. The motor should be connected as shown in Fig. 202, and a suitable brake or dynamometer provided (§§ 303 to 305). One independent circuit includes the field winding with its regulating rheostat and an ammeter; the other circuit is formed through the armature of the motor, with an ammeter and a starting and regulating rheostat. The latter is needed if the supply voltage is not steady. A voltmeter is connected across the armature terminals. The readings may be conveniently recorded on a data sheet similar to the one shown below.

	Arm. Amps.	Field Amps.	Volts	R. P. M.	Torque
Inst. No........					
Constant.......					

Begin the test with the highest load, say 25 per cent overload, and at the highest field current. Read the armature amperes, field amperes, terminal voltage, speed, and brake load. Shift the brushes a few steps in each direction and take the same readings; note sparking, if any. Reduce the load in steps to zero, and take a sufficient number of readings for plotting the curves. The field current and the terminal voltage must be kept constant throughout the whole test. Make similar runs at a few other values of field current. Also make runs with other values of line voltage, say 10 per cent above and a like amount below normal, in order to see the influence of this factor on the performance of the motor.

Report. For each value of field current, plot the performance curves shown in Fig. 205. On the same curve sheet, indicate the effect of varying the supply voltage and of shifting the brushes.

282. No-Load Saturation Curve of a Shunt-Wound Motor. — When it is desired to obtain experimentally a no-load saturation curve (§ 229) of a direct-current machine, and no driving motor is available, the machine under test may be driven as a motor at no load, and a saturation curve obtained in this way. The machine is connected as in Fig. 202, the field circuit being regulated independently of the armature circuit. In performing this test it is of importance to select the proper speed — one that can be maintained within wide limits of field current.

The best way is to begin with the highest possible value of the field current, and at the highest available voltage at the armature terminals. The value of the speed obtained under these conditions is then maintained at lower values of the field current, by lowering the armature voltage accordingly. The values of the voltage read at the armature terminals are practically equal to the induced e.m.f., but if greater accuracy is desired the ohmic drop through the armature may be subtracted from the readings.

It is not important to obtain experimentally a saturation curve at exactly the speed for which it is to be used. The induced voltage at no load is strictly proportional to the speed, so that the observed values may be easily re-computed for any other desired speed.

283. EXPERIMENT 12-F. — No-Load Saturation of a Shunt-Wound Motor. — The purpose of the experiment and the connections are explained in the preceding section. Begin with the highest possible value of field current, and with the full line voltage, or even overvoltage, applied at the armature terminals. Read the field current, the armature current, the armature volts, and the speed. Reduce the field current in steps, and at each step introduce more resistance into the armature circuit; or vary the armature voltage in some other way (§ 274), so as to keep the speed nearly constant. Read the instruments at each step. Before leaving the laboratory, measure the resistance of the armature, including the brushes. For this measurement use approximately the same values of current as were read during the test.

Report. Correct the observed voltage readings for the *ir* drop. Correct the induced e.m.f.'s so obtained for the effect of incorrect speed, if any. Plot the corrected voltages against field current as abscissæ, at a constant speed. Show on a few readings that the induced voltage is proportional to speed.

284. Dynamic Braking. — Dynamic braking (Fig. 214) is used for stopping or slowing down a motor, and consists in re-connecting it so as to convert it into a generator and make it dissipate its stored kinetic energy in the form of I^2R heat in an external resistance. This change is accomplished by means of two switches, *a* and *b*, which in this case are shown as contactors, operated by individual electromagnets. When the switch *a* is closed and *b* is open, the machine runs in the usual way as a motor. With the switch *a* opened and *b* closed, and with the field circuit connected to the line as before, the motor armature will generate an e.m.f. and force a current through the resistance *R*.

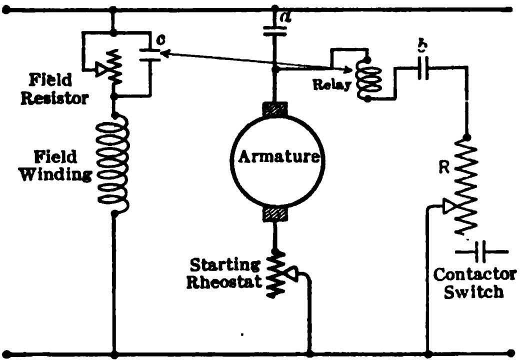


FIG. 214. Connections for dynamic braking.

The energy required for the I^2R loss in the resistance is supplied at the expense of the stored mechanical energy in the armature, flywheel, gears, lifted load, etc. This causes the motor to slow down and ultimately to stop. In many cases dynamic braking is utilized only for slowing down, as it is most effective at high speeds, and a mechanical brake is applied to stop the machine.

An adjustable-speed motor may be running at a high speed with its field weakened, when the operator desires to stop the motor. The braking effect is strongest with the full field, and therefore a contactor switch *c* is sometimes provided to short-circuit the field rheostat. This switch may be made to operate automatically by a relay in series with the brake resistance, so that *c* is closed as soon as a current begins to flow through *R*.

The advantages of dynamic braking over a mechanical brake are: (a) The stored energy is largely dissipated in an external resistance which may be remotely located, while a friction brake develops heat close to the motor; (b) A quicker reduction of speed is obtained, because a time element is involved in the setting of the friction brake; (c) The motor shaft is relieved of a considerable strain to which a mechanical brake subjects it; (d) The wear and tear and the upkeep expense of the mechanical brake are reduced, when the latter is used for the final stop only. On the other hand, dynamic braking involves certain additional complications and expense; and if it is used frequently, the motor capacity must be somewhat greater, to take care of the additional heating of the armature.

Dynamic braking is also used to bring the motor quickly from full speed to a desired lower speed. In this case the braking resistance *R* (Fig. 214) is connected in parallel with the armature, and the two are connected to the line in series with the starting rheostat. The voltage drop in the latter reduces the voltage at the motor terminals to a value below that of its counter-e.m.f., so that the armature generates a current which circulates through the resistance in parallel with it. The armature quickly comes to a lower speed, at which the regular motor connections may be re-established, or the circuit opened and a mechanical brake applied.

In the case of a series motor, dynamic braking is often used on unbalanced hoists for lowering the load, but it is necessary to excite the fields separately. The series-field winding is connected across the line in series with a resistance, thus converting the machine into a separately excited generator (Fig. 176).

The so-called regenerative control, or pumping-back of power into the line, is somewhat similar in principle to dynamic braking. The power, however, is not wasted, but is recovered for use in other devices connected to the same line. A notable example is the regeneration of power by an electric train going down grade. The series fields are excited from a separate low-voltage generator, and the speed can be nicely controlled within wide limits. Air brakes need be used for stops only. See also Chapter XIV.

285. EXPERIMENT 12-G. — Dynamic Braking and Regenerative Control of a Shunt-Wound Motor. — For the purpose and the method of dynamic braking see the preceding section. The experiment can be better performed on a motor connected to a heavy fly-wheel, as otherwise the time of stoppage may be too short for reliable readings. Connect the motor as in Fig. 214, using a double-throw switch in place of the contactors *a* and *b*. Provide an ammeter in the brake resistor circuit and one in the field circuit. Connect a voltmeter across the armature and have a good tachometer for reading instantaneous speeds. Special recording instruments (§ 62) should be used for accurate results. (1) Bring the motor up to speed at no load with the full field current.

Record the field current and the speed, and change the armature circuit quickly from the line to the brake resistor. Read load amperes, armature volts, and the instantaneous speed as accurately as possible, every few seconds until the motor stops, or at least until it slows down to a low speed at which it can be readily stopped by a mechanical brake. Note the position of the regulating handle on the brake resistor. (2) Repeat the test with a few different values of brake resistance. (3) Perform similar runs with different values of field resistance. (4) Make a few runs in which the field resistance is " in " when the armature is connected to the line, and is short-circuited at the same time that the armature is switched over to the brake resistor. This may be done by hand or with an automatic magnet switch *c* shown in Fig. 214. (5) Vary the brake resistance during the process of retardation in such a way as to keep the armature current as nearly constant as possible, thus obtaining the maximum braking action. (6) Try the arrangement described in the preceding section for bringing the motor quickly to a lower speed, with the starting resistance in the circuit. Adjust the values of the resistances for the best results. (7) Bring the machine to its highest safe speed as a motor and then, without disconnecting the armature from the line, quickly increase the field current. Observe the reversal of the armature current due to the generator action. A zero-center ammeter in the armature circuit is convenient for such a test. (8) Measure the resistance of the armature and of the brake resistor for the points used in the test.

Report. (1) Plot the readings against time as abscissæ, or tabulate them. (2) Give the best value or values of brake resistance for use with this motor in regular operation, and prove these values from your test data. If the brake resistance should be short-circuited in steps during the process of retardation, give the values of step resistances and show how this may be done automatically by magnet switches, actuated either by the armature voltage or by the load current. (3) Give the diagram of connections and the results of the test specified under (6) above. (4) Consider theoretically the case of a compound-wound motor (§ 298) and explain what should be done with the series field during dynamic braking. (5) Give the general theory and the equations from which the time of stoppage or of speed reduction for a given shunt-wound motor could be predetermined by calculation. The reduction in the kinetic energy during an infinitesimal interval of time is equal to the I^2R loss in the brake resistance plus the losses in the motor itself (§ 331).

THE SERIES-WOUND MOTOR

286. The diagram of connections of a series-wound motor is shown in Fig. 203, and the performance characteristics in Fig. 215. The fundamental property of such a motor, at a constant line voltage, is that its torque varies greatly and inversely with the speed. The motor is capable of exerting quite a high starting torque without an excessive current, but as the load torque is reduced the motor speeds up practically without limit. Unless precautions are taken, a series-wound motor without load will run away, and the armature is likely to burst because of the excessive centrifugal force.

The reason for this behavior of the series motor lies in the fact that the armature current also excites the fields. At no load, the law of the conservation of energy requires the motor current to be small. This small current produces but a weak magnetic field, while the counter-e.m.f. must be practically equal to the applied voltage, the IR drop in the machine being very small. Hence, the motor armature speeds up until the counter-e.m.f. induced by a weak field reaches the required value. This relationship may also be seen from eq. (3) in § 267. At no load, e is practically equal to the constant applied voltage E. Consequently, the product of r.p.m. times the flux Φ must be constant. But Φ is small, being excited by a small no-load current. Hence the speed must be high.

The speed of a series-wound motor thus varies automatically with its load torque, and this characteristic makes the motor particularly well adapted for railway, crane, and hoist service. Constant speed is not essential for such service; on the contrary, it is desired that the speed should automatically decrease as the load torque increases. Otherwise, fluctuations in the power supply and the mechanical stresses in the motor itself would make its operation more difficult.

287. Torque Characteristics. — The electromagnetic torque of an electrical machine is proportional to the product of the armature current and the air-gap flux, other factors remaining unchanged, and is independent of the relative speed between the conductor and the field. This is a fundamental law of interaction between an electric current and a magnetic field (§ 264). The same relationship is also represented by eq. (8) in § 267.

In a series-wound motor the field is excited by the same current that flows through the armature. Hence, without saturation in the iron parts, the torque would be proportional to the square of the current. Such is nearly the case at high speeds and light loads when the current is small. At start, when the machine takes a heavy current, the magnetic circuit is highly saturated, and the torque varies more nearly as the first power of the current.

Thus, in a series-wound motor both the speed and the torque are definite functions of the current, and therefore there is a definite relationship between the speed and the torque, the so-called *torque-speed* characteristic. If values of speed be plotted as abscissæ and the corresponding values of torque as ordinates, the maximum ordinate is at zero speed, and the torque drops rapidly as the speed increases. The torque approaches the axis of abscissæ at high speeds. This curve is used in plotting time-speed curves of electric trains and in other acceleration problems.

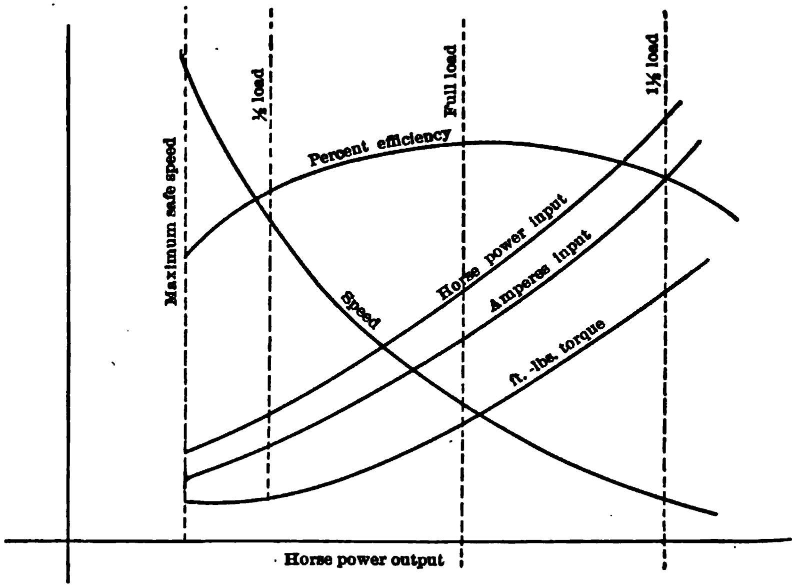


FIG. 215. Performance curves of a series-wound motor.

288. Speed Control of a Series Motor. — Since the series-wound motor is inherently a variable-speed machine (Fig. 215), the speed control in this case means an external change (made by a resistance or otherwise) in order to obtain a different speed characteristic. The two methods most commonly employed for this purpose are: a resistance in series with the motor circuit, and a resistance shunted around its field winding (Fig. 203). When two or more motors are connected to the same load, as for example in an electric car or locomotive, two ranges of speed are obtained by connecting the motors in series and in parallel. In special cases, such as in testing, the armature may also be shunted by closing switch S_1. All these methods will now be considered more in detail.

(1) *Series Resistance.* This method is wasteful of power, requires large, well-ventilated rheostats, and is used only with comparatively small motors for intermittent service, such as crane and hoist work. The torque of a series-wound motor is a function of the current only. The speed of the motor depends upon the voltage at the armature terminals, so that a resistance in series with the motor makes it possible to obtain a desired torque at a lower speed. The power loss is in the same proportion as the voltage drop in the resistance. As the load torque varies, the current taken by the motor also varies, so that the voltage drop across the series resistance fluctuates with the load. Thus, the motor operates at a variable terminal voltage, and its inherent variable-speed characteristics are accentuated. A counter-e.m.f. motor-generator set (Fig. 210), when applied to a series motor, accomplishes the same purpose without so much power waste. Moreover, it can be used both to cut down and to boost the line voltage.

(2) *Shunt around the Field Winding.* The effect of shunting the field winding by a resistance (Fig. 203) is to weaken the field flux and consequently to speed up the motor. The counter-e.m.f. being practically constant and equal to the applied voltage, a weaker flux means a higher speed. With a given load torque the motor draws more current from the line with the field shunted, because the torque is proportional to the product of the armature current and the field flux. The shunting of the series field is used in railway motors for running at a high speed on level stretches. Such motors are provided with interpoles for better commutation.

(3) *Series-Parallel Connection of two Motors.* When two series-wound motors are connected in series across a constant-voltage line, each is working at one-half of the applied voltage, so that with a given current the counter-e.m.f. is only about one-half that at full voltage. Therefore, with a given torque, the motor speed is reduced to one-half. This connection is used on electric cars for starting and for running at lower speeds.

(4) *Shunted Armature.* Just as shunting the field winding speeds the motor up, so shunting the armature by a resistance slows it down. This is because the field flux becomes relatively stronger. This method is wasteful of power, because the armature voltage is nearly equal to the line voltage. Thus, for example, in a 500-volt motor the voltage drop across the field may be of the order of 20 volts, leaving 480 volts across the armature. One ampere shunted around the field winding represents a loss of only 20 watts, while 1 ampere shunted around the

armature means a loss of 480 watts. For this reason, this method is hardly ever used in actual service. For its use in testing see § 290.

289. Performance Characteristics from an Ampere-Speed Curve. — The direct method of obtaining the performance characteristics of a series-wound motor (Fig. 215) is by loading the motor on a Prony brake (§ 303) or on a dynamometer (§§ 304, 305). With large motors the brake becomes bulky, inaccurate, and difficult to handle, and a transmission dynamometer of suitable characteristics is seldom available. In such a case the performance curves shown in Fig. 215 may be quite accurately computed from the so-called ampere-speed curve and the losses, both of which are determined experimentally.

The ampere-speed curve of a series-wound motor is obtained by belting or otherwise connecting the motor mechanically to a steady load, and running the motor under the desired conditions of current, voltage, and field strength. The results of such a test are plotted in the form of curves, with current as ordinates against speed as abscissæ. Each curve refers to a particular value of terminal voltage, with the field winding shunted or not, as the case may be (Fig. 203). Such a test can be performed quite accurately, the load being steady and there being no brake to keep adjusted while the speed is being measured.

There is a definite relationship between any two ampere-speed curves taken at two different terminal voltages: with the same current (that is, at the same field excitation) the speeds vary as the induced counter-e.m.f.'s. For example, let the speed of a motor be 1200 r.p.m. at a certain current and with 220 volts at its terminals. Let the ohmic drop in the machine at this current be 15 volts. The speed at the same current and at the terminal potential difference of 180 volts can be computed as follows: The counter-e.m.f. is 205 volts in the first case and 165 volts in the second. With the same field strength in both cases, the counter-e.m.f.'s are proportional to the speeds. Thus, the unknown speed is 1200 × 165/205 = 966 r.p.m.

Similarly, there is a definite relationship between two speeds with the field shunted and unshunted. Let, for example, the current in the foregoing illustration be 150 amperes, the resistance of the armature 0.06 ohm, and that of the field winding 0.04 ohm. Let the field winding be shunted by such a resistance that the armature current is 200 amperes when the field current is 150 amperes. The ohmic drop in the machine is $200 \times 0.06 + 150 \times 0.04 = 18$ volts, and the counter-e.m.f. is 202 volts. Hence the speed under these conditions will be $1200 \times 202/205 = 1183$ r.p.m. This computation is not quite exact, because at 200 amperes the armature reaction is greater than at 150 amperes, and consequently the net field flux is somewhat weaker.

290. Losses in a Series-Wound Motor. — These losses consist of the I^2R loss in the armature and field windings, the iron loss, and friction. The copper loss is usually computed from the resistance of the windings, and the iron loss plus friction (or the total no-load loss) may be determined from a simple test based upon the following considerations:

The friction and windage loss depends upon the speed only. The iron loss, or the armature core loss, consists of hysteresis and eddy currents. It depends upon the speed and upon the field excitation, and increases with both. Therefore, the motor must be run at no load, at the same speed, and with the same field current as at a certain desired load. The power input into the armature is then almost entirely used for overcoming the no-load losses and is a measure of these losses.

For this purpose the motor is run with the armature shunted by a variable resistance (Fig. 203), as is mentioned in § 288 under (4). By regulating the line rheostat and that in parallel with the armature, it is possible to adjust the speed and the field current to correspond to one of the points on the ampere-speed curve. The power input into the armature, corrected for the I^2R loss, will give the sum of the no-load losses. The subject is treated more in detail in §§ 321 and 322, but the above information is sufficient for calculating the performance characteristics.

When the power input into the motor is known from the ampere-speed curve, the output is obtained by subtracting from the input the computed copper loss and the measured no-load losses. The efficiency is equal to the ratio of the output to the input. The useful torque developed by the motor on the shaft is computed from the output, by means of eq. (6) or (7) in § 267.

291. EXPERIMENT 12-H. — Ampere-Speed Characteristics of a Series-Wound Motor. — The purpose of the experiment and the use of ampere-speed curves are explained in the two preceding sections. This experiment and the one following should be performed on the same motor.

(1) Wire up the motor, as in Fig. 203, and connect it to a suitable steady load. Start the motor in the usual way with the starting rheostat, and adjust the load so as to make the motor draw the heaviest safe current. The terminal voltage must be kept constant throughout each run. Read amperes, armature volts, and speed. Reduce the load in steps and take similar readings at each step. (2) Make similar runs at one or two other values of terminal voltage. (3) Make a few runs at the same terminal voltage as in the first run, but with the field wind-

ing so shunted as to reduce the field current by a definite percentage in each run. (4) Measure the resistances of the armature and field.

Report. (1) Plot the observed values of current and armature voltage to speed as abscissæ. (2) Show that the curves taken at different terminal voltages agree among themselves. (3) Show that the curves with the field shunted and unshunted also agree among themselves. (4) Compute an ampere-speed curve for a different assumed terminal voltage and with a different amount of shunting.

292. EXPERIMENT 12-I. — Efficiency of a Series-Wound Motor from its Ampere-Speed Curve and No-Load Losses. — The purpose and the method of the experiment are described in § 289. The experiment should be performed on the same motor as the preceding one, and the ampere-speed curves should be drawn in advance. All the runs specified below are to be performed at no load.

(1) Connect the motor as in Fig. 203. Select a point on an ampere-speed curve corresponding to the lowest speed and the highest current. Adjust the resistance in the main circuit and that in parallel with the armature until the motor runs at the same speed and with the same field current as on the selected point on the ampere-speed curve. When the field winding is not shunted, the field current is the same as the armature current, but with the shunted field it is important to have the correct field current, because the no-load losses depend on it. Having adjusted the speed and the field excitation to the exact values they have at the selected point on the ampere-speed curve, read the armature amperes and the voltage between the armature terminals. Take similar readings for a number of points on the same ampere-speed curve. In the same manner duplicate some points on the other ampere-speed curves, if performance characteristics are desired at other values of terminal voltage, and with shunted fields.

Report. For each set of readings, compute watts input into the armature and subtract the armature I^2R losses, if appreciable. Plot curves of no-load losses and of the total copper loss to correspond with each ampere-speed curve. From these data compute the performance characteristics shown in Fig. 215. Plot them either to output, to torque, or to speed as abscissæ.

293. EXPERIMENT 12-J. — Brake Test on a Series-Wound Motor. — The purpose of the experiment is to obtain directly the performance curves of the series motor, as shown in Fig. 215. Precautions should be taken to keep the motor from running away. With a shunt motor the brake can be safely released, since the speed of the motor is practically the same at no load as when loaded. In a series motor the speed increases enormously as soon as the load is taken off, and either the armature, the commutator, or the bearings may be damaged if the motor be allowed to run at this speed. For this reason, always *open the electric circuit before releasing the brake;* or at least insert a sufficient resistance to keep the speed down.

As an additional precaution against running away, an underload circuit-breaker may be connected in the circuit; when the load is taken off and the current falls below a certain limit, this device automatically opens the circuit. However, the student should not rely absolutely on such a circuit-breaker, since it may " stick " just when it is expected to act. It is best to have one man of the section stand near the main switch, and open the circuit if the motor should reach a dangerous speed. The terminal voltage should be kept as nearly constant as possible, and if necessary a regulating rheostat should be used for this purpose. The same data sheet may be used as in § 281.

Begin the test with the highest load, take readings of amperes, volts, speed, and torque. Then reduce the load by approximately equal steps, until the safe upper limit of the motor speed is reached.

Take a few readings with the field weakened by about 10 and 20 per cent by means of a shunt resistance, and read both the armature and the field current. Take a few runs with the supply voltage about 10 per cent above and below its normal value. Measure the resistance of the armature, including the brushes, and also that of the field winding.

Report. Plot the performance curves shown in Fig. 215, using the output, the torque, or the speed as abscissæ. Check theoretically the observed values of speed, with the field winding shunted and unshunted, by computing the corresponding counter-e.m.f.'s (§ 289).

294. EXPERIMENT 12-K. — No-Load Saturation Curve of a Series-Wound Motor. — This curve may be obtained in any one of the following three ways: (1) By driving the machine as a generator with the field excited from an independent source (§ 229). (2) By driving the machine as a motor at no load, with the field separately excited; the test is then identical with that described in §§ 272 and 282. (3) By driving the machine as a series motor at no load, with the armature shunted, as in Experiment 12-I, and its speed kept constant.

295. Starting Torque of Series-Wound Motor. — In some kinds of service for which the series motor is used, a knowledge of its starting torque is of particular importance. This information may be obtained by means of a Prony brake or transmission dynamometer, in the way in which values of torque are obtained with the motor running, except that at standstill the friction is somewhat indefinite. One has to rock the brake back and forth, and either get the limits of torque or the average value. Values of starting torque are often required at high currents, which the motor can stand only for a few seconds without overheating. Such readings must be taken promptly, and the motor must be allowed to run light between the readings, to cool the windings.

The torque at a high value of current may also be estimated by computation from the measured torque at a moderate current, provided that the no-load saturation curve of the motor is known (§ 294). According to eq. (8), § 267, the electromagnetic torque is proportional to the product of the current and the flux. Let a torque T_1 be exerted between the armature and the field with a current of 50 amperes flowing through the machine, and let it be required to determine the torque at 100 amperes. Let the following data also be known from the no-load saturation curve, taken at a constant speed:

Field Current Amperes	*Armature e. m. f.* Volts
50	72
100	90

The torque T_2 at 100 amperes is then determined from the expression

$$T_2/T_1 = (100/50) \times (90/72).$$

More generally

$$T_2/T_1 = (I_2\Phi_2)/(I_1\Phi_1) = (I_2e_2)/(I_1e_1) \quad . \quad . \quad . \quad . \quad (10)$$

where e_2 and e_1 are the induced voltages taken from a no-load saturation curve of the machine at the exciting currents I_2 and I_1 respectively.

It must be clearly understood that T_1 and T_2 are the values of the total electromagnetic effort between the field and the armature, and not the values of the useful torque available on the shaft. At standstill the latter values are smaller by the amount of static friction which must be measured or estimated separately.

A source of inaccuracy in formula (10) lies in the armature reaction (§ 358) which makes the value of the flux, when a torque is exerted, different from that with the same field excitation and with no armature current. It would therefore be more accurate to use the values of the induced voltages from an ampere-speed curve (§ 289) where the armature reaction is present at its full value. However, since the starting torque usually must be much higher than the actual requirements, no particular accuracy is necessary, as a rule, in its determination, and the additional complication of introducing ampere-speed curves is hardly warranted.

296. EXPERIMENT 12-L. — Static Torque Test on a Series-Wound Motor. — The purpose of the experiment is a direct study of the starting torque of a series motor with a brake. Begin the test with the heaviest possible current and take readings promptly so as not to overheat the machine. Estimate the static friction by measuring the pressure which the brake arm exerts on the scale with the current off. Drive the motor as a generator by means of another available machine, and take its no-load saturation curve. If preferred, the no-load saturation curve may be obtained by one of the other two methods mentioned in § 294. Measure the resistances of the windings.

Report. Plot the values of starting torque against amperes as abscissæ. Give the data on the estimated static friction of the machine. Plot the no-load saturation curve and show how closely the points on the torque curve observed experimentally check with eq. (10). Explain the effect of the armature reaction on the torque at low and at high saturation and at the knee of the saturation curve. Assuming the torque to vary as the n-th power of the current, give the values of n for small, medium, and high values of the current.

297. Motor for Starting Automobile Engine. — Motors used for cranking automobile gasoline engines are of the series type designed to give a particularly heavy starting torque, although their rating for continuous service is very small. In so far as the motor itself is concerned, its tests are not different from those performed on any other series motor and described above. It is customary to plot the performance characteristics of such a motor to speed as abscissæ rather than to its output, as shown in Fig. 215.

There is one feature which distinguishes the actual service performance of an automobile starting motor from that of a series motor connected to a large constant-potential source of supply. The automobile motor is operated from a storage battery of limited capacity, and without any starting or regulating resistance in series. Before the armature acquires any considerable speed and counter-e.m.f., the motor draws a very large current which is limited only by the low internal resistance of the battery (§ 535) and of the motor itself. This current, and the voltage at the motor terminals, depend on the kind and size of battery used, on the state of its charge, the temperature, etc.

Thus, performance curves of an automobile starting motor, obtained with a constant-voltage supply, will not give a true picture of its performance with a battery. On the other hand, the performance obtained with a battery may be misleading, unless the condition of the battery is known. In practice, such tests are made with a certain agreed type and size of battery, when the latter is well charged. Motors which under such conditions do not develop a certain specified torque are rejected.

THE COMPOUND-WOUND MOTOR

298. An ordinary shunt-wound motor is entirely satisfactory in most cases where an approximately constant or adjustable speed is required with variable load, as for example in machine-tool drive. There are, however, cases of considerable practical importance, where the characteristics of the shunt-wound motor can be improved by providing it with an additional series winding, similar to that of a compound-wound generator. Such a motor is called a compound-wound motor; the series winding can be connected either to strengthen or to weaken the field produced by the shunt winding. In the first case the motor is said to be *cumulatively* compounded; in the second, *differentially* compounded.

299. Cumulative Compounding. — Cumulative compounding of a shunt motor is used when the motor is frequently subjected to heavy short overloads, for example when driving a punch and shears, a planer, or an elevator. With a shunt-wound motor, when a punch touches a sheet of metal, or when a planer tool begins a new cutting stroke, there is a rush of current into the motor armature to provide the necessary torque. With a cumulatively compound-wound motor this rush of current is reduced because the field is automatically strengthened when an overload occurs, so that the same torque is obtained with a smaller current This is because the electromagnetic torque, or the tangential force between the armature and the field, is proportional to the product of the field flux and the armature current (§ 267). If the field becomes stronger when an overload occurs, the armature current need not be as large for the same torque.

The speed of a cumulatively compounded motor fluctuates more than that of a shunt-wound motor, because of the above-mentioned automatic strengthening of the field on overloads. This is an advantage rather than a drawback, in that the output is automatically reduced on overloads as compared to that which it would be at a constant speed.

A compound-wound motor possesses operating characteristics intermediate between those of a shunt-wound and a series-wound motor. By properly selecting the relative number of ampere-turns in the two windings, the characteristics may be made to approach those of either of the two simpler types of motors. It is even possible to adjust the characteristics during the operation. Thus, a compound-wound elevator motor may be started with a large number of series turns to get a high torque, and then part of the series winding may be short-circuited to give a higher speed and more of a constant-speed characteristic of a shunt-wound motor. For further details regarding field windings see § 243.

300. EXPERIMENT 12-M. — Effect of Cumulative Compounding on a Motor. — The experiment is conducted essentially as an ordinary load test on a shunt motor (§§ 280 and 281). In order to see more clearly the influence of the series winding, it is advisable to provide different degrees of compounding, either by dividing the series winding into sections, or by connecting a variable shunt around it (Fig. 204).

(a) Begin with the heaviest load and no compounding, that is, use the shunt winding only; adjust the load so as to have the armature current say 25 per cent above the normal rating. Read amperes, volts, speed, and field current; also the torque if a brake or dynamometer is used.

(b) Introduce part of the series winding into the circuit, and take readings with the same input or brake torque as before. Increase the number of series ampere-turns, with the same load, and read as before. Take altogether four or five sets of readings to illustrate the action of the series winding at a certain input or load.

(c) Take a lighter load and again take readings with different degrees of compounding for several steps down to no load. The results obtained will show the influence of compounding on the current taken by the motor, on its efficiency, and on speed variations as compared to an ordinary shunt motor. If it is possible to keep the brushes stationary they should not be shifted during the whole test, as this would change the armature reaction, and consequently the speed of the motor. Should it be necessary to move the brushes because of an excessive sparking, this must be noted in the results, and a record made of the number of commutator bars by which the brushes were shifted.

(d) The behavior of the motor under actual fluctuating load conditions should be observed, both with and without the series winding. A lathe is convenient for this purpose; a piece of work is put in it, such that the tool cuts during only a part of the revolution, thus giving the motor a fluctuating load. No *exact* measurements can be attempted under these conditions unless a special recording ammeter and a tachograph are available. In the absence of such, the current is read every few seconds, and speed observed as closely as possible with an ordinary tachometer. The results when plotted to time as abscissæ will give an approximate idea of the general performance of the motor under sudden overloads, with and without the series winding.

Report. The results of the tests should be plotted to either torque, output, or armature amperes as abscissæ, all on the same sheet, so as to show the influence of compounding on speed, efficiency, input, and output of the motor.

301. Differential Compounding. — There are cases in which a nearly *constant speed at all loads* is very essential. This is the requirement, for instance, in some textile mills, where one motor operates a large number of spinning or weaving machines. The speed of the motor should not depend on the number of machines in actual operation, for every change in speed affects the quality or the design of the product. Motors used for such purposes are provided with a shunt winding and also with a demagnetizing, or *differential* series winding, for the following reason:

A shunt-wound motor usually slows down a few per cent as the load increases, and in order to bring up the speed to its former value, it is necessary to weaken the field of the motor by the field rheostat. In a differentially compounded motor this is done automatically. When the load increases the armature current also increases; thus a larger current flows through the series winding. Since this winding is so connected as to oppose the shunt-field winding, the field strength is reduced, and the motor speeds up. By a proper adjustment of the number of turns of the series winding, a fairly constant speed can be secured throughout the working range of the motor. By a further increase in the number of turns the speed of the motor may be made to increase with the load.

It is not possible to obtain an absolutely constant speed at all loads because of the saturation of the iron parts. The speed depends upon the *flux*, while the compounding action is proportional to the number of *ampere-turns*. When the iron is far from saturation the flux is proportional to the ampere-turns, so that theoretically a constant speed may be had at all loads. With the usual degrees of saturation this condition does not hold true, and a motor compounded so as to give the same speed at full-load as at no load, runs at somewhat lower speeds at partial loads. This result of saturation is observed in a generator compounded so as to give the same voltage at full load as at no load; it then gives too high a voltage at intermediate loads (§ 242).

A differentially-compounded motor does not start as easily as an ordinary shunt motor because the starting current weakens the field and thereby reduces the starting torque. If an excessive starting current is allowed to flow through the series field, the action of the latter may even become stronger than that of the shunt field and the motor will have a tendency to start in the wrong direction. For this reason differentially-wound motors are usually provided with a switch for short-circuiting the series winding during the starting period. After a considerable speed has been attained, and the current has dropped to a lower value, this switch is opened, and the motor runs up to its rated speed.

With the development of satisfactory centrifugal speed regulators, the field of application of differentially-compounded motors has been narrowed down to special cases.

302. EXPERIMENT 12-N. — Effect of Differential Compounding on a Shunt-Wound Motor. — The purpose and the theory of differential compounding are explained in the preceding section. The motor under test is connected to a steady load, such as a generator or a blower, either directly or through a dynamometer (§§ 304 and 305). A Prony brake is not so suitable for this experiment because of the difficulty of keeping the load constant while the speed is being read. If the output cannot be measured by any of the above mentioned means, the speed readings may be referred to the values of armature current plotted as abscissæ (the ampere-speed curve, § 279).

(1) Disconnect the series winding altogether. Begin the runs with the largest armature current and the highest shunt-field current practicable. Read both currents and the line voltage, and measure the speed as accurately as possible. Keep the shunt-field current constant and reduce the load in steps to zero, taking similar readings at each step.

(2) Introduce part of the series winding so as to oppose the shunt field, or if it is not practicable to subdivide this winding, connect all the series turns in the circuit and arrange to shunt them by a variable resistance. Repeat the runs performed in (1), with several different values of effective series turns.

(3) Run the motor as in (1) and (2) above, but at a much lower value of the shunt-field current. Make such runs at two or three different values of exciting current to determine the effect of the saturation upon speed regulation.

(4) Determine the number of turns in the series winding (§ 243).

Report. (1) Plot the observed values as ordinates against the armature amperes as abscissæ. For each value of the shunt-field current state the number of series turns which gives the best speed regulation between no load and full load. Calculate the percentage of speed deviation for some of the runs, especially the maximum percentage of deviation from the desired speed with the high and low values of the shunt-field current. Explain the difference theoretically.

BRAKES AND DYNAMOMETERS

303. The Prony Brake. — The performance curves shown in Figs. 205 and 215 can be obtained directly by loading the motor and measuring the input, output, and speed. The simplest device for loading a motor is a Prony brake (Fig. 216); it is extensively used with small and medium size motors. In the form shown in the sketch, it consists of an iron band *ab* lined with soft wood, heavy canvas, or regular brake lining. The band embraces the pulley *P* of the motor, and is fastened to the beam *k*, the end of which rests on the scale *S*.

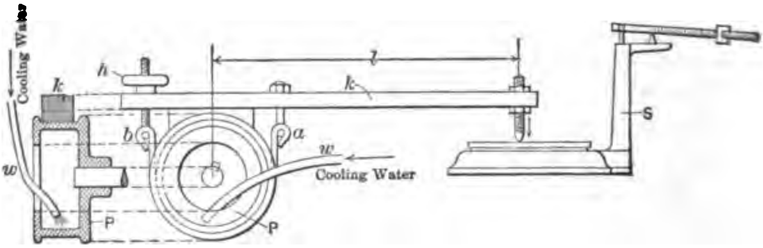


FIG. 216. A Prony brake with water-cooled pulley.

When the motor armature revolves, friction is developed between the brake lining and the pulley; the power of the motor is thus converted into heat. The brake pressure is regulated by the handwheel *h*, and in this way various loads may be obtained. The turning moment, or the *torque*, as it is called, is proportional to the pressure exerted and is measured on the scale. In most cases it is necessary to carry away the heat developed by friction, in order to prevent burning the brake lining. The pulley shown in the sketch is cooled by a stream of water from the pipe *w*; the water is kept against the inner surface of the face of the pulley by centrifugal force and the flange prevents it from being spilled.

The brake arm exerts a certain pressure on the scale even with the loose band, because of its weight. This unbalanced weight, or tare, must be subtracted from the scale reading to obtain the net pressure. Let F be the *net* pressure on the scale at the end of a lever of length l. Then the torque $T = Fl$, in meter kilograms, or in foot-pounds, according to the units used. The corresponding output, P, is computed from formula (6) or (7), § 267.

For loading very large motors the so-called water-brake is used; power is absorbed in it by driving water through paths of high resistance. The resultant friction heats up and even evaporates the water, while cold water is being continually supplied from the mains.

Eddy-current brakes are used to some extent with small motors. Such a brake usually consists of a copper disk mounted on the motor shaft, and a set of electromagnets placed near the disk and supported on knife-edges or ball bearings. When the disk revolves, eddy currents are induced in it as it cuts through the field produced by the electromagnets. The electromagnets tend to follow the disk, as in the classical Arago experiment, but are prevented by a lever which rests on a scale, as in Fig. 216; the pressure on the scale measures the torque. The load is varied by regulating the exciting current of the electromagnets; artificial cooling of the disk is sometimes necessary. An eddy-current brake gives a steadier load than is possible with a mechanical brake.

304. Transmission Dynamometer. — One objection to the Prony brake is that the friction between the brake lining and the pulley continually varies, so that the load is not quite steady while the speed is being measured; this limits the accuracy of the test. Another disadvantage is that a considerable amount of energy must be dissipated in a limited space. It is much more convenient to load the motor on a steady output, such as an electric generator, a pump, a blower, etc., provided that this output can be readily measured. One method is to use a generator whose efficiency is known from a previous calibration. Another way is to connect the load to the motor under test through a transmission dynamometer, such as is shown in Figs. 217 and 218. This dynamometer does not absorb any power, but merely measures the torque between the driving and the driven shafts.

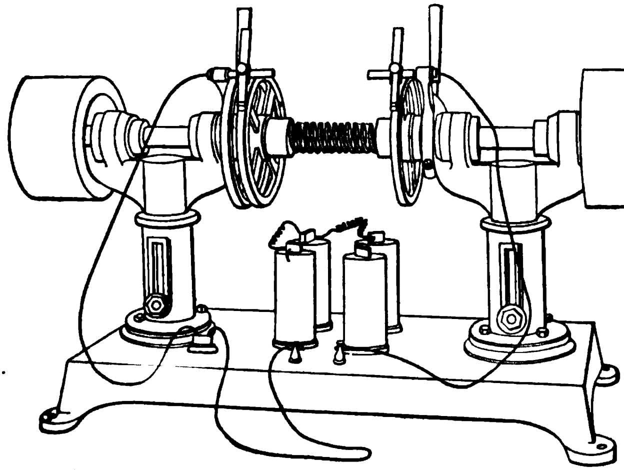


FIG. 217. A transmission dynamometer (see Fig. 218).

The torque is measured by the angle of torsion of the spiral spring previously calibrated in pounds or in kilograms. This angle is measured electrically, as shown in the sketches. With the machine stationary, an electric circuit is established through a few dry cells, a telephone receiver, the dynamometer spring, brushes b_1 and b_2, and the corresponding contacts on two insulating rings. The brushes are set in such a position that they come under the contacts simultaneously when the spring is not under torsion. The circuit is then closed and this fact is recognized by a click in the telephone receiver. When the dynamometer is revolving, carrying a load, the spring is twisted and the contacts are no longer closed simultaneously; the click in the telephone disappears. The left-hand brush is then shifted along the contact ring until the click is heard again. The angle of shift is read on the stationary index, and is evidently equal to the angle by which the spring has been twisted. The transmitted power is calculated from the formula:

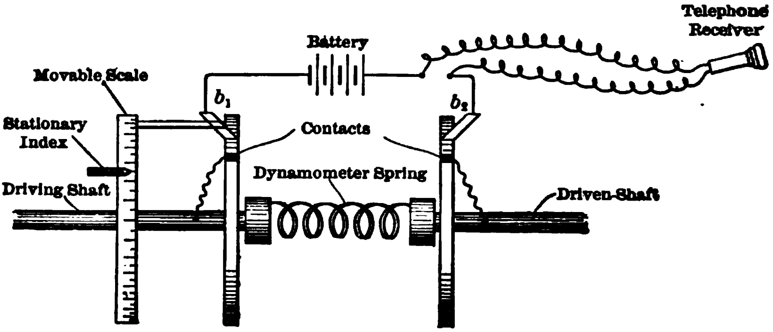


FIG. 218. Electrical connections in the dynamometer shown in Fig. 217.

$$\text{power output} = \text{constant} \times \text{angle} \times (\text{r.p.m.}) \quad . \quad . \quad . \quad . \qquad (11)$$

The spring is previously calibrated by weights, and the torsion constant calculated. Several springs of different rigidity may be used with the same dynamometer to increase its useful range. The dynamometer may be direct-connected to the two machines or it may be provided with one or two pulleys, and one or both machines may be belted to it.

305. Electromagnetic Dynamometer. — This dynamometer (Fig. 219) is essentially a direct-current machine which may act either as a generator or as a motor. Its revolving part is coupled or otherwise connected to the machine under test, while the stationary part, instead of being fastened to the bed-plate, is supported from the shaft on separate bearings. The stator is carefully balanced so that its center of gravity lies as nearly as possible in the vertical plane passing through the center line of the shaft. When the dynamometer field is properly excited, and a current is flowing through its armature, a tangential electromagnetic effort is exerted on the stator, tending to set it in rotation. This torque is opposed by a lever arm pressing on a scale, as in Fig. 216. This pressure is read on the scale and the torque is computed as with the Prony brake.

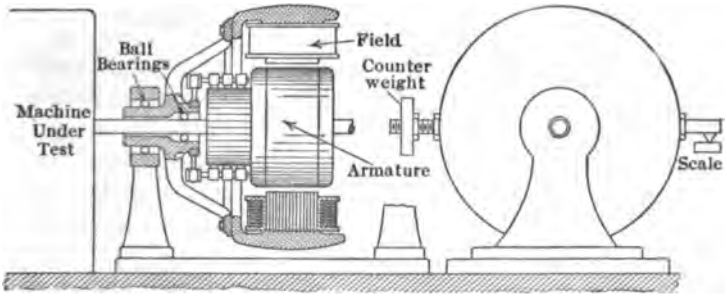


FIG. 219. An electromagnetic dynamometer.

The total torque exerted between the stationary and the revolving parts of the dynamometer includes that due to the regular armature currents, eddy currents and hysteresis, as well as the torque corresponding to the inner bearing and brush friction. The windage torque is not recorded on the scale, and is either neglected or estimated from a separate test.

When the machine under test is an electric motor or a prime mover, the dynamometer is made to act as a generator, either dissipating the electrical energy in a resistance or pumping it back into a line. When the machine under test is an electric generator, a pump, or any machine requiring mechanical energy to drive it, the dynamometer is made to act as a motor receiving electrical energy from the line.

An earlier form of this dynamometer, the so-called *cradle* dynamometer, consists of an ordinary electrical machine whose frame is mounted on a " cradle " supported on trunnions. The deflection of the frame is counterbalanced by weights moved along an arm. Such a device can be easily rigged up in a laboratory.

A Calibrated Machine. Any ordinary direct-current machine may be used for the same purpose as the above described dynamometer, without having its stator frame supported on bearings, and without the scale. It is only necessary to calibrate the machine carefully as to its efficiency and losses at different loads and speeds, by one of the methods described in Chapter XIII. Then, when the input is known, the output may be found from a curve, or vice versa. Of course, there will be inaccuracies due to the variations in the core loss with the temperature, ageing of the iron, variable friction, and additional load losses due to the armature reaction and commutation that are difficult even to estimate. For this reason the dynamometer arrangement described above is more accurate, since all the factors just enumerated are taken care of automatically in the mechanical force measured on the

TROUBLES IN DIRECT-CURRENT MACHINES

306. Troubles in an electrical machine may be either of mechanical or of electrical nature. Mechanical troubles, such as unbalancing, defective bearings, etc., can be detected with comparative ease; they are remedied in the same way as in any non-electrical machine. Purely electrical troubles require more special knowledge for their detection; some of the symptoms and the detection of the more important electrical troubles are described below.

The three parts of a direct-current machine that are liable to troubles of electrical nature, are: the armature, the commutator, and the field magnets; these will be considered separately. It is hardly possible to give detailed and complete instructions for locating all kinds of troubles, since they often occur in a manner and place in which they are least expected. Sound judgment and an instinct bred of experience enable a trained man to locate a trouble in a few minutes, where a beginner may spend several days. For insulation measurements see Vol. II.

307. Troubles in Field. — The principal troubles in the field circuit are as follows: The winding may be partly or totally short-circuited, or it may be grounded to the frame. The coils may be connected in such a way as to produce two adjacent poles of the same polarity. The winding may be broken (open circuit). The machine may have lost its residual magnetism, or it may have it in the wrong direction.

All these troubles can be detected by comparatively simple means. A short-circuit of low resistance may be detected by measuring the resistance of the winding with a Wheatstone bridge, or by the drop-of-potential method. A short-circuit of comparatively high resistance is best detected with alternating current. The faulty coil is removed from the pole piece and placed in a core-type transformer (Fig. 245) with the upper part of the core removable. The primary coil of the transformer is connected to an available source of d.c. supply, while the field coil under test serves as a secondary. When this coil is free from short-circuit, the primary current is not affected by its presence. Any short-circuit will cause an induced secondary current, and thus will increase the primary line current, read on an ammeter. A partial short-circuit may be burned out by this method and thus made easier to locate.

If the winding is grounded to the frame, an ordinary incandescent lamp connected to a source of power will light up between the winding and the frame; or else the ground can be detected by a Wheatstone bridge, a galvanoscope, a megger, etc. If the ground resistance is high, it is measured with a voltmeter, as follows: Let the unknown ground resistance be x and the known resistance of a voltmeter be R. The voltmeter is first connected directly across a constant source of voltage; let the indication be E. Then it is connected to the same source in series with x. Let the new indication be e. The resistance x acts as a voltmeter multiplier (§ 37) and we have

$$E/e = (R + x)/R \qquad (12)$$

from which

$$x = R(E - e)/e \qquad (13)$$

One ground connection may not interfere with the operation as long as the rest of the circuit is thoroughly insulated from the ground. But should another point touch the ground a short-circuit is produced, with all its harmful effects. An open circuit, or a loose connection, is easily traced out with a test lamp. Wrong polarity can be detected by bringing a magnetic needle near consecutive poles. A loss of residual magnetism is usually discovered when the generator fails to excite itself, although a broken or wrong connection has the same effect.

A portable incandesent lamp is sometimes used in testing the polarity of the field coils in a multipolar machine. This is done by placing a lighted carbon-filament lamp between the pole tips. According to the direction of the magnetic flux the loops of the filament will either draw closer together or become separated. In this way a dead pole can be detected. Another test of the polarity of field coils consists in passing a current through the field winding and placing two ordinary iron nails on two adjacent poles. If the polarity is right the nails will be attracted, and if wrong they will be repelled. A dead coil may be detected in a similar manner.

308. Troubles in Armature. — There are four principal faults to look for in an armature, viz., a short-circuit, an open circuit, grounded winding, and a reversed coil. The usual method of locating a short-circuit (Fig. 220) consists in passing a current through the stationary armature and measuring the voltage drop between each two adjacent commutator segments by means of a milli-voltmeter, or a low-reading voltmeter, V. At the place of a short-circuit, as between a and b, the drop is almost zero, or at least lower than that between the other segments. After the trouble has been thus located, it remains to determine whether the short-circuit is in the coil *c* or between the two commutator bars. This is done by disconnecting the coil from the commutator and trying the voltage drop again between the same two commutator bars and between the ends of the coil. An open circuit is also tested by means of a low-reading voltmeter. In this case the deflection suddenly rises when passing over the fault. A ground is detected by the same means as in a field coil (§ 307).

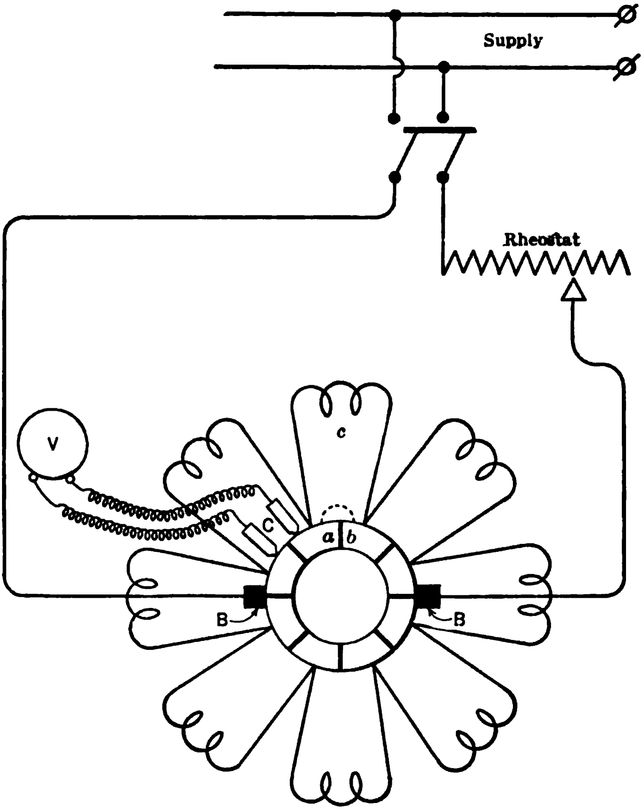


Fig. 220. Location of a fault in an armature with a low-reading voltmeter.

A modification of the foregoing test consists in using alternating current, instead of direct current, and applying a telephone receiver instead of a milli-voltmeter. Where the winding and the commutator are clear of short-circuit, a distinct humming is heard in the receiver; the humming ceases when passing over the short-circuited bars. A clear understanding of the armature connections is necessary in order to apply this test intelligently (See Armature Windings, Vol. II). Under certain conditions, in a multipolar armature, there may be other " silent " coils besides the damaged one.

A core-type transformer is often used for locating short-circuited armature coils. The principle is the same as that described above for field coils. Armature coils may either be tested individually or assembled on the armature core. In the latter case the arrangement is such (Fig. 221) that the total voltage induced in a perfect winding is equal to zero. The total width *a* between the transformer poles is smaller than the coil pitch, so that no voltage is induced in the coils in position 1, while the voltages induced in pairs of coils, such as 2 and 3, neutralize each other. However, should there be a local short-circuit between the turns in coil 2, a current would flow through it; and if the transformer is large enough, the heat in the coil may be sufficient to be detected, or may partly burn the insulation.

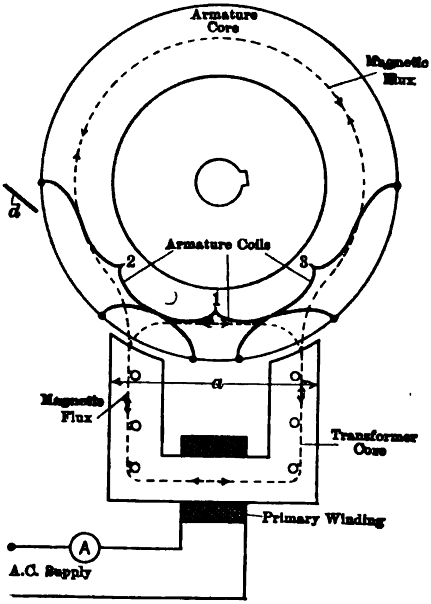


Fig. 221. A transformer for locating a short-circuited armature coil.

The armature is slowly turned on its support so as to bring all the coils in succession under the influence of the transformer. If the short-circuit current is not large enough to be detected by heating or by the increase in the transformer current, a piece of metal, or even a screw-driver, is passed around the commutator so as to short-circuit different coils in succession. A spark will be drawn from good coils, while the short-circuited coil will show no spark. However, the absence of sparking may also indicate an open-circuited coil. To decide the question, a light piece of sheet iron, shown at *d*, is held near the coil under test. The short-circuit current will create a local flux and the piece of iron will vibrate rapidly. No such vibration occurs with an open-circuited coil. A telephone receiver may also be applied to successive commutator bars to detect a faulty coil.

An *open-circuited* or short-circuited coil can sometimes be detected by inspection, especially when the machine is started as a motor. With a short-circuited coil the armature will not start on the first few rheostat points, and, when it does start, will take an excessive current. The defective coil will be warmer than the rest of the winding. With an open-circuited coil a vicious spark is visible under the brushes as the coil undergoes commutation, that is, passes from one group of coils to another. The corresponding commutator bars will be found burned and roughened. An open circuit may also be detected with a test lamp, or by a bar-to-bar test described above for locating a short-circuited coil. With a clear understanding of the armature connections the reader will be able to predict changes in the distribution of current or voltage due to an open-circuit, and will plan his measurements accordingly.

A *ground,* or an electrical connection between an armature coil and the iron core, can often be located with a test lamp. With a low-resistance armature and a dead ground it may be necessary to short-circuit the commutator with a bare copper wire and to send a heavy current between it and the shaft of the machine through the grounded coil. This will create around the faulty coil a local magnetic field which can be detected with a small piece of sheet iron. A telephone receiver between successive commutator bars may also be used.

A *reversed armature coil* is one in which the leads to the commutator are reversed. By passing a large current through the armature, this coil is made to create in the corresponding slots a magnetic flux of wrong polarity. This polarity can be determined with a compass needle or with a small bar of magnetized steel, placed in succession over different slots.

A more accurate test for a reversed coil is as follows: The armature is placed on two horses, and a field coil is mounted over it. The field coil is excited with direct current, a milli-voltmeter is connected to two adjacent commutator bars, and then the field circuit is quickly opened. This will induce a voltage in the armature coils, and the milli-voltmeter will show a transient deflection or " inductive kick." By connecting the milli-voltmeter in succession to different coils, a coil will be found for which this deflection is a maximum. After this, each coil is investigated in this position as the armature is being rotated on the horses. A reversed coil will show a " kick " in the opposite direction. A faulty coil, for example, one with a wrong number of turns, will cause a deflection of wrong magnitude. If the armature is large or has single-turn coils, or if the milli-voltmeter is not sensitive enough, a greater deflection can be obtained by reversing the foregoing method.

A heavy current is passed through the armature and this circuit is opened to give the kick. The milli-voltmeter is connected across the field coil which is held in the same position as before. This coil is made of a large number of turns of fine wire, to give a deflection of sufficient magnitude.

309. Commutator Troubles. — These are usually manifested in excessive heating and sparking; sometimes the cause lies in the commutator itself, and sometimes in the armature winding, as explained above. The principal commutator troubles proper are: rough surface, a high bar, high insulation between bars, a wrong setting of the brushes, and a short-circuit between the bars. A rough surface, or a high bar, is usually detected by an inspection of the commutator as soon as the trouble has been noticed. A wrong setting of the brushes is discovered by shifting the brushes until the position of minimum sparking is found. Old machines usually require shifting of the brushes with various loads because of a large armature reaction. More modern machines generally have a permanent setting from no load to full load. In a machine with commutating poles (§ 268) the brushes must be set on the geometric neutral.

A short-circuit between the bars is the most serious trouble and requires immediate attention; if the bars *a* and *b* (Fig. 220) are short-circuited they short-circuit the coil *c*. As this coil revolves in a strong magnetic field, heavy currents are produced in it, and the coil is usually burned out in a short time. The methods of locating this trouble are the same as those of locating a short-circuited armature coil and are described in the preceding section.

310. Some Causes of Sparking. — To give an idea of the variety of causes, both electrical and mechanical, that lead to poor commutation and sparking, a list is quoted below prepared by a prominent manufacturer of carbon brushes. See also §§ 364 to 378 on commutation and brushes.

1. Brushes off electrical neutral. — Shift to neutral by trial, or set on neutral by means of voltmeter.
2. Brushes spanning too many bars. — Trim down faces of brushes for short distance back from end or, if holders are clamp type, order thinner brushes.
3. Brush studs not parallel with the commutator bars. — Bend the brush studs or grind or shim under the bolts which fasten the studs to the yoke.
4. Incorrect brush spacing. — Check the spacing by counting the number of bars between studs or by placing a strip of paper around the commutator with divisions marked off equal to the number of studs,

and correct the spacing by rotating the brush studs or the brush holders on the studs.

5. Brushes tight in brush holders. — Clean the holders with gasoline, and if brushes are still tight, sandpaper them down or file out the holders carefully.

6. Brush pressure too low. — Pressure should be 1¾ to 2¼ pounds per square inch cross-section for stationary motors and generators, 2½ to 4 pounds for elevator and mill motors, 3 to 5 pounds for crane motors, 4 to 7 pounds for railway motors.

7. Too low contact drop of brush. — Consult a brush manufacturer.

8. Insufficient abrasive action of brushes. — Use a commutator stone or more abrasive brushes.

9. High mica. — Use abrasive brushes or a commutator stone, or undercut the mica.

10. Chattering. — Due to rough and dirty commutator, high mica, high bars, and flat spots.

11. Poor adjustment of interpoles. — Consult the manufacturer of the machine.

12. Overloads. — Undercut the mica, use low-friction brushes and check up all causes for short-circuit currents, such as: (a) Brush off neutral. (b) Faulty brush spacing. (c) Too thick brushes. (d) Unequal air-gaps. (e) Crooked brush studs. (f) Too low contact drop of brushes. (g) Unbalanced armature winding.

13. Open circuit in armature coil. — Rewind that part of the armature.

14. Loose end connection. — Scrape and resolder all defective connections.

15. Worn bearings. — Shim or renew the bearings.

16. Unequal air-gaps. — Shim the short poles or grind off the faces of the long poles, or if from worn bearings, see paragraph above.

17. Short-circuit currents between brush studs caused by unbalanced armature winding. — Consult the manufacturer of the machine.

18. Eccentric commutator on high-speed machine. — Turn or grind.

19. Poor belt lacing. — Re-lace or, still better, use a continuous belt.

20. Pound of reciprocating engine driving the machine.

21. Unstable foundation.

22. Cross currents between generators operated in parallel driven by reciprocating engines, due to variation in angular speed of engines. — Use heavier fly-wheel.

311. EXPERIMENT 12-O. — Locating Troubles in a Direct-Current Machine. — The purpose of the experiment is to become familiar with the various methods described in §§ 307 to 310. Since an inexperienced person may spend a long time before finding a short-circuit or a ground, it is advisable to let the student himself provide various faults in the machine, observe its behavior, and then perform the measurements described above for locating that particular trouble. In this way more can be accomplished within a shorter time, and various faults can be studied systematically. A fault sometimes causes different symptoms, according to whether the machine is running as a generator or as a motor; therefore, the machine should be tested as both whenever practicable.

(a) Begin the experiment by short-circuiting part of the field winding and operate the machine as a generator and as a motor; note the abnormal symptoms observed. Then do the same with the other troubles enumerated in § 307: make wrong connections, produce an opposite residual magnetism, etc. Record all the symptoms observed, and make clear to yourself how the real cause could be located by gradually eliminating all other possible causes.

(b) Produce successively in the armature the faults described in § 308 and observe the symptoms accompanying each. Care should be taken not to damage the machine, as these faults are usually accompanied by vicious sparking and heavy currents. The necks of two commutator bars may be provided with small screws for placing an artificial short-circuit between them. For short-circuiting, use a small fuse wire which would melt before the coil could be damaged. It is interesting to observe the effect of two or more coils short-circuited simultaneously. Show how the coils can be located in this case by the drop-of-potential method.

(c) Produce some of the commutation faults described in § 309. Be careful not to damage the machine, and in each case make clear to yourself the exact appearance of the trouble, as distinct from sparking due to some other cause.

(d) After having had sufficient practice with the above faults, the student should feel enough confidence in himself to locate some prearranged troubles, without knowing their place or nature. One man of the section, or the instructor, may be asked to arrange one or more faults in the machine, for the others to locate. This will make the experiment more profitable and interesting.

Report. Describe the observed faults and the methods used in testing and locating them. Where more than one method is possible, state the advantages and the disadvantages of each, and the scope of its application. Write brief instructions for an electrical repair shop as to what to do and in what order, when a small direct-current machine is sent in as defective and when no other information is available as to the exact nature of the trouble.

CHAPTER XIII

LOSSES IN DIRECT-CURRENT MACHINERY

312. A DIRECT-CURRENT machine can be operated either as a generator or as a motor. In the former case it transforms the mechanical energy of the prime mover into electrical energy available at its terminals; in the second case it converts the electrical input from the line into mechanical energy available on its shaft. In both cases this transformation of energy is necessarily accompanied by losses in the machine itself.

These losses cause the output of the machine to be less than the input; the ratio of the two is called the *efficiency* of the machine. Efficiency is thus indirectly a measure of the losses. By definition

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} \quad \ldots\ldots\ldots\ldots \quad (1)$$

This expression holds true for either a generator or a motor. The efficiency may also be expressed in terms of the output and the losses, thus:

$$\text{Efficiency of generator} = \frac{\text{electrical output}}{\text{electrical output} + \text{losses}} \quad \ldots \quad (2)$$

$$\text{Efficiency of motor} = \frac{\text{mechanical output}}{\text{mechanical output} + \text{losses}} \quad \ldots \quad (3)$$

The efficiency of a motor may also be expressed in the following form:

$$\text{Efficiency of motor} = \frac{\text{electrical input} - \text{losses}}{\text{electrical input}} \quad \ldots \quad (4)$$

313. Direct and Indirect Methods of Determining Efficiency. — The experimental methods used for determining efficiency differ according to whether it is expressed explicitly in terms of both the input and the output, as in formula (1), or only in terms of one of these and the losses, as in formulæ (2), (3), and (4). The first method is sometimes called *the direct method* of determining efficiency and the second, *the indirect*. Both are used in practice, and there are cases in which one or the other is preferable.

When both the input and the output are to be measured directly, an actual load test is necessary, and this is the principal drawback of the method. While the *electrical* input or output can usually be measured quite accurately with comparatively simple means, the measurement of *mechanical* power requires a brake, a transmission dynamometer, or similar devices which are neither easily operated nor very accurate even with a small machine, and are hardly applicable to large machines. Moreover, the waste of power unavoidable with such a test is objectionable, and in many cases even prohibitive, because it may be impossible to get the necessary source of power. Besides, the direct method gives only the sum total of the losses and not the separate losses. The results obtained cannot be very accurate, because the losses are determined as a difference between two large and not very different quantities — the input and the output. A small error in either of these may result in a considerable error in the computed magnitude of the losses. These objections do not apply to the opposition or "loading-back" tests described in the next chapter.

The *indirect* method is therefore usually employed, and by its use the segregation of the losses into the separate components is made possible. The machine under test is run at *no load*, and the power necessary for driving it is measured. This power is used entirely for overcoming the losses, and from it the core loss and the friction in the machine can be computed. There are three ways in which a machine may be run at no load, viz.,

(1) The machine may be driven electrically, as a motor.

(2) The machine may be driven mechanically, by a motor.

(3) The stored kinetic energy of the revolving part of the machine may be utilized (the retardation method).

All these methods, with suitable modifications, are applicable to alternating-current machines.

314. Losses in a Direct-Current Machine. — The losses in an electrical machine, running as a motor or as a generator, may be subdivided into the following three classes:

(a) The I^2R heat loss in the armature, commutator, brushes, field windings, rheostats, shunts, etc. Most of these losses can be computed from the measured ohmic resistances of the windings. The additional copper loss due to commutation and to eddy currents in the armature conductors is either neglected or estimated by means of known empirical coefficients for the type of machine under test.

(b) Iron loss, or core loss, consisting of hysteresis and eddy currents in the armature core and a small additional loss due to eddy currents in the pole faces. These losses depend upon the magnetic flux and the speed of the machine. If the efficiency is desired for one particular speed and field current only, the core loss may be combined with the friction loss and the machine may be run at no load under the desired conditions. Otherwise, these losses are preferably separated, as is explained below.

(c) Mechanical losses, viz., bearing friction, brush friction, and windage or air resistance. These losses depend only upon the speed of the machine. With a belt drive the friction depends on the tension of the belt. This increase in friction can hardly be taken into account, and ordinarily is not charged to the electric machine.

315. Example of Computation of Efficiency from Losses. — Let it be required to calculate the efficiency curve of a 50 horse-power 220-volt shunt-wound motor. Suppose, then, that at no load the motor takes 10.6 amperes (armature current), that the field current which the motor requires is equal to 5.8 amperes, and that the resistance of the armature, found by measurement, is 0.0377 ohm. Neglecting the armature copper loss, the iron loss and friction equals 220×10.6 watts $= 2.33$ kilowatts. The excitation loss is 220×5.8 watts $= 1.275$ kilowatts. The total loss independent of the load is $2.33 + 1.275 = 3.605$ kilowatts. The copper loss in the armature depends upon the load; since we do not know what current the motor takes at an output of 50 horse-power, we may find it by trials. A still better method is to construct the whole efficiency curve from no load to one and a quarter load and then take from this curve the point corresponding to the rated load.

An ideal 50 horse-power motor would take, at full load, $50 \times 746/220 = 170$ amp. The real motor will probably take 190 or 200 amperes. We therefore construct the efficiency curve for points, say, between 100 amperes and 250 amperes. The full load point will surely lie on this curve. Take, for example, the 150-ampere point. The total electrical input is $(150 + 5.8) \times 220 = 34.28$ kilowatts; the copper loss in the armature is $150^2 \times 0.0377 = 0.848$ kilowatt. The total loss is $3.605 + 0.848 = 4.453$ kilowatts.

$$\text{The efficiency} = \frac{34.28 - 4.453}{34.28} = 87 \text{ per cent.}$$

$$\text{The output} = \frac{34.28 - 4.453}{0.746} = 40.1 \text{ horse-power.}$$

In this manner the efficiency can be calculated for various values of output. The efficiency at an output of 50 horse-power is then found from the curve plotted. The efficiency of a generator is computed by a similar method.

For accurate results it is necessary to drive the machine at no load under conditions as nearly identical as possible with those under load.

In the case of a generator, the speed must be equal to the expected speed of the prime mover, which may be slightly different at different loads. The field current must have the same value as is expected at the load for which the efficiency is to be computed; in other words, the armature voltage must be equal to the desired terminal voltage of the machine under load, plus the ohmic drop in the series circuit. Because of the armature reaction (§ 358), these factors are difficult to estimate theoretically. Therefore, it is advisable to precede the no-load test by a load test under the desired conditions of terminal voltage or field current (§§ 235 and 236). From these tests the induced armature voltage can be accurately computed and the no-load test performed under more nearly correct conditions. For additional load losses, see § 316 below.

In the case of a motor, the difficulty lies in estimating the actual speed under different loads and with different values of field current. To obtain more accurate results, the no-load test should be preceded by taking an ampere-speed curve (§ 279). Then, for any desired current input, the counter-e.m.f. at the proper speed can be computed and the motor run at no load under the same conditions.

316. Conventional Efficiency and Directly Measured Efficiency. — In the foregoing computation of the efficiency from the losses, the following assumptions are made: (1) With given values of exciting current and speed, the core loss is the same at full load as it is at no load. (2) The copper loss in the loaded armature is equal to the computed I^2R loss, where R is the ohmic resistance measured with the armature stationary. (3) The voltage drop between the commutator and the brushes is the same in a loaded machine as in the machine standing still. (4) There are no commutation losses.

In reality all these assumptions are only approximately correct. The armature reaction (§ 358) weakens the flux under one-half of the pole, and strengthens it under the other half. The core loss increases nearly as the square of the maximum flux density to which a particular part of the laminations is cyclically subjected. All armature teeth undergo cycles of magnetization corresponding to the increased flux density, so that the core loss in the teeth increases appreciably. On account of this non-uniform flux distribution, eddy currents are induced in the armature conductors, causing an additional copper loss not taken into account in the computed value of I^2R. There is also some additional loss due to a non-uniform distribution of current under the brushes, commutation spark, copper loss in the short-circuited armature coils (§ 365), and eddy currents in the pole shoes.

These *additional load losses* (Zusätzliche Verluste), or *stray load losses*, cannot be either computed or measured directly, but their existence has been proved experimentally by a comparison of the efficiencies of the same machine determined by the direct and indirect methods (§ 313). The efficiency computed from the losses comes out appreciably higher than that determined from the ratio of the output to the input.

Eliminating from the load test the friction, the computed I^2R loss, and the core loss at no load, the remainder will represent the stray load losses enumerated above. Certain numerical coefficients have been suggested, varying from 1.1 to 1.3, by which the measured no-load losses should be multiplied to obtain the corresponding losses under full-load conditions. Any such coefficient is but a crude empirical correction which should be applied with great caution and only to types of machines for which it has been determined experimentally. Some idea of commutation losses may be obtained by running the machine with the armature short-circuited and noting the increase in power necessary to drive it, over and above that required on open circuit.

The American Institute of Electrical Engineers in its Standards recognizes a *conventional* efficiency computed from the no-load losses, and the *directly measured* efficiency determined from an input-output test. The opposition or " circulating-power " tests described in Chapter XIV are particularly well adapted to determining the efficiency under the actual load conditions. At the same time, it is recognized that the conventional efficiency can be determined much more readily, and as long as competitive machines are always tested and compared by the same method, there is seldom an occasion for determining the efficiency from an actual load test.

Before beginning any of the tests described in this chapter it is advisable to let the machine run idle for at least half an hour to get the bearings warmed up and the friction more nearly constant. This precaution may not be always practicable in a students' laboratory because of the limited time available, but is advisable in more accurate work.

317. Determination of the Total No-Load Loss in a D.C. Machine Driven Electrically. — This method is applicable to a machine with either a series or a shunt winding, or with both. The machine is connected for running as a shunt-wound motor (Fig. 202) and the shunt-field winding is excited in the usual way. The series winding, if needed for excitation, should not be left in series with the armature circuit, but separately excited like the shunt winding, preferably from an independent low-voltage source. In a compound-wound machine the series winding may be left out entirely if the required field strength can be obtained with the shunt winding alone. For a modification of this method especially adapted to the series motor, see § 321.

The machine is run at any desired values of speed and armature voltage by regulating the field rheostat and the rheostat in the main circuit. The power input into the armature is measured as the product of the armature current times the voltage at the armature terminals. This input goes to overcome the core loss and friction, plus a small copper loss due to the current itself. Knowing the armature resistance, the latter loss may be easily corrected for. The efficiency is computed from these readings as is explained on the example in § 315.

318. EXPERIMENT 13-A. — Efficiency from Losses, Machine Driven Electrically. — The purpose of the experiment and the method are explained in § 317. Connect the machine under test as in Fig. 202, and run it at no load and at a high speed for a sufficient time to warm up the bearings. If the efficiency is desired for the machine acting as a generator, take a load characteristic either at a constant voltage (§ 236) or at a constant field current (§ 235). If the machine is to be used as a motor, take one or more ampere-speed curves (§ 280). In either case measure the resistance of the armature, including the brushes; also that of the series-field winding and any other resistance that is in series with the armature, and is an integral part of the machine, such as the interpole winding (§ 268) and the compensating winding (§ 370). From these data, compute in the laboratory the values of the e.m.f. induced in the armature for the points for which the losses are to be measured.

Start the machine in the usual way as a motor, and bring it up to the desired speed at the required value of the armature voltage. Measure accurately the armature current at no load and note the values of speed, voltage, and field current. Take such readings for several points on the desired characteristic of the machine.

If it is not practicable to take a load characteristic or an ampere-speed curve, estimate the induced voltage in the case of a generator, and the speed under load for a motor, from the terminal voltage and the ohmic drop in the machine, disregarding the armature reaction.

Report. Plot the load characteristics or the ampere-speed curves, if any were taken. If not, show how the necessary data for the no-load runs were estimated. Explain the reason for determining the losses at the particular values of field current or voltage and speed selected. Correct the armature input at no load for the I^2R loss, or show that it was negligible. Compute the total losses for a number of values of armature load current and plot the corresponding efficiency curves against either the input or the output as abscissæ.

319. Determination of the Total No-Load Loss in a D.C. Machine Driven Mechanically. — There are cases in which the machine under test cannot be run as a motor, so that the test described in §§ 317 and 318 is not practicable. For example, the machine to be tested may be wound for 500 volts and only a 110-volt source of supply may be available. In such cases (Fig. 222) the machine is belted to a small auxiliary motor and driven at the right speed and excitation, at no load. The input into the driving motor is equal to the losses in the machine under test, less a correction for the losses in the driving motor itself.

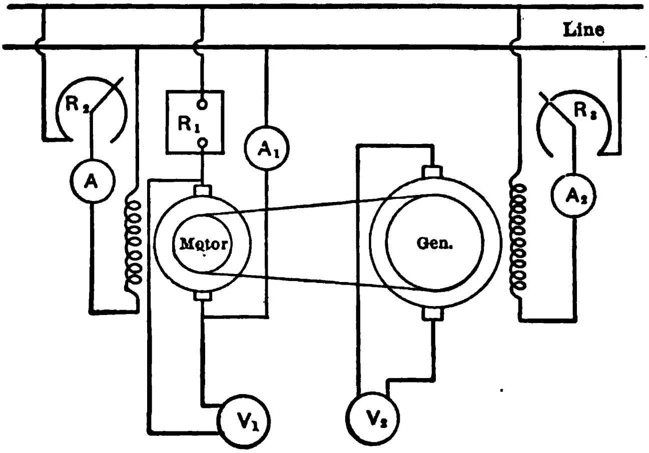


FIG. 222. Separation of losses in a direct-current machine. The machine is driven mechanically as a generator.

The losses in the driving motor are taken into account as follows: Let the input into the armature of the auxiliary motor, in one of the runs, be 5 amperes at 104 volts, or 520 watts. Suppose that when the same motor was run alone (with the belt off) it took an armature current of 1.3 amperes at 101 volts, with the same excitation and at the same speed in both cases. Let the resistance of the motor armature, as found by measurement, be 0.16 ohm. Then the iron loss and friction in the motor amount to $131.3 - 1.3^2 \times 0.16 = 131$ watts. Therefore the iron loss and friction in the machine under test are

$$520 - 131 - 5^2 \times 0.16 = 385 \text{ watts.}$$

It will be seen from these figures that the correction for the I^2R loss is small when the driving motor is running idle, and in many cases may be neglected altogether. The rating of the driving motor must be small as compared to that of the machine under test, in order that the driving motor may be run at a point near its maximum efficiency. In this case the correction for its own losses is comparatively small, and the result will be more accurate. The correct values of speed and of field excitation at which the machine under test must be driven, are determined or estimated in the same manner as in the case of a machine driven electrically.

Instead of estimating the losses in the driving motor, its output may be determined directly by either of the following two methods:

(a) *The Prony Brake.* After having taken the necessary readings with the machine under test, disconnect the latter, and apply a Prony brake (§§ 281 and 303) to the driving motor. Adjust the load in such a way as to duplicate the former input readings, that is, the same field current, the same armature current, and speed. From the brake reading, the output can be directly computed, and the losses in the motor need not be known. The driving motor may be of either direct or alternating-current type, and the theory of its operation has no bearing on the test. It is only necessary to duplicate the input readings with the machine under test and with the brake. The brake output is equal to the input into the machine under test.

(b) *A Transmission Dynamometer.* The machine under test is connected to the driving motor through a transmission dynamometer (§ 304). The input into the machine under test is read directly, and it is not even necessary to know the electrical input into the driving motor.

320. EXPERIMENT 13-B. — Efficiency from Losses, Machine Driven Mechanically. — The experiment is performed as explained in § 319. The diagram of connections is shown in Fig. 222; the driving motor is at the left, the machine under test at the right. The fields of both machines are excited separately, so that the power loss in the field windings does not enter into the calculations.

(1) Drive the machine under test at the speed or speeds at which it is to be operated under load, and at the required values of field excitation or armature voltage, as the case may be. For each run read the input into the armature of the driving motor, its field current, and speed. (2) At the end of the experiment take off the belt and run the driving motor alone for the same values of speed and field current. (3) Measure the armature resistances of both machines.

Report. Explain the reasons for selecting the particular values of field current and speed for the various runs. (2) Tabulate the input readings into the driving motor. Tabulate the input readings with the driving motor running alone, and from these figure out the losses in the machine under test. (3) Plot these losses and the computed copper loss against the input or the output as abscissæ, and plot the efficiency curves to the same abscissæ. State sources of inaccuracy in the test and in the computations.

321. Efficiency of a Series-Wound Machine from Losses. - The machine may be driven either electrically or mechanically (§§ 317 and 319), and if the field winding be excited from an independent source, the tests do not differ in any respect from those described above for a shunt-wound machine. It is possible, however, to drive the machine electrically as a motor at no load, with the field winding in series with the armature (Fig. 203), provided that a resistance is connected in parallel with the armature, as shown. By shunting the major part of the field current through this resistance it is possible to run the motor with a high field flux and at a moderate speed. The armature current is just sufficient to supply the core loss, and the friction and windage losses. Thus, these losses can be measured at different speeds and at different values of field excitation. The efficiency of the machine may then be computed under any desired load conditions (§§ 289 and 290).

In § 317, where a similar method is described, the operating characteristics of the machine are supposed to have been determined experimentally, in the form of an ampere-speed curve. This ampere-speed relationship may also be obtained by a simple computation from a test with the shunted armature, as is illustrated in the following numerical example.

Let a 220-volt series motor have an armature resistance of 0.4 ohm and a series field resistance of 0.3 ohm. Let it be required to obtain its performance characteristics at a load current of 30 amperes. At this current, the counter-e.m.f. is $220 - 30(0.4 + 0.3) = 199$ volts. The motor is run at no load with the armature shunted, and the two rheostats are so adjusted that the field current is 30 amperes when the voltage between the brushes is slightly over 199 volts, say 200 volts. Let the armature current under these conditions be 3 amperes. Then the counter-e.m.f. is $200 - 3 \times 0.4 = 198.8$ volts, which is close enough to the desired value, 199. Let the speed be 1250 r.p.m. Neglecting the effect of the armature reaction, we now have the same speed and the same core loss as with the machine loaded at 30 amperes. The no-load losses are equal to $3 \times 198.8 = 596.4$ watt. The useful output at this current is $220 \times 30 - 596.4 - 30^2 \times 0.7 = 5374$ watts. The efficiency is $5374/6600 = 81.4$ per cent, and the torque (§ 267) is $974 \times 5.37/1250 = 4.18$ met.-kg.

By performing similar computations and measurements at other assumed values of current, complete performance curves of the motor can be plotted.

322. EXPERIMENT 13-C. — Efficiency of a Series-Wound Machine from the Losses. — The purpose and the method are explained in the preceding section. (1) Measure the resistances of the armature and the field, and compute the values of the counter-e.m.f. for different values of load current to be used. (2) Connect the motor as in Fig. 203 and run it at no load and at the corresponding values of field current and counter-e.m.f. At each step measure the field amperes, armature amperes, armature volts, and speed. Be sure to measure the actual field current and to compute the voltage drop in the armature for the corresponding load current. As a check on the method, connect the motor to a suitable load and take an ampere-speed curve, as in § 291.

Report. Give the values of the measured resistances and the computed values of counter-e.m.f. Plot field current, no-load losses, and copper losses against speed as abscissæ. For the same abscissæ compute and plot the input, the output, efficiency, and torque. Plot the observed ampere-speed curve and explain the discrepancy, if any, with the computed one. Using the available test data, explain, by means of an example, how an ampere-speed characteristic could be computed for operation with the field winding shunted in a given ratio. See § 288 under (2). Explain how the test and the computations would be modified if the machine under test were intended for use as a generator, and the characteristics were desired under those conditions.

EXPERIMENTAL SEPARATION OF LOSSES

323. The designer of electrical machinery — and sometimes the user — is not satisfied to know the sum total of the losses, but desires to know the values of the separate component losses described in § 314 and shown in Fig. 223. The same losses are shown in the following table.

Total loss	(1) Copper loss	Copper loss in armature; Copper loss in field.
	(2) Core loss	Hysteresis loss; Eddy-current loss.
	(3) Mechanical losses	Bearing friction; Brush friction; Windage.

A knowledge of the separate losses is of importance for the following reasons:

(a) Knowing the losses as functions of the load, speed, etc., it is possible to determine the rating of the machine for different classes of service (intermittent load, variable load, constant load, etc.).

(b) The magnitude of the separate losses is a check on the quality of materials, workmanship, and design.

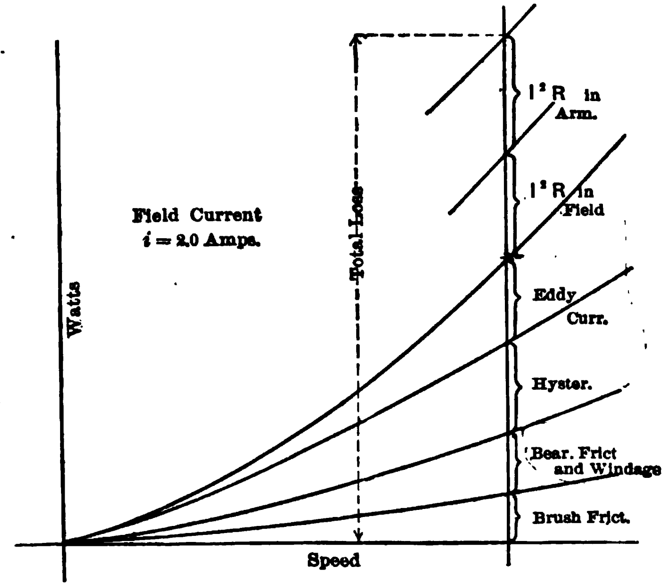


FIG. 223. Separate losses in a direct-current machine.

(c) Knowing the values of the principal losses, the designer is enabled to vary the dimensions of the machine, still keeping its efficiency as high as competition may require, and preserving reasonable limits of temperature rise.

Of the three kinds of losses, only those mentioned under (2) and (3) above need be discussed here. The copper loss, I^2R, is readily calculated from the measured resistances of the windings. However, see § 316 in regard to additional load losses.

324. Core Loss and Mechanical Losses. — Frictional losses (3) depend only on the speed of the machine, and may be plotted as in Figs. 223 and 224. In ordinary practical testing, the bearing friction is seldom separated from the windage.

The core loss depends upon the speed and the field current, and is usually represented by a set of curves, as shown in Fig. 225. The components of the core loss — hysteresis and eddy currents — are shown separately in Figs. 226 and 227. It is important for the manufacturer to have eddy currents separated from the hysteresis loss. A high hysteresis loss means that the iron used is of inferior quality, or the flux density is too high. A large eddy-current loss may indicate that the laminations are too thick, or not sufficiently insulated from each other. Thus the remedy is quite different in the two cases.

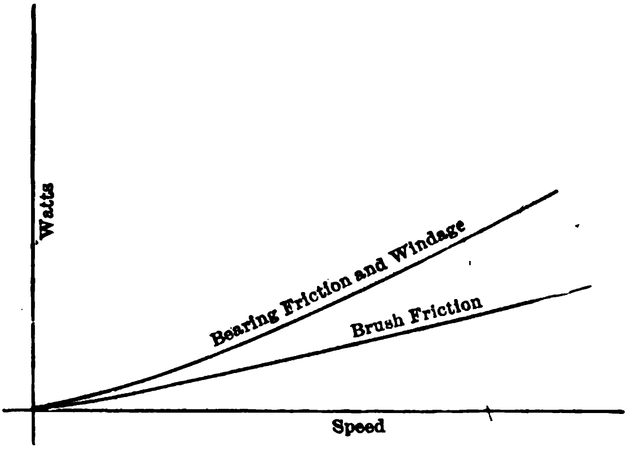


FIG. 224. Brush friction separated from bearing friction and windage.

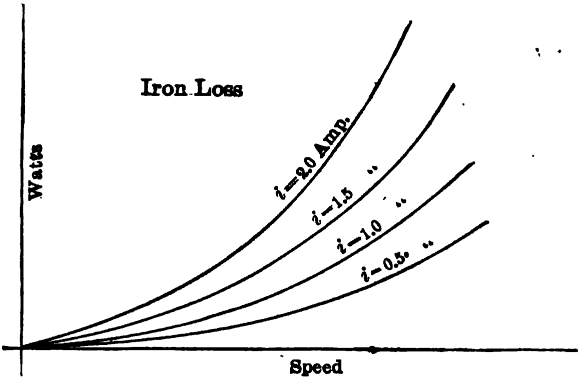


FIG. 225. Curves of iron loss as a function of speed and of exciting current.

The sum of the losses (2) and (3), or, as it is sometimes called, the total no-load loss (Fig. 228), is determined by measuring the power necessary for driving the machine at no load. The three possible methods of doing this are the same as are enumerated in § 313 under the indirect methods of measuring efficiency. In particular, the reader is advised to be[illegible] thoroughly familiar with §§ 317 to 320 before proceeding any further. The total loss having been obtained under different conditions of speed, excitation, friction, etc., the individual losses are computed mostly by subtraction. For example, if it takes P_1 watts to drive a machine with the brushes pressing on the commutator and P_2 watts with the brushes lifted, the brush friction evidently is equal to $(P_1 - P_2)$ watts.

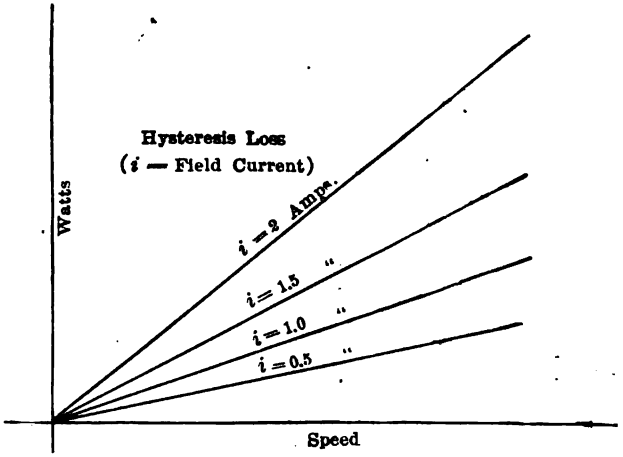


FIG. 226. Curves of hysteresis loss as a function of speed and of exciting current.

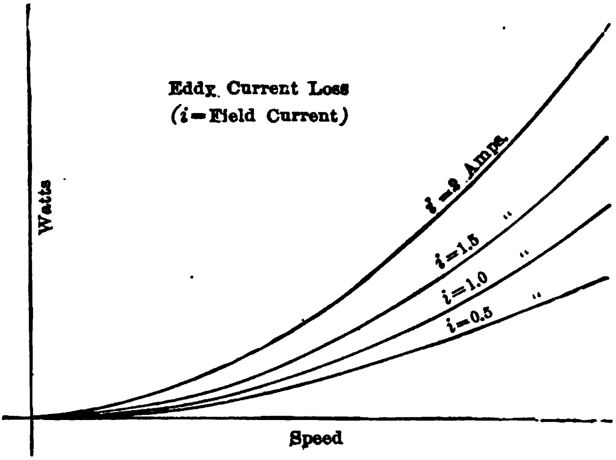


FIG. 227. Curves of eddy-current loss as a function of speed and of exciting current.

The tester's skill consists in running the machine under such conditions that from the data obtained the individual losses may be separated with the least amount of labor and with the greatest possible accuracy. Before performing any of the following experiments, §§ 334 to 337 should be carefully studied and each experiment planned with the view to the subsequent separation of the losses by computation.

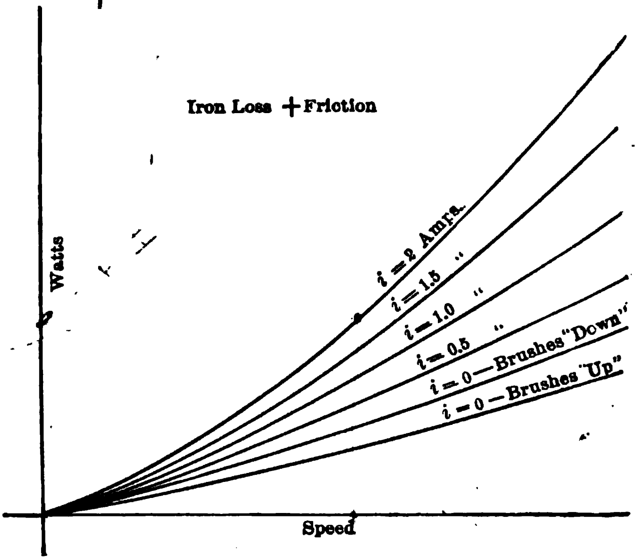


FIG. 228. Total no-load losses as a function of speed and of exciting current.

325. EXPERIMENT 13-D. — Separation of Losses, Machine Driven Electrically. — The purpose of the experiment is to separate the total losses in a direct-current generator or motor into the components shown in Fig. 223. For general instructions see §§ 323 and 324. The machine under test is connected as shown in Fig. 202 and driven electrically, as a motor at no load, within as wide limits of field current and speed as is feasible. The machine should first be run for at least half an hour at a high speed in order to attain constant friction conditions. Lubrication must, of course, be kept the same during the whole test.

(a) Begin the test with the strongest field current possible and at the maximum speed that can be obtained with this field current, i.e. at a reasonable over-voltage. Read armature volts and amperes, field amperes, and speed. Then, *keeping the field current constant at this maximum value*, reduce the speed in 8 to 10 approximately equal steps to its lowest possible limit, by introducing resistance in the armature circuit; take similar readings at each step. The values of the product "armature volts times armature amperes" (with a small correction for the armature I^2R loss) will give directly the data for the upper curve in Fig. 228.

(b) Make similar runs with smaller values of field current, to correspond to the other curves on the same sheet. If a separation of losses is to be made, as in Fig. 232, use at least six different values of field current and take the last value as small as possible, so as to obtain the required range of speed.

(c) The two lower curves in Fig. 228 cannot be taken experimentally since the machine requires an appreciable field current to run as a motor. The friction can be determined from the other curves by calculation (§ 334). The separation of the brush friction from the bearing friction and windage cannot be performed very accurately because at least two brushes must lie on the commutator during the test. The only possible way is to lift as many brushes as possible, and from the decrease in the input to figure out what the reduction in power would be if *all* brush friction were eliminated. This test should be performed at a low field current, because then the friction constitutes a larger percentage of the total losses.

(d) Measure the resistances of the windings.

Report. Plot watts input into the armature (less the armature I^2R loss) against speed as abscissæ, as in Fig. 228. Separate the friction from the iron loss as explained in § 334; separate the hysteresis loss from the eddy-current loss as shown in Fig. 232. Do this at least for the normal excitation of the machine, although preferably for all the curves shown in Fig. 225, so as to show the influence of the value of field current on the iron losses. Plot all the losses at a chosen field current to speed as abscissæ (Fig. 223). Show how the efficiency at full load and at the rated speed may be computed from these data, for the machine running as a generator and as a motor.

326. EXPERIMENT 13-E. — Separation of Losses, Machine Driven Mechanically. — The purpose of the experiment is to separate the total losses in a direct-current generator or motor into the components shown in Fig. 223. For general instructions, see §§ 323 and 324. The machine under test is belted or direct-connected to a small auxiliary motor and driven at no load, at various speeds and with varying field excitation. The connections are shown in Fig. 222. It is preferable to excite the machine under test from a separate source. The input into the armature of the driving motor represents the sum of iron loss and friction in both machines, plus a small I^2R loss in the motor armature itself (§ 319).

(a) The machines should first be run for at least thirty minutes at a high speed in order to warm up the bearings and make the friction constant. Then excite the machine under test to its highest possible value, and raise the speed to the highest safe limit. Read all the instruments, as shown in Fig. 222, and also the speed of the set. Reduce the speed in steps, either by increasing the field current of the driving motor or by inserting resistance in its armature circuit; both methods can be used simultaneously if so desired. It is sometimes preferable to keep the field current constant, because it is then easier to correct the data for the core loss in the driving motor. The field current of the machine under test must be kept constant during each run, readings being taken at each speed. This test corresponds to the upper curve in Fig. 228, after the losses in the driving motor have been subtracted from the volt-ampere input into its armature, as is explained at the end of § 315.

(b) The same run should be repeated at several lower values of field current in the machine under test, until a sufficient number of curves has been obtained (five or six). The last curve is taken with the field circuit opened and corresponds to the frictional loss alone.

(c) Lift the brushes from the commutator, so as to eliminate their friction; the difference in the input with the brushes " down " and " up " gives the value of the brush friction. If the machine has several pairs of brushes it is advisable to lift them up in pairs, to see that the friction is reduced in proportion to the number of brushes removed.

(d) Take off the belt and run the driving motor alone at the same speeds and with the same values of exciting current as before, in order to determine its own losses. The loss in the belt can be only roughly estimated. The arrangement should be such as to make this loss as small as possible. Under normal conditions it is not over 2 per cent of the power transmitted.

(e) Measure the armature resistances of both machines.

Report. The requirements are the same as in § 325, except that the iron loss is separated from the total loss directly, by using the experimentally determined friction curve. The method shown in Fig. 231 may be used as a check.

327. Separation of Losses in a Series-Wound Motor. — The reader is supposed to be familiar with the tests and computations described in §§ 321 and 322, and only additional information is given below.

The no-load losses in a series motor depend on two independent variables: the field current and the speed. With a given field strength, the speed is proportional to the induced e.m.f., or very nearly to the voltage at the armature terminals. Thus, it may be said that the *iron loss and friction depend on the field current and on the voltage at the* *armature terminals.* For the purpose of plotting curves, one of the variables must be kept constant during each run. It is convenient to keep the armature voltage constant for each curve, and to vary the field current by means of the rheostats shown in Fig. 203. Data for several loss curves should be taken, within as wide limits of armature voltage and field current as possible, and plotted against speed as abscissæ.

In order to separate the friction from the iron loss, a second set of runs is made, without the shunting resistance around the armature, and with the armature and the field simply connected in series. The terminal voltage must be kept sufficiently low; otherwise the motor will run away. When running at no load under such conditions, the motor takes very little current, and its field is so weak that the iron loss can be neglected and the whole input into the armature assumed to be equal to the friction loss. Note that this is not true with a shunt motor, since there the field has its full value at no load as well as at any other load, and the iron loss is not negligible under any circumstances. The values of friction given by this curve are subtracted from the ordinates of the curves previously taken, and new curves are plotted giving the iron loss separately as a function of speed and field excitation. The hysteresis loss is then separated from the eddy-current loss, as is explained in §§ 335 to 337. Incidentally, the first series of runs gives not only the total losses, but also data for ampere-speed curves corresponding to various terminal voltages.

328. EXPERIMENT 13-F. — Separation of Losses in a Series-Wound Motor. — The purpose and the method of conducting the experiment are explained in the preceding section. Connect the machine as a series motor (Fig. 203) with a resistance shunted across the brushes and another for regulating the main current. Have an ammeter in the field circuit, another in the armature circuit, and connect a voltmeter across the brushes. Run the motor at a high speed for at least thirty minutes to warm up the bearings.

(a) For the first series of runs (iron loss + friction) shunt the armature so that a greater part of the current passes around it through the shunt resistance. Run the machine at no load, and adjust the conditions so as to have a full rated voltage or more across the armature terminals, and a field current of a value at least 25 per cent above the rated current of the motor. Read field amperes, armature amperes, volts across the armature, and speed. Keeping the field current constant, reduce the voltage at the motor terminals in steps until the motor almost stops, and take readings at each step. Make similar runs with lower values of field current.

(b) For the second series of runs (friction alone) remove the shunting resistance, and run the motor light throughout the possible range of speeds. A considerable resistance must be inserted in the main circuit at all times to keep the motor from running away. Take numerous readings of amperes, volts, and speed.

(c) Measure the resistances of the armature and of the field windings.

(d) The ampere-speed curve of the machine may be taken experimentally, or it may be computed from the runs with the armature shunted. With these data the performance characteristics of the machine can be predetermined under any desired conditions of speed, terminal voltage, armature current, or field current.

Report. Plot the power input into the armature to speed as abscissæ, correcting the voltage readings, if necessary, for the IR drop in the armature; each curve must refer to a constant field current. The friction curve may be plotted on the same curve sheet. Subtract the friction from the total loss. Separate the hysteresis from the eddy currents, as in §§ 335 to 337. Compute an ampere-speed curve, as in § 321, and show how the efficiency can be determined for any desired input or speed.

RETARDATION METHOD

329. Instead of driving a machine electrically or mechanically for separating the losses, it is possible to bring it up to the highest safe speed and shut off the power; the machine will then gradually slow down to a standstill, converting its stored kinetic energy into the losses. While the machine is slowing down, instantaneous values of speed are read every few seconds or are recorded continuously. Such runs are made with various values of field current, and the results plotted as speed-time or *retardation* curves (Fig. 229).

The form of these curves depends upon the relative magnitude of the iron loss and the friction, and on their variation with speed. A method is given in § 331 for converting these curves, by calculation, into those shown in Fig. 228. Subsequently the losses can be separated, as explained in §§ 334 to 337.

This method is more applicable to large machines in which the stored energy of the armature is sufficient to maintain the rotation during a comparatively long time (several minutes), so that instantaneous speeds can be taken with considerable accuracy. If the machine has been brought up to speed mechanically, by an auxiliary motor, the belt is thrown off. If set in rotation electrically, the armature circuit is broken.

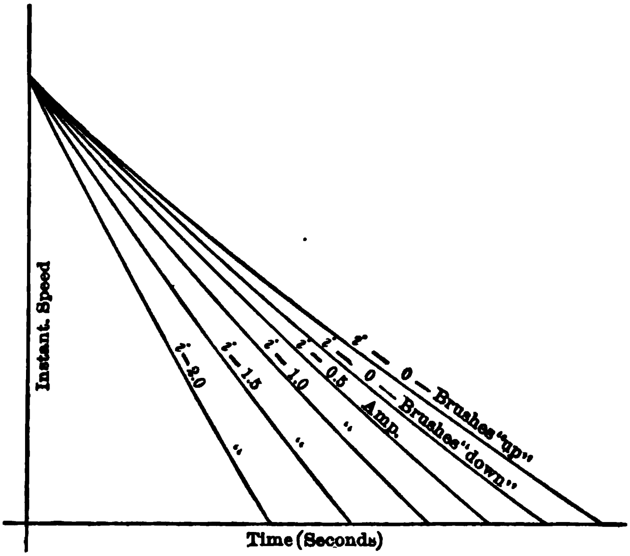


Fig. 229. Retardation or speed-time curves at various values of field current.

330. Measurement of Instantaneous Speeds. — The principal difficulty in performing a retardation test lies in an accurate measurement of speed; otherwise the method is simple and reliable, especially with large machines. Several methods have been used or proposed for measuring instantaneous speeds, and the principal ones are enumerated or described below.

(1) A small magneto-generator, belted to the machine and connected to a sensitive voltmeter, is a convenient device for the purpose.

(2) A centrifugal speed-indicator or a vibrating-reed tachometer may be used, provided that the rate of deceleration is not too great to cause an appreciable time lag in the scale indications.

(3) A recording tachometer or an oscillograph may be used to draw a retardation curve directly.

(4) An ordinary speed-counter may be used to record the total number of revolutions from the moment at which the connection to the external power is broken. This speed-counter is read every two or three seconds without stopping it; the difference of each two consecutive readings gives the total number of revolutions during these two or three seconds. From these readings the average speed corresponding to the middle instant of each time period can be determined.

(5) A voltmeter is left across the armature terminals, and read every two or three seconds. If the field current is kept absolutely constant the induced voltage is exactly proportional to the speed. Before breaking the connections to the source of external power, the coefficient between the speed and the voltage must be determined, so as to be able to convert the observed voltage-time curve into a speed-time curve. This method is not reliable with the field winding opened, because the voltage induced by the residual magnetism is quite low and the brush contact drop is relatively large even at no load.

(6) It is important to measure the speed accurately during the first part of the retardation curves, just after the power is shut off, because the values of the losses are usually desired at these speeds. Sumpner's differential voltmeter method is convenient for the purpose. The machine is speeded up as a motor, and the armature circuit opened on one side of the line only, by a single-pole switch. A low-scale direct-reading voltmeter is connected across the switch terminals and shows practically zero as long as the switch is closed. When the switch is opened and the motor slows down, the voltmeter measures the difference between the line voltage and the e.m.f. induced in the machine under test. The line voltage being constant, and the induced e.m.f. proportional to the instantaneous speeds, the voltmeter readings are proportional to the drop in speed.

(7) The speed may also be read stroboscopically. For this purpose several concentric circles are drawn on a cardboard or metal disk and on the circumference of each circle a number of uniformly spaced projections are marked, the number being different for each circle. The disk is mounted on one end of the shaft of the machine under test.

An electrically driven tuning fork is provided with an aluminum wing on each prong, and each wing has a slit in it. When the fork is stationary the slits overlap, so that the rotating disk can be observed. When the tuning fork is set in vibration, the slit is alternately closed and opened, permitting periodic glimpses of the rotating disk. At a definite speed of the motor, the projections on one of the circles seem stationary when viewed through the tuning fork. Knowing the frequency of the fork and the number of projections, the observer may compute the speed. An alternative method is to determine empirically the speeds at which different figures on the disk appear stationary. After the machine has been brought to a high speed, the observer views the disk through the tuning fork, and marks the instants at which various figures become stationary. These instants may be marked electrically on a chronograph drum. For details see D. Robertson, " Separation of the No-Load Stray Losses in a C.-C. Machine by Stroboscopic Running-Down Methods," Institution of Electrical Engineers (British), Journal, Vol. 53, 1915, pp. 308 to 328.

(8) In addition to the speed, the rate of decrease in speed, or the deceleration, may also be measured, since it enters into the final formulæ discussed in the next section. A simple method consists in connecting an electrostatic condenser in series with a galvanometer, across the armature terminals of the machine under test. As long as the speed and the voltage are constant, the charge on the condenser also remains constant, and the galvanometer deflection is zero. As the speed of the machine becomes lower, the voltage induced in the armature decreases and consequently the condenser charge is reduced in the same proportion. The current which now flows through the galvanometer is proportional to the rate of decrease of the charge or voltage with the time. But the voltage being proportional to the speed, the galvanometer current is proportional to the rate of change in speed with the time, or to the deceleration of the machine.

331. Theory of the Retardation Method. — The kinetic energy stored in the armature at a certain speed is equal to $\frac{1}{2}K\omega^2$, where K is the moment of inertia of the revolving part, and ω is its angular velocity. The latter is proportional to the speed n expressed in r.p.m., so that the stored energy may also be represented in the form $\frac{1}{2}Cn^2$, where C is a constant proportional to the moment of inertia of the revolving part.

Let P, expressed in power units, be the value of the total losses (iron loss and friction) corresponding to this speed. We then have

$$-d\left(\tfrac{1}{2}Cn^2\right) = P\,.\,dt \quad \text{. . . . .} \quad (5)$$

which means that the decrease in kinetic energy during an infinitesimal interval of time dt is equal to the work performed by the armature in overcoming the losses P during the same interval. Differentiating, we obtain $-Cndn = P\,.\,dt$, or

$$P = -C\,.\,n\,.\,dn/dt \quad \text{. . . . .} \quad (6)$$

It is shown below that ndn/dt is numerically equal to the length of the *subnormal* BD (Fig. 230) to the retardation curve of the machine. Thus the foregoing equation becomes

$$P = C\,.\,BD \quad \text{. . . . .} \quad (7)$$

The instantaneous total loss P is proportional to the length of the subnormal to the retardation curve.

Proof that $-ndn/dt = BD$: The ratio $-dn/dt$ is equal to the trigonometric tangent of the angle ACB, the line AC being tangent to the retardation curve at the point A. If AD is normal to the curve, the angle DAB is equal to the angle ACB. From the triangle DAB we have

$$DB = AB \cdot \tan DAB = -n \cdot dn/dt \quad . \quad . \quad . \quad . \quad . \quad (8)$$

which proves the proposition. The "minus" sign is necessary because dn/dt is negative (the speed decreases with the time), while DB is positive. The shape of the revolving part of an electric machine is too complicated to allow its moment of inertia K to be calculated with sufficient accuracy. Therefore K, or the constant C which is proportional to it, is eliminated by performing an additional experiment, as is explained in the next section.

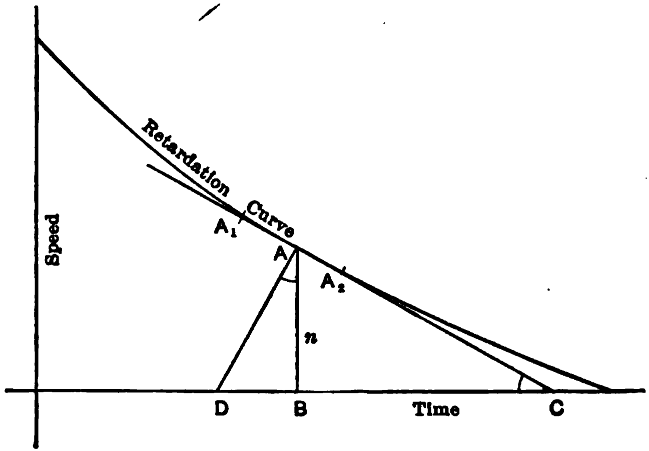


FIG. 230. Analysis of a retardation curve.

Thus, by using eq. (7), the curves in Fig. 229 are transformed into those in Fig. 228 by drawing normals and measuring the corresponding subnormals. The latter, to a certain scale, represent iron loss + friction. Some error is introduced on account of the uncertainty of the exact direction of the normal, but with reasonable care, reliable results are obtained. A convenient way to draw a normal is to take two points, A_1 and A_2 (Fig. 230), at small equal distances from A and to draw AD perpendicular to A_1A_2. If it is preferred to compute P analytically, then writing eq. (5) for a finite interval of time between the points A_1 and A_2, we get

$$\tfrac{1}{2} C (n_1^2 - n_2^2) = P (t_2 - t_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

From this equation P is calculated and is assumed to represent the losses at the point A, for the instant $t = \tfrac{1}{2}(t_1 + t_2)$.

332. Elimination of Moment of Inertia of Revolving Part. — The constant C which enters in the preceding formulæ, and which is proportional to the moment of inertia of the revolving part of the machine, may be eliminated by one of the following four methods:

(1) *The value of the loss P is determined for one point by an independent method.* The machine under test is driven electrically or mechanically (§§ 317 to 321) at a certain speed n, and with a certain field current. The value of the total loss P is determined from the power input into the armature. Then a retardation test is made, using the same field current and starting at a speed somewhat higher than n. The subnormal DB, measured at the point corresponding to the speed n, will give directly the scale for P, since $P = C \times BD$. If, for example, P was found to be 600 watts, and $DB = 2.5$ cm., then the scale is 600/2.5, or 240 watts loss per cm. length of the subnormal.

(2) *The moment of inertia is changed by a known amount.* Two retardation runs are performed in succession with two different fly-wheels on the motor shaft. Both fly-wheels must be of nearly the same weight and shape in order to have the friction and windage nearly the same, but their moments of inertia must differ from each other by a considerable amount. The excess of moment of inertia, or rather of the corresponding constant, ΔC, of one of the fly-wheels over that of the other, must be known. Let the subnormals to the two retardation curves at a certain speed be BD and B_1D_1 respectively. Then, according to eq. (7), we have

$$P = C \,.\, BD = (C + \Delta C) \,.\, B_1D_1 \quad . \;.\; .\; .\; . \quad (10)$$

or, solving for C,

$$C = \Delta C \,.\, B_1D_1/(BD - B_1D_1) \quad . \;.\; .\; .\; .\; . \quad (11)$$

Because of the necessity of having two fly-wheels of known moment of inertia, this method is of limited application. Moreover, the retardation tests performed with these fly-wheels may give somewhat higher values of friction and windage than should be properly charged to the machine under test. In practice it is not always necessary to have two separate fly-wheels; adjustable weights can be moved along the spokes so as to change the moment of inertia of a fly-wheel by a known amount, without changing its weight; or else one retardation test may be performed without any fly-wheel and another with a plain disk of known moment of inertia.

(3) *A known additional load is put on the machine.* This load, p, may be obtained either by closing the armature on a resistance and keeping the output constant with a wattmeter, or by applying a Prony brake with a definite torque. With either method p can be kept constant through a limited range of speed only. This is sufficient,

however, since only one point on the retardation curve is necessary. According to eq. (7) we then have

$$P = C \,.\, BD$$
$$P + p = C \,.\, B_1D_1$$

where B_1D_1 is the subnormal to the new retardation curve. Eliminating P we get:

$$C = \frac{p}{B_1D_1 - BD} \quad \dots\dots\dots\dots \quad (12)$$

A disadvantage of this method is that the additional load should make the retardation curve much steeper in order that the difference between BD and B_1D_1 in eq. (12) may be measured with sufficient accuracy. This means that in the second run the machine may have to be stopped rather quickly, and time intervals or speeds cannot be measured as accurately as in the first run.

(4) *The machine is left connected to the power supply.* The armature is brought up to the highest possible speed and then, with the terminals left connected to the power supply, a fairly large resistance is suddenly inserted in series with the armature. While the revolving part is slowing down, power is drawn continuously from the line, so that the armature slows down at a lower rate than if it were disconnected from the line.

The slope of the retardation curve is determined at a certain selected speed n. The armature input e_1i_1 is also read at that speed. A similar test is made with a lower series resistance, and the armature is made to accelerate past the same speed n, by drawing more power from the line. The slope of the acceleration curve and the armature input e_2i_2 are again determined at the speed n. We then have

$$e_1i_1 = P - C \,.\, B_1D_1 \quad \dots\dots\dots \quad (13)$$
$$e_2i_2 = P + C \,.\, B_2D_2 \quad \dots\dots\dots \quad (14)$$

The first of these equations expresses the fact that during the retardation period the electrical input into the armature goes to overcome the excess of the total loss P over and above the amount supplied by the kinetic energy of the revolving part. Similarly the second equation states that during the acceleration period the armature input serves partly to overcome the total loss P and partly to accelerate the revolving masses. Subtracting eq. (13) from eq. (14) and solving the result for C, we obtain

$$C = \frac{e_2i_2 - e_1i_1}{B_1D_1 + B_2D_2} \quad \dots\dots\dots\dots \quad (15)$$

333. EXPERIMENT 13-G. — Efficiency and Separation of Losses by the Retardation Method. — The purpose and the method of conducting the experiment are explained in §§ 329 to 332.

(a) Have the machine wired so that it can be brought up to speed electrically as a motor (Fig. 202). Run the machine for about thirty minutes to attain steady conditions of lubrication. Bring it up to the highest safe speed, measure this speed, and then suddenly open the circuit (both armature and field), allowing the machine to slow down to a standstill. Measure instantaneous retardation speeds by one of the methods described in § 330. Repeat the run several times until you have learned to read speeds quickly; a metronome may be of assistance.

The armature circuit can be safely opened by tripping a circuit-breaker, but when opening the shunt-field circuit special precautions must be taken in view of the high inductance of the field windings. It is well to provide a special *field-discharge* switch, which in opening the line circuit simultaneously closes the field winding upon itself, through a suitable resistance. In this way an easy path is offered for the inductive discharge, and the danger of breaking down the insulation is minimized.

(b) Bring the machine up to speed again; open the circuit and lift the brushes at the same moment. Take a retardation curve without the brush friction. Repeat this run several times until you get reliable results.

(c) Make several retardation runs, leaving the field circuit closed and opening only the armature circuit. Runs should be made with five or six values of exciting current. The value of the field current while the machine is slowing down need not necessarily be the same as in speeding it up. It is advisable, however, to keep it the same, because otherwise it takes too long for the current to settle to the desired value, on account of the high inductance of the field windings. In order to obtain high speeds, apply voltages to the armature higher than the rated voltage of the machine.

(d) To determine the constant C of the revolving part, use one of the methods described in § 332.

(e) Before leaving the laboratory, measure the resistance of the armature winding.

(f) Sometimes the conditions are such that the machine under test cannot be brought up to the required high initial velocity electrically, as a motor, and has to be speeded up by an auxiliary motor. To obtain some experience and skill in this method, belt the machine under test to a suitable small motor, bring the set up to a high speed and throw off the belt. Take a few retardation curves by this method.

Report. Plot the observed retardation curves (Fig. 229), and ca culate C as in § 332. Convert the retardation curves into the l curves shown in Fig. 228, according to the directions given in § 33 The remaining requirements are the same as in Exps. 13-D and 13-E Compare the results obtained in parts (a) and (f) of the experime and state the advantages and the disadvantages of the two metho of bringing the machine up to speed, especially in the case of a hyp thetical very large machine — not the one tested.

SEPARATION OF LOSSES BY COMPUTATION

334. Separation of Friction from Iron Loss. — It is mentioned i § 325 that when the curves shown in Fig. 228 are obtained by driv the machine electrically as a motor, it is impossible to obtain data the friction alone by test. The friction in this case is computed b applying the self-evident principle that *the friction loss at any spee the limit towards which the total loss is tending* when the excitation a the applied voltage are reduced to zero, the speed being kept co stant. In order to make use of this principle, the curves of to loss in Fig. 228 are replotted to induced e.m.f.'s as abscissæ (Fi 231), each curve referring to a certain constant speed. With th machine running at no load, the voltage at the armature termin is practically equal to the induced counter-e.m.f. of the machin

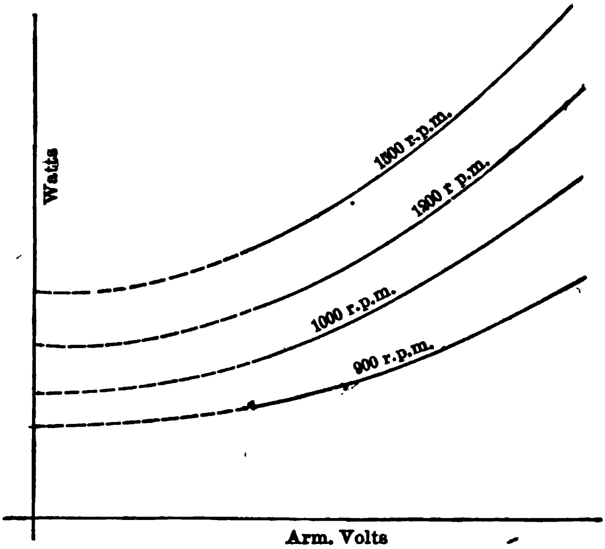


FIG. 231. Extrapolation of curves of total loss, to separate the friction from the core loss.

but a correction for the IR drop in the armature can be readily made if so desired. These curves are then produced to the axis of ordinates as shown by the dotted lines; the intercepts give the values of friction at the corresponding speeds. Plotting these values of friction against speed as abscissæ (as in Figs. 224 and 228) the required friction curve is obtained.

This method is based upon the fact that at a constant speed and at no load the field flux is proportional to the induced armature voltage. Thus, for each curve in Fig. 231, the points, going from the right to the left, represent values of the losses at decreasing values of the field flux. In the limit, at the intercept with the axis of ordinates, the applied voltage, and hence the flux, is zero, so that no iron loss is present and the loss is due to the friction alone. Here, and in the rest of the chapter, "friction" is meant to include the air friction, or windage, as well (Fig. 224).

The dotted parts of the curves in Fig. 231 are obtained either according to the judgment of the eye, or analytically, by assuming them to be parabolæ. In the later case the general equation of the curves may be written in the form

$$P = Q + kE^2, \quad \dots\dots\dots\dots \quad (16)$$

where P is the total loss, Q is the friction loss, and E the induced voltage; k is a constant whose numerical value does not enter into the calculations. In this equation the core loss at a constant frequency is assumed to vary as the square of the induced e.m.f., that is, as the square of the field flux. Writing this equation for some two points on one of the curves, we have:

$$\left.\begin{aligned} P_1 &= Q + kE_1^2 \\ P_2 &= Q + kE_2^2 \end{aligned}\right\} \quad \dots\dots\dots\dots \quad (17)$$

Eliminating k and solving for Q, we get

$$Q = \frac{P_1 - P_2(E_1/E_2)^2}{1 - (E_1/E_2)^2} \quad \dots\dots\dots \quad (18)$$

This value of Q gives the point at which the curve meets the axis of ordinates.

A more accurate solution. For some machines the exponent of E for the curves in Fig. 231 differs considerably from 2, and is different for different parts of each curve. Equation (16) is then replaced by a more general expression

$$P = Q + kE^2 + aE^n \quad \dots\dots\dots \quad (19)$$

This formula contains four parameters, Q, k, a, and n. The curve which it represents can therefore be made to pass through four points on the given experimental curve. Taking a derivative of both sides of eq. (19) with respect to E and multiplying the result by E, we get

$$P'E = 2\,kE^2 + naE^n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

where

$$P' = dP/dE \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Eliminating aE^n between eqs. (19) and (20), gives

$$nP + (2 - n)\,kE^2 = nQ + P'E \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

To eliminate nQ, this equation is written for two points on the given curve, say points 1 and 2. Subtracting the results from each other we get

$$n\,(P_1 - P_2) + (2 - n)\,k\,(E_1^2 - E_2^2) = P_1'E_1 - P_2'E_2 \quad . \quad (23)$$

A similar equation can be written for two other points on the curve, say 3 and 4, thus

$$n\,(P_3 - P_4) + (2 - n)\,k\,(E_3^2 - E_4^2) = P_3'E_3 - P_4'E_4 \quad . \quad (24)$$

These are two simultaneous linear equations which can be readily solved for the unknown quantities n and $(2 - n)k$; the friction loss, Q, is then computed from eq. (22) applied to one of the four points on the curve. The values of the derivative P' for the four points (watts per volt) can be estimated with sufficient accuracy by laying a straightedge tangent to the curve at the desired point.

335. Separation of Hysteresis and Eddy-Current Losses.—The curves shown in Fig. 228 give the total losses and the friction loss for different values of speed and field current. Subtracting the friction loss from the total losses gives the core losses (hysteresis and eddy currents) for various values of speed and exciting current (Fig. 225). The next step is to separate the hysteresis loss from the eddy currents. This is done on the basis of the fact that, *with a given excitation, the power loss due to hysteresis is proportional to the speed, while that due to the eddy-currents increases as the square of the speed* (§§ 209 and 210). Thus, at a certain speed of n revolutions per minute, and at a constant excitation, the total iron loss

$$p = Hn + Fn^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

where H and F are two constants characterizing hysteresis and eddy-current (Foucault) losses respectively at the given field current. A small letter p is used here to distinguish the core loss from the total loss previously denoted by P.

This equation can be used for the separation of the losses in two ways, purely analytical and graphico-analytical. Both solutions are given below, and the reader is also referred to § 223 for a somewhat similar treatment.

336. Analytical Separation of Iron Losses. — Select one of the curves in Fig. 225 and apply eq. (25) to two points taken sufficiently far apart and corresponding to two values of speed, n_1 and n_2:

$$\left.\begin{aligned} p_1 &= Hn_1 + Fn_1^2 \\ p_2 &= Hn_2 + Fn_2^2 \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (26)$$

These equations can be readily solved for H and F. After this, the straight line Hn representing the hysteresis loss (Fig. 226) can be drawn, and the parabola Fn^2 may be plotted, giving the eddy-current loss (Fig. 227). The same procedure is then repeated for the other curves in Fig. 225, and in this way the hysteresis may be separated from eddy currents, for various desired values of speed and field current.

The disadvantage of this purely analytical method is that in selecting two points on a curve the assumption is made that these points are absolutely correct; in other words, that any two other points on the same curve would give exactly the same values of H and F. In reality, the values of H and F vary somewhat for different pairs of points selected on a curve. This means a tedious process of calculating H and F for several combinations of points and taking average values, unless the method of least squares is used, with its involved computations.

337. Graphico-Analytical Separation of Iron Losses. — The graphico-analytical method described below permits of taking simultaneously into account as many points on a curve as desired; in this way the most probable values of the separated losses are obtained in a comparatively simple manner. Dividing both sides of eq. (25) by n, we get

$$p/n = H + Fn \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (27)$$

This is the equation of a straight line between p/n and the speed n. A line DC of this kind is shown in Fig. 232. In order to draw this line, one of the curves in Fig. 225 is chosen and several of its ordinates are divided by the corresponding abscissæ. The values of (p/n) so obtained are marked with crosses in Fig. 232, as ordinates for the corresponding values of speed. The most probable straight line, DC, is then drawn through these points according to the judgment of the eye. Producing this line to the axis of ordinates, a horizontal line AB is obtained, which subdivides the ordinates of DC into the constant part MN equal to the hysteresis coefficient H, and the variable part NP corresponding to the eddy currents, and equal to Fn.

Knowing H, the hysteresis loss is obtained by multiplying H by the desired values of n. Knowing Fn, the eddy-current loss is found by multiplying values of Fn by the corresponding values of n. The separation of the core losses for one particular value of field current is thus completed.

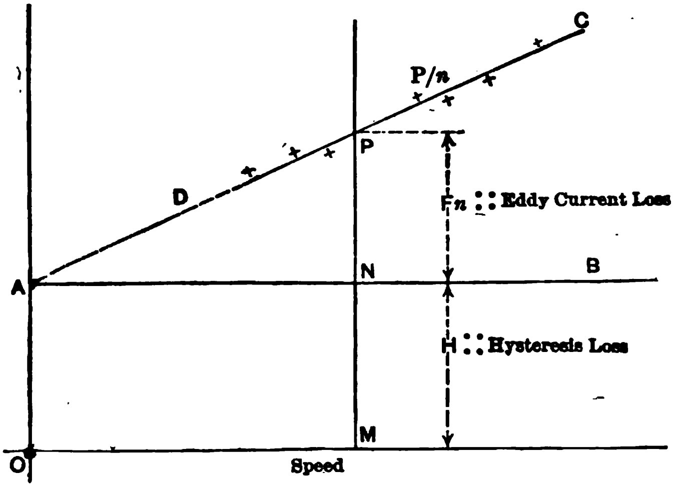


FIG. 232. Graphico-analytical method of separation of hysteresis from eddy currents.

CHAPTER XIV

DIRECT-CURRENT MACHINERY — OPPOSITION RUNS

338. TESTS for determining the efficiency, voltage regulation, and temperature rise in electric generators and motors, may be conveniently performed without the expenditure of much energy, when two machines of about the same size are available. The machines are then loaded to their full capacity *on each other*, no power being wasted in outside resistances, or in a Prony brake. The machines are connected together, both mechanically and electrically (Figs. 234 and 235), and driven so that one machine acts as a generator, the other as a motor. The motor drives the generator, while the latter supplies electric power back to the motor. This is called the opposition or the "pumping-back" method of testing machines.

If the machines were ideal — without losses — the set would be self-contained and would require no power from the outside to drive it. In reality, the losses in both machines have to be supplied either by an auxiliary motor, a booster, or by connecting the set to a source of electrical supply. The power supplied to the set from outside covers only the losses in both machines, and herein lies the economy of the method. Let it be required, for example, to test two 500-kilowatt generators at full load. Assuming the efficiency of each machine to be 90 per cent, the source of power would have to supply $500 \div (0.90)^2 =$ 618 kilowatts if one machine were to drive the other in the ordinary way, the second machine being loaded on resistances. When running the same two machines in opposition, the line has to supply only approximately 10 per cent of the 500 [illegible] 500 kilowatts, or only 100 kilowatts. If a small driving motor or a booster is used (Fig. 234), the losses in these machines must also be supplied from the line; but even then the power demand is much smaller than with an ordinary load test.

In a generator or a motor driven in opposition with another machine, the electrical conditions may be so adjusted as to have a full-load current and the rated voltage. Therefore the opposition or the pumping-back method is used for determining the efficiency, regulation, and temperature rise of a machine, since these three tests require the machine to be run under full-load conditions.

339. A Mechanical Analog of Opposition Runs. — The following analog (Fig. 233) may make clearer the underlying principle of all the opposition methods described below. Let it be required to determine the efficiency of a large crane or hoist under full-load conditions, when there is not enough power to drive it at full load, but when another identical hoist is available. Let the two machines be belted, as shown in the sketch, so that when hoist No. 1 lifts its weight P_1, hoist No. 2 lowers its weight P_2. The weights are equal and correspond to the rated capacity of the hoists; the whole system is mechanically balanced. Without friction, a very small force would cause a movement in either direction. In reality quite a considerable force is necessary to overcome the friction in both machines, and to start the movement.

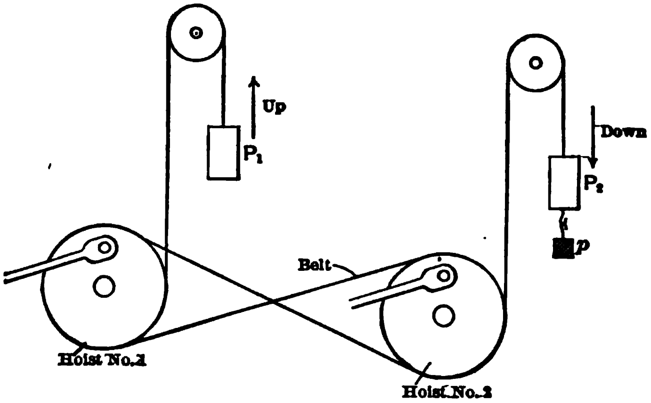


FIG. 233. A mechanical analog illustrating the electrical opposition methods.

This extra power may be supplied either by driving one of the shafts or by adding an extra weight p to the load of one of the hoists. The first solution is analogous to the auxiliary motor shown in Fig. 234, the second solution corresponds to the case of electric power being supplied from the line, as in Fig. 235. Let each of the large weights on the hoists be equal to 10 tons, and let an additional weight of 2 tons be required to move the system at its rated speed. This corresponds to a friction loss of 1 ton per hoist; the average load is 11 tons, consequently the efficiency of either hoist is $(11 - 1)/11 = 91$ per cent.

340. Classification of Opposition Methods. — The methods by which two electrical machines may be run in opposition, are usually classified according to the way in which the losses are supplied. The copper loss may be supplied either electrically or mechanically, and the iron loss + friction may also be supplied either electrically or mechanically. This gives the four combinations shown in the following table:

	Iron loss and friction supplied mechanically	Iron loss and friction supplied electrically
Copper loss supplied electrically	(1) Blondel	(3) Kapp — Parallel Potier — Series Hutchinson — Series and Parallel
Copper loss supplied mechanically	(2) Hopkinson	(4) Impracticable (?)

Blondel's method (1) is theoretically the most rational, because the electrical loss is supplied electrically, while the mechanical losses are supplied mechanically; its disadvantage is that it requires a booster set and an auxiliary motor (Fig. 234). Omitting the booster set and supplying all the losses by the auxiliary motor, gives Hopkinson's method (2); this method is as good as Blondel's for the determination of temperature rise and of voltage or speed regulation, but gives only approximate results for efficiency.

On the other hand, omitting the auxiliary motor in the Blondel scheme, gives the three methods marked (3) in the table (Fig. 235). All the losses are supplied electrically, either by a booster or directly from the line, or by both methods. The correct method among these three is that of Hutchinson, in which the copper loss is supplied by the booster and the iron loss + friction from the line.

Kapp omits the booster and supplies all the losses from the line; Potier does not use the line and supplies all the losses by the booster. These two methods are simpler than Hutchinson's and just as good for the measurement of temperature rise and of voltage or speed regulation, but give only approximate results for efficiency. The method (4) has never been worked out in practice, for it does not seem rational to supply electrical losses mechanically, and vice versa. It may be possible, however, that a simple and satisfactory arrangement of this kind could be devised. All the above methods will now be described more in detail.

341. Losses Supplied Electrically and Mechanically. — We shall begin with Blondel's method, because it is more correct theoretically, and because all the other methods may be deduced from it. The connections are shown in Fig. 234. Two identical machines under test are belted or coupled together; their armatures are connected electrically in opposition. The shunt fields should be excited separately, from the line, whenever possible. The series fields and the interpole windings, if any, are left connected as in the regular operation. The combination of the armature, series field, and interpole winding is referred to below as the *series circuit* of the machine. A small auxiliary motor is belted to one of the machines, and supplies the power for overcoming the iron loss and friction of the set. A booster is connected in series with the armatures, and generates the voltage necessary for overcoming the resistance drop in the series circuits of the two machines. The booster is shown in the diagram driven by a separate motor.

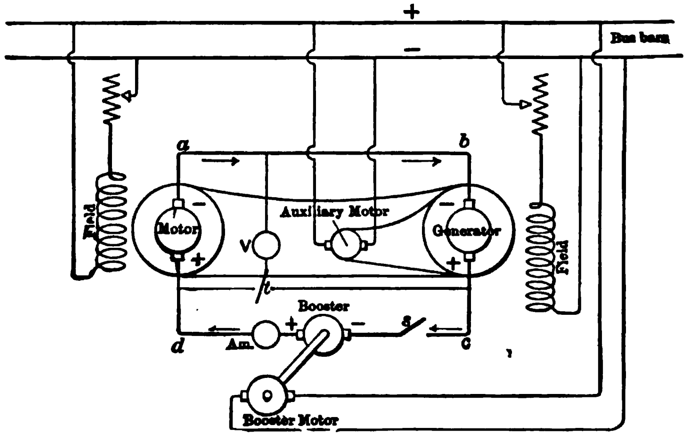


Fig. 234. Blondel's opposition method of testing two direct-current machines.

The set is started and brought up to speed by the auxiliary motor with the switch *s* open. Then the fields of both machines are excited to the required value, and when the voltmeter *V* gives the same reading on both sides, the switch *s* is closed. Very little current flows between the two machines, the armatures being in opposition. By exciting the booster so as to add its voltage to that of the generator, any desired current may be made to circulate between the two machines. The adjustment of the brushes and other similar precautions are the same as in a regular load test.

By regulating the field rheostats of the two machines, the speed of the auxiliary motor, and the booster voltage, any desired conditions of current, voltage, and speed of the set under test can be established. When both machines are equally excited, the booster supplies only the copper loss in the series circuits of the two machines, because the electromagnetic torque between the field and the armature in one machine is equal and opposite to that in the other machine, so that no excess of power is left on the belt for overcoming the mechanical losses. The induced e.m.f.'s being also equal and opposite, the current is made to circulate only by the voltage supplied by the booster. Therefore, the copper loss is supplied entirely by the booster; consequently the iron loss and friction must be supplied by the auxiliary motor.

342. Details of Blondel's Method. — The test may be performed for the purpose of determining (a) the efficiency of the machine, (b) its voltage regulation, and (c) its temperature rise. The machine may be intended for use as a generator or as a motor. All these cases will be considered separately.

(*a*) *Efficiency.* Let the machine under test (Fig. 234) be intended to be used in regular service as a generator. Then its speed and voltage during the test are adjusted to the rated values, and the load current is taken larger than the specified value by the amount of the field current. During the test, the latter is supplied separately, but in regular operation it must be supplied by the armature itself. The other machine must in this case be identical with the machine under test, or else its efficiency must be known. Assuming it to be identical, its fields must be excited to the same value in order to have the same iron loss in both machines. The armature current is adjusted to a proper value by varying the speed and the field current of the booster.

Let P be the power output, in watts, supplied by the auxiliary motor, I the current circulating between the machines, E the rated voltage of the generator, i the exciting current during the test, and e the voltage between the positive terminals of the two machines under test. Voltage e is practically equal to that between the armature terminals of the booster, except for a small drop in the connection cd, the switch, and the ammeter. The resistance of the connection ab must be kept as low as possible. We have then:

useful output of the generator $= E\,(I - i)$;
iron loss + friction in each machine $= \frac{1}{2}\,P$;
copper loss in the series circuit of each machine $= \frac{1}{2}\,eI$;
shunt excitation loss in each machine $= Ei$;

the latter includes the loss in the field rheostat.

$$\text{Efficiency of the generator} = \frac{E\,(I - i)}{EI + \frac{1}{2}\,P + \frac{1}{2}\,eI} \quad . \; . \; . \quad (1)$$

The output of the auxiliary motor is determined from its input a losses (§ 319); all the other quantities which enter into this expressi are measured directly during the test.

Now let the machine under test be intended for service as a moto With the same notation as before, we have:

$$\text{Efficiency of the motor} = \frac{EI - \frac{1}{2}P - \frac{1}{2}eI}{E(I + i)} \quad . \quad . \quad . \quad (2$$

The denominator of this expression represents the total input into th armature and shunt field. The numerator is equal to the input int the armature less the core loss, friction, and the copper loss in the seri circuit. If the motor is rated in terms of current or power input, th circulating current I can be adjusted directly to its proper value. If however, the rating is given in terms of output, it may be necessar to make several trials before the value of I is found for which th numerator of formula (2) has the desired value.

(*b*) *Voltage or Speed Regulation.* The adjustment is the same as before, but the machines need not be alike. Having obtained the desire load conditions on the generator under test, reduce the booster voltage or increase that of the other machine until the circulating current is reduced to zero, or, more correctly, to the value of the field current. This corresponds to throwing the load off the machine, as is required for the test on voltage regulation (§ 233). Measure the terminal voltage under these conditions. Another way of determining the voltage at no load is to open the switch between the generator and the motor and to drive the generator at no load by the auxiliary motor. The percentage of voltage rise is by definition equal to the regulation of the generator.

When the machine under test is intended to be used as a motor, the percentage regulation refers to the speed drop between no load and full load. In this case, the voltage across the motor terminals is kept constant by means of the booster field. After the full-load readings have been taken, the speed of the auxiliary motor is adjusted until the circulating current is reduced to zero, or, more correctly, to that which the motor under test takes at no load. Another way of accomplishing the same result is to throw off the belt between the two machines and to run the motor at no load.

(*c*) *Temperature Rise.* The adjustments are the same as before: the machine under test must be run at the same current which its armature carries at the specified load, at the same field current, and at the rated speed. The other machine may have any characteristics, provided that it can carry the load required by the first machine. If the two machines are identical, the temperature run can be made on both simultaneously.

Before beginning the run, the resistances of the armature and the field, while cold, must be measured. The machines are then run the required number of hours at the specified load, and hot resistances are measured in order to calculate the temperature rise (§ 8). The temperatures of the armature iron and of the commutator are measured by thermometers, thermo-couples, etc. Sometimes the temperature of the windings is also measured by thermometers, as a check on resistance measurements. See also § 351.

343. EXPERIMENT 14-A. — Efficiency, Regulation, and Temperature Rise by the Blondel Opposition Method. — The machines are connected as in Fig. 234; instructions for starting the set are given in the preceding article. The runs for efficiency, regulation, and temperature rise may all be performed without changing the connections. Perform these tests, assuming first that the machine under test is running as a generator and then as a motor.

(*a*) Begin the experiment by measuring the cold resistances of the generator and of the auxiliary motor. Bring the set up to speed and adjust the current at about 25 per cent overload. Take the necessary readings, and then gradually reduce the armature current to zero, so as to obtain complete curves of efficiency and voltage regulation. Take a similar set of readings for speed regulation as a motor. Read the terminal volts E, the differential voltage e, the field current, the circulating amperes, the speed, and the input into the small auxiliary motor. The latter reading should comprise armature amperes, volts, field amperes, and speed. At the end of the test take off the belt and run the auxiliary motor light, in order to determine its own losses.

(*b*) Belt the motor again, bring the set up to speed, and adjust the conditions for the temperature run, so as to obtain an appreciable temperature rise in an hour or less. An average machine ought to be able to stand at least 50 per cent overload in current and 25 per cent overpotential for this period of time. At the end of the run, measure hot resistances, and determine the temperature rise in accordance with the methods prescribed by some competent national or international organization.

Report. Plot curves of efficiency, voltage regulation, and speed regulation, using the circulating current as abscissæ. On the same curve sheet, plot curves of output, for the machine working as a generator and as a motor, using the same abscissæ. Give the data on temperature rise and compute the final results.

344. Losses Supplied Mechanically — Hopkinson's Method. — The above-described Blondel method of running two machines in opposition requires an auxiliary motor and a booster set. If it is desired to do away with the booster, the current between the two machines has to be circulated by weakening the field of one, or by strengthening the field of the other. In this case the auxiliary motor (Fig. 234) supplies the copper loss as well as the iron loss and friction of the set. This method is simpler than Blondel's, but the iron loss is not the same in the two machines, the fields being differently excited. This is of no consequence for regulation and temperature tests, as in this case the two machines need not even be identical. The method gives only approximate results for efficiency, but efficiency is usually determined from the losses and not from an opposition test.

When a temperature run is required on both machines, each may be run as a motor half of the time with the other acting as a generator. The set should be run at such a terminal voltage, slightly different from the normal, that the *average* core loss in the two machines will be equal to the core loss for each in actual operation. The voltage must be above the rated voltage if the machines are intended for operation as generators, and below the rated value for motors.

If P is the output of the auxiliary motor, the total losses in each machine may be assumed to be approximately equal to $\frac{1}{2}P$ and expressions (1) and (2) for the efficiency become:

$$\text{Efficiency of the generator} = \frac{(I - i)\,E}{IE + \frac{1}{2}P} \quad . \; . \; . \; . \; . \quad (3)$$

$$\text{Efficiency of the motor} = \frac{EI - \frac{1}{2}P}{E\,(I + i)} \quad . \; . \; . \; . \; . \quad (4)$$

345. EXPERIMENT 14-B. — Efficiency, Regulation, and Temperature Rise by the Hopkinson Opposition Method. — The method is explained in § 344; the directions for performing the experiment are similar to those in § 343.

346. Losses Supplied Electrically — Hutchinson's Method. — The losses may be supplied electrically by the three methods enumerated under (3) in the table above. The theoretically correct method for determining efficiency is that of Hutchinson, where the losses are supplied partly in series, partly in parallel. The connections are shown in Fig. 235. The method is similar to that of Blondel (Fig. 234), except that the auxiliary motor is dispensed with, and the iron loss + friction is supplied from an external source of electric power, through the leads *pa* and *qd*. The set is started from the line with the rheostat *R*, using the left-hand machine as a motor, the switch *s* being open.

When the machines are up to speed, the generator field is excited, and when the voltages of the two machines are approximately equal, the switch *s* is closed. After this, the required current is established between the two machines by suitably exciting the booster.

With two identical machines, the field current must be the same in both, at least for the efficiency test, in order to have the same iron loss. The copper loss is supplied mainly by the booster, as in the Blondel method. The line *pq* supplies the iron loss and friction, and also a small part of the copper loss. The correction is easily applied, as is shown below, and does not impair the accuracy of the method.

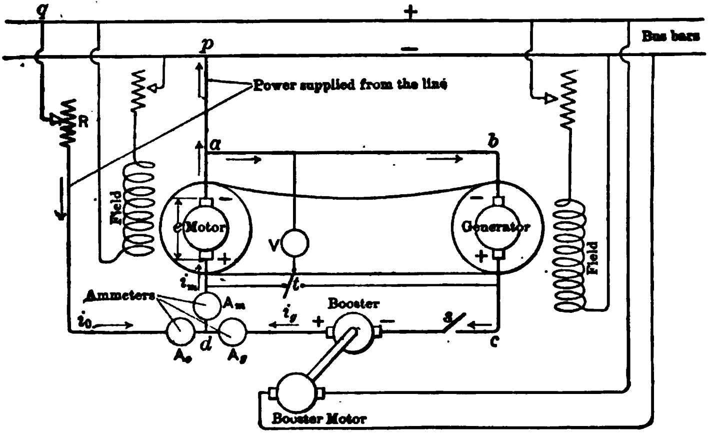


FIG. 235. Hutchinson's opposition method of testing two direct-current machines.

Kapp's method is obtained from that of Hutchinson by omitting the booster; all the losses are supplied directly from the line. A current is made to circulate between the two machines by weakening the motor field. This method has the advantage of greater simplicity, and is just as satisfactory for regulation and temperature runs. It does not give quite correct values for efficiency, both the iron loss and the copper loss in the two machines being different.

Potier's method is obtained from that of Hutchinson by omitting the connections to the line and supplying all the losses from the booster. The field of the motor must in this case be stronger than that of the generator, in order to obtain an additional torque for overcoming the iron loss + friction of the set. With this method the copper loss in both machines is the same, but the iron loss is somewhat different; the same remark applies to it as to Kapp's method.

347. Efficiency by Hutchinson's Method. — Let Ei_0 be the power delivered to the set from the line (Fig. 235), and ei_g that delivered by the booster, where E and e have the same meaning as in § 342. Then the expression $(Ei_o + ei_g)$ represents the total losses of the set, except the shunt excitation. Assuming these losses to be the same in both machines, the loss in each is equal to $\frac{1}{2}(Ei_o + ei_g)$ and the efficiency may be computed. The assumption of equal losses is not quite correct because the currents i_g and i_m in the two machines are somewhat different. If greater accuracy is required, the copper loss should be figured out separately; that is to say, with the notation shown in the diagram and in § 342, we have

$$Ei_o + ei_g = i_g^2 r + i_m^2 r + P \quad . \; . \; . \; . \; . \; . \; . \quad (5)$$

This equation expresses the fact that the power Ei_o supplied from the line, plus the power ei_g delivered by the booster, is equal to the copper loss in the series circuits of the two machines, plus their iron loss and friction P. The resistance r is that of the armature of either machine, including the brushes and the series field, if any. The fields being equally excited, the voltages induced in the two armatures are equal and opposite. Therefore, the differential voltage e between the points c and d must be equal to the voltage drop in the series circuits of the two machines. In other words,

$$e = i_g r + i_m r \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

The current i_o supplied from the line is the difference between the motor current i_m and the generator current i_g, that is,

$$i_o = i_m - i_g \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

Substituting the values of e and i_g from eqs. (6) and (7) in eq. (5), we get

$$P = Ei_o - i_m i_o r \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (8)$$

In other words, the iron and friction loss, P, of the set is equal to the power Ei_o supplied from the line, less the correction term $i_m i_o r$. The iron and friction loss in each machine is $\frac{1}{2} P$. Knowing r, the copper loss in the series circuit of either machine may easily be calculated from the ammeter readings. The efficiency can thus be determined, as in the Blondel method, taking the excitation loss into account.

348. EXPERIMENT 14-C. — Efficiency, Regulation, and Temperature Rise by the Hutchinson Opposition Method. — The theory and directions are given in §§ 346 and 347. The order in which the experiment is performed and the requirements for the report are about the same as in § 343. At the end of the test disconnect the line at p and q and run the set by supplying all the losses from the booster;

this is Potier's method, mentioned in § 346. Then disconnect the booster and supply all the losses from the line; this is Kapp's method mentioned in the same article.

349. Opposition Test on Series Motors. — The methods given in the table (§ 340) may be used with corresponding changes for testing series motors, more particularly railway motors. Such motors are tested in large quantities, and it pays to provide special arrangements in order to facilitate the handling of machines. In order to understand the details of these methods, and especially the necessary computations, the student should be familiar with the treatment of the series motor given in Chapters XII and XIII.

One of the opposition methods sometimes used for such motors is similar to that shown in Fig. 234, except that the motors, instead of being belted together, are geared to a countershaft. This shaft is driven by an auxiliary motor, which supplies the mechanical losses. The fields are connected in series with the armatures, and a booster supplies the copper loss.

A simpler method, which is a modification of Kapp's method, is shown in Fig. 236. The motors are coupled together and are connected electrically in opposition. The motor to the left is started from the line and drives the other machine, which acts as a generator and supplies current to the water rheostat; the switch S_1 is open. When a desired speed has been reached, and the voltmeter V_1 shows nearly zero, the switch S_1 is closed. By increasing the resistance of the water rheostat, the generator is made to send part of the current back into the motor, reducing the current drawn from the line. Finally the circuit-breaker S_2 is opened, and the machines run in opposition.

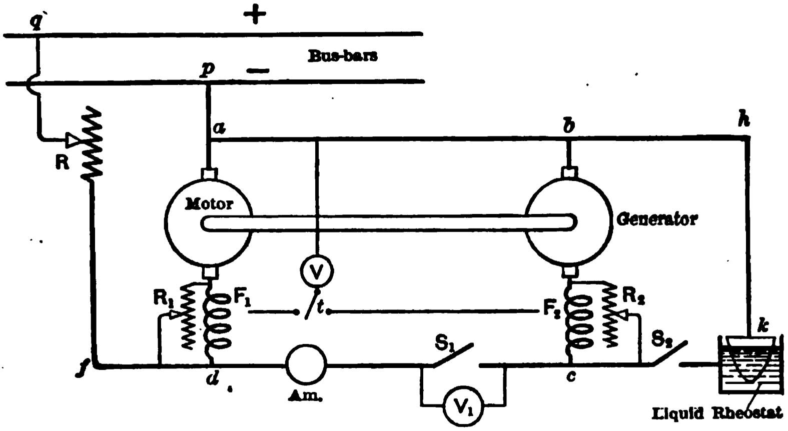


FIG. 236. An opposition test on two series-wound motors.

Shunt rheostats R_1 and R_2 are provided around the field windings F_1 and F_2, in order to regulate the speed of the set and the current circulating between the machines. The machine with a weaker field acts as a motor, the other as a generator. To equalize the conditions during the temperature run, each machine is made to act half of the time as a generator, and half of the time as a motor. The computations for efficiency are similar to those given above for shunt- and compound-wound machines.

350. EXPERIMENT 14-D. — Opposition Test on Two Series Motors. — Two identical railway motors are connected according to either scheme described in § 349 and run at several speeds within their operating range, the purpose of the test being to obtain ampere-speed and efficiency curves. Make clear to yourself, in advance, the details of the proposed method of computing the results in order not to omit taking any necessary data during the test. After the preceding test, a heavy overload should be put on the motors for about an hour (temperature run). Measure cold and hot resistances, and also determine the temperature rise by thermometers or by some other approved method. See also the instructions for the other experiments in this chapter.

Report. Plot ampere-speed and efficiency curves of the motor; give the values of temperature rise by resistances and by thermometers.

351. Heat Tests and Temperature Measurements. — An accurate heat run and a careful determination of the hottest spots in the core, in the windings, and on the commutator, are of the greatest practical importance in acceptance tests of electrical machinery and apparatus, both revolving and stationary. For this reason, very detailed instructions as to loading, temperature measurements, and the highest permissible temperature limits are specified in the Standards of the American Institute of Electrical Engineers. These standards and rules are frequently revised with the progress of the art, such as better quality of insulation, more accurate apparatus for measuring temperatures, and new industrial conditions. For this reason it is preferred in this work not to give any detailed instructions for heat tests and temperature measurements. The reader is advised to consult the latest edition of the Institute Standards, or the corresponding rules issued by other national bodies in this country and abroad.

CHAPTER XV

ARMATURE REACTION AND COMMUTATION

352. THE purpose of the experiments described in this chapter is to illustrate to the student (a) the distortion of the magnetic field in a direct-current machine due to the armature current, (b) the influence of this distortion upon the commutation, and (c) the production of a correct field for commutation by means of commutating poles (interpoles) and a compensating winding. An outline of the elements of commutation is added for a more advanced study. Much of this subject is usually treated in the theory of electrical machinery, without experiments, but it is thought that the first-hand information which the student obtains in the laboratory will help him to appreciate more thoroughly the importance of these factors in the design and operation of direct-current machinery. When undertaking a study of this chapter the student is supposed to be familiar with the first part of Chapter VII (The Magnetic Circuit) and with a few simple tests on generators and motors, described in the preceding chapters on direct-current machinery.

MAGNETIC FLUX AT NO LOAD

353. Total Flux in Armature and in Field Structure. — Most of the magnetic flux excited by the field winding of a direct-current machine finds its way between any two adjacent poles through the armature iron, and constitutes the useful flux of the machine. Some of the flux passes from pole to pole directly through the air, without entering the armature iron (Fig. 238). This part of the flux induces no e.m.f. in the armature conductors and is known as the *stray or leakage flux*. Both the total flux in the field poles and the useful flux through the armature may be measured *with the machine stationary*, by a modification of the ballistic method shown in Figs. 131 and 136.

An exploring or secondary winding of a known number of turns is placed on one of the pole-pieces near the yoke, to include the leakage flux, and another exploring coil is placed on the armature in such a way as to be linked with the total useful flux. The first exploring coil may be placed on the yoke instead of the pole-piece in which case it will measure but one-half of the flux per pole (except in a bipolar machine with undivided magnetic circuit, where it will measure the total flux per pole). The machine is excited with a current through the regular field winding, and a ballistic galvanometer (§ 158) is connected to one of the secondary test coils, for example, to that on the field pole. When the primary circuit is opened, or the current in it is reversed (to eliminate the effects of the residual magnetism), a current impulse is sent through the ballistic galvanometer and the deflection is read on the scale. Knowing the galvanometer constant, the number of turns of the secondary winding, and the resistance of the secondary circuit, the observer can compute the change in the flux. The field is then excited again to exactly the same value, and the ballistic galvanometer is connected to the exploring winding on the armature. Its deflection is again noted when the field is destroyed or reversed. From this experiment the useful armature flux may be calculated as above, and the difference between the two will give the leakage flux.

With a large machine it is hardly practicable to open the field circuit suddenly, or to reverse the current in it, on account of the high self-induction of the winding. The induced e.m.f. may reach several thousand volts, which is dangerous to the operator, and may damage the insulation of the winding. In such a case the flux should be varied in steps. For this purpose an adjustable resistance is connected in series with the field winding, and is short-circuited by a switch. By suddenly opening the switch the flux is changed by the desired amount, through the decrease in the field current, and the ballistic impulse noted. In this way the flux may be reduced to zero in several steps, or reversed, and the sum of the galvanometer readings used to compute the total flux. If a fluxmeter is available (§ 161) the field may be varied slowly, and a total magnetization curve taken.

The armature flux may also be computed from eq. (1), § 227, by driving the machine at no load and at a constant speed, and measuring the induced voltage. This method is practicable only when the number of armature conductors and the number of armature circuits in parallel are known.

354. Leakage Coefficient. — The leakage coefficient, by definition, is the ratio of the total flux per pole to the useful flux in the armature. This ratio varies in modern machines from 1.15 to 1.40, according to the type and proportions of the machine and the saturation in the iron. A leakage coefficient of 1.30 means that out of every 130 lines of force in the field frame, only 100 actually pass through the armature; the rest find their path directly from pole to pole through the air.

It is important to keep the leakage flux as low as possible, since any increase in the total flux means a disproportionate increase in field copper, on account of saturation of the iron (§ 169). Moreover, a machine with a large leakage gives a poor voltage regulation because the armature reaction deflects the flux more easily into the leakage paths. The leakage coefficient increases with the saturation, because the reluctance of the useful path in the armature increases while that of the leakage paths in the air remains constant. With the same useful flux, the leakage is larger when the machine is loaded than at no load, because the magnetomotive force between the poles is larger.

The leakage coefficient may be determined as explained in § 353 by measuring the fluxes in the field and the armature ballistically and taking their ratio. The galvanometer need not be calibrated; with the same number of turns on the armature and field poles, the ratio of the deflections gives directly the value of the leakage coefficient. If the number of turns is different, then both readings must be reduced to the same number of turns.

A compensation (or zero) method may also be used for measuring the leakage coefficient; with this method no ballistic galvanometer is required. A smaller number of exploring turns is placed on the pole-piece than on the armature; the two exploring coils are connected in opposition, and in series with an ordinary low-reading voltmeter of the moving-coil type (in place of a ballistic galvanometer). When the flux is suddenly changed, the e.m.f.'s induced in the two coils are in opposite directions, and the voltmeter " kick " records the difference of the two. The ratio of the numbers of turns in the two coils is varied until the voltmeter needle remains at zero when the flux is suddenly increased or decreased. Then the inverse ratio of the numbers of turns is equal to the ratio of the fluxes, or to the leakage coefficient.

Let, for example, an exploring coil of 20 turns be put on the armature. Let us assume that with 15 turns on the field, the voltmeter gave a small deflection in one direction; with 16 turns a small deflection in the opposite direction. The ratio of the fluxes, or the leakage coefficient, is between $20/15 = 1.33$ and $20/16 = 1.25$. The true value may be estimated by noting the relative values of the deflections. This method is not as accurate as the ballistic method, but is more convenient and accurate enough for some practical purposes.

When a low-reading voltmeter is used for flux measurements, its moving system must be slightly weighted in order to increase its moment of inertia. Otherwise the pointer may move back and forth while the flux is being changed. This is because the flux often varies at different rates in the field and in the armature.

Commutating poles (§ 268) somewhat increase the leakage coefficient of the main poles by providing a low-reluctance path between adjacent pole tips. Therefore, in studying magnetic leakage experimentally, it is advisable to determine its value with and without the commutating poles, and also with the latter unexcited and excited with different numbers of ampere-turns.

355. EXPERIMENT 15-A. — Ballistic Measurement of Magnetic Flux in a D.C. Machine. — The purpose of the experiment is to obtain a magnetization curve of a direct-current machine at standstill, and with no armature current; also to determine its magnetic leakage under these conditions. The theory is explained in the two preceding sections.

(1) Place an exploring winding on one of the field poles and another on the armature along the center lines between two adjacent poles; preferably use fine wire. Demagnetize the machine thoroughly by repeated reversals of the field current, while reducing its value to zero; or use an alternating current and decrease it gradually to zero. Connect a ballistic galvanometer to the exploring winding on the field pole and increase the exciting current in steps from zero to the maximum practicable value. At each step read the field current and the ballistic throw. Be sure that the total resistance of the galvanometer circuit is known at each step, as it may be necessary to vary this resistance for some steps to obtain the best sensitivity of the galvanometer. Reduce the maximum current in steps, and if possible go over the whole hysteresis loop (§ 179).

(2) Repeat the experiment with the galvanometer connected to the exploring coil on the armature, using as nearly as possible the same values of exciting current. If a fluxmeter is available (§ 161) perform a similar run, using this instrument in place of the ballistic galvanometer.

(3) For a few values of the field current determine the leakage coefficient directly, using the zero method and a low-reading voltmeter. Use a very high, a medium, and a low exciting current in order to observe the effect of saturation upon the value of the leakage coefficient.

(4) Increase the pole arcs by adding iron bars or strips to the pole tips. It is not necessary to fasten them securely, as they will be held by magnetic attraction. Observe the increase in magnetic leakage due to the closer proximity of the pole tips. Similarly investigate the effect of excited and unexcited interpoles upon the leakage between the main poles.

(5) Before leaving the laboratory make a sketch of the magnetic circuit of the machine, especially of the form of the pole-pieces, as nearly as possible to scale. Count the number of turns on both auxiliary windings, and ask for the galvanometer constant.

Report. Give the diagram of connections used, the arrangement of the exploring coils on the machine, and an approximate shape of the magnetic circuit. Against values of exciting current as abscissæ, plot values of the total flux in the poles and in the armature. To the same abscissæ plot the ratio of the two, or the leakage coefficient. Check a few values of the latter with the data obtained with the zero method. Show the effect of bringing the pole tips closer together upon the value of the total flux, the armature flux, and the leakage coefficient. Analyze the data regarding the effect of the commutating poles upon the leakage.

356. Distribution of Flux in the Air-Gap. — For some purposes it is of interest to know not only the total flux in the air-gap of a machine, but also the space distribution of flux density (Fig. 237). In this sketch the armature circumference is developed into a straight line which is used as the axis of abscissæ. The two adjacent brushes of opposite polarity are shown midway between the poles, and the distance between them is divided into 180 electrical degrees. Another scale is also shown, marked in commutator segments, of which there are 30 per pole in this particular machine. Against these abscissæ, the corresponding values of flux density, found by experiment or computation, are plotted as ordinates.

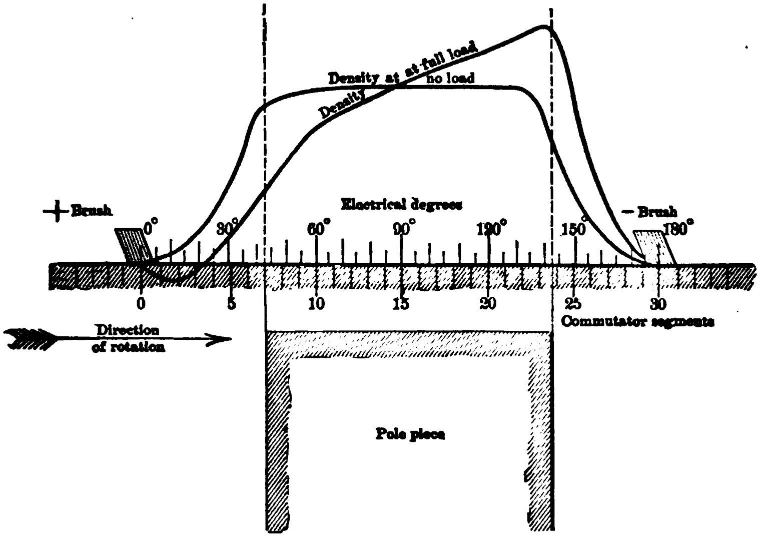


FIG. 237. Flux density distribution under a pole, with the generator at no load and loaded.

At no load the flux density under the pole is practically uniform, and gradually decreases to zero in the neutral zone midway between the poles, where the brushes are placed. When a current is flowing through the armature, it becomes a source of magnetomotive force, or *armature reaction*, which distorts the original flux distribution. The full-load flux distribution shown in Fig. 237 corresponds to the case of a generator, the flux being shifted in the direction of rotation of the armature. With a motor, since the armature currents flow in the opposite direction, the field is shifted against the direction of rotation. The armature reaction is treated more fully in §§ 358 to 360 below.

Curves, such as are shown in Fig. 237, are of importance in the study of armature reaction, magnetic leakage, commutation, etc., and it is therefore desirable to become acquainted with experimental means for obtaining the necessary data. The principal methods are as follows:

(1) *Narrow test coil.* A long narrow test coil, connected to a ballistic galvanometer, is inserted in the desired place in the air-gap and then is rapidly withdrawn. Similar results may be obtained by leaving the coil in position and varying or reversing the field. In either case, the galvanometer deflection is a measure for the total flux through the coil, and consequently for the average flux density within its area. The coil is moved from place to place and the deflections plotted against its positions.

(2) *Wide test coil.* A test coil of a width equal to the pole pitch may be used instead of the narrow one. The wide coil is placed on the armature in such a position that its sides are opposite the centers of two adjacent poles. In this position there is as much flux entering the coil from the N pole as from the adjacent S pole, and the total flux through it is zero. Therefore, if this coil be connected to the ballistic galvanometer and the field current changed or reversed, there will be no current impulse through the galvanometer. Let the test coil be shifted by a small electrical angle α, so that more flux enters it from one pole than from the other. Now if the flux be reversed, the galvanometer deflection will be a measure of the excess flux over the angle 2α. Consequently, the galvanometer deflection is proportional to the flux density near the center of the poles. Thus, by moving the coil in small steps, the *increase* in the galvanometer throw each time is a measure for the corresponding flux density. One of the armature coils may be used as such a test coil, by being disconnected from the rest of the winding and connected to a galvanometer or fluxmeter. Instead of reversing the flux, the armature may be held in a position against a strong spring and then allowed to turn quickly by a small angle. The throw of the ballistic galvanometer due to the cutting of the lines of force

within this small angle is again a measure of the average flux density over the angle of rotation.

(3) *A bismuth spiral* (§ 202) could be used for exploring the field, although this method is hardly ever applied in practice.

(4) *Double-pilot brush.* The method mostly used in practice is that of measuring the e.m.f. induced in the armature coils as they pass a certain point in the air-gap. With the preceding methods the armature is stationary, while here the machine must be driven as a generator or as a motor. Two small brushes are mounted on a temporary brush-holder, and the distance between them is adjusted to be equal to the distance between the centers of two adjacent commutator segments. The brushes are insulated from each other, and are connected to a low-reading voltmeter. When the armature is revolving, the voltage between these two " pilot " brushes is equal to that at the terminals of any armature coil which comes into the proper position with respect to the field. In other words, the pilot brushes measure the voltage induced in a certain place of the field; this voltage is proportional to the average flux density over the width of one armature coil. When the machine is being driven as a motor the regular brushes are used for the load current.

By gradually moving the pilot brushes around the commutator, the flux density distribution is measured over the whole pole pitch, and the results plotted as in Fig. 237. The same experiment, repeated with the armature loaded, gives a different distribution, because of the weakening and distorting action of the armature ampere-turns. In this case the observed voltmeter readings must be corrected for the voltage drop in the coil, in order to obtain the true induced e.m.f.'s. This correction is added to the readings in the case of the generator and subtracted in the case of the motor. Further refinements of this method are possible whereby the interval of time, during which the voltmeter (or a galvanometer) is in the circuit, is considerably shortened, and the flux density averaged over a smaller angle.

The double-brush method requires an extra brush-holder on the machine, and a divided sector for measuring angles. A simpler arrangement, sufficiently accurate for practical purposes, consists of a template made of cardboard, with holes for voltmeter points. The holes are spaced so as to correspond to a certain number of commutator bars. The template is laid around the commutator, as closely as possible, but without touching it, and is securely fastened to the brush-holder. When the armature is revolving, voltmeter points are inserted in each two adjacent holes and the voltage measured. Ordinary metal points may scratch the commutator; it is better to use hard drawing pencils.

(5) *Single-pilot brush.* Instead of measuring the voltage induced in an armature coil by means of two pilot brushes, the same voltage may be measured with a single-pilot brush, by taking the difference of the two voltmeter readings between this pilot brush and one of the regular brushes of the machine. For example, let the voltage measured between the negative brush of the machine and the pilot brush, in a certain position, be 72 volts. Shift the pilot brush by one commutator segment towards the positive brush. If the new reading is 75 volts, then the voltage induced in this particular coil is 3 volts. This method is more suitable for obtaining a curve of voltage distribution along the commutator than for the flux density curve. The ordinates of the latter curve are proportional to the values of the slope of the first curve ($B = \text{const.} \times de/d\alpha$). Conversely, the voltage-distribution curve is the integral, or the area curve, of the flux-density curve.

(6) *Oscillograph.* If an oscillograph (Vol. II) is available, the curve of flux-density distribution may be readily obtained by taking a curve of the voltage induced in an armature coil. For this purpose two small slip rings are mounted on the armature shaft, with soft brushes bearing on each. These slip rings are connected to an armature coil or to two adjacent commutator segments, while the brushes are connected to the oscillograph. The curve traced by the latter gives the voltage induced in that particular armature coil. The instantaneous values of voltages are proportional to the elementary fluxes cut by the conductors, so that the same curve gives the flux-density distribution. A correction for the IR drop is necessary when a load current flows through the coil. This method is very convenient when many distribution curves have to be taken for a special study.

357. EXPERIMENT 15-B — Magnetic Flux Distribution in the Air-Gap of a D.C. Machine at No Load. — The purpose of the experiment is to obtain the flat-top curve of flux-density distribution at no load, shown in Fig. 237. For a similar test on a loaded machine, see Exp. 15-C. The various available methods of measuring flux densities are described in the preceding section. Either two movable pilot brushes, or a template and voltmeter points would ordinarily be used. Bring the machine to full speed, run it at no load as a generator or as a motor at normal voltage, with the regular brushes in the geometric neutral.

(1) Read volts between consecutive commutator segments and note the corresponding angular positions of the pilot brushes. Read also the voltage between one of the regular brushes and a pilot brush. Keep the speed, the terminal voltage, and the field current constant, and

note their values. Take similar sets of readings with the field highly saturated, and then with a very low saturation.

(2) Add strips of iron to the pole tips and again take readings for one of the field-distribution curves, at the same speed and voltage.

(3) Before leaving the laboratory, count the commutator segments, and note the number of poles. Make a sketch of the pole shoe; measure its axial length and peripheral width; also the radial length and the average diameter of the air-gap. If possible, find the total number of armature conductors and the number of parallel circuits in the armature.

Report. Check the sum of voltmeter readings between the pilot brushes against the terminal voltage of the machine. Also check single-pilot and double-pilot readings, and make readjustments and corrections where necessary. Plot curves of flux-density distribution. Sketch the position of the pole shoe over the curves. Explain the effect of saturation and of the added iron strips upon the shape of the curves. For a few points, show that the slope of the single-pilot brush curve is proportional to the ordinates of the double-pilot brush curve, and explain the reason. If the total number of armature conductors is known, compute the flux per pole, using eq. (1) in § 227. Dividing this flux by the pole-pitch area will give the average flux density under the pole, and consequently the scale for the plotted curves.

ARMATURE REACTION

358. A current flowing through the armature winding of a direct-current machine (Fig. 238) is a source of magnetomotive force which influences the value and the space distribution of the flux due to the field winding. In this sketch the surface of the armature is developed into a plane, so that the relations hold true for any number of poles. The directions of the armature and field currents are shown in the usual way by dots and crosses, assuming the machine to be acting as a generator. The " belts " of armature currents between consecutive sets of brushes are in opposite directions.

In studying the armature reaction of a direct-current machine, it is not necessary to consider the magnetomotive force of the individual armature coils, but only that of groups of coils between adjacent brushes. The function of the commutator consists in maintaining *stationary groups* of coils between adjacent sets of brushes while in reality *individual* coils are revolving (§ 364). The armature reaction, or the magnetic action of the armature currents upon the field, is due to these *quasi-stationary* groups of coils, and therefore is practically independent of the velocity of rotation. There is a small effect of the currents in the coils undergoing commutation, but this effect is neglected in the present discussion.

The armature reaction depends upon the position of the brushes. In Fig. 238 the brushes are shown in the geometric neutral, and the armature reaction is known as *transverse.* In Fig. 239 the brushes are shifted, and in addition to the transverse reaction there is also a *direct* armature reaction. These two components will now be considered separately.

359. Transverse Armature Reaction. — Let the brushes be placed on the geometric neutral. In a real machine the armature conductors are connected at the ends, as is shown by the slanted straight lines (see Vol. II, Armature Windings). This is necessary in order to form a continuous symmetrical winding closed on itself, and one in which the same relationship with respect to the poles and the brushes is preserved as the armature revolves. However, it is easier to understand the magnetomotive force of the armature conductors, if we imagine these conductors grouped at each instant into "concentric" coils, also shown in the figure. Neither the electric nor the magnetic relations are thereby affected in so far as the armature reaction is concerned, except that this arrangement holds true for an instant only.

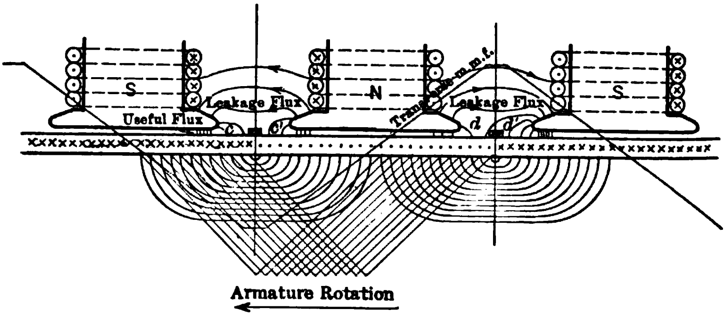


Fig. 238. Armature reaction in a direct-current machine; brushes in the geometric neutral zone.

We now have a wide fictitious armature coil with a magnetomotive force acting *up* along the axis of the brush *dd'*, and a similar one acting *down* along the axis of the brush *cc'*. This kind of armature reaction, with its poles between the main poles, is known as the *transverse reaction.*

Comparing the directions of the magnetomotive forces of the armature currents with those of the exciting field winding, we shall see that the flux is strengthened under the leading pole tips and weakened under the trailing tips. The armature currents in a direct-current *generator* thus tend to shift the magnetic flux of the machine in the direction of rotation of the armature. In a *motor*, with the same field polarity and with the same direction of rotation, the armature currents flow in the opposite directions, and the flux is shifted against the direction of rotation.

Theoretically, neglecting the effect of saturation, the flux is strengthened on one side of a pole as much as it is weakened on the other. But, on account of the saturation of the pole tips and of the armature teeth, the magnetomotive force required to produce a given increase in flux will produce a much greater decrease when applied in the opposite direction, especially in the region of the " knee " of the saturation curve. Therefore one result of field distortion is the weakening of the field. In a direct-current machine the induced electromotive force is proportional to the total flux, but is independent of its space distribution; formula (1) in § 227. Therefore, the transverse armature reaction reduces the induced e.m.f. of the machine only indirectly, due to the saturation of the magnetic circuit.

360. Direct Armature Reaction. — Let now the brushes be shifted in the direction of rotation (Fig. 239) in order to improve the commutation (§ 228). The armature reaction is partly "transverse," that is, along the center lines of the spaces *E* and *W*, and partly *direct*, along the axes of the main poles. In order to separate these two components, we divide the total belts of armature currents into fictitious coils *a* and *a'* which exert a purely transverse or cross-magnetizing action, and *b*, *b'*, and *b''*, which oppose the magnetomotive force of the field coils and constitute the so-called *direct armature reaction*. It will be seen that shifting the brushes in the direction of rotation weakens the field flux while shifting them against the direction of rotation strengthens it.

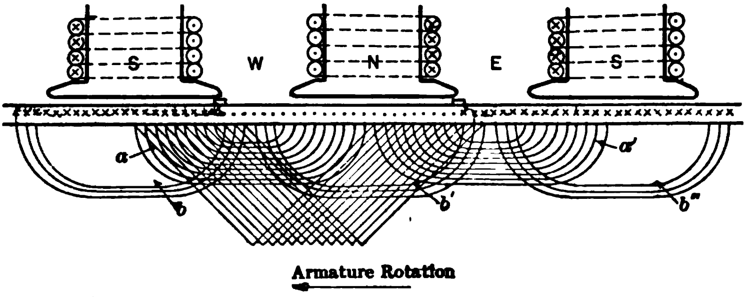


FIG. 239. Armature reaction in a direct-current machine; brushes shifted from the geometric neutral zone.

The foregoing conclusions have been reached by considering the machine to be operating as a generator. When it is working as a motor, with the same field polarity and with the same direction of rotation, the direction of the armature currents is opposite to that shown in Fig. 239, because the electromagnetic torque is now exerted in the direction of rotation. Thus, for a motor we have the opposite conclusions from those reached for the generator. All these results are tabulated below.

	Generator	Motor
Transverse reaction *shifts* the field	in the direction of rotation	against the direction of rotation
Direct reaction *weakens* the field when the brushes are shifted	in the direction of rotation	against the direction of rotation
Direct reaction *strengthens* the field when the brushes are shifted	against the direction of rotation	in the direction of rotation

The requirements for sparkless commutation are such that the brushes have to be shifted in the same direction in which the transverse reaction shifts the main field, unless the transverse magnetomotive force has been reversed by commutating poles (§ 268), by a compensating winding (§ 370), or by both.

The armature reaction influences the performance of a machine in various ways: It modifies the magnitude and the space distribution of the flux, it affects the terminal voltage of a generator, or the speed of a motor, and it affects commutation. While these consequences are but different aspects of the same fundamental phenomenon, they are taken up separately in the experiments that follow, in order to present to the student the individual effects of each more clearly. The commutation problem is treated separately at the end of the chapter.

361. EXPERIMENT 15-C. — Magnetic Flux Distribution in the Air-Gap of a Loaded D.C. Machine. — This experiment should be performed on the same machine and with the same pilot brushes as the preceding experiment, in order that the results may be compared. Plan all the runs keeping in mind the following two requirements: first, the results must be comparable to those in Exp. 15-B; second, only one factor must be varied at a time, the others being left unchanged.

(1) With the brushes in the neutral position (or shifted as little as possible) run the machine heavily loaded as a generator, at the same speed and field current as in the first run of the preceding experiment.

(2) If possible also run the machine loaded, using first a high value and then a low value of field current; these values should be the same as at no load. (3) If the machine has no commutating poles, repeat one of the runs with the brushes shifted as much as possible. (4) Add iron strips to the pole tips and repeat one of the runs with the brushes in the neutral, and one with the brushes shifted. (5) Make some of the characteristic runs with the machine operating as a motor, loaded heavily on a generator, a blower, or some such steady load.

Report. Plot the data in such a form as to bring out the influence of each factor, *viz.*, flux-density distribution with an armature current *vs.* that at no load; distribution with a brush shift *vs.* no brush shift; effect of low saturation *vs.* high saturation; generator *vs.* motor, etc. In each case show that the observed change in the shape of the no-load distribution curve could have been foreseen theoretically.

362. EXPERIMENT 15-D. — Effect of Armature Reaction on the Terminal Voltage of a Shunt-Wound Generator without Interpoles. — The purpose of this experiment is to investigate the influence of armature reaction and of brush shift upon the terminal voltage of a generator. The computations for the report require some of the data taken in Exp. 15-C, so that this experiment, the preceding two, and the one following should be performed on the same machine.

(1) With the generator running at its rated speed, excite the field as strongly as possible. Set the brushes on the geometric neutral and load the machine to the limit imposed by commutation. Read the field current, armature current, speed, and terminal volts. With a low-reading voltmeter determine the voltage between the heel and the toe of one of the brushes and check on a few other brushes; note whether the toe has a higher or a lower potential than the heel.

(2) Shift the brushes in steps by small known angles, both forward and backward, and at each step adjust the field current and the load current to the same values as at first. Take readings similar to those above. (3) Repeat the entire test with as low a value of field current as commutation will permit.

(4) Before leaving the laboratory, be sure to have accurate data for the no-load saturation curve of the machine and the armature resistance.

Report. (a) Plot curves of terminal volts *vs.* angles of brush shift, in electrical degrees. (b) For any set of readings, estimate the value of the armature reaction as follows: Add the *ir* drop in the armature to the terminal voltage, in order to obtain the voltage actually induced in the armature. Let the corrected voltage correspond to the ordinate *ab′* in Fig. 177, and let the field current be *Oa*. The corresponding voltage at no load is *ab*. Thus the armature reaction in terms of field current is equal to *ca*. In other words, if *Oa* is the field ampere-turns, then *ca* is the *equivalent* opposing armature ampere-turns. Perform such computations for a few points and plot or tabulate the results in such a way as to bring out clearly the effects of saturation and of brush shift. (b) Show that the general character of voltage changes could have been predicted from the theory of armature reaction. (d) Explain the nature of the voltage between the toe and the heel of the brush, and give the observed data of the value and the polarity of this difference of potential.

363. EXPERIMENT 15-E. — Effect of Armature Reaction on the Speed of a Shunt-Wound Motor Without Interpoles. — The purpose of this experiment is to investigate the influence of armature reaction and of brush shift upon the speed of the motor. For this experiment it is preferable to connect the motor to a generator, a blower, a centrifugal pump, or some such steady and adjustable load. The runs and the requirements for the report are essentially the same as in the preceding experiment, except that there the speed is kept constant and the voltage is allowed to fluctuate, while with the motor the terminal voltage is kept constant, and variations in speed are noted.

COMMUTATION AND COMMUTATING POLES

364. Stationary Groups of Armature Coils. — The function of the commutator in a direct-current machine may be briefly explained as follows (Fig. 238): Consider a group of armature coils in series between two sets of brushes of opposite polarity. All these coils are under the influence of a magnetic flux of the same polarity so that the e.m.f.'s induced in them are added. Consider a short interval of time during which the armature has moved by one commutator segment. The electrical conditions at the end of the interval are precisely the same as at the beginning of it, and hence the induced e.m.f. is the same. During the interval, the induced e.m.f. remains practically the same, except that one out of the many coils leaves the group under consideration, and another takes its place. Thus, the induced voltage between the brushes is slightly fluctuating, the frequency being equal to that at which the commutator segments pass under the brushes.

It may thus be said that by means of the commutator and stationary brushes, *groups of coils* under the individual poles are kept practically stationary, in spite of the fact that in reality *individual coils* are revolving. Therefore, a commutator is an indispensable part of a direct-current generator or motor in which conductors are connected in series within the machine itself, forming a closed winding. The latter limitation is necessary on account of the so-called homopolar machine which is not considered in this discussion.

365. Current Reversal in a Coil Undergoing Commutation. — In order to understand the cause of sparking under the brushes, the necessity for brush shift, and the purpose of commutating poles (interpoles), the student should familiarize himself with the elementary processes which take place in the coils undergoing commutation. In Figs. 240 and 241 the wavy lines marked 1, 2, 3 represent part of the armature winding at three different instants of time. The coil A, which is shown heavier, is the one in which the reversal of current is to be studied. The peripheral width of the brush B is assumed to be less than the width of one commutator segment or at most equal to it. The groups of coils marked L and R are respectively those to the left and to the right of the coil under commutation.

At the instant (1) the coil A, denoted in this position as A_1, belongs to the L group of coils, and the current is flowing through it to the right, as is shown by the arrowheads on top. This current reaches the brush through the commutator lead f_1' and the commutator segment. At the instant (3) the same coil, now marked A_3, belongs to the R group, and the current flows through it to the left, as shown by the lower arrowheads. Thus, while the armature has moved by one commutator segment to the right, the current in the coil under consideration has been completely reversed. The time interval (1-3) is exceedingly small; the student can take as an example a machine with which he is familiar and, knowing the number of commutator segments and the rated speed, can compute the fraction of a second that it takes for the armature to move by one commutator segment.

When an electric current is forced to reverse rapidly, the e.m.f. of self-induction is brought into play, that is, the electrical inertia of the magnetic flux linked with the current in the coil under consideration (§ 131). This magnetic flux is denoted in Fig. 240 by ϕ_1 and ϕ_3 respectively, and being produced by the changing current, must be reversed with the reversal of the current. In so doing, the flux induces a counter-e.m.f. which opposes the reversal of the current in A_3, and impedes the flow of current from the part R_3 of the armature winding through A_3, f_3, and the commutator segment to the brush.

Should this inductive e.m.f. be sufficiently high, the current from R_3 may take a "shunted" path to the brush, through the lead f_3', the wrong commutator segment, and finally into the toe of the brush in the form of a spark across the insulation. Thus, the fundamental cause of sparking is a high counter-e.m.f. induced in the coil under commutation. This e.m.f. forces part of the armature current to reach

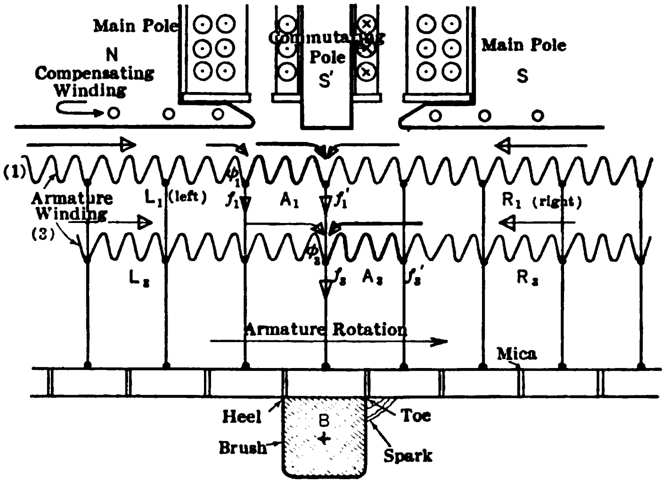


FIG. 240. Armature currents at an instant when the brush is in contact with one commutator segment.

the brush through an air path and the adjacent commutator segment which is no longer in contact with the brush.

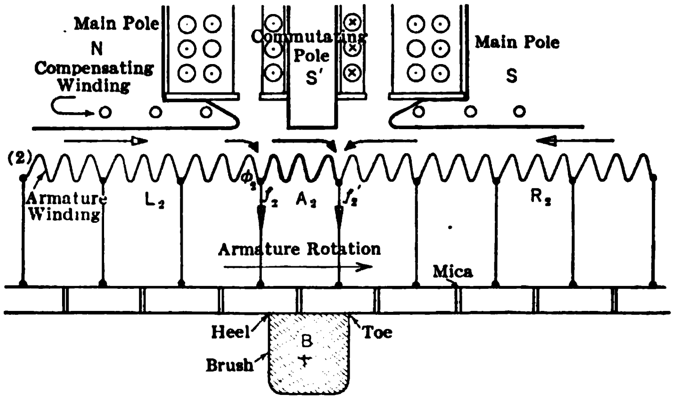


FIG. 241. Armature currents at an instant when the brush bridges two commutator segments.

The voltage across the spark, or the so-called *sparking voltage*, is not equal to the e.m.f. induced in the coil, on account of the voltage drop in the local circuit through the coil and the brushes. In a mathematical theory of commutation it is necessary to consider the variable resistance of this local circuit and the law of change of the current in the coil A during the interval of time (1-3). For the major part of this interval the coil is short-circuited by the brush, as shown in the position A_2 (Fig. 241). In this position the armature current is conducted to the brush through two leads, f_2 and f_2', and the coil A_2 forms a local circuit through these leads, the two commutator segments, and the brush. Thus the law of reversal of the current in A_2 depends not only upon the e.m.f. induced in it but also upon the contact resistance of the two parts of the brush. The contact area under the toe decreases, that under the heel increases with the time. The phenomenon is further complicated by the fact that the brush is often made wider than one commutator segment in order to reduce the current density. In this case two or more coils undergo commutation at the same time, so that their mutual inductance and the division of current among them must be taken into account.

366. Factors which Affect Commutation. — The simplified description of the phenomenon of commutation given above should be sufficient for a clear understanding of the following additional statements concerning it:

(1) *Coil inductance.* Any arrangement which reduces the inductance of the coils undergoing commutation improves the commutation. As such may be mentioned: smaller number of turns per commutator segment, shorter armature, properly designed end connections, fractional-pitch winding, etc.

(2) *Direction of brush shift.* The e.m.f. of inductance in the coil may be neutralized by the inductive action of an external field of proper magnitude and direction. Since the purpose of such a field is to facilitate the reversal of current, the direction of the "reversing" field, *in a generator*, must be the same as that to which the R (or the leading) group of coils is subjected. For this reason in a generator the brushes, if shifted at all, are shifted in the direction of rotation of the armature, anticipating the reversal. *In a motor*, the current flows against the induced e.m.f., so that the field to which the coil has been subjected previous to the commutation period helps the reversal. For this reason in a motor without interpoles, the brushes are shifted against the direction of rotation, if a shift is necessary to improve the commutation.

(3) *Armature reaction.* In a generator the effect of the transverse armature reaction (§ 359) is to strengthen the field under the *trailing* pole tip and to weaken it under the *leading* tip. In Fig. 238 the symmetrical fringing field without the armature reaction is shown at cc' and one distorted by the armature reaction at dd'. This distortion of

the field makes it more difficult to bring the coil under commutation within the influence of the proper reversing flux density. Moreover, when the brushes are shifted in the direction of rotation, a demagnetizing action of the armature upon the field is produced (§ 360), and the whole field flux is further shifted in the direction of rotation. Thus, in a machine without interpoles, and with a strong armature reaction, it sometimes happens that no sparkless position of the brushes can be found at a heavy load. The same is true for the motor, considering the opposite direction of brush shift.

(4) *Contact resistance.* The effect of the reactive-e.m.f. in the coil (Fig. 241) is to make the lead f_2' take more than its share of current as compared to f_2, resulting in a higher current density under the toe of the brush, and, with a sufficiently high local current density, likewise in a spark. A brush of high contact resistance remedies this tendency to some extent, and makes it easier to obtain sparkless commutation. Theoretically, with a very high contact resistance the parts of the current conducted through f_2 and f_2' respectively should be nearly in the ratio of the corresponding contact areas under the brush. Thus, the current density under the heel and toe portions of the brush is more nearly equalized and a better commutation results. In practice other factors limit the range of application of this remedy.

(5) *Resistance leads.* An effect somewhat similar to that of increasing the contact resistance of the brushes is obtained by inserting resistances into the commutator leads, such as f_1, f_2, etc. Because of the resultant increase in the resistance of the local circuit of the short-circuited coil, the local current is cut down, and this tends to equalize the current density under the brush. This remedy has been applied mainly in alternating-current commutator motors.

367. Commutating Poles. — As is stated in the preceding section, the reactive e.m.f. in the coil undergoing commutation may be neutralized by subjecting the coil to the influence of an external field. Shifting the brushes for this purpose has practical limitations, including the inconvenience of changing their position with changes in the load. *It is much simpler to bring the necessary reversing field to a coil undergoing commutation than to try to bring the coil to the field.*

The commutating pole (or interpole) S' (Figs. 240 and 241), placed mid-way between the two main poles, is such a solution of the commutation problem. In a generator its polarity is the same as that of the leading pole S. In a motor the brushes are shifted backwards and consequently the commutating pole in the same position should be excited with the N polarity. Since the required commutating field should be proportional to the current to be reversed in coil A, the commutating poles are excited by a winding connected in series with the armature. This winding is marked "auxiliary field winding" in Fig. 242. The magnetic circuit of the commutating poles is kept at a low saturation, in order to make the required flux density as nearly as possible proportional to the load current.

The necessary number of ampere-turns on a commutating pole is much higher than that required to produce the same flux density in the air-gap of an unloaded machine. It must be remembered that the transverse armature reaction (§ 359) has its maximum magnetomotive force midway between the main poles, that is, under the center of the commutating pole, so that it is necessary first to compensate for these ampere-turns and then to set up a commutating field in the right direction.

In a machine provided with interpoles, there is no occasion for shifting the brushes from the geometric neutral. Should the brushes, for some reason, be shifted, the effect will be similar to a cumulative or differential compounding, because the flux of each commutating pole will then be added to or subtracted from that of the adjacent main pole acting upon the same group of armature conductors.

Inductive Shunt. In a newly designed machine it is rather difficult to predetermine the exact number of turns for the commutating pole. For this reason a larger number of turns is sometimes provided than is necessary (Fig. 242) and an adjustable shunt is connected across the winding. When the machine is being tested this shunt is adjusted to obtain the best possible commutation at all loads. In a machine designed for rapid fluctuations or reversals of the armature current (as in some drives in steel mills), care must be taken to divide the variable current properly between the interpole winding and its shunt, under all conditions. The winding being much more inductive

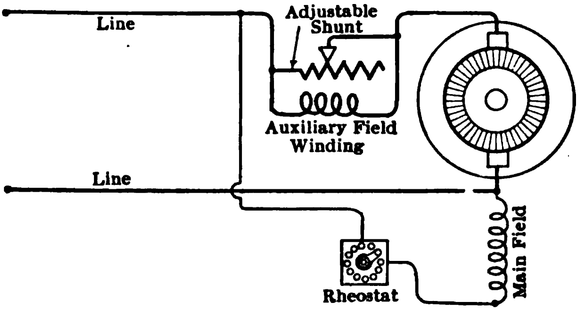


FIG. 242. Diagram of connections of a shunt-wound machine with interpoles.

than an ordinary resistance shunt, a rising current would flow in preference through the shunt, while a decreasing current would tend to continue through the inductive path. This would provide a commutating field of wrong magnitude and might result in sparking, or even in flashing over, between the brushes. For this reason, shunts are sometimes made inductive in the form of coils, and their ratio of resistance to inductance is adjusted to agree with that of the interpole winding.

It is hardly fair to test the same generator with and without commutating poles. A machine designed with commutating poles has a relatively much stronger armature (measured in ampere-turns), and its other constants are different from what they would be were the machine designed without interpoles. The following experiment, however, will help the student to see clearly the influence of the commutating poles.

368. EXPERIMENT 15-F. — A Study of Commutating Poles in a Direct-Current Generator. — The purpose of this experiment is to investigate the effect of commutating poles (Fig. 240) upon the commutation and voltage regulation of the machine. The generator should be connected as for an ordinary load test (Fig. 180) and driven by an adjustable-speed motor.

(1) *Load Test.* For each speed in the following runs the field rheostat should be set to give the desired voltage at the maximum load and the brushes set exactly on the geometric neutral. (a) With the commutating poles properly connected, run the machine at the maximum feasible speed, and obtain the necessary data for plotting the external characteristic at that speed (§ 235), that is, read terminal volts, load amperes, and the field current. Repeat the run at the lowest and at one or two intermediate speeds, including the rated speed. (b) Cut out the interpoles and, using the same values of speed as before, repeat the preceding experiment in so far as sparking at the brushes will permit. Note the quality of commutation on your data sheet as indicated in § 228. (c) Take a few readings with the reversed polarity of the winding on the commutating poles, and note the voltage regulation and the quality of commutation.

(2) *Brush Shift.* With the commutating-pole windings in the circuit, load the machine to a safe limit at the rated speed. Shift the brushes by steps, both forward and backward, as far as sparking will permit. Keep the load current and the terminal voltage constant, and at each step record the field current, the angle of brush shift in electrical degrees, the voltage between the toe and the heel of a brush, and the quality of commutation. (b) Take similar readings at no load and

at one or two intermediate loads. Take one or two sets of readings with the brushes shifted, and in the neutral position, without readjusting either the field rheostat or the load rheostat, so as to find the effect of the brush shift in either direction upon the terminal voltage and the load current. (c) Cut out the commutating pole windings and repeat the preceding test.

(3) *Effect of Inductive Shunt.* Arrange some connections to shunt the interpole winding at will by a non-inductive or by an inductive shunt, preferably through a double-throw switch. Observe the effectiveness of each in taking care of commutation with sudden fluctuations of the load and with slow changes.

(4) *Limits of Commutating Flux Density.* Provide a variable non-inductive resistance as a shunt to the interpole winding. Find by eriment the best value of such resistance to take care of commu- n from no load to a reasonable overload. Find what fraction of the mature current flows through the interpole winding at this resistance. vestigate the limits within which the shunted resistance and the arma- ure current may be varied (a) without appreciably affecting the quality of commutation and (b) still permitting commutation without excessive sparking. Measure the voltage between the heel and the toe of a brush with each of the various arrangements tried. Excite the interpoles from a separate source so as to have a fairly strong commutating field at no load. Find out the effect of such a field in either direction upon the quality of commutation at light loads. Observe whether or not the spark changes from the heel of the brush to the toe when the commutating field is reversed.

Report. (1) For each speed plot the external characteristics of the machine with the commutating-pole winding in and out. Indicate the results with the commutating winding reversed. Mark on the curves the quality of commutation. (2) For each load used in part (2) of the experiment tabulate or represent graphically the effect of each angle of brush shift upon the voltage drop and sparking. (3) Show theoretically that according to the direction of the brush shift the commutating-pole winding has a cumulative or differential compounding effect, and check this statement with the observed readings. (4) Explain in detail why an inductive shunt is more effective than a non-inductive one in taking care of sudden fluctuation in the load current; substantiate this explanation with your experimental data. What is the ideal proportion between the resistance and inductance of a shunt for the interpole winding and why? Would you use highly saturated commutating poles or only slightly saturated? Should there be any relationship between the degree of saturation in the interpoles and that in the iron core of the inductive shunt? (5) Tabulate the results of shunting the commutating pole winding by various resistances, and explain why the commutation remains apparently perfect with quite a wide range of commutating flux. Give the data on the test with separately excited interpoles, and from the theory of commutation explain the observed findings as to sparking.

369. EXPERIMENT 15-G. — A Study of Commutating Poles in a Direct-Current Shunt-Wound Motor. — This experiment is similar to the preceding one, except that in each run the applied voltage is kept constant, instead of the speed, and fluctuations in speed are noted, whereas in the generator variations in the terminal voltage are observed. A steady load, such as an electric generator, is preferable for this experiment. It is not necessary to refer the characteristics of the motor to the outputs as abscissæ, unless a transmission dynamometer or a calibrated generator is available. It is sufficiently accurate to refer the speed and other readings to the power input into the armature as abscissæ, correcting for the I^2R loss. Some results are preferably plotted to the armature current as abscissæ.

370. Compensating Winding. — In a machine subjected to violent overloads, rapid reversals of current, etc., it is very important to remove the transverse armature reaction (§ 359) as completely as is possible, in order to be assured of satisfactory commutation under all conditions. In many machines there is not enough space between the main poles to provide an interpole winding sufficient for this purpose. In such a case the transverse armature reaction is *compensated* by a distributed winding placed in holes in the main pole-shoes (Fig. 241). This distributed compensating winding is connected in series with the armature and its ampere-turns are so computed as to be equal and opposite to those of the armature. Thus, most of the armature magnetomotive force is removed, and the commutating poles need be provided with comparatively few turns to create the proper commutating field. The compensating winding is quite expensive and is therefore seldom used. For most purposes the required compensation is satisfactorily obtained by using interpoles alone.

371. EXPERIMENT 15-H. — A Study of Compensating Winding in a Direct-Current Machine. — The theory and purpose of the compensating winding are explained in the preceding section. In practice a machine with a compensating winding would always be provided with interpoles, so that it is difficult to separate the study of one from the other. The directions given for the two preceding experiments should therefore be carefully studied in planning this experiment. Pilot brushes (§ 356) will be found useful in ascertaining the extent of compensation of the transverse armature reaction. Curves shown in Fig. 237 should be taken under various load conditions (a) with both the interpoles and the compensating winding in the circuit, (b) with only one of them in the circuit and (c) with both cut out. A curve of flux-density distribution at no load should be plotted on the same sheet for direct comparison. In connection with this study, the quality of commutation should be investigated, and also the voltage regulation in the case of the generator and speed regulation for the motor. The effect of sudden fluctuations in the load and even of sudden reversals of the armature current may be made a subject for further study. Finally the relative proportion of ampere-turns upon the interpoles and in the compensating winding may be varied and the effect of these variations upon the characteristics of the machine and the quality of commutation determined.

372. Experimental Study of Commutation. — A complete theory of commutation includes a great many factors, each of which has been a subject of extensive experimental and analytical researches. Most of this work is outside the scope of this book. Only the principal factors and methods of testing are mentioned below. See also § 375 below on testing of brushes, and the end of Chapter XII under "Troubles in Direct-Current Machines."

(a) *The contact voltage drop* of various grades of carbon and graphite brushes as a function of mechanical pressure, speed, temperature, direction of current, its density, and frequency; the critical current density for each grade of brush. This subject is conveniently studied on separate collector rings, although the results may not be directly applicable to a commutator subdivided into segments and with a rapidly varying current density. A direct study on an actual machine or on a special revolving commutator is also possible but much more difficult.

(b) *The physical operating condition of brushes.* Under this heading the following topics may be studied: temperature of the heel and of the toe, in the positive and in the negative brushes; distribution of energy losses on the face of the brush; the relation between these losses, the width of the brush, and the temperature rise on the commutator; carrying of copper by the current from the commutator to the brush; life of brushes with various degrees of sparking; effect of undercutting the mica.

(c) *Inductance*, self and mutual, of the coils undergoing commutation may be studied by sending a high-frequency alternating current through a stationary armature and measuring the impedance of the coils in different combinations. Eliminating the ohmic drop, the reactance, and hence the inductance, may be computed. It must be remembered that a closed circuit containing the field winding will act as th secondary of a transformer, and that considerable eddy currents may be induced in the pole-pieces; both of these will reduce the true inductance that would be obtained by testing the armature entirely removed from the field structure.

(d) *Magnetic fields in the commutation zone.* A coil of very fine wire is placed in the same slots with an armature coil, so as to form a very close magnetic coupling with it. It is subjected to the same magnetic fields as the armature coil, but has no current flowing through it. The ends of the test coil may be connected to an oscillograph through two small slip rings, and the effect of the fields studied directly from the shape of the curve of induced e.m.f. Stationary test coils may also be mounted in the commutating zone for a study of the oscillating field produced by the currents in the short-circuited coils, as distinct from the field due to the transverse armature reaction. This has an important bearing upon the question of the effect of the transverse armature reaction on current reversal. Pulsations of current and voltage in the external circuit, pulsations of field current, eddy currents in pole faces, and such similar problems may also be studied in this connection. The effect of commutating poles, their flux density, width, possibility of over-commutation, etc. are also interesting topics for study.

(e) *The spark voltage.* The distribution of potential along the contact surface of the brush, and the voltage at different points between the brush and the commutator segment, are important factors in determining the quality of commutation. With a perfect reversal of current, the current density at all points under the brush is the same, and consequently the voltage drop across the contact surface is constant. Any departure from these two conditions characterizes a nonuniform distribution of current density under the brush. From six to ten points may be marked on the lateral surface of the brush, and the voltage measured between these points and the corresponding points on the commutator, sharp copper contacts being used. A low-reading voltmeter will give average values of these voltages, while an oscillograph will indicate their actual variation with the time. When using an oscillograph it is preferable to mount an auxiliary slip ring connected to one of the commutator segments and to take the voltage curve between the slip ring and one of the brushes.

(f) *Time of reversal.* Theoretically, the time t of current reversal in an armature coil is determined by the duration of the short-circuit, that is, by the relation $vt = b - m$, where v is the linear velocity of the commutator surface, b is the peripheral width of the brush, and m the thickness of one mica insulation. Actual oscillograms sometimes show a considerably shorter period of time, due probably to imperfections in brush contact. An experimental study of this subject is of considerable practical interest, especially the effect of careful grinding of the brushes to fit the commutator surface.

(g) *Oscillograms of current.* With perfect commutation the current in a short-circuited coil should vary with the time from the full value in one direction to that in the opposite direction, according to the straight-line law. The degree of departure from this law is one of the criteria of commutation, showing the effect of inductance, external fields, and other disturbing factors. To obtain a curve of current reversal, a low-resistance ammeter shunt is connected into the circuit of one of the armature coils, and potential leads are taken from it through two slip rings to an oscillograph. It is also advisable to have a similar ammeter shunt in one of the commutator leads, to get the distribution of the total current along the commutator segments, as a check on the first curve.

(h) *Commutation on short-circuit.* With a very large machine it is sometimes difficult to provide a load of sufficient magnitude to investigate the quality of commutation on a heavy overload. The same overload current may be easily produced by running the machine with the armature short-circuited and the field excited from a separate source. The physical conditions of commutation are essentially the same as at a higher terminal voltage, except that the armature reaction has relatively a much greater influence because of the weakened external field. Nevertheless, such a test is of great practical importance, especially if one finds out the amount of allowance for more difficult conditions of commutation under short-circuit, and the percentage of reduction in the criterion or in the limiting value of the current. In other words, if under the actual load conditions serious sparking begins at 500 amperes, at what current would the same degree of sparking take place on a short-circuit?

373. Study of Commutation on an Equivalent Circuit. — The principal difficulties in studying commutation on an actual machine are due to the rotation of the armature. Moreover, with the coils embedded in slots it is not easy to eliminate or to modify the individual factors and thus to estimate their effect. For this reason some of the best investigations upon commutation have been made on an equivalent circuit (Fig. 243) in which only the commutator C revolves, while two identical stationary storage batteries, E_1 and E_2 in parallel, replace the e.m.f.'s induced in the L and R groups of coils (Fig. 240). The alternate commutator segments are connected to two slip rings, s_1 and s_2, which in turn are connected to the batteries through adjustable resistances, R_1 and R_2, and " ballast " inductances, L_1 and L_2. The purpose of the adjustable resistances is to enable one to balance the two currents in case the batteries give slightly different e.m.f.'s; the inductances are intended to keep the currents in the two battery circuits practically constant, and unaffected by the rapid fluctuations of the current in the coil undergoing commutation.

Only one brush, B, is used, and it is held against the commutator by means of a special brush-holder at A with an adjustable pressure. The peripheral width of the brush must be less than, or at the most equal to, the width of one commutator segment. The external circuit, or the load, is imitated by the resistance R_3 and inductance L_3.

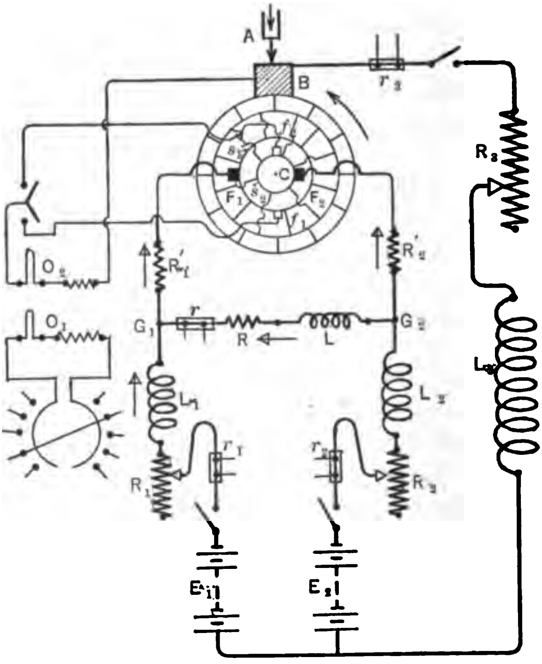


FIG. 243. An equivalent circuit with adjustable constants for studying commutation.

The coil undergoing commutation is represented by a shunted combination of resistance R and inductance L between points G_1 and G_2. As the brush B touches alternate commutator segments, the current through this shunted circuit is forced to flow in the alternate directions, thus imitating the action of an armature coil which is being transferred from one group of coils to the other.

The oscillograph shunts are marked r, r_1, r_2, r_3 and are used for taking curves of currents in the four main branches of the circuit. By means of a radial switch the oscillograph element O_1 may be connected at will to any of these shunts. The voltage curves between the brush and the commutator segments are taken with the second oscillograph element O_2, using separate brushes f_1 and f_2 so as to avoid the voltage drop in the main current brushes F_1 and F_2.

With this device, some of the factors and problems in commutation, enumerated above, can be studied much more conveniently and accurately than on an actual machine; the magnitude of the individual factors may be readily varied or their influence even eliminated.

374. EXPERIMENT 15-I. — Study of Commutation in a Direct-Current Machine. — The theoretical elements of commutation are explained in §§ 364 to 368; the various factors to be studied experimentally are enumerated in § 372. Most of this study is rather advanced and requires the use of an oscillograph and of special apparatus (§ 373), and more time than an average student can devote to the subject. Nevertheless, a thoughtful experimenter can derive considerable benefit even from a superficial study of commutation with limited means. In particular, some of the simpler problems enumerated in § 372, under (a), (b), (d), (e), and (h), may well be attempted, in order to make clear to oneself the physical nature of the problem and the experimental difficulties involved.

BRUSHES AND THEIR TESTING

375. The two most common processes of manufacturing carbon brushes are termed the *squirting*, or extruding, *process* and the *molding*, or machining, *process*. In the former of these the material in the form of pulp is forced under pressure through a metal die and then cut off to the desired length and baked at a high temperature to carbonize the bond and permanently set the material. This method is used in making the cheaper grade of carbons, which do not have the strength to resist the breaking and chipping in service. The other process consists in molding the material into blocks under heavy pressure and then baking it. Carbons are cut from these blocks and machined to exact size.

The information on carbon and graphite brushes, given below, has been compiled mostly from the data furnished by the engineers of the

National Carbon Company, to whom the author wishes to express his sincere appreciation.

376. Brush Composition. — According to their composition, brushes may be divided into the following classes:

Amorphous carbon brushes. The amorphous carbon brush is hard and has the lowest current-carrying capacity of any of the various grades. It is used in a few cases where commutating conditions are very severe, demanding a tough brush, but its field of application is not large.

Carbon-and-graphite brushes. The addition of graphite makes the brush softer, increases the current-carrying capacity, decreases the coefficient of friction, and reduces the contact drop. A great variety of characteristics is found in the various grades of brushes falling in this class, so that its field of application is quite large.

Electro-graphitic brushes are somewhat harder than the preceding class of brushes, but as a rule are non-abrasive or practically so. They have better lubricating properties and higher current-carrying capacities than the first two classes of brushes, and as a class, better commutating characteristics.

Pure graphite brushes. Brushes of this composition are the softest brushes and have the highest current-carrying capacity of any of the brushes, with the exception of the metal-graphite type. On account of the softness of these brushes, as low a spring tension should be used as is consistent with satisfactory operation. Brushes of this composition have good lubricating qualities, making them suitable for high commutator speeds, and it is in this field that they find their most extensive application.

Brushes impregnated with a lubricating material. The pore space in a carbon brush is about 25 per cent by volume, and in this type of brush the carbon is impregnated with a lubricant. The coefficient of friction and some other characteristics of the brushes are improved by this treatment.

Metal-graphite composition brushes. Brushes composed of a combination of metal and graphite have the highest carrying capacity that it is possible to obtain, except in all-metal brushes. These brushes are used on plating generators, on direct-current machines of very low voltage, and on collector rings where the carrying capacity is too high for carbon or graphite brushes.

377. Electrical Characteristics of Brushes. — The following electrical characteristics of brushes and tests are of practical importance:

(1) *Specific resistance.* This is the resistance in ohms of a cube whose edges are 1 centimeter or 1 inch long. This value is determined

by measuring the resistance of a long rod before it is cut into brushes of the regular size. Any of the methods mentioned in Chapter I for low-resistance measurements may be used.

(2) *Contact drop*. This is sometimes improperly called contact resistance and is the value in volts of the difference in potential between the contact surface of the brush and the commutator segment. This voltage seems to be more of the nature of counter-e.m.f. and varies but slowly with the current density within wide limits of the latter. The apparatus for determining the contact drop consists of a copper ring driven by a variable-speed motor, the brushes being held firmly against the ring by holders which permit accurate measurements of the brush pressure. The current used flows from the brush to the ring and from the ring to the brush in such a way as to represent the conditions of positive and negative brushes of a direct-current machine. Curves of current and voltage are plotted at different speeds, brush pressures, and current densities. A pyrometer is used to give the necessary temperature data on both the brushes and the ring.

There is a definite relation between friction and contact drop. Contact drop decreases with an increase of brush pressure, while the friction increases with the pressure. There is a definite point at which the decrease in contact drop is overbalanced by the loss due to the increase in friction. Therefore, in the determination of brush tension both of these losses should be considered, and the value chosen must effect a compromise between these two losses. The pressure should seldom be less than 1¾ pounds per square inch. For crane motors, mining locomotives, railway motors, and similar rough and intermittent service, the pressure ranges from 4 to 7 pounds per square inch (§ 310).

While the specific resistance of a brush has some effect on the commutation of a machine, its effect is not as marked as that of the contact drop. A brush of high specific resistance tends to reduce the short-circuit currents on the brush face to some extent, but this effect is not very important. A more important advantage is the tendency to reduce short-circuit currents between the individual brushes of a stud, which currents may be caused by improper brush spacing. The chief disadvantage of high specific resistance is the high heat-loss due to the passage of current. This increases the temperature of the commutator and consequently decreases the rating of the machine. On the whole, the disadvantages of a brush of high specific resistance would greatly outweigh the advantages, were it not for the fact that high specific resistance and high contact drop usually go hand in hand.

In making tests on both the specific resistance and the contact drop it is well to investigate their values up to considerably higher temperatures than would be expected in normal operation. In fact, it is advisable to go up to the point of "glowing," because it is under such abnormal conditions that a good brush may save a machine, or at least tide over an emergency.

(3) *Current-carrying capacity.* The carrying capacity is the current density, in amperes per square centimeter or square inch of contact surface, that a brush can carry continuously without serious heating. In other words, the current-carrying capacity is fundamentally limited by the temperature reached on the contact face. For intermittent service, such as cranes or elevators, the brush may sometimes be rated at from 50 per cent to 75 per cent higher. When giving a figure for carrying capacity, the brush engineer not only considers the actual load current, but also the short-circuit currents in the coils undergoing commutation (§ 365), and the heating produced by friction and contact drop. Carrying capacity, while primarily an electrical characteristic, is subject to pronounced influence by all of the mechanical or physical characteristics of brushes having any influence on the temperature. These mechanical or physical characteristics are: coefficient of friction, abrasiveness, density, and thermal conductivity. All of these are considered below.

(4) *Arcing and burning tests.* In selecting brush stock it is well to test it for its ability to withstand disintegration due to severe arcing and burning. Almost any arrangement that will sustain an arc is suitable for such a test, provided that the physical quantities which enter into the test can be measured and that the conditions can be repeatedly reproduced with a fair degree of accuracy.

378. Mechanical Characteristics of Brushes. — The following mechanical characteristics of brushes and tests are of practical importance:

(1) *Coefficient of Friction.* The apparatus for determining brush friction consists of a copper slip-ring very accurately balanced and driven by an adjustable-speed motor. The brush-holders and studs are also accurately balanced on a pin bearing, the friction of which is so slight as to be negligible, so that only the actual brush friction is recorded on the fine spring balances used. Thermometers are so suspended as to measure the temperature of the brush as nearly as possible to the brush face. Friction tests are run at different speeds and at different values of brush tension for each grade of brush under comparison.

Atmospheric conditions, composition of the brush, brush tension, current density, peripheral speed, composition of commutator or slip ring, condition of commutator, i.e., whether slotted or flush, and temperature, are factors which have more or less effect upon the coefficient of friction. Therefore, the results obtained on a smooth slip ring must be used judiciously and not generalized to other conditions of operation.

The hardness of a brush does not indicate its coefficient of friction. The former depends upon the hardness of the particles composing the brush and upon the strength of the binder holding these particles together. Friction depends on the structure of the particles making up the brush and upon the impurities in the carbon or graphite which give the brush an abrasive or cutting quality. It is, therefore, not true to say that a hard brush will wear a commutator faster than a soft brush. In fact, a soft brush will wear a commutator more rapidly than a hard brush with the same percentage of abrasive material, because of the fact that the soft brush wears away more rapidly, thus permitting the abrasive material to feed down on to the commutator at a more rapid rate. It is possible to get a high friction without abrasive action, but it is impossible to get abrasive action without a high friction. The most common impurities which cause abrasive action are mica, quartz, silica, carborundum, and iron oxide.

A film of oil between the brush and the commutator, besides acting as a lubricant, has the property of increasing the contact drop. However, the porosity of the brush and the high temperature at the brush face usually cause the oil film to disappear very quickly, and the injurious effect of oil on mica makes it dangerous practice to lubricate a commutator. On undercut commutators the practice is even more dangerous, because of the collection of dust and dirt in the slots which may result in flash-overs or short circuits.

(2) *Hardness.* The mechanical hardness of a brush is determined with an instrument called a scleroscope, in which a steel weight falls from a constant height upon the article whose hardness is to be measured, and then rebounds up a graduated scale, the height of the rebound being measured. This, of course, does not give the actual hardness, but a relative figure which is useful for purposes of comparison.

(3) *Mechanical Strength or Toughness.* The usual strength test on brush stock is really a flexure test. The test piece is supported on two knife-edges and a third knife-edge presses on the middle. A weight is moved along a graduated lever arm and the pressure is increased until the piece breaks. This force, properly correlated with the dimensions of the prism under test, gives the unit strength or toughness of the material.

(4) *Abrasiveness.* Resistance to abrasion, or scouring action of the carbon brush, is the ability to withstand the wear due to friction. This term should not be confused with hardness. Hardness relates to com-

pactness of the mass as a whole, while abrasiveness relates to the cutting action given a brush by the more or less gritty particles in the brush mixture. The abrasive action of a brush depends largely upon the composition of the ash which forms a part of the brush. The amount of ash can be determined by heating the brush in a special furnace to a temperature which will burn out all of the carbon. Silica, mica, and carborundum form the major part of the abrasive material found in the ash of carbon brushes. In general it may be said that an abrasive brush should be used where the mica between the commutator segments is very hard and is not undercut, or where excessive reactance voltage creates a serious tendency toward high mica. An abrasive brush will keep the mica level with the commutator and will prevent the sparking and chattering due to high mica. Except in cases where there is a tendency toward high mica or heavy sparking which roughens the commutator, pronounced abrasive qualities in a brush are undesirable.

There seems to be no reliable test for abrasiveness. Efforts have been made to develop an apparatus to test this characteristic, but these have failed to produce results that check with actual experience with the different grades of carbons under service conditions. This is attributed to the influence of the commutating currents on disintegration at the brush face, which cannot be reproduced in a testing machine. Brushes which have shown decided abrasive characteristics in operation will sometimes glaze over on the test machine and run for a long period of time without wear.

(5) *Maximum peripheral speed.* Most types of brushes operate satisfactorily only as long as the linear speed of the commutator surface does not exceed a certain value. Therefore, when a brush is to be used on a high-speed surface, it is advisable to determine the speed limit or to ascertain it from the maker of the brush.

(6) *Vibrator test.* Railway motor brushes are subjected to a special test in a vibrator to determine their ability to resist breakage due to chattering. This test must be made at least as severe as in the actual service, and only the stock that will stand this test should be specified or used.

(7) *Density.* The real density of the material of a brush is the weight of the carbon exclusive of the pores compared to the weight of an equal volume of water. The average real density of brushes is approximately 2.00. The apparent density is the weight of the carbon with the pores compared to the weight of an equal volume of water. The average apparent density of brushes is approximately 1.5. The average pore space in brushes amounts to about 25 per cent. A low-density brush, because of its low inertia, will more readily follow the irregularities of

the rotating element and for this reason is particularly adapted to very high peripheral speeds.

(8) *Thermal conductivity.* This characteristic is of importance because it determines the amount of heat that can be conveyed away from the brush face to the holder and to the frame of the machine; consequently it determines in part the temperature of the commutator. Graphite has a higher thermal conductivity than most forms of carbon, and if it is possible to replace a carbon brush with a suitable graphite one, the temperature of the commutator may be reduced by several degrees. Thermal conductivity may be expressed either in the absolute or in the relative measure. The absolute conductivity of a brush material is the number of watts which will flow through one cm. cube at the temperature difference of 1° C. between the opposite sides. Relative conductivity is expressed in per cent of the thermal conductivity of some standard material.

CHAPTER XVI

THE TRANSFORMER

379. A STATIONARY transformer, in the usual sense of the word, is a device based on electromagnetic induction, for converting alternating-current power from one voltage to another. For example, a lamp load in a house may be 10 amperes at 100 volts. By means of a transformer (Fig. 244) of suitable design, this power can be obtained from a 2000-volt supply by drawing only 0.5 ampere. The product is 1000 volt amperes in either circuit, and no power is lost except a small amount in the transformer itself.

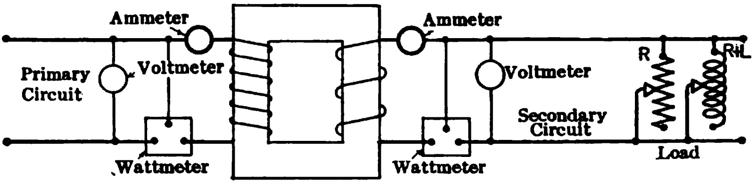


FIG. 244. Diagram of connections for a load test on a transformer.

A transformer (Figs. 245 to 247) consists of a laminated iron core and of two windings, primary and secondary, in an inductive relation to each other. One of the windings is connected to the source of power, the other to the load. The winding connected to the supply should always be called the primary, the other the secondary, although in practice these terms are sometimes used rather loosely. Sometimes it is preferable to call them the high-tension winding and the low-tension winding respectively, because a transformer can usually be connected either so as to raise the voltage or to lower it. For an exception in the case of instrument transformers, see §§ 416 and 424.

380. Core-Type and Shell-Type Transformers. — There are three ways of arranging the windings and the core relatively to each other. The so-called *core-type transformer* (Fig. 245) consists of one iron core and of two sets of coils, one on each of its legs. The *shell-type transformer* (Fig. 246) consists of one set of coils, and each side is surrounded by an iron core. The combination type or the *distributed-core type transformer*, shown in Fig. 247, permits of a somewhat better utilization of the copper and iron. Electrically the three types are equiva-

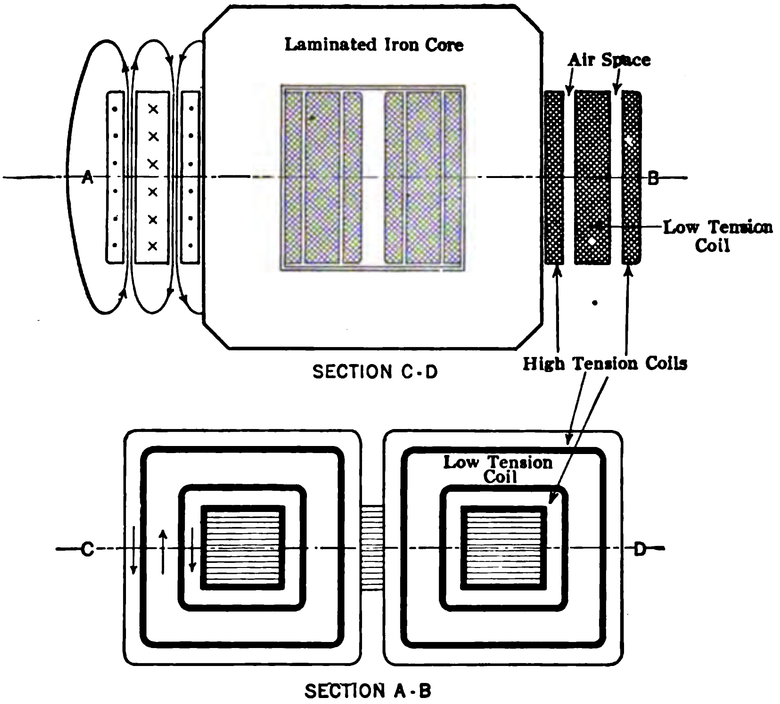
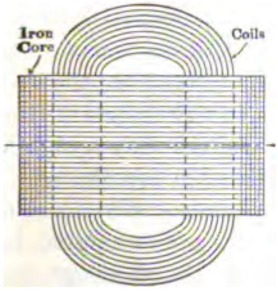
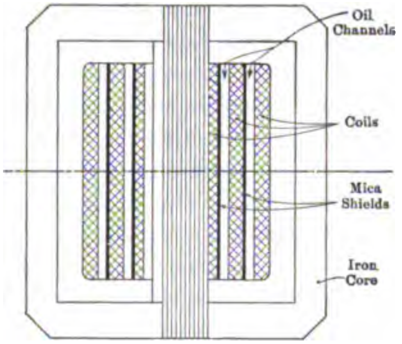
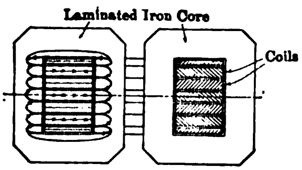


Fig. 245. A core-type transformer.







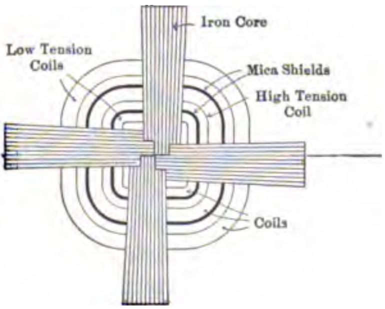


Fig. 246. A shell-type transformer with a single iron core above, and with two separate cores below.

Fig. 247. A combination-type or distributed-core transformer.

lent, the difference being mainly mechanical and determined by the convenience and cost of manufacture.

Transformers are usually immersed in oil, which keeps the windings and the core cooler in operation, and also protects the insulation from moisture. In smaller sizes the oil is cooled naturally through the containing steel case. In larger sizes it is usually necessary to increase the heat radiating surface of the case by making it corrugated or by adding circulating pipes. In very large transformers the oil is cooled by circulating water in pipes placed within the transformer tank. When several transformers are to be cooled by water, it is sometimes preferred to circulate the oil in pipes in a separate water tank, or under a spray. Another method of cooling is by circulating air. No oil is used, the transformer case being left open at the top and bottom; air is blown through the core and the windings by a suitable blower.

The following are the principal tests performed on standard transformers, either by the manufacturer or by the customer. The purpose of these tests is mainly to check the guaranteed performance characteristics, the quality of materials, and the correctness of winding and assembly.

381. Introduction to Transformer Testing. — A student working with a 2200-volt transformer must remember that *it is dangerous* to come in contact with its high-voltage side; great care should be exercised in regard to touching any switches, wires, or instruments connected to the high-tension side. Rubber matting should be put on the floor for protection, so as to insulate the tester from the ground, and he should not touch either the walls, pipes, or any objects that might be connected to the ground. In addition, rubber gloves should be provided for the person who reads the high-tension instruments (Fig. 244) so that he could not touch them by mistake. *Directions of the instructor should be followed exactly,* and no changes attempted before consulting him. An electrical engineer has to handle high-tension circuits in his practical work, and it is advisable for him to get some experience with such circuits before leaving college. He should know, however, that a mistake or negligence may prove fatal, and should act accordingly.

(1) *Voltage-Ratio Test* at no load, to determine whether the transformer has been wound correctly, and the leads and taps properly brought out. Part of the checking of the connections is the so-called *polarity test* (§ 408) to determine which terminals are to be connected together for parallel operation.

(2) *Resistance of the Windings.* This measurement is usually done by the drop-of-potential method (§ 2) and serves several purposes,

viz., (a) It is a check on the winding and its assembly; (b) The results may be used in computing the efficiency and the voltage regulation of the transformer; (c) If performed before and after the heat run it gives the data for the computation of the temperature rise; (d) A comparison of the I^2R loss computed from the resistance measurements with that determined from the wattmeter test (§ 385) gives an idea of the extent of eddy currents in the conductors.

(3) *Core Loss and Magnetizing Current.* This test is performed at no load to determine the amount of power lost in hysteresis and in eddy currents in the iron core, and the current which the transformer takes at no load. The results are used to check the quality of the iron and to compute the efficiency of the transformer. The temperature rise may also be estimated from this test and the preceding one. The upper limit of the no-load or magnetizing current is often prescribed by the contract, and is also a check on the design, the quality of the iron, and its assembly.

(4) *Short-Circuit Test,* to determine the impedance drop and the copper loss. The results are used in computations of the voltage regulation and efficiency. This test also serves as a check on the winding, its connections, and soldered joints.

(5) *Heat Run.* The purpose of this test is to determine the maximum temperature which the principal parts of the transformer reach after a certain number of hours of operation under prescribed conditions. This test is of great practical importance.

(6) *Test of Insulation, to Determine its Strength and Condition.* This includes a high-potential test between each winding and the core, between the two windings, and sometimes between the sections or turns of the high-tension winding. The insulation resistance between the primary and the secondary winding is also sometimes measured. These tests are described in Volume II.

(7) *Load Test.* This test is avoided as much as possible because of the expense and difficulties of providing a suitable artificial load, especially for a large transformer. If required for the heat run mentioned under (5) above, it is preferable to use the so-called opposition or loading-back method (§ 391) in which power is circulated between two transformers.

382. Voltage Ratio. — Consider a transformer (Fig. 244) connected on its primary side to a suitable source of alternating-current supply, and with its secondary winding open, that is, disconnected from the load. The alternating current in the primary winding excites in the core an alternating flux, which induces alternating e.m.f.'s in both the primary and the secondary windings. The two windings being placed on the same iron core and subjected to the same alternating flux, the voltages induced *per turn* will be the same in both. Since the turns in each coil are connected in series, the total induced voltage per coil is proportional to its number of turns. We thus have the following fundamental relationship: *In a transformer the voltages induced, e_1 and e_2, by the useful flux are in the same ratio as the numbers of turns, n_1 and n_2, in the corresponding windings.* Mathematically expressed,

$$e_1/e_2 = n_1/n_2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

This expression holds true either for the instantaneous or for the effective values of the induced e.m.f.'s, e_1 and e_2. For a relationship between the induced voltage, the flux, and the number of turns, see eq. (6) in § 220.

In the usual transformer, the primary voltage drop is very small, so that the primary applied voltage, E_1, is practically equal and opposite to the primary induced e.m.f., e_1, even at full-load. Similarly, the voltage drop in the secondary winding being small, the secondary terminal voltage is nearly equal to the secondary induced e.m.f., e_2. Therefore, we have *approximately*

$$E_1/E_2 = n_1/n_2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

Equations (1) and (2) become practically identical at no load.

383. EXPERIMENT 16-A. — Voltage Ratio in a Transformer at No Load. — The purpose of this experiment is to prove that the ratio of the induced voltages is equal to the ratio of the numbers of turns. It is advisable to have a transformer in which the turns may be actually counted and their number varied at will, either by interconnecting individual coils or by using different taps on the same coil. A voltmeter is used on the primary side and another on the secondary side, unless the voltages are such that the same voltmeter may be used with a double-throw switch. An electrostatic voltmeter (§ 53) is sometimes suitable for the high-tension side. It is also advisable to have an ammeter in the primary circuit in order to get an idea of the order of magnitude of the no-load current taken by the transformer under different conditions. For a special study of magnetizing current, see § 398.

(1) Select an arrangement of windings for which the number of turns is known and apply a voltage of known frequency to the primary terminals. Read the primary and secondary voltages and the primary current. Raise the applied voltage in steps and also reduce it below the first value, to see if the ratio of voltages remains independent of the applied voltage.

(2) Keep the applied voltage constant and vary its frequency. This

can best be done by having an alternator driven by an adjustable-speed motor. Note the increase in the magnetizing current as the frequency is decreased.

(3) Make a few check runs with a different arrangement of windings or with different taps on the same coils. Observe that the magnetizing current increases as the primary number of turns is reduced, the voltage and the frequency being kept constant.

(4) Select two secondary coils and designate the terminals of one *a*, *b*, and those of the other *c*, *d*. Connect *a* to *c* by a jumper and measure the three voltages *ab*, *bd*, and *cd*. Keeping the same primary voltage, disconnect the jumper at *c* and put it to *d*. Measure the voltages *ab*, *bc*, and *cd*. Show from the readings that in one case the coils were connected cumulatively, and in the other case in opposition (§ 408).

(5) Take two coils of the same number of turns and, by the method described in (4) above, find which terminals should be connected if these two coils are to be operated in parallel. Actually connect them in parallel and show that the terminal voltage is the same whether one or both coils are in the circuit. Change the number of turns in one of the coils by a few per cent, and show by means of an ammeter that a considerable circulating current is produced if an attempt is made to operate such coils in parallel.

Report. (a) By means of curves or a table, show the effect of the applied voltage and of the frequency upon the ratio of transformation. (b) Show that eq. (2) is nearly correct for most of the readings, and discuss the departures, if any. (c) Plot curves showing how the magnetizing current varies with the applied voltage, the frequency, and the primary number of turns. Explain these variations by the theory of an iron core excited with alternating current (§ 224). (d) From the experiment with two secondary coils, describe what is meant by their "polarity," and explain how transformer coils may be connected in series and in parallel.

384. Ratio of Currents on Short-Circuit. — Consider again the same transformer (Fig. 244) connected on the primary side to a comparatively low source of alternating voltage (not over 5 per cent of the rated voltage), and on the secondary side short-circuited through an ammeter of negligible resistance. It is found experimentally that under such conditions, with a core of low reluctance, and with a winding of low-leakage reactance (§ 406), the currents, i_1 and i_2, in the primary and secondary circuits very closely satisfy the condition

$$i_1 n_1 = i_2 n_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

both for the instantaneous and for the effective values of the currents; n_1 and n_2 are the corresponding numbers of turns. In other words: *In* *a short-circuited iron-core transformer, with low leakage, the currents are inversely as the numbers of turns in the two windings.*

This simple relationship follows directly from the fact that under such conditions the flux in the core is quite small. The applied voltage being low, the counter-e.m.f. has to be low, and since this e.m.f. is induced by the flux in the core, the flux must be small. The secondary winding being short-circuited, even a small induced voltage causes a considerable current flow in it. This current would produce a large flux, were it not for the primary current which automatically adjusts itself to a value almost equal and opposite to that in the secondary, because the resultant number of ampere-turns needed for a small flux is almost equal to zero; eq. (3) follows directly. The applied voltage must be kept low, so as not to overheat the windings. This voltage is used to circulate the two currents, overcoming the ohmic resistance and the leakage reactance of the two windings.

One of the practical uses of the short-circuit test is to determine the equivalent resistance and reactance of the windings (§ 385). It is also used to check the design and the soundness of manufacture and assembly. Sometimes it is used to determine the temperature rise in the windings, although the results should be applied judiciously, because practically no heat is generated in the core on account of a low flux density, and therefore the core helps to conduct the heat away from the windings.

385. Equivalent Impedance, Resistance, and Reactance from Short-Circuit Test. — The secondary output on short-circuit being equal to zero, and the core loss negligible, practically the whole primary input is expended in the I^2R loss in the windings. This loss may be computed from the resistances of the windings, or it may be measured directly on a wattmeter connected in the primary circuit. With transformer coils made of small-size conductors, the results check within a few per cent, but with very large copper bars used for heavy currents, the actual copper loss comes out considerably higher than the computed copper loss. This is due to eddy currents and to a non-uniform distribution of alternating currents through the cross-section of the conductor (skin effect). This increase in the copper loss must be taken into account in computing the efficiency of a transformer.

It has been found experimentally that a short-circuited transformer behaves like an impedance coil consisting of a resistance and a reactance in series (§ 138). While the energy is transmitted into the secondary circuit by induction and not by conduction, the result is the same as if the secondary winding were done away with and the impedance of the primary winding increased by a certain amount. This subject is considered theoretically in § 402, but for the present the student may regard the foregoing statement and the following conclusion as experimental facts:

In a transformer with a core of low reluctance, and near the point of short-circuit, resistances and reactances can be transferred from one circuit to the other by being multiplied by the square of the ratio of the numbers of turns. In other words, a resistance, r_1, in the primary circuit is equivalent to a resistance r_2 in the secondary, if the two satisfy the condition

$$r_1/r_2 = (n_1/n_2)^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

A similar relationship holds for reactances, viz.,

$$x_1/x_2 = (n_1/n_2)^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

Because a short-circuited transformer behaves like an impedance coil with a very low flux density in the core, the current is proportional to the applied voltage. Moreover, one may speak of the *equivalent* impedance, reactance, and resistance of the transformer, in the sense in which these terms are applied to a choke coil. In practice, these three quantities are determined from a short-circuit test as follows:

(a) *The Equivalent Impedance*, z_{eq}. The effective value of the applied voltage E_1 on short-circuit is divided by the corresponding current I_1, that is,

$$z_{eq} = E_1/I_1 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

(b) *The Equivalent Resistance*, r_{eq}. Let the wattmeter reading in the primary circuit be P watts, with the secondary short-circuited. The equivalent resistance is computed from the expression

$$P = I_1^2 r_{eq} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

If a wattmeter test is not feasible, the resistances R_1 and R_2 of the windings are measured with direct current, and the equivalent resistance computed by means of formula (4), namely

$$r_{eq} = R_1 + R_2 (n_1/n_2)^2 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (8)$$

(c) *The Equivalent Reactance*, x_{eq}. Knowing the equivalent impedance and resistance, one may determine the equivalent reactance either graphically or analytically, as for any choke coil (§ 138), that is, from the relationship

$$x_{eq}^2 = z_{eq}^2 - r_{eq}^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

386. EXPERIMENT 16-B. — Short-Circuit Test on a Transformer. — The purpose of the experiment is to check the ratio of the currents, and to determine the equivalent impedance and resistance of the transformer, as is explained in the preceding two sections. Connect a low-resistance ammeter across the secondary terminals, using as short and heavy leads as possible, and an ammeter in series with the primary winding. Use a low-range voltmeter across the primary terminals and connect a wattmeter to measure the input into the primary circuit (Fig. 244). Provide an arrangement for regulating the primary applied voltage, such as a rheostat or an auto-transformer (§ 395). A separate alternator, whose excitation may be varied at will, is also quite convenient.

(a) Apply a primary voltage sufficient to produce a maximum safe current in both windings of the transformer. Read both ammeters, the wattmeter, and the voltmeter. Reduce the voltage in steps and take similar readings at each step. (b) Insert in the secondary circuit a small known non-inductive resistance, sufficient to affect the primary current. Take a set of readings as above. Cut out the resistance and insert an adjustable resistance in the primary circuit. Find the value of this resistance that will make the instrument readings the same as before. Repeat with a few different values of resistance. (c) Perform a similar test with reactances. (d) If the ratio of turns is not known from the preceding experiment, determine it by reading a few sets of no-load voltages. (e) Measure the resistance of both windings with direct current.

Report. (1) Plot the currents against the values of applied voltage as abscissæ, and explain the reason why both curves are straight lines. (2) Show that the ratio of the currents is inversely as the ratio of the numbers of turns. (3) To the same abscissæ plot the wattmeter readings, and also the computed total I^2R loss. Explain any considerable discrepancy between the two. (4) Compute the values of the equivalent impedance, resistance, and reactance of the transformer. (5) Show from the data on the inserted resistances and reactances that eqs. (4) and (5) hold true.

387. Vector Diagram of an Ideal Transformer. — An ideal transformer is defined as one in which the resistances and the leakage reactances of both windings are equal to zero, and the reluctance of the magnetic circuit is so low that it takes a negligible magnetomotive force to excite the magnetic flux for normal operation.. The vector diagram of such a transformer is shown in Fig. 248 and will be readily understood on the basis of its behavior on open circuit (§ 382) and on short-circuit (§ 384). The particular ratio of turns selected in the figure is $n_1/n_2 = 2$. The diagram shows the following properties of an ideal transformer:

(a) The primary terminal voltage E_1 is equal and opposite to the primary induced e.m.f., e_1, because of the assumption of zero impedance drop in the primary winding.

(b) The secondary terminal voltage E_2 is equal to the secondary induced voltage e_2, because by assumption the impedance drop in the secondary winding is zero.

(c) The induced voltages e_1 and e_2 are in the ratio of the respective numbers of turns and are in time phase with each other, being induced by the same flux.

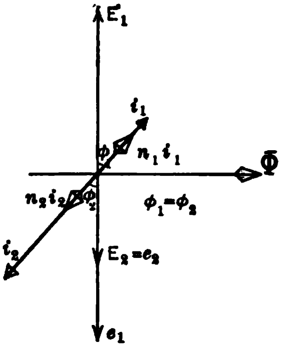


FIG. 248. A vector diagram of an ideal transformer.

(d) The primary and secondary m.m.f.'s, n_1i_1 and n_2i_2, are equal and opposite because the magnetic core by supposition requires a negligible magnetizing current. Hence, the currents are inversely as the numbers of turns, and are in phase opposition with each other.

(e) The induced voltages, e_1 and e_2, lag by 90 degrees behind the common flux Φ. This follows directly from the fundamental law of induction (§ 157). Let the flux vary with the time t according to the sine law, $\phi = \Phi \sin 2\pi ft$, where f is the frequency of the supply. The induced e.m.f., e_0, per turn of either the primary or the secondary winding is equal to $-d\phi/dt$. Differentiating, we find $e_0 = -2\pi f\ \Phi\ \text{Cos}\cdot 2\pi ft$. Hence, the induced e.m.f. varies according to the cosine law, and passes through zero and maximum values one quarter of a cycle later than the flux.

(*f*) The primary and the secondary power factors are equal to each other, because the two terminal voltages are in phase opposition, and so are the two currents. The corresponding phase angles are denoted in the figure by ϕ_1 and ϕ_2.

(g) The power input into the primary circuit is equal to the secondary output, no power being lost in an ideal transformer. For the currents we have $I_1n_1 = I_2n_2$, and for the voltages $E_1/n_1 = E_2/n_2$; besides, $\text{Cos}\ \phi_1 = \text{Cos}\ \phi_2$. Multiplying these three expressions, term by term, we get $E_1I_1\ \text{Cos}\ \phi_1 = E_2I_2\ \text{Cos}\ \phi_2$.

Figure 248 contains vectors of four different kinds of physical quantities, viz., voltages, currents, flux, and m.m.f.'s. Four different kinds of arrowheads are used to distinguish these quantities. This notation is more or less standard and facilitates the reading of a vector diagram.

In a good modern transformer, the impedance drop in either winding at full load amounts to but a small percentage of the induced voltage. Also, the magnetizing ampere-turns constitute but a small percentage of the full-load ampere-turns of either winding. Hence, the diagram shown in Fig. 248 may be said to represent *approximately* the relations in an actual transformer, except at light loads. An exact vector diagram of an actual transformer is deduced in § 401, but the foregoing diagram of an ideal transformer is sufficiently accurate for the solution of many practical problems, except those involving voltage drop and regulation.

388. Voltage Drop and Regulation. — The transformer windings possess a certain amount of resistance and of leakage reactance, both of which cause an internal voltage drop. Thus, with a constant applied primary voltage, the secondary terminal voltage depends upon the value of the load current and upon the power factor of the load. As the load fluctuates, the secondary voltage also varies. To maintain satisfactory service these fluctuations must not exceed a certain limit, and for this reason the determination of the voltage regulation of a transformer is of great practical importance.

Let, for example, a transformer have a ratio of turns equal to twenty to one. When the primary winding is connected to a 2200-volt supply the secondary voltage at no load is 110. Let the voltage drop in the primary winding, due to its impedance, be such that when subtracted vectorially from 2200 volts it leaves 2160 volts to be transmitted into the secondary. With the ratio of transformation equal to twenty to one, the secondary induced voltage will be 108 volts. From this value the secondary voltage drop has to be subtracted vectorially, leaving about 106 volts at the secondary terminals. Thus, as the load varies between zero and its rated value the secondary terminal voltage fluctuates between 110 and 106 volts. This is probably as wide a range as is permissable, and a wider range would not give satisfactory service to users of incandescent lamps.

According to the definition adopted by the American Institute of Electrical Engineers, the *percentage of voltage regulation* of the foregoing transformer is

$$(110 - 106)/106 = 3.8 \text{ per cent.}$$

This is " the percentage ratio of the change in the quantity occuring between the two loads, to the value of the quantity at either one or the other load, taken as the normal value " (1921 Standards, Rule 3535). In the foregoing example the full-load voltage of 106 is taken as the normal value.

More generally, let a transformer be intended for operation at a certain *normal* load P_n, and at a *normal* secondary voltage E_n. Let such a load and voltage be actually produced, and let the corresponding primary terminal voltage be E_1. Now let the load be changed to some other desired value, P_2, *keeping* E_1 *constant*, and let the new secondary voltage be E_2. Then, according to the foregoing definition

$$\text{percentage regulation} = (E_2 - E_n)/E_n \quad . \quad . \quad . \quad . \qquad (10)$$

As such a load, P_2, a zero load is usually chosen, so that E_2 becomes the secondary voltage at no load.

The voltage variation on the secondary side, with a constant primary voltage, may be determined either by the *direct method*, i.e., by actually loading the transformer, or it may be calculated from the measured resistances and reactances of the transformer windings. The second, or *the indirect method*, is always preferred in practice, because it gives more accurate results and involves but a small expenditure of power. This method is described in §§ 403 and 405. In order, however, that the student may observe a transformer in actual operation and be able to judge about the order of magnitude of voltage fluctuations, he should be given an opportunity to determine the regulation by actually loading a transformer on ohmic and inductive resistances, as described in the next three experiments. It is advisable to determine the voltage regulation of the same transformer by the direct and the indirect methods in order to be able to check the results.

389. EXPERIMENT 16-C. — Load Test on a Transformer with Non-inductive Load. — The purpose of this experiment is to measure directly the secondary voltage as a function of the load, and to determine deviations from the ideal ratios of the currents and voltages. This experiment is mainly illustrative of the properties of a transformer, because in practice the simpler and more accurate short-circuit test is used instead (§ 403).

The diagram of connections is shown in Fig. 244 except that the inductive branch of the load is omitted. The wattmeters are not absolutely necessary, but are desirable as a check. The difference between the secondary voltage at no load and at full load is rather small, and for this reason a transformer with comparatively poor regulation should be selected, and the primary and secondary voltages measured with the utmost care and accuracy. The primary voltage must be kept absolutely constant and a regulating rheostat may be inserted in the primary circuit for this purpose. The readings may be recorded on a data sheet like the one shown in the next experiment.

Begin with the highest safe current and adjust the prim v gs, the to the desired or rated value, to be kept constant througho he losses Read carefully the volts, the amperes, and the watts in b nt method Reduce the load in steps and take similar readings at eac ay the prinally open the load circuit and read the magnetizing current on a low-range ammeter.

Report. To load current as abscissæ, plot the secondary terminal voltage, voltage drop, and per cent voltage drop or regulation. To the same abscissæ plot also the primary current and the primary and secondary volt-amperes. Show how nearly these curves are in accord with the equations, the vector diagram, and the theory given in § 387.

390. EXPERIMENT 16-D. — Load Test on a Transformer and the Effect of Power Factor on Voltage Regulation. — This experiment is an extension of the preceding one, to include inductive loads. The diagram of connections is shown in Fig. 244 and a sample data sheet below. For the inductive part of the load, an adjustable choke coil, a single-phase induction motor, or a synchronous motor may be used. For obtaining a leading current a set of static condensers or an over-excited synchronous motor may be used. The runs may be arranged either at a constant power factor and varying current, or at a constant current and varying power factor, according to the laboratory facilities. The primary terminal voltage must be kept constant throughout the experiment. A power-factor meter will save time in making adjustments, although the value of the power factor in the final setting should be checked by the wattmeter readings.

In either case begin with the heaviest possible current and the lowest value of the power factor. Then either keep the current constant and increase the power factor in steps to unity and again reduce to lower values with the current leading, or else keep the power factor constant and gradually reduce the current to zero.

	Inductive Load —— Amps. Constant						
	Prim. Amps.	Prim. Volts	Prim. Watts	Sec. Amps.	Sec. Volts	Sec. Watts	Power Factor
Inst. No....							
Constant...							

n
Mot
tain norm
a load an
primary te

Report. Show that at a constant power factor the voltage drop is nearly proportional to the current. Show that with the same current the voltage drop is greater at lower values of power factor, and that a voltage rise instead of drop may take place with a sufficiently large leading current. Explain these facts theoretically, and show that the test results approximately confirm the equations and the vector diagram in § 387. Show that the efficiency of the transformer is so high that it is hardly feasible to determine it from the ratio of the wattmeter readings. A small inaccuracy in one of the readings may make the output appear greater than the input. A clear understanding of this fact will explain the necessity of computing the efficiency from the losses (§ 393).

391. Two Transformers Connected in Opposition. — Sometimes a transformer must be tested under full load or overload conditions, as for example in order to determine its temperature rise. A small transformer may be loaded on resistances or reactances without much difficulty, but with a large transformer, or with one wound for an extremely high voltage, such a test may be difficult and expensive. When two similar transformers are available, one transformer can be loaded on the other (Fig. 249) so that the load energy circulates between the two. In this case no power is wasted except that necessary for supplying the losses in both transformers. This is the so-called "pumping-back" method, also known as the "stray-power" or "opposition" method. It is used in testing generators and motors, as well as transformers (Chap. XIV).

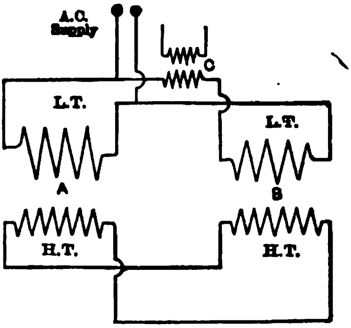


FIG. 249. Two transformers connected in opposition.

In Fig. 249, *A* and *B* are two transformers under test. Their low-tension windings are connected in parallel to a power supply, and the high-tension windings are connected *in opposition* to each other. As the transformers are identical, no current can flow through their secondaries, and but a small magnetizing current flows through the low-tension windings. At the same time full iron loss takes place in the cores of the transformers since they are subjected to the full rated voltage. Now, if a full-load current be made to circulate in the windings, the transformers will be under the same conditions in regard to the losses and voltage regulation as under actual full-load. A convenient method of circulating a load current is to open one of the circuits, say the primary, and to introduce a source of alternating current at a comparatively low voltage. This auxiliary source is shown in the figure as a transformer *C*. By regulating the voltage of this source, a full-load current can be produced in the primaries of the transformers under test; this current causes a full-load current to flow in the secondaries which therefore need not be opened. This is particularly convenient when the secondary coils are wound for a high voltage.

In this test the transformers are under actual load conditions as far as the losses and heating are concerned; at the same time, the energy supplied from outside is merely sufficient to cover the losses. For example, with two 1000-kw. transformers of a full-load efficiency of about 98 per cent, the total expenditure of power is only about 40 kw. When but one transformer is available, the same test can sometimes be performed by connecting the two halves of each winding in opposition.

When an opposition test is performed for a determination of the temperature rise only (§ 351), it is not strictly necessary for the heating current to be an alternating current of the right frequency; it may be a current of any frequency, or even a direct current. If no suitable alternating current is available, both the high-tension and the low-tension circuits are opened, and sources of direct current, sufficient to produce the desired currents in the windings, are introduced. As long as the I^2R losses have the required values the final temperature of the transformers will be the same as under actual load conditions. See, however, § 385 in regard to the difference between the true ohmic and the effective resistance, and the corresponding difference between the computed and measured copper loss.

When the opposition test is used for a determination of the voltage regulation, the current furnished by the auxiliary source *C* must not only be of the correct frequency, but must have the proper phase displacement with respect to the secondary terminal voltage of one of the transformers. In this case the transformer *C* may be in the form of a phase-shifter or induction regulator. The proper phase position is found by means of an ammeter, a voltmeter, and a wattmeter (or a power-factor meter) connected in the circuit of one of the transformers under test. One transformer delivers power to the other, so that in one of them the phase angle between the secondary terminal voltage and the secondary current is less than 90 degrees, while in the other it is greater than 90 degrees.

392. EXPERIMENT 16-E. — Opposition Test on Two Transformers. — The purpose of this experiment is to learn how to circulate electric power between two comparatively large transformers by furnishing the losses from a limited power supply (§ 391). This method of loading may be used for heat runs, for voltage regulation, for a determination of the efficiency, or any other purpose for which a transformer has to be tested under full-load conditions. In practice the opposition method is widely used for heat runs. The connections are shown in Fig. 249. In this particular experiment it is desired to determine the voltage regulation and the efficiency, as well as the temperature rise. Therefore, an ammeter and a voltmeter should be connected in the secondary circuit, if possible. Ammeters, voltmeters, and a wattmeter should be used in the primary circuit to read the following quantities: the applied voltage, that furnished by the booster transformer, the currents furnished by both sources of supply, the power input from the source C, and the core loss.

(1) *Voltage Regulation.* The circulating current must be of the same frequency as that of the applied voltage; the phase of the circulating current must be adjustable, in order that it may be possible to determine the secondary voltage at any desired power factor. Read the secondary voltage when the desired current circulates between the transformers, and then read the same voltage at no load. The percentage of regulation is computed according to the definition in § 388. Take readings with the current lagging, leading, and in phase with the voltage.

(2) *Efficiency.* In this case also, the circulating current must be of the frequency of the applied voltage, and the voltage phase of the booster transformer must be so adjusted that the circulating current is at a desired phase angle with the terminal voltage of the transformers under test. This can be ascertained by a wattmeter so connected as to measure the input into one of the transformers under test. The core loss is read on the wattmeter in the main supply circuit, and the copper loss on a wattmeter connected in the secondary circuit of the booster transformer.

(3) *Heat Run.* Measure the resistance of the windings while cool, and then put a safe overload on the transformers, both in current and in voltage, to produce an appreciable temperature rise, say within an hour. This is permissible when the purpose of the run is to obtain some experience rather than the actual temperature rise (which requires several hours). Try circulating a current from the same source as the applied voltage, then a current from a different source and of a different frequency, and also a direct current. In the case of direct current, make sure that the m.m.f.'s of the primary and secondary windings are equal and opposite. Try a reversed or a wrong direct

current in one of the windings and observe the effect on the core loss and on the magnetizing current. At the end of the run, measure the resistances of the windings again, and determine the temperature rise according to the A.I.E.E. rules.

Report. (1) Give the results of the voltage regulation run, and show that they check qualitatively with the theory given in §§ 387 and 388. Compare the accuracy of this method with that of determining the voltage regulation from a short-circuit test.

(2) Describe the arrangement whereby the phase of the circulating current was altered and measured. Give the efficiency computations, and compare the accuracy of this method with that of computing the efficiency from the losses (§ 393).

(3) Give the results of the heat run, and explain how the current was circulated between the transformers. Describe and explain what happened when a wrong direct current was circulated through one of the windings.

393. Efficiency from Losses. — By definition, the efficiency of a transformer is the percentage ratio of its net power output to its power input. Let the output of a transformer be 1200 kw. and the corresponding input 1223 kw. Then the efficiency at this particular load is $1200 \div 1223 = 98.1$ per cent. If there were no losses in the transformer, the input would be equal to the output. In this particular example the losses amount to $100 - 98.1 = 1.9$ per cent of the input. This shows the close relationship between the efficiency and the losses.

It may seem at first that the simplest method of measuring the efficiency of a transformer would be to have it actually loaded and to read the output and the input by means of wattmeters in the primary and in the secondary circuits; their ratio would give directly the efficiency of the transformer. However, this direct method has serious drawbacks, and is hardly ever used in practice. It involves the use of a wattmeter on the high-tension side, and requires the utmost accuracy in calibrating and in reading the wattmeters, because the values to be read differ by but a few per cent. Moreover, with large transformers, the amount of power required for a full-load test is not always available. Therefore, it is customary to measure the losses in a transformer separately (at no load), and then to *calculate* the efficiency. This can be done by much simpler means and with a considerably greater accuracy than is possible with the direct method.

The losses in a transformer consist of the I^2R or copper losses in both windings, and of the iron loss or *core loss* (hysteresis and eddy currents). The copper loss depends only on the load, and can be easily calculated for any load, if the effective resistances of both windings are known. These resistances are usually measured with direct current, by means of a Wheatstone bridge or by the drop-of-potential method. With conductors of considerable cross-section, the eddy currents and a non-uniform distribution of current over the cross-section may cause the actual I^2R loss to be considerably higher than that figured out from the direct-current resistance measurement. In this case it is preferable to measure the actual copper loss with a wattmeter on short-circuit (§ 386). The iron loss depends on the magnitude of the magnetic flux, and is practically independent of the load. This follows from the fact that with a constant impressed voltage the flux is also nearly constant (neglecting a small primary drop). Therefore the iron loss can be determined at no load and assumed to be the same at all loads.

As an illustration, let it be required to determine the efficiency of a 2200 to 110-volt, 5-kva. transformer, at $\frac{3}{4}$ load. Let the resistances of the windings be 10.7 ohms and 0.026 ohm respectively, and let the iron loss, determined by a wattmeter at no load, equal 96 watts. At three-quarters of the rated load the output of the transformer is 3.75 kva., or 34.2 amperes at 110 volts. The primary current is equal to $\frac{1}{20}$th of this, or 1.71 amperes; thus we have:

$$\begin{array}{lcr}
\text{Primary copper loss} & = \overline{1.71}^2 \times 10.7 = & 31.2 \text{ watts.} \\
\text{Secondary copper loss} & = \overline{34.2}^2 \times 0.026 = & 30.4 \text{ watts.} \\
\text{Iron loss} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & = & 96 \text{ watts.} \\
\hline
\text{Total losses at } \frac{3}{4} \text{ load} & & 157.6 \text{ watts.}
\end{array}$$

$$\text{Efficiency} = \frac{3750}{3750 + 157.6} = \text{about 96 per cent.}$$

In this way, data may be obtained for any desired load, and an efficiency curve plotted from no load up to the maximum feasible overload.

394. EXPERIMENT 16-F. — Efficiency of a Transformer from its Losses. — The experiment comprises three distinct measurements:

(1) Ratio of transformation;
(2) Resistances of the windings;
(3) Core loss.

The ratio of transformation is determined by voltmeters, as in § 383. The resistances of the windings are measured with direct current, usually by the drop-of-potential method. As the resistance of the low-tension winding is much smaller than that of the high-tension winding, it may be necessary to use different ammeters and voltmeters. Always remove the voltmeter leads when opening or closing the D.C. circuit, especially on the high-tension side; the inductive kick may damage the instrument. As a check on the resistance measurement, determine the I^2R loss with a wattmeter, as in § 386.

The core loss is measured by a wattmeter, as in § 218. It is not necessary to measure it from the high-tension side; 110 volts applied on the low-tension side will produce the same flux and give the same core loss as 2200 volts applied on the high-tension side. Take curves of core loss and magnetizing current from zero voltage to about 50 per cent over-potential. In performing this measurement, be sure to have the high-tension side open. Also remember that high-tension terminals become alive the moment an A.C. voltage is applied to the low-tension terminals; in order to avoid the possibility of a fatal mistake, have the high-tension leads taped up.

Report. Give the ratio of transformation and the resistances of the windings. Check the calculated I^2R loss with that measured with the wattmeter, and use the higher value in the efficiency computations. Plot curves of core loss, corrected if necessary for the I^2R loss due to the exciting current. To kilovolt-amperes as abscissæ plot the copper loss, the iron loss, per cent total losses, and the efficiency. Show that the efficiency is a maximum, or the per cent total loss a minimum, when the two losses are equal. On the same curve sheet, indicate by little crosses or circles a few values of efficiency, when the supply voltage is 10 per cent above or below the rated value; select these points within the range of highest efficiencies. Similarly, indicate the general effect of departures in the frequency of the supply upon the losses, efficiency, and rating of the transformer. Do not forget that the flux is a function of the frequency, as well as of the applied voltage.

395. The Auto-Transformer. — An auto-transformer, or single-winding transformer, is shown in Fig. 250. The primary or line voltage is applied between the terminals A and B, and power is taken to the load off the terminals B and C of the same winding. The two parts of the winding are mounted on the same iron core, and are in the closest possible magnetic interlinkage with each other. The currents in the

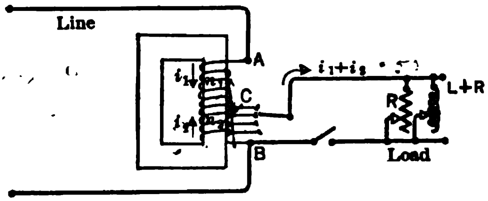


FIG. 250. An auto-transformer connected to a load.

parts AC and BC of the winding may be quite different from each other, and for this reason the corresponding coils are made of conductors of different cross-section, in order to save weight and space.

Consider first the conditions at no load. The applied voltage causes a magnetizing current to flow through the winding, thus producing an alternating flux in the core. This flux induces, between the terminals A and B, a counter-e.m.f. practically equal and opposite to the applied voltage. The drop of potential is uniformly distributed over the winding so that any desired fraction of the total voltage may be taken between one of the terminals and an intermediate point, or between two intermediate points. Thus, for example, if the total number of turns is 100 and the applied voltage is 1000, the voltage per turn is 10 volts. Singling out 40 consecutive turns anywhere in the winding will give a secondary e.m.f. of 400 volts. It saves an extra terminal to use one of the primary terminals, such as B, for the secondary load also. Different secondary voltages are obtained by using leads to different turns, or different taps, as they are sometimes called.

Now let the load switch be closed and let a considerable current flow in the secondary circuit. With a closed magnetic core and moderate saturation in iron, the necessary magnetizing ampere-turns are negligible as compared to the primary or secondary ampere-turns. The latter two are therefore practically equal to each other, and with the notation in Fig. 250 we have

$$n_1 i_1 = n_2 i_2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (11)$$

Here n_1 and n_2 are the numbers of turns in the parts AC and CB of the winding respectively, and the difference in the meaning of n_1 here and in a two-winding transformer (§ 384) should be carefully noted.

According to the first Kirchhoff law, the load current is equal to the sum of the currents in the transformer itself, or

$$I_l = i_1 + i_2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (12)$$

The fundamental voltage relationship expressed by eq. (2) becomes

$$E_1/E_2 = (n_1 + n_2)/n_2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (13)$$

where E_1 is the line voltage and E_2 is the load voltage. Equations (11), (12), and (13) comprise the principal relationships in an auto-transformer when the magnetizing current and the internal voltage drop within the transformer itself can be neglected. Under these simplified conditions, the vector diagram is identical with Fig. 248, although i_2 and n_1 have a different meaning. The load current may be added to the [illegible] by drawing a vector equal to $i_1 + i_2$ in the direction of i [illegible]

In the foregoing example, the part AC consists of 60 turns and BC of 40 turns. If the load current is, for example, equal to 50 amperes, then 20 amperes are supplied from the line and 30 amperes by the secondary winding. Under these conditions the primary and the secondary m.m.f.'s are equal and opposite, since $20 \times 60 = 30 \times 40$. The power furnished from the line is equal to that consumed in the load because $1000 \times 20 = 400 \times 50$, the assumption being that no energy is lost in the transformer itself.

Theoretically, an auto-transformer requires less copper than an ordinary two-winding transformer, as may be seen from the following estimate. Let the amount of copper required per ampere-turn in the foregoing auto-transformer be A. The total amount of copper then is

$$(20 \times 60 + 30 \times 40)A = 2400\,A.$$

With an ordinary transformer the corresponding amount of copper would be

$$(20 \times 100 + 50 \times 40)A = 4000\,A.$$

Thus, in this particular case, over one-third of the amount of copper can be saved by using windings connected *conductively*, instead of *inductively*.

Auto-transformers are widely used for starting induction motors (§ 475), for voltage regulation, and for other purposes where the difference between the primary and the secondary voltage is not too great. A disadvantage of the auto-transformer is that it is impossible to insulate the high-tension coils from the low-tension circuit. This is inadmissible in many cases on account of the danger to life or property. For an application of the theory of the ordinary transformer to the auto-transformer, see the last paragraph in § 405.

396. EXPERIMENT 16-G. — Load Test on an Auto-Transformer. — The purpose of the experiment is to illustrate the relations stated in the preceding article. The experiment is performed in a manner similar to Experiment 16-C above. Use an auto-transformer with regulating taps (Fig. 250), or one of the windings of an ordinary transformer, if it is provided with taps. An auto-transformer may also be improvised by connecting the two windings of an ordinary transformer in series.

(a) Determine the ratio of the voltages at no load. Connect the two outside points of the windings to a source of supply and measure the voltages between the various taps. Also apply a [illegible] across a part of the winding, and read the resulting higher volt[illegible] currents [illegible] the total winding. In so doing, be careful to have enough turns in the part of the winding connected across the line, so as not to draw an excessive current.

(b) Determine the ratio of the currents on short-circuit (§ 386). Read the currents i_1, i_2, and $i_1 + i_2$ shown in Fig. 250; also the corresponding line voltage and the power input.

(c) Choose a suitable tap and load the transformer as in Fig. 250. Have ammeters connected so as to read the primary, the secondary, and the load current. There should also be a voltmeter and a wattmeter in both circuits. One voltmeter, one ammeter, and one wattmeter, in connection with a polyphase board (§ 58), are usually sufficient for the purpose. Begin with a maximum permissible load, and reduce it in steps to zero. At each setting read all the currents, voltages, and watts.

(d) Repeat the same experiment with a few other taps, taken nearer to A or to B. In the latter case, be careful not to overload the part BC of the winding, since the current i_2 increases inversely as the number of turns.

(e) Before leaving the laboratory, ascertain the number of turns between the various taps. Measure the resistances of the windings and the core loss.

Report. (1) Draw a diagram of connections in the auto-transformer and mark on it per cent voltage at different taps. (2) Plot i_1 and i_2 to load amperes as abscissæ, and show that the conditions (11) and (12) are approximately fulfilled. (3) Plot curves of secondary voltage against load as abscissæ, to bring out the influence of the power factor. Explain the observed differences in per cent regulation with different taps. (4) Compute the efficiency from the losses and check with that obtained from the wattmeter readings. (5) Analyze the results of the short-circuit test; check the secondary voltage measured in the load test with that computed from the transformer impedance (§ 405).

CHAPTER XVII

THE TRANSFORMER (*Continued*)

397. Magnetizing Current. — When the secondary or load circuit of a transformer is open, the primary current serves merely to maintain the transformer flux and is therefore called the magnetizing or exciting current. In usual power transformers it amounts to but a small percentage of the full-load current, and is therefore neglected in the approximate diagram shown in Fig. 248. However, the value of the magnetizing current is of importance for some purposes. For example, the designer checks his computations by comparing the actual and the computed values, and indirectly checks the quality of iron and assembly. The operating engineer desires to keep the magnetizing current to a minimum, because during the periods of small power-demand on the system the magnetizing currents of numerous transformers constitute an appreciable load of very low power factor.

With the secondary circuit open, a transformer becomes identical with the Epstein apparatus (§ 219) and the theory given there also applies here. The following experiment on magnetizing current is intended to be an application of the theory, and should be performed accurately and intelligently so as to enable the student to compare his results with theoretical conclusions.

The experiment may be performed using either winding, whichever gives more convenient values of currents and voltages. The reader is warned, however, that as soon as a comparatively low voltage is applied to the low-tension winding a dangerous difference of potential may exist between the high-tension terminals. Before commencing the experiment these should be taped or otherwise made inaccessible. The initial rush of magnetizing current depends upon the state in which the iron was left by the preceding application of voltage, and upon the part of the voltage wave at which the circuit is closed. It is sometimes possible to obtain an initial inrush of magnetizing current equal to several times its normal value.

The vectorial relations at no load may be understood by reference to Figs. 248 and 251. The primary impedance drop at no load is usually so small that it can be neglected altogether. Therefore $E_1 = -e_1$, and the flux vector Φ is perpendicular to both. Assuming low saturation in iron, no hysteresis, and no eddy currents, the magnetizing ampere-turns, $i_0 n_1$, are in phase with Φ. Since the magnetizing current, i_0, flows through the primary winding, its m.m.f. corresponds to n_1 turns, and, with an ideal core, is in phase with the flux and in a lagging phase quadrature with the primary applied voltage E_1.

Because of saturation and hysteresis in iron, the magnetizing current is not sinusoidal even when the flux and the applied voltage vary according to the sine law. In this case the actual magnetizing current is *approximately* replaced by an *equivalent sine-wave current* so that a vector diagram may be used. Because of the power loss in hysteresis and eddy currents the vector i_0 of the equivalent magnetizing current is not in phase with the flux but has an energy component in phase with the applied voltage E_1, and leads the flux by the angle α.

Should the applied voltage be non-sinusoidal, the wave-form of the magnetizing current may still further depart from the sine law. The flux and the voltage are connected by the relationship $e = - d\phi/dt$, while the flux and the magnetizing current are connected by the saturation curve of iron. To study the relationship between these three quantities draw some waves of flux as functions of time, say a flat-topped, a sinusoidal, and a peaked one. The slope at each point is proportional to the induced voltage which is practically equal to the terminal voltage. The corresponding values of magnetizing current can be also plotted to the same abscissæ, so that finally one obtains the flux, the e.m.f., and the magnetizing current as functions of time. By actually performing such a construction it will be found that a flat-topped voltage wave corresponds to a peaked flux wave, and vice versa. A peaked flux wave means higher flux densities, a larger magnetizing current, and a greater core loss, and should therefore be avoided.

398. EXPERIMENT 17-A. — Magnetizing Current in a Transformer. — The purpose of the experiment is a study of the magnetizing current as a function of the number of turns, applied voltage, its frequency, and wave-form. The theory is explained in the preceding section. The transformer is wired up on the primary side as in Fig. 244; the secondary terminals are left open and should be taped up. To observe the transient current, close the circuit several times in succession, holding one of the leads in the hand and observe the difference in the spark. This phenomenon is particularly noticeable at higher values of saturation in the core.

(1) Begin with the highest voltage to which the transformer may be safely subjected, and read amperes, volts, and watts. Reduce the applied voltage in steps and take similar readings at each step.

(2) If an alternator driven by a variable-speed motor is availabl make similar runs at different frequencies.

(3) To investigate the effect of the number of turns, take a few readings at a constant voltage and frequency, using different taps. Th two halves of the winding can also be used in series and singly.

(4) Before leaving the laboratory, determine the number of turns i the winding with which the experiment was performed, the dimension of the iron core, and the resistance of the winding.

(5) If an oscillograph is available, a few characteristic curves of magnetizing current should be taken to show the steady as well as the transient conditions.

(6) If an alternator is available, in which the wave-form of the induced voltage may be varied at will, investigate the effect of a flat-topped and of a peaked voltage-wave upon the shape and magnitude of magnetizing current. An ammeter will show the difference in the effective values and an oscillograph may be used to ascertain the actual wave-form.

Report. (1) Correct the voltmeter and wattmeter readings for the effect of the ohmic drop and copper loss, as in § 220, at least for the readings for which this correction is appreciable. (2) Plot amperes and watts against the corrected voltages as abscissæ, at different frequencies. (3) From the data taken with different numbers of turns, show that the general character of variation of the magnetizing current with the frequency, number of turns, and voltage, is in accordance with the theory of the transformer. Pick out the data for which the ratio of the voltage to the frequency is the same. Show that for such data the magnetizing current is approximately equal, and give a theoretical reason. (4) From the known dimensions of the core and number of turns, compute the actual flux densities and magnetizing volt-amperes per unit volume of iron, for the extreme conditions obtained during the test (§ 224). (5) Give the results of the test on the initial rush of current, and show theoretically that the initial value of magnetizing current may be large when the residual magnetism is in the wrong direction. (6) Give the results of the oscillograph test and of the runs in which the wave-form of the applied voltage was varied. Explain your findings and give theoretical reasons for the results.

399. Performance Diagram with Ohmic Resistances Taken into Consideration. — In the performance diagram of an ideal transformer (Fig. 248) the magnetizing current and the ohmic resistances of the windings have been neglected. We shall now take these factors into account, and later shall also consider the influence of the leakage reactance (§§ 400 and 401).

Let the secondary circuit be closed through a given load impedance which causes the current i_2 to lag behind the terminal voltage E_2 by a known angle ϕ_2 (Fig. 251). Let it be required to construct a vector diagram giving the two terminal voltages, the two currents, and the magnetic flux in their proper magnitude and phase position.

First we shall assume the magnetic coupling between the primary and the secondary circuit to be perfect, that is, the two windings to occupy practically the same space. This condition may be obtained approximately by winding the two coils at the same time, so that one is embedded in the other. With such a winding, any magnetic line of force due to the current in a primary coil always links with the corresponding number of turns of a secondary coil, and there is no flux which links with either one or the other coil alone. It is then said that the transformer possesses no magnetic leakage, and the windings have no leakage reactance. For such an ideal transformer the diagram shown in Fig. 251 may be explained as follows:

Let Φ be the vector of magnetic flux and let e_1 and e_2 be the e.m.f.'s which are induced by it in the primary and secondary windings respectively. These vectors are proportional to the flux and lag behind it by 90 degrees. The absolute value and the position of the secondary terminal voltage, E_2, are determined by the condition that this voltage is equal to the secondary induced e.m.f., e_2, less the ohmic drop in the secondary winding. The latter drop is shown in the diagram in phase with the secondary current, so that the terminal voltage E_2, plus the drop i_2r_2, is equal to the secondary induced e.m.f., e_2.

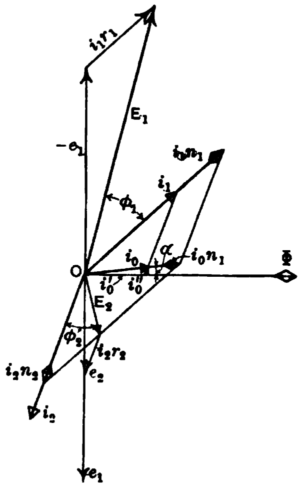


Fig. 251. The performance diagram of a transformer, with the ohmic resistances (but not reactances) taken into consideration.

The primary current, i_1, is determined by the condition that its m.m.f., i_1n_1, plus that of the secondary current, i_2n_2, together produce a number of ampere-turns just sufficient for maintaining the flux Φ. The latter number of turns, i_0n_1, is determined by the reluctance of the magnetic circuit and the core loss. With moderate values of saturation, i_0n_1 is proportional to the flux; otherwise it is taken from the saturation curve of the core. Moreover, this m.m.f. is not quite in phase with the flux which it produces, but leads it by an angle α, because of hysteresis and eddy currents.

Thus the primary m.m.f., i_1n_1, must be equal to a geometric sum of i_2n_2 and i_0n_1 in order to give the required exciting m.m.f. Dividing i_1n_1 by the primary number of turns, n_1, gives the primary current i_1.

The applied primary voltage, E_1, opposes the counter-e.m.f., e_1, induced in the primary winding, and is greater than that e.m.f. by the amount i_1r_1 necessary for overcoming the primary ohmic drop. We therefore draw the vector $-e_1$ equal and opposite to e_1 and from its end draw a vector equal to i_1r_1, in phase with the primary current. The resultant gives the applied voltage E_1. The diagram is now completed and if a sufficient number of quantities are given the remaining ones may be determined. The simplest case is one in which the secondary terminal voltage and current are given, because, then the diagram can be constructed step by step. Should the primary voltage be given instead, the problem is solved as is shown in the following sections.

Apart from the purely numerical relations, the reader should clearly understand the principle of action and reaction which determines the directions of the currents and voltages in such a way as to comply with the law of the conservation of energy. In accordance with this principle, the secondary and the primary m.m.f.'s are drawn practically in phase opposition to each other, and the e.m.f. induced in the primary winding is shown almost in opposition to the applied voltage. If this were not so, the transformer would act as an amplifier of energy: the secondary current would help the primary in building up the flux, and the latter would induce an e.m.f. that would assist the primary voltage in further increasing the current. The energy delivered by the transformer would thus continue to grow indefinitely in contradiction to our every-day experience. A clear understanding of the principle of reaction, namely, that a flux created by an applied alternating voltage always opposes it, and that a secondary current opposes the primary current, will materially assist the student in drawing a transformer diagram.

400. Magnetic Leakage in the Transformer.—We have assumed so far a perfect magnetic coupling between the primary and the secondary windings. In reality there is a space between the two (Figs. 245, 246 and 256). Neglecting the magnetizing current, the primary and secondary currents are in phase opposition, and the instantaneous directions of the currents are shown in the usual way by dots and crosses. It will thus be seen that each half of the primary coil, with the adjoining half of the secondary coil, forms a fictitious coil whose m.m.f. is in the opposite direction from that of the next fictitious coil. These combined m.m.f.'s produce the leakage fluxes, as shown in the figures, linked either with the primary winding alone or with the secondary winding alone. The currents in these windings vary with the time approximately according to the sine law, hence the leakage fluxes vary according to the same law.

In accordance with the fundamental law of induction, the fluxes induce, in the corresponding windings, e.m.f.'s in lagging time quadrature with the currents themselves. The effect is the same as if the coupling between the coils were perfect, but a reactive coil were connected in series with the primary circuit of the transformer and one in series with its secondary circuit. The amount of inductance in these equivalent coils is called the primary and the secondary leakage inductance of the transformer, respectively.

An imperfect transformer with magnetic leakage may thus be replaced by a perfect one, with the imperfections taken outside and represented by fictitious reactance coils of proper magnitude. The effect of the primary inductance is to consume part of the applied primary voltage. The remainder balances the e.m.f. induced by the useful flux. Consequently, the primary leakage affects the value of the useful flux and hence the value of the secondary induced e.m.f. With a lagging load current, this e.m.f. comes out smaller than that which would be induced in an ideal transformer, with the same applied voltage. The effect of the voltage drop in the secondary inductance is to reduce the secondary terminal voltage still further.

Strictly speaking, the leakage inductance reduces the applied primary voltage and the induced secondary e.m.f only with a lagging secondary current. When the secondary current leads the voltage, the leakage reactance may boost the voltage, as is explained below. It is only because in practice the lagging current predominates that the leakage fluxes in a transformer have come to be considered as causing an additional drop and not a rise in voltage.

401. The Complete Vector Diagram of the Transformer. — The vector diagram shown in Fig. 251 needs a correction for the effect of the leakage reactances described in the preceding section. The complete vectorial relations are shown in Fig. 252 for a lagging secondary current. This diagram differs from Fig. 251 only by the addition of the vectors marked i_1x_1 and i_2x_2 respectively, representing the reactive voltage drop in the transformer. These vectors are drawn in leading quadrature with the corresponding currents and are equal and opposite to the e.m.f.'s induced by the leakage fluxes. They represent the components of the applied primary and of the induced secondary voltage which are consumed in overcoming the effect of the leakage fluxes.

It will be seen from the geometry of the figure that with a lagging secondary current the reactance drop increases the difference between the induced and the terminal voltages. It is said, therefore, that on inductive load the internal magnetic leakage of the transformer affects the voltage regulation adversely. With a constant applied voltage and a constant secondary current, the secondary voltage reaches its minimum when the power factor of the load is zero.

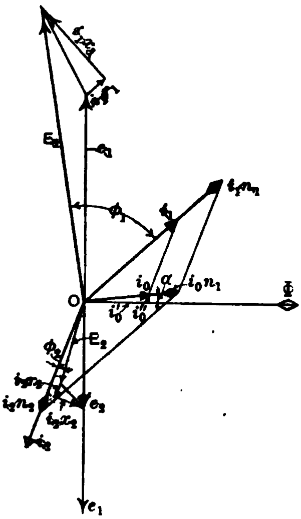


Fig. 252. A complete vector diagram of a transformer.

It is instructive to draw a similar diagram for a leading load current. It will be found that the internal reactance of the transformer raises the secondary voltage, and E_2 reaches its maximum when the load current is purely reactive. Such a case arises, for example, when a transformer feeds a long transmission line at no load, supplying its charging current. Under these conditions there may be a partial resonance and a dangerous rise in voltage (Vol. II).

Sometimes the leakage reactance of a transformer is purposely made large to protect it against violent short-circuits on the load side.

402. Simplified Transformer Diagram. — In the usual type of transformers used on power and lighting circuits, the proportions are always so chosen that the no-load current is but a small fraction of the full-load current. This is very desirable from the operating point of view, in order not to load the generating machinery and the lines unnecessarily with a reactive magnetizing current which brings no income, and causes an extra I^2R loss and voltage drop.

Assuming the magnetizing ampere-turns, i_0n_1 in Fig. 252, to be negligible in comparison with the primary and the secondary ampere-turns, these become equal to each other, and the parallelogram of the m.m.f.'s degenerates into a straight line with the currents i_1 and i_2 in phase opposition (Fig. 253). We thus have the simple relationship

$$i_1n_1 = i_2n_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

which, together with the other fundamental relationship, viz.,

$$e_1/e_2 = n_1/n_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

materially simplifies the theory of the transformer for most practical purposes.

It is now possible to combine the primary and the secondary internal voltage drops into one shown by the cross-hatched triangle. For this purpose all the voltage vectors in the secondary circuit, that is, the figure $OA_2B_2e_2$, are multiplied by the ratio of n_1 to n_2 and turned by 180 degrees, giving the figure $OA_2'B_2'e_1$. We thus obtain the figure $OA_2'\,KA_1$ which shows directly the relationship between the primary and the secondary terminal voltages, and from which the induced voltages have been eliminated. *The secondary terminal voltage* OA_2' appears in this figure not in its real magnitude, but *reduced to the primary circuit,* which means multiplied by the ratio of the numbers of turns.

The combined or equivalent ohmic drop, i_1R_1, is equal to the sum of e_1B_1 and $A_2'B_2'$, or

$$i_1R_1 = i_1r_1 + i_2r_2(n_1/n_2) \quad . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

Taking into consideration eq. (1), we may also write

$$i_1R_1 = i_1[r_1 + (n_1/n_2)^2r_2] \quad . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

Thus, *the equivalent resistance of a transformer, reduced to the primary circuit, is*

$$R_1 = r_1 + (n_1/n_2)^2r_2. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

By a similar reasoning it can be shown that *the equivalent reactance is*

$$X_1 = x_1 + (n_1/n_2)^2\,x_2 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

Therefore, when the magnetizing current is negligible compared to the load current, we have the following important result: *Resistances and reactances can be transferred from the primary into the secondary circuit, or vice versa, by being multiplied by the square of the ratio of the number of turns* (§ 385). Not only can the internal impedance of a transformer be thus transferred from one circuit to the other, but an external impedance as well.

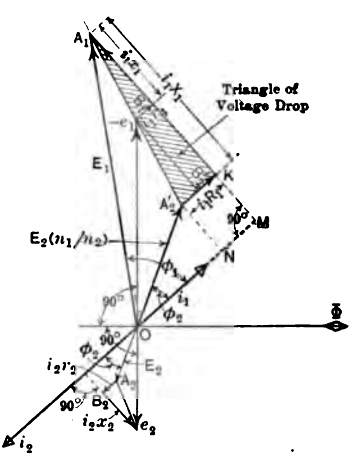


FIG. 253. A simplified transformer diagram; the cross-hatched triangle represents the total drop.

Let R_1 be a resistance inserted into the primary circuit, and R_2 another resistance connected in series with the secondary circuit. The percentage voltage drop caused by the first resistance is $p_1 = i_1R_1/e_1$ and that caused by the second resistance is $p_2 = i_2R_2/e_2$. With the assistance of eqs. (1) and (2) we find that $p_1 = p_2$ when $R_2 = R_1(n_2/n_1)^2$. In other words, the resistance R_1

causes the same per cent drop in the primary circuit as a resistance $R_1(n_2/n_1)^2$ in the secondary circuit. In problems in which voltage drop and regulation alone are considered, these two resistances may be used interchangeably. A similar rule applies to reactances.

The analytical solution for the relations represented in Fig. 253 is as follows: In the triangle OMA_1

$$E_1^2 = \overline{OM}^2 + \overline{MA_1}^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

But $OM = OA_2' \operatorname{Cos}\phi_2 + A_2'K$, and $MA_1 = OA_2' \operatorname{Sin}\phi_2 + KA_1$.

Hence,

$$E_1^2 = [E_2(n_1/n_2)\operatorname{Cos}\phi_2 + i_1R_1]^2 + [E_2(n_1/n_2)\operatorname{Sin}\phi_2 + i_1X_1]^2. \quad (8)$$

The relations represented in Fig. 253 and by eqs. (5) and (6) are shown in the form of the so-called *approximate equivalent diagram of transformer connections* in Fig. 254. The transformer windings are assumed to be devoid of resistance and of leakage reactance, and these imperfections of both windings are shown concentrated in the primary circuit, causing a difference between the voltages E_1 and $E_2(n_1/n_2)$. Instead of being concentrated in the primary circuit, the resistances and the reactances may also be concentrated in the secondary circuit.

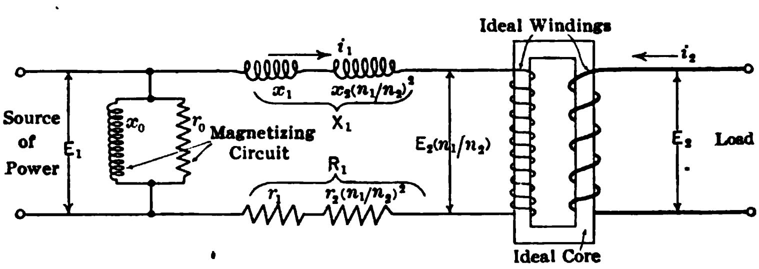


FIG. 254. The approximate equivalent diagram of transformer connections.

The magnetizing current is taken into consideration *approximately* by means of a reactance x_0 in parallel with a resistance r_0. The reactance x_0 allows a quadrature or reactive current to flow, equal to the magnetizing current of perfect iron at no load. The resistance r_0 allows an in-phase or energy current to flow, to represent the loss due to the hysteresis and eddy currents. Usually this "exciting circuit," consisting of r_0 and x_0, can be omitted altogether.

403. Example of Predetermination of Regulation. — After these general considerations it will be shown, by means of an example, how to predetermine the voltage regulation of a transformer from a short-circuit test. First of all, determine as accurately as possible the ratio of transformation at no load; let it be 2200 to 110 volts. Then measure the ohmic resistances of both windings, using direct current (drop-of-potential method). Let the resistance of the primary be 1.05 ohm, and that of the secondary = 0.002 ohm. Now short-circuit the secondary (low-tension) terminals and apply a voltage at the primary terminals sufficient to produce a considerable current in the windings. It is advisable to short-circuit the secondary winding through an ammeter in order to be able to read the current in it; in computing the results, the resistance of the ammeter must be added to that of the secondary winding. Let the applied pressure be 120 volts, and let the primary and secondary currents be 25 amperes and 500 amperes respectively. The currents are almost exactly in the inverse ratio of the no-load voltages. If the resistance of the secondary ammeter is 0.001 ohm, the total secondary resistance is 0.003 ohm, or, reduced to the primary circuit, $0.003 \times 20^2 = 1.2$ ohm. Thus, the total equivalent ohmic resistance of the transformer (including the ammeter) reduced to the primary circuit, is $1.05 + 1.2 = 2.25$ ohms. The equivalent resistance of the transformer itself is $1.05 + 0.002 \times 20^2 = 1.85$ ohm.

Now with reference to Fig. 116 the total applied voltage is used partly for overcoming the equivalent ohmic drop of the transformer and partly for overcoming its equivalent reactance. The first drop is equal to $2.25 \times 25 = 56.2$ volts; therefore the reactive drop is

$$\sqrt{(120)^2 - (56.2)^2} = 105.7 \text{ volts.}$$

Hence, the equivalent reactance, reduced to the primary circuit, is $105.7/25 = 4.23$ ohms.

Let it be required to predetermine the voltage regulation of this transformer at the secondary terminal voltage of 110 volts and at an output of 70 kw., the power factor of the load being equal to 80 per cent (lagging). The secondary current is

$$\frac{70{,}000}{110 \times 0.80} = 796 \text{ amperes,}$$

or, reduced to the primary circuit, $796/20 = 39.8$ amperes. The voltage at the primary terminals of the ideal transformer (Fig. 254) is 2200 volts. The equivalent ohmic drop is $1.85 \times 39.8 = 73.6$ volts; the equivalent reactive drop is $4.23 \times 39.8 = 169$ volts. We now construct the diagram, as in Fig. 253, and find that the voltage E_1 equals 2370 volts. The same result follows from the formula (8), namely,

$$E_1^2 = (2200 \times 0.80 + 73.6)^2 + (2200 \times 0.60 + 169)^2$$

from which $E_1 = 2370$ volts. At no load this voltage produces a secondary e.m.f. of $2370/20 = 118.5$ volts; therefore, by definition, the regulation of the transformer at this particular load is

$$\frac{118.5 - 110}{110} = 7.7 \text{ per cent.}$$

404. EXPERIMENT 17-B. — Voltage Regulation of a Transformer from Short-Circuit Test. — The experiment is performed as explained in the directions for Exp. 16-B. It is expected, however, that a more advanced student will obtain better data and will apply the results to a predetermination of voltage regulation, as explained in the preceding section. If possible, at least one set of very accurate readings of terminal voltages should be obtained on actual load, preferably on an overload at a low power factor (most unfavorable conditions), so as to check the predetermined regulation with that observed experimentally.

It is also desired that the student verify the above deduced law (§ 402) that resistances and reactances can be transferred from the primary circuit into the secondary by multiplying them by the square of the ratio of the numbers of turns. This can be tried either in connection with the short-circuit test, or during the load test.

Report. Construct Kapp's diagram (§ 405) and plot percentage regulation with full-load current, to power factor as abscissæ. Show how closely the predetermined regulation checks with the observed. Give the experimental results on the transfer of resistances and reactances from the primary into the secondary circuit, and check them with the theory.

405. Kapp's Diagram. — It is a tedious process to construct the diagram shown in Fig. 253 separately for different values of load current and power factor. With a given value of current, the triangle A'_2KA_1 is independent of the power factor, and the only variable is the angle ϕ_2 between the secondary terminal voltage and the current. Dr. Gisbert Kapp therefore suggested a modification of the above diagram (Fig. 255) with the triangle of voltage drop as a basis. In Fig. 255 the triangle OKC is the same as the triangle A'_2KA_1 in Fig. 253. In either case it is constructed from the results of a short-circuit test. The current vector i is vertical and is in phase with the ohmic drop OK. The primary line voltage E_1 being constant, a semicircle is described with C as a center and with a radius equal to E_1. To find the secondary voltage E_2 (reduced to the primary circuit) a vector OE is drawn at the angle ϕ_2 corresponding to the power factor of the load. The length

OE gives the value of the secondary terminal voltage corresponding to this power factor.

With a purely inductive load, the secondary voltage is a minimum, and is represented by the vector OE_x perpendicular to i. With a non-inductive load, the secondary terminal voltage is equal to OE_r in phase with the current i. In the case of a pure capacitive load, the secondary voltage is equal to OE_c; it is larger than the applied voltage E_1 (reduced to the same number of turns). This is due to a partial resonance between the capacitance of the load and the inductance of the transformer itself (see index to Vol. II).

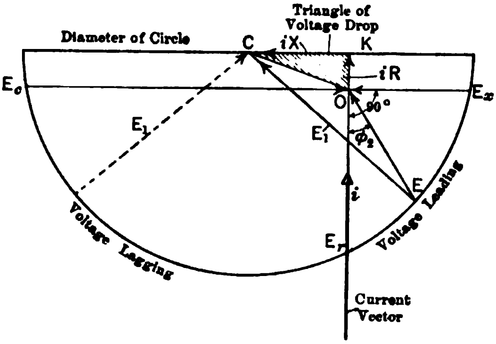


FIG. 255. Kapp's diagram of voltage drop and regulation of a transformer.

If the secondary voltage, and not the primary, is constant, a semicircle must be drawn from *O* as a center. Vectors from *C* to an intersection with this semicircle will then give the values of the primary voltage necessary for maintaining the required secondary voltage with a varying power factor. For other values of current, the triangle *OKC* must be changed in proportion.

The difficulty in applying Kapp's diagram to an actual transformer is that the triangle *OKC* is comparatively small, and therefore the difference between *OE* and *CE* cannot be measured with sufficient accuracy. An analytical computation, according to eq. (8), is somewhat involved. In various electrical handbooks and in theoretical works, the reader will find charts, tables, and approximate formulæ by means of which the voltage regulation may be predetermined from a short-circuit test with less labor.

The preceding theory of voltage regulation applies to the auto-transformer (§ 395) provided that i_2 is taken to be the load current and not the current in the secondary part of the transformer itself. In other words, i_2 in the foregoing diagrams stands for $i_1 + i_2$ in Fig. 250. The equivalent resistance is computed from the wattmeter reading on short-circuit, and the equivalent impedance is the ratio of the primary volts to the primary amperes on short-circuit. As a matter of fact, when predetermining the voltage regulation of a transformer from a short-circuit test, it is not necessary to know whether the transformer has two separate windings or only one. All tests and computations can be performed on an auto-transformer as if it had two separate windings.

406. The Effect of Subdivision of Windings upon the Leakage Inductance. — Consider two transformer windings (Fig. 256) of the same total number of turns. In the arrangement (a) shown to the left, one primary coil P is used and one secondary coil S of approximately the same thickness. Since at any considerable load the primary and the secondary ampere-turns are practically equal, the required amount of copper in P and in S is approximately the same, assuming the same current density and the same space utilization in both. The leakage flux is shown by curved lines, and the instantaneous currents by dots and crosses. With such "concentrated" or "bunched" coils the magnetic leakage is a maximum and the voltage regulation is poor.

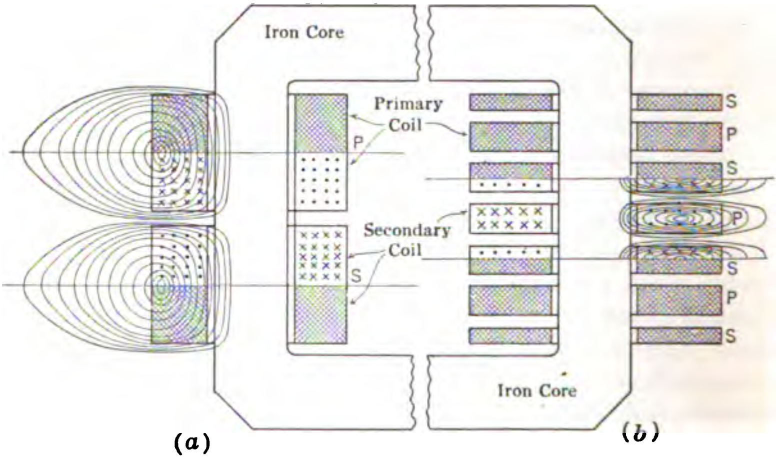


FIG. 256. The effect of subdivision of windings upon leakage fluxes.

To reduce the leakage m.m.f.'s, the windings are subdivided into sections, as shown at (b) to the right. The primary and the secondary coils are placed alternately, with "half coils" at the ends. With such an arrangement, each half of a primary coil with an adjacent half of the secondary coil forms a leakage m.m.f. which excites a leakage flux. These fluxes are smaller than at (a) because their exciting m.m.f.'s are smaller, and moreover each flux is linked with fewer turns and therefore induces a lower e.m.f. On the other hand, there are more e.m.f.'s connected in series, so that the reactive drop decreases nearly inversely as the number of sections and not as its square.

The inductance of a coil occupying a certain space is equal to $\mathcal{P}n^2$, where $\mathcal{P}$ is the equivalent permeance of the magnetic circuit of the coil and n is its number of turns. Let the leakage inductance of the primary coil to the left be

$$L = \mathcal{P}n^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Assuming the permeance of the leakage space between each two coils in the arrangements (a) and (b) to be the same, the inductance of each primary coil to the right is

$$L' = \mathcal{P}(n/q)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where q is the number of sections into which the primary winding is subdivided. There are q sections in series, so that the total inductance of the subdivided primary winding is

$$qL' = q\mathcal{P}(n/q)^2 = \mathcal{P}n^2/q = L/q \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

In reality $\mathcal{P}$ is not quite the same with the subdivided winding, but the above deduction shows roughly the effect of subdivision of a transformer winding upon its leakage inductance. At no load, with only a small magnetizing current flowing through one of the windings, the leakage fluxes are negligible and the induced voltages are the same in both cases, so that in this respect the subdivided and the bunched windings are equivalent.

For the so-called interlacing of transformer coils in the three-wire and T-connection, see §§ 410 and 411.

The reader is warned against a hasty conclusion that the voltage regulation of a transformer with q sections is q times better than that of one with bunched windings. With reference to Fig. 253 it will be readily seen that reducing the length of the vector i_1X_1 to one-half by no means reduces the arithmetical difference between OA_1 and OA'_2 to one-half. Especially at high values of load power factor, the influence of the reactive drop on the voltage regulation is quite small, as may be seen from an examination of the figure. At a very low power factor the reactive voltage drop is nearly in phase with the terminal voltage, so that the influence upon the regulation exerted by it is nearly in proportion to its value. On short-circuit the leakage reactance practically determines the current, and therefore the best method of studying the leakage reactance of a transformer is on short-circuit (§ 403).

For the purpose of this study it is best to have a special transformer consisting of a comparatively large number of identical coils, with two half-coils at each end. While in practice only one half-coil would be used at each end, it is better to have two, in order that the student may determine how much the leakage flux is increased when full coils are used at the ends. For the principal readings it is convenient to use a one-to-one ratio of transformation, in order that the secondary quantities may be transferred directly into the primary circuit.

One of the windings is short-circuited through a low-resistance ammeter, and an alternating voltage is applied to the other, just sufficient to circulate a safe large current in both windings. An ammeter, a voltmeter, and a wattmeter are connected on the primary side.

The ratio of primary volts to amperes will give the total equivalent impedance of the transformer. The wattmeter indication represents practically the I^2R loss only, because the useful flux on short-circuit is quite small and the core loss is negligible. Dividing the wattmeter reading by the square of the supply current gives the equivalent effective resistance of the transformer (§ 385).

Knowing the hypotenuse and one of the sides of the impedance triangle (Fig. 115), the other side is easily found either by computation or graphically, and thus the total equivalent leakage reactance of the transformer is ascertained. Its effect upon the voltage drop and regulation is determined as is explained in § 402.

407. EXPERIMENT 17-C. — Study of Transformer Leakage. — The purpose of this experiment is to study the influence of the arrangement and subdivision of the transformer coils upon its leakage inductance. The method is explained in the preceding section. The frequency of the supply circuit must be accurately known for each reading. See § 93 regarding wattmeter readings at low power factor.

(1) Begin with the most unfavorable conditions, that is, with all the primary coils concentrated on one side of the core, and all the secondary coils on the other. Bring the voltage up so as to give the maximum safe value of the current, and read the instruments; note also the frequency of the supply. Take one or two check readings at reduced voltages.

(2) Now " sandwich in " one of the windings between the two halves of the other, and repeat the readings. If the same values of current or voltage are used as before, the effect of the rearrangement will be brought out more clearly.

(3) Subdivide the windings still more, always keeping in mind the general arrangement indicated above, that is, there must be as small an m.m.f. as possible in the space between two coils, and this m.m.f. must be balanced in such a way as to make the primary and the secondary ampere-turns equal. This will necessitate the use of half-coils at the ends. Try also one or two combinations with the full coils at the ends, and note the increase in the leakage reactance.

(4) For a few characteristic arrangements of the coils, determine the actual voltage drop in the transformer under load, so as to be able to compare it with that computed from the short-circuit test. For this purpose load the transformer on reactances, on an unloaded induction motor, etc., so as to have as low a power factor as possible. This is desirable because on an inductive load the reactive voltage drop is much more nearly in phase with the line voltage, and therefore affects it much more than at a high power factor. Since the voltage drop constitutes but a small percentage of the terminal voltage, this test and especially the voltage readings must be taken quite accurately to avoid disappointment in working up the data.

(5) To increase the accuracy of the preceding run, a differential method of measuring voltages may be used. For example, a primary and a secondary terminal may be connected together and the difference between the potentials of the two other terminals measured with a low-reading voltmeter. With a one-to-one ratio of transformation this reading ought to be nearly zero at no load.

(6) Before leaving the laboratory, measure carefully, with direct current, the resistances of all the coils used, and if possible determine the number of turns in each, at least relatively.

Report. (1) Show schematically the various arrangements of the coils used during the experiment, and refer to them in the report by number only. (2) Show that the primary and the secondary amperes were in the inverse ratio of the number of turns, and check or reject the readings in which this condition was not fairly closely fulfilled. (3) From the primary volt and ampere readings compute the corresponding impedances. Calculate the effective resistances from the wattmeter readings and check with the values obtained with direct current; see § 402, eq. (5). (4) Determine the corresponding leakage reactances and correct where necessary for the frequency. Show that the relative values of these reactances are such as might be expected theoretically. (5) For the arrangement of the coils for which a load test was performed, compute the per cent voltage regulation and compare with that obtained from the test.

408. Transformers in Parallel. — When two or more transformers (Fig. 257a) are connected in parallel on the primary side only, each secondary furnishing its own load, the individual transformers may have entirely different characteristics and even different ratios of transformation. If, however, the secondaries are also to be connected in parallel (Fig. 257b), to supply a common load, the characteristics of the individual transformers must be such that the total load output is divided among them in a fairly correct proportion from no load to a reasonable overload. See in this connection the discussion on the parallel operation of direct-current machines (§ 248).

When the ratio of the no-load voltages is the same for all the transformers, the load may be transferred to the primary circuit and the transformers themselves replaced by their equivalent impedances (Fig. 257c). It will now be seen that if the output is divided properly, say at full load, it will also be divided in the right proportion at any other load. The required condition is that the vectors of voltage drop in all the transformers must constitute the same fraction of the applied voltage and form the same phase angle with it. This means that all the transformers must have the same per cent ohmic drop and the same per cent reactive drop at their respective rated currents.

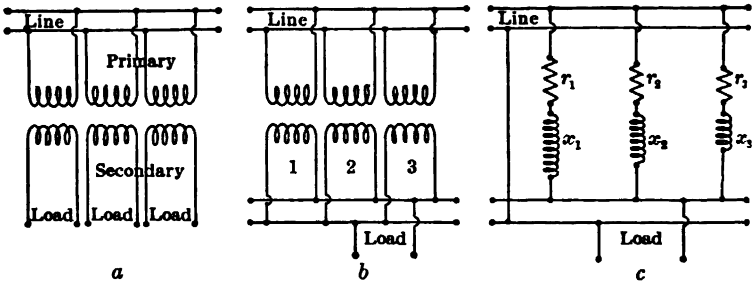


FIG. 257. Transformers in parallel.

If one of the transformers has a smaller internal drop than the others it will tend to take a greater share of the total load. The division of the load is improved by adding a proper amount of external resistance or reactance, or both, in series with that particular transformer. When the transformation ratios of the individual transformers are slightly different, an exact correction requires an auto-transformer which acts as a booster for the transformer with the lower ratio.

Polarity Test. When the secondaries of two or more transformers are to be connected in parallel, proper leads or terminals must be used to avoid a short-circuit. Therefore, before connecting in parallel, the transformers must be tested and marked as to their polarity. Let the primary terminals of a transformer be arbitrarily marked a_1 and b_1, and let the secondary terminals, a_2 and b_2, be defined as follows: At any instant when the primary induced e.m.f. is of such a direction that the potential of a_1 is above that of b_1, the potential of a_2 is also above that of b_2. When the polarity of all the transformers has been determined and marked in this manner, the terminals bearing the same letter are connected to the same side of the line, to insure an agreement in the polarity on the secondary side.

A simple method of testing for polarity consists in connecting the primary in series with the secondary. Let the supposed terminal b_2 be connected to a_1 and an alternating voltage applied between a_1 and b_1. The voltage measured between b_1 and a_2 should be higher than that between b_1 and a_1; otherwise the unknown terminal is a_2 and not b_2.

Another method consists in testing the secondary of a transformer against that of another of known polarity. The primaries are connected in parallel to the same source of supply. On the secondary side one of the terminals of the standard transformer, say a_2, is connected to one of the terminals of the transformer under test. If this latter terminal happens to be also a_2, then the voltage between the two remaining free terminals will be zero; otherwise it will be equal to twice the secondary voltage of each transformer. This can be ascertained either with a voltmeter or with an incandescent lamp in series with a protective resistance or reactance.

409. EXPERIMENT 17-D. — Transformers in Parallel. — The purpose of this experiment is to study the connections, operation, and division of load between two transformers in parallel. The transformers should be of nearly the same ratio of numbers of turns, but preferably of different rating. The ratio of active turns in one of the transformers should be made adjustable by a small amount, by providing a tap on one of its coils. The percentage ohmic and reactive drop should also be different for the two.

(1) Test the polarity of the transformers, as is explained in the preceding section, so as to know which secondary terminals are to be put together.

(2) Connect the transformers in parallel, both on the primary and on the secondary side, as in Fig. 257b, and provide a suitable load. Have an ammeter in the secondary circuit of each transformer and one in the load circuit. Adjust the load to such a value that one of the transformers is carrying its rated current, while the other is underloaded. Read the three ammeters. Select a few different loads, at different values of power factor, and take similar readings.

(3) Perform a short-circuit test on each transformer, as in § 403, and add a sufficient amount of external resistance and reactance in the circuit of one of the transformers or both until, with the same current, the voltmeter and the wattmeter readings are equal. Then the two transformers have the same per cent ohmic drop and the same per cent reactive drop.

(4) Connect the two transformers again in parallel, as in (2) above,

and show that they divide the load much more nearly in proportion to their ratings.

(5) Change slightly the ratio of one of the transformers by using a tap on one of its windings. Show that with unequal voltage ratios the transformers may be made to divide the load properly at some one value of the current, but that they will not divide it in the right ratio at any other current. Connect an auto-transformer to act as a booster for one of the transformers, and adjust it so that the transformers will divide the total current properly at all loads.

Report. (1) Give data on the transformers used and describe the polarity test. (2) Give the numerical data showing which transformer took a disproportionately greater share of the load. Draw a few vector diagrams of the three currents to show how much the two transformer currents were out of phase with each other, and also how the ratio of the component currents varied at different loads. (3) Give the test data and the computations to show how the external resistances and reactances were adjusted to get the same per cent internal drop in both transformers. (4) Give the data to prove that, with such an adjustment, the transformers divided the load properly. (5) Show that with the different ratio of transformation the division of the load was different at different values of the total current and explain this fact theoretically. Give data to indicate how much an additional auto-transformer improved the distribution of the load.

410. Three-Wire A.-C. Distribution. — The three-wire distribution shown in Fig. 258 has the advantage of considerable copper economy over the ordinary two-wire system. It permits the use of lamps and other current-consuming devices of voltage e_2, say 110 volts, with the copper cross-section in the two external conductors corresponding to that required by the voltage $2e_2$. This cross-section is but one-quarter of that required at the voltage e_2 for the same voltage drop and with a given power consumption. When the load on both sides is balanced, no current flows through the middle wire. This wire carries only the current corresponding to the difference of the loads on the two

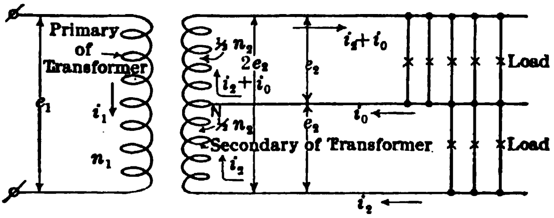


FIG. 258. Three-wire distribution from the secondary of a transformer.

outside conductors, and can therefore be of comparatively small size. The direct-current three-wire system is described in § 255.

In an alternating-current three-wire distribution, the total secondary voltage is obtained from an ordinary transformer which steps down the transmission voltage e_1 to the desired value $2e_2$. The middle, or neutral wire is connected to the middle point N of the secondary winding, and thus is kept at all times at a potential midway between the potentials of the two outside conductors.

With the notation shown in the figure, the current through one-half of the secondary winding is i_2, that through the other half is $i_2 + i_0$, where i_0 is the current in the neutral. Neglecting the magnetizing current of the transformer, the primary and the secondary ampere-turns must be equal (§ 402), so that the currents are connected by the relationship

$$i_1n_1 = \tfrac{1}{2}\,n_2i_2 + \tfrac{1}{2}\,n_2(i_2 + i_0) \quad . \; . \; . \; . \; . \; . \; . \quad (12)$$

or

$$i_1n_1 = (i_2 + \tfrac{1}{2}\,i_0)n_2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (13)$$

This relationship holds true algebraically for instantaneous values of the currents, and geometrically for their effective values. When the power factor of the load is different on the two sides of the neutral, the three m.m.f. vectors in the foregoing equation form a triangle.

Because of the internal drop in the transformer itself, the secondary voltage with an unbalanced load is somewhat lower on the side which is more heavily loaded. Care must therefore be taken to subdivide or to " interlace " the transformer windings on the two sides, so as to minimize this effect, as is shown below. Sometimes an automatic voltage regulator is used on each of the two external conductors to keep the voltage constant and independent of the load.

Interlacing of Coils. Due to the difference in the currents in the two halves of the secondary winding, the impedance drop is also different. The resultant unbalancing of the two secondary voltages is remedied in part by using adjacent secondary coils on the opposite sides of the neutral. Let there be, for example, eight secondary coils, numbered consecutively from 1 to 8. Instead of connecting the coils 1, 2, 3, 4 in series on one side and the coils 5, 6, 7, 8 on the other side of the neutral, the coils 1, 3, 5, 7 are used for one circuit and 2, 4, 6, 8 for the other. In this way the leakage fluxes due to those coils are distributed symmetrically and the reactive drop in the two circuits is equalized. A similar interlacing of coils is used in one of the transformers in the T-connection (Vol. II).

411. EXPERIMENT 17-E. — Currents and Voltages in a Transformer on a Three-Wire System. — The diagram of connections is shown in Fig. 258. The primary voltage must be kept constant, and if necessary a regulating rheostat must be provided. Insert ammeters to read all three secondary currents, and a voltmeter to read the voltages. Connect an ammeter, a voltmeter, and a wattmeter on the primary side.

(1) Provide a suitable non-inductive load and distribute it as evenly as possible between the two sides of the secondary circuit, so as to have as nearly as possible a balanced load. The current in the neutral should then be almost equal to zero. Read all the voltages, currents, and watts.

(2) Reduce the load on one side in steps to zero, keeping the current on the other side constant. At each step read all the voltages, currents, and watts as before.

(3) Connect a wattmeter on one side of the secondary circuit, bring the load again to the same value as at the beginning of the preceding run, and leave the load on the side without the wattmeter constant. On the other side make the load more and more inductive, keeping the current itself constant. In other words, keep the currents in the two outside wires constant and equal to each other, but vary the phase relation of one with respect to the other. At each step read carefully the watts, the voltages, and the currents, including that in the middle conductor.

(4) If the transformer coils may be connected in different combinations (§ 406) investigate the effect of the interlacing of the secondary coils upon the voltage balance, as is explained in the preceding section. Take readings of the two secondary voltages under the two extreme conditions of load unbalancing, namely, (a) with a maximum load on one side and a zero load on the other side, and (b) with the two currents equal in magnitude but of widely different power factor. For each of these combinations use the two extreme arrangements of the secondary coils, namely, the one in which the consecutive transformer coils are all on the same side of the circuit, and the other in which every second coil belongs on the opposite side of the circuit.

(5) Before leaving the laboratory, measure accurately the ratio of transformation at no load.

Report. (1) Plot the results of the first run against the current in the neutral as abscissæ. Check eq. (13) by using the ratio of the no-load voltages for the ratio of the numbers of turns. Show that the current in the neutral is equal to the arithmetical difference of the currents in the two outside conductors. (2) Against the current in the neutral as abscissæ, plot all ampere, volt, and watt readings, and the values of power factor. Show from the results that eq. (13) now holds true geometrically. (3) Show in detail the arrangements of coils used in experimenting with the effect of interlacing. Give the data to indicate the amount of unbalancing of the two secondary voltages in two comparative runs. Explain the observed facts theoretically by drawing a sketch of the transformer with its leakage fluxes, as in Fig. 256.

412. Constant-Current Transformer. — Ordinary alternators which supply current for light and power are essentially *constant-potential* machines. There are a few cases in which a *constant current* is required with a varying load. Such is the case of street lighting with series-type incandescent or arc lamps. A transformer (Fig. 259) which converts constant-voltage a.c. energy into constant-current a.c. energy, and thus allows a series circuit to be fed from an ordinary alternator, is called a constant-current transformer.

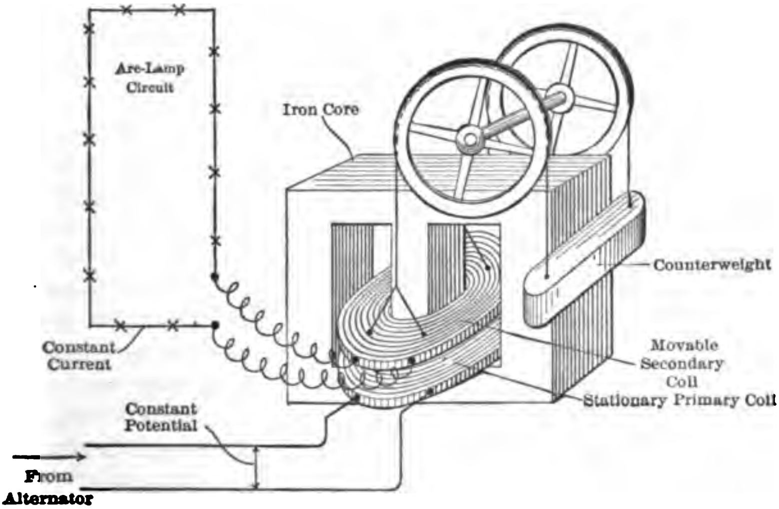


FIG. 259. A constant-current transformer connected to a constant-potential supply circuit.

A constant-current transformer (Fig. 259), like any ordinary transformer, consists of two windings placed on an iron core. The essential feature of the transformer is that its secondary coil is movable, while in an ordinary constant-potential transformer both coils are stationary. The stationary primary winding is connected to the alternator circuit; the movable secondary coil is connected to the load circuit in which the current is to be maintained constant and independent of the number of lamps in operation. When the load circuit is open, the secondary coil rests on the primary, since only part of its weight is balanced by the counter-weight. Closing the load circuit causes secondary currents to flow in the movable coil, and it is repelled upward. The farther it is repelled, the lower becomes the electromotive force induced in it, because the magnetic leakage between the two coils increases as the distance between the coils increases (Fig. 256). The counter-weight is so adjusted that with any position of the movable coil the secondary current remains of practically the desired constant value. When the number of lamps, or the resistance of the circuit, decreases, the current tends to increase; this increases the repulsion between the coils, and the movable coil is repelled farther upward. The useful flux is thereby reduced and the induced current again decreases.

A more accurate explanation is as follows: The primary and the secondary coils with their currents form an m.m.f. similar to that shown by dots and crosses in Fig. 256. This m.m.f. produces a leakage flux which passes in horizontal planes between the coils, from the middle part to the two outside legs of the core. This flux exerts a vertical force upward on the movable coil, the force being proportional to the current in the coil and to the flux density. But the latter is also proportional to the current, the saturation of the magnetic circuit being comparatively low. Thus, the electromagnetic force of repulsion is proportional to the square of the current in the movable coil.

This force is balanced by the excess weight of the movable coil over that of the counter-weight. This downward force being constant in all positions of the coil, the electromagnetic upward pull must be also constant if the coil is to remain in a stable equilibrium. In high positions of the movable coil the useful flux which links with the two coils is smaller (looser coupling); consequently the secondary induced e.m.f. is also smaller. Thus, when the resistance of the external circuit is reduced the coil moves upward because its current is greater than can be balanced by the counter-weight. The coil reaches a higher position in which a smaller e.m.f. induces a normal current and remains there as long as the conditions in the external circuit remain unchanged.

In an experimental study of a constant-current transformer it is sometimes desired to find the phase angle between the primary and the secondary voltages or currents. The phase angle between the voltages can be found as follows: Let the primary terminals of the transformer be denoted A_1 and B_1; the secondary terminals A_2 and B_2. Connect B_1 and B_2 together; measure the voltages A_1-B_1, A_2-B_2, and A_1-A_2. The vectors of these voltages form a triangle from which the phase relation may be determined.

The phase relation between the currents can be determined by using temporarily a common conductor for one side of the primary and secondary circuits, with an ammeter in this line. The primary amperes, the secondary amperes, and the common amperes are read at a desired load. This again gives a triangle of vectors from which the phase relation may be determined. The phase angle between the primary voltage and current is obtained from a wattmeter reading; the secondary phase angle is computed in a similar manner. As a check on the relations thus obtained, a wattmeter may be connected to read the watts corresponding to the primary volts and secondary amperes, and then vice versa.

Large constant-current transformers are sometimes provided with two stationary and two movable coils, mounted on the same core. One set of coils is independent of the other, and each secondary is connected to a different load circuit. Each movable coil is provided with a separate counter-weight.

413. EXPERIMENT 17-F. — Study of a Constant-Current Transformer. — The theory and the connections are described in the preceding section. The secondary voltage in such a transformer may be quite high, and great care must be exercised to protect the live parts and the instruments. A high-resistance rheostat may be used as the equivalent of a lamp load. An ammeter, a voltmeter, and a wattmeter should be connected in both the primary and the secondary circuit.

(a) Begin the test with the secondary circuit open; then close it on a high non-inductive resistance, and reduce the resistance in approximately equal steps to zero. At each step read all the instruments, and note the positions of the secondary coil on a scale marked on the core.

(b) Determine the limits of current within which the transformer (with a suitable counter-weight) is capable of regulating for constant current. For example, with a transformer rated at 6 amperes, try the regulation at 5 amperes and at 8 amperes.

(c) Investigate the effect of the power factor of the load upon the performance of the transformer.

(d) Determine how promptly the transformer acts with sudden changes in resistance.

(e) Measure the voltage induced in the movable coil in its various positions, with the secondary circuit open; also with the secondary circuit closed on a constant resistance.

(f) For a few values of the load determine the phase relation between the primary and the secondary volts; also between the corresponding amperes, as is explained in the preceding section.

(g) Measure the core loss and the resistances of the windings.

(h) Find out about the methods used in practice to keep a series circuit closed when one of the lamps burns out.

Report. (1) Compute the values of the non-inductive resistance used in the first run, by dividing the secondary volts by the secondary amperes. Plot all the readings to the values of this resistance as abscissæ. (2) Check a few values of the primary watts by adding the computed losses to the secondary output. (3) Give data showing the current regulation at different values of the secondary current. If the regulation was not quite good, give theoretical reasons for the observed curve. (4) Explain theoretically the effect of the power factor of the load upon the performance of the transformer, and give the observed data in support of the explanation. (5) Explain the action of the transformer with sudden changes in the load. (6) Construct a few vector diagrams showing the phase relations between the primary and the secondary currents and voltages. Check these diagrams by means of wattmeter readings. (7) Describe some methods used in practice for keeping a series circuit closed when one of the voltage-consuming devices becomes open-circuited.

CHAPTER XVIII

INSTRUMENT TRANSFORMERS

414. INSTRUMENT transformers (Figs. 50 and 51) are used in connection with alternating-current ammeters, voltmeters, wattmeters, watt-hour meters, relays, etc., to change a line current or voltage to a more convenient value and to insulate the instruments from a high-tension circuit. Such transformers are liable to cause small errors in instrument indications, and it is sometimes of importance to know how to calibrate an instrument transformer and to find the magnitude and the influence of its errors.

Potential transformers and current transformers are considered separately below, although the theory and the methods of calibration are described in the same order for both. The reader who will familiarize himself with the treatment of potential transformers will find that of current transformers quite similar to it. A general knowledge of the construction, operating characteristics, and performance diagram of the transformer is presupposed, and sufficient information on these topics will be found in the preceding two chapters.

POTENTIAL TRANSFORMERS

415. When an alternating voltage exceeds 110 or 220 volts, it is usually safer, more convenient, and more economical to connect a voltmeter, the potential circuit of a wattmeter, a potential relay, etc., not directly to the line, but through a small transformer (Fig. 51). Such a transformer is called a *potential* or *voltage transformer*. The primary of a potential transformer is connected to a high-tension alternating-current supply, and therefore must be insulated for the full line voltage. For high voltages such a transformer is usually of the oil-immersed type, and in its essential construction details does not differ from a power transformer (Chapter XVI), except for its small size and nominal rating (15 volt-amperes not being uncommon).

In order that a voltmeter, connected through a potential transformer, may indicate the true primary voltage, either it must be calibrated together with the transformer, or else the voltage ratio of the transformer itself must be accurately known. In a good modern potential transformer the internal voltage drop is very small and the ratio of the terminal voltages remains sensibly constant and almost equal to the ratio of the numbers of turns, as long as the transformer is not overloaded on the secondary side.

There are practical cases in which it is necessary to determine the actual ratio of transformation to a fraction of one per cent, under the actual load conditions, that is, with the same measuring instruments connected to the secondary winding as in the actual operation. Such a case arises, for example, when one or more potential transformers are used in connection with a watt-hour meter which measures large amounts of energy delivered to a municipality or to an extensive industrial plant, and where even a small error would lead to a considerable difference in the monthly bill. When using a potential transformer with a watt-hour meter or a wattmeter, it is of importance to observe the proper *polarity* (§ 408). Otherwise the meter would run backwards, or a polyphase meter might indicate the difference instead of the sum of the amounts of energy consumed in the two phases. See Vol. II, chapters on polyphase relations.

A transformer designed to give a ten-to-one ratio of terminal voltages when excited from the high-tension side, will not give an exact one-to-ten ratio when excited from the low-tension side. This is because one of the windings is usually provided with compensating turns to correct for the losses in the transformer itself.

Besides an error in the ratio of voltages, a potential transformer also introduces a small error in the phase angle of the measured voltage. It will be seen from Fig. 252 that the primary and the secondary terminal voltages are out of phase with each other, because of the internal drop in the transformer itself. In a well-designed potential transformer, this error in angle amounts to but a fraction of a degree, and is of no consequence when the transformer is used with a voltmeter or a relay. But when it is used in connection with a wattmeter or a watt-hour meter, this error affects the phase angle between the current and the voltage, and consequently the indication of the meter. At high values of power factor the error is negligible. For example, $\cos 3° = 0.9986$, $\cos 4° = 0.9975$, and the phase error of one degree affects the indication only by about 0.1 of one per cent. On the other hand, at lower values of power factor an error of one degree leads to a considerable error in measured power. For example, $\cos 59° = 0.515$, $\cos 60° = 0.500$, the error being 3 per cent. This is more than is usually permissible in metering energy. See also §§ 93 and 95.

It is of practical importance to know how to measure accurately, in a potential transformer, the errors in voltage ratio and in phase angle. Usually the errors involved are very small and such measurements require accurate instruments and a skilled observer, except where only a rough check is desired. The principal methods in use and their theory are described below.

416. Errors in Ratio and in Phase Angle. — The various methods used for the determination of the ratio and the phase angle in a potential transformer may be best explained by reference to Fig. 260. $OA = e$ is the vector of the unknown secondary voltage of the transformer under test, and $OB = e'$ is a reference voltage. It is required to determine the magnitude of e in terms of e' and the phase angle α between the two. Let BC be perpendicular to OA. Since angle α is always small, not over 2 degrees in an extreme case, OC is nearly equal to OB, the value of cos 2° being 0.9994. Thus, the *algebraic* difference between OA and OB is practically equal to $AC = m$, or

$$e - e' = m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

AC may be considerably smaller than the *geometric* difference AB between the two vectors. If $BC = n$ is known, then

$$\sin \alpha = n/e' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Thus, when m and n are known, the unknown voltage e and its phase displacement α against the reference voltage e' can be computed from eqs. (1) and (2).

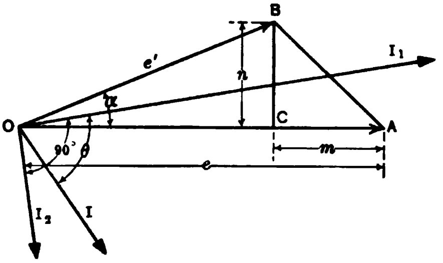


FIG. 260. The general principle of measurement of the ratio error and of the phase angle in a potential transformer.

In practice e' is usually the secondary voltage of another potential transformer of the same nominal voltage ratio as the transformer under test, the two transformers being connected in parallel on the primary side. This reference transformer must be carefully calibrated in an especially equipped laboratory, both as to its ratio and as to its phase angle. The ratio and the phase angle of the transformer under test can then be determined in their relation to those of the standard transformer, by comparatively simple means. Such methods are known as the *relative methods* of testing potential transformers (§§ 417 to 420). On the other hand, the *absolute methods* (§ 421) are those by means of which the ratio and the phase angle are determined directly by comparison of the primary with the secondary voltages of the transformer under test.

Let, for example, the voltage ratio of the standard transformer be

0.3 of one per cent high and let the ratio of the transformer under test be 0.1 of one per cent below that of the standard transformer. Then the absolute error of the transformer under test is 0.2 of one per cent (high). Similarly, if the phase error of the standard transformer is 20 minutes (lagging) and that of the transformer under test is 30 minutes (lagging with respect to the standard transformer), then the absolute error of the transformer under test is 50 minutes (lagging).

417. Comparator Voltmeter. — A convenient device for determining the ratio and the phase angle of a potential transformer is the so-called comparator voltmeter, shown in Fig. 261. This voltmeter is of the electro-dynamometer type (Fig. 40), except that the stationary coils are not excited in series with the movable coil. In this respect the device more nearly resembles the wattmeter shown in Fig. 81. The separate source is an " exciting transformer " which may be connected to the same phase 1 as the transformer under test, or to phase 2 in which the voltage wave is in time quadrature with that in phase 1. The potential transformer under test and a standard potential transformer are connected in parallel on the primary side and in opposition on the secondary side, through the movable coil of the voltmeter. Each of the two transformers under comparison must be provided with a suitable " load; " that is, it must be connected to one or more measuring instruments or their equivalent impedance, in order to have the same terminal voltages as in actual service.

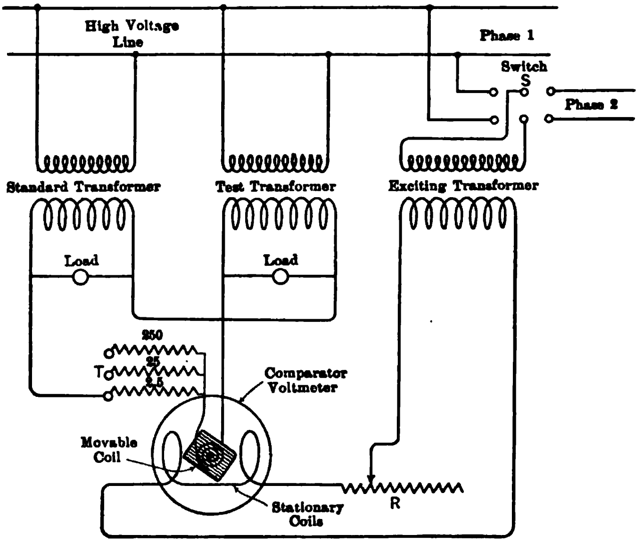


FIG. 261. The comparator voltmeter for calibration of potential transformers.

The three terminals of the comparator voltmeter, denoted by T, are connected to the movable coil through different series resistances, and the markings 2.5, 25, and 250 refer to the values of full-scale deflection of the instrument, in volts. For transformers of the same nominal ratio the 2.5-volt scale is usually sufficient; if not, the 25-volt scale is used. The 250-volt scale is used in preliminary trials only, to protect the instrument. The steps of the resistance R are marked for different secondary voltages, from 95 to 125, so that the specified constant current (about 0.2 ampere) can always be made to flow through the field winding. Because of the separate field excitation, the scale divisions are practically uniform, and not crowded at the lower part of the scale as in the ordinary dynamometer-type voltmeter.

First the switch S is thrown to the left and the voltmeter deflection read. Since the exciting transformer is connected to the same phase as the other two transformers, and the resistance R is non-inductive, the field current is practically in phase with the secondary voltages of the two transformers under comparison. This current is marked I_1 in Fig. 260. A dynamometer-type instrument measures the average torque between the current in the stationary winding and the in-phase component of the current in the movable coil (§ 90). The current in the movable coil is due to the differential voltage BA, and because of the large resistance in series with this coil, the current is practically in phase with BA. Thus, with a constant field current, the deflection of the instrument is proportional to the component of BA in phase with I_1. Because of the small magnitude of the angle between I_1 and e, this component of BA is practically equal to m. Knowing m and e' (practically equal to OC), the unknown voltage e may be computed from eq. (1).

Then the switch S is thrown to the right, bringing the field current practically in quadrature with e and e'. The new current is denoted in Fig. 260 by I_2. By the same reasoning as before, we find that the voltmeter deflection is practically equal to n; n being known, the angle α is computed from eq. (2).

The auxiliary phase not in quadrature with the main phase. Even if a two-phase supply is not available, a quadrature electromotive force may be improvised by using a T-connection (Vol. II). However, should it be impossible to get a quadrature voltage, the field current I for the second reading may be displaced with respect to I_1 by a known angle θ (Fig. 260). For example, if one of the remaining phases of a three-phase supply is used, the angle θ can be made practically equal to 60°. In this case the voltmeter measures the component of the voltage BA in phase with I. The projection of BA on I is equal to the sum of the projections of m and n on I. Remembering that the directions of OA and I_1 practically coincide, we get

$$BA \cos (BA,I) = m \cos \theta + n \sin \theta \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In this expression the left-hand side represents the actual voltmeter reading; m is known from the first reading and the angle θ is known. Therefore, n may be computed, and then α determined from eq. (2). For $\theta = 60°$, the foregoing expression becomes

$$n = 1.16 \, [BA \cos (BA,I) - 0.5 \, m] \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Since m is small as compared to e or e', a considerable error in its value will affect the value of the transformer ratio but very little. For this reason the angle θ need not be known very accurately. Thus, the differential method has the advantage that only moderate accuracy in measured quantities is required.

Signs of errors. The scale of the comparator voltmeter is marked to indicate whether the voltage of the transformer under test is higher or lower than that of the standard transformer. As a check, it may be noted that adding a secondary load on the transformer (especially an inductive load) lowers its secondary terminal voltage. If the addition of such a load increases the voltmeter reading with current I_1, then the standard transformer has a higher voltage, and vice versa. As to the phase angle, the reversed secondary voltage *at no load* usually leads the primary voltage. The addition of a non-inductive load always tends to cause the secondary voltage to lag. Such an increase in load should be tried with the current I_2, and if the reading n increases, then e was originally lagging behind e', and vice versa.

418. The Wattmeter Method. — The comparator voltmeter described above is a special instrument and may not always be available. The preceding method can be used with an ordinary dynamometer-type indicating wattmeter, provided that its deflections can be made of sufficient magnitude and also provided that certain precautions are observed. The instrument selected should be of low current-range, say 2 or 5 amperes, and its stationary coils may have to be overloaded to obtain a magnetic field of sufficient strength. The potential coil always possesses some unavoidable inductance, so that sufficient resistance must be used in series with it to bring the current approximately in phase with the differential voltage to be measured. The wattmeter must be calibrated in volts with a given field strength. The use of a wattmeter for the calibration of potential transformers antecedes that of the comparator voltmeter, and the latter has been developed in order to overcome the limitations and inaccuracies involved in the use of an ordinary wattmeter.

419. The Use of Two Watt-hour Meters. — This method is a modification of that shown in Figs. 260 and 261; the connections are shown in Fig. 262. The two potential transformers to be compared are connected to the potential circuits of two induction-type watt-hour meters (§ 120) *A* and *B*. The series coils of these meters are supplied with current from an auxiliary transformer connected to a non-inductive load. Let the auxiliary transformer be first connected to phase 1: We then have the conditions corresponding to the use of current I_1 in Fig. 260, except that the voltages e and e' are here used directly, and not their difference *AB*.

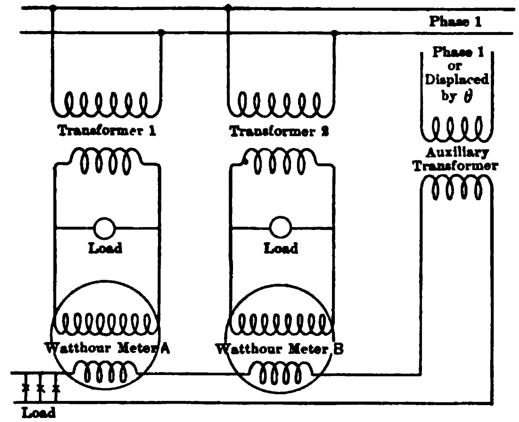


FIG. 262. The use of two watt-hour meters for calibration of potential transformers.

Let a_1 be the number of revolutions of meter *A* during a chosen interval of time, when connected to transformer 1. Let b_2 be the corresponding number of revolutions made during the same interval of time by meter *B* running on transformer 2. Let p and q be the constants of these meters. Then

$$R_2/R_1 = pa_1/qb_2 \quad \ldots\ldots\ldots \quad (5)$$

where R_1 and R_2 are the voltage ratios of the two transformers. In order to eliminate p and q the meters are interchanged. If the new readings are b_1 and a_2, we have

$$R_2/R_1 = qb_1/pa_2 \quad \ldots\ldots\ldots \quad (6)$$

Multiplying these equations term by term, and extracting the square root, we find

$$R_2/R_1 = \sqrt{a_1b_1/a_2b_2} \quad \ldots\ldots\ldots \quad (7)$$

Usually the two ratios, R_1 and R_2, differ but little from each other, and the following *approximate formula*, derived from the preceding one, may be used:

$$(R_2 - R_1)/R_1 = \tfrac{1}{2}\left[\frac{a_1 - a_2}{a_2} + \frac{b_1 - b_2}{b_2}\right] \quad \ldots\ldots \quad (8)$$

It is possible so to arrange the experiment that $a_1 = a_2$, in which case the computations are still further simplified. The approximations involved will amount to less than 0.1 of one per cent in ratio for differences in formula (8) not exceeding 3 per cent, and to less than 0.01 of one per cent in ratio for differences not exceeding 1 per cent.

Another set of readings is then taken at a known angle of phase displacement θ between the current in the series coils of the watt-hour meters and the voltage in phase 1. If the auxiliary transformer is connected to one of the remaining phases of a three-phase supply, θ is nearly 60°. Let the phase angle of transformer 1 be α_1, the reversed secondary voltage leading the primary voltage by this angle. Then, with a lagging current, the meter A is being actually operated at a phase displacement of $\theta + \alpha_1$, instead of θ. The expression for the true energy of the load, as measured by the two meters, is

$$\frac{a_1 R_1\, p \cos\theta}{\cos(\theta + \alpha_1)} = \frac{b_2 R_2\, q \cos\theta}{\cos(\theta + \alpha_2)} \quad \ldots\ldots\ldots \quad (9)$$

In this formula $a_1 R_1 p$ is the value of the energy recorded by meter A, and $\cos\theta / \cos(\theta + \alpha_1)$ is the correction for the phase-angle error; the right-hand side of the equation has a similar meaning for meter B. Interchanging the meters we get a similar expression for the true energy:

$$\frac{b_1 R_1\, q \cos\theta}{\cos(\theta + \alpha_1)} = \frac{a_2 R_2\, p \cos\theta}{\cos(\theta + \alpha_2)} \quad \ldots\ldots\ldots \quad (10)$$

Eliminating p and q from eqs. (9) and (10), we obtain

$$\frac{\cos(\theta + \alpha_1)}{\cos(\theta + \alpha_2)} = \frac{R_1}{R_2}\sqrt{\frac{a_1 b_1}{a_2 b_2}} \quad \ldots\ldots\ldots \quad (11)$$

If the phase angle of one of the transformers, say α_1, is known, and the ratio R_1/R_2 has been previously determined from either eq. (7) or (8), then eq. (11) can be solved for the unknown phase angle α_2 of the transformer under test. The angles α_1 and α_2 are ordinarily very small, and the following *approximate* formula, derived from the preceding one, may be used:

$$(\alpha_2 - \alpha_1)_{\text{in minutes}} = \frac{3438}{\tan\theta}\cdot\left[\frac{a_1 - a_2}{2a_1} + \frac{b_1 - b_2}{2b_1} - \frac{R_2 - R_1}{R_1}\right]. \quad (12)$$

420. EXPERIMENT 18-A. — Calibration of a Potential Transformer. — The purpose of the experiment is to become familiar with one or more of the methods of calibration described in the preceding sections. The diagrams of connections and the general procedure are sufficiently explained there. The ratio and the phase angle of the transformer under test should be determined for a light load, for the rated load, and for an overload; also for a practically non-inductive load and for one at a low power factor. In view of the small magnitude of the quantities to be measured, it is of particular importance to make the connections and to take the readings with the utmost care. Before the test and during the experiment, it is desirable to investigate the principal sources of inaccuracy and errors, and to estimate their possible effect upon the results. This applies in particular when an ordinary wattmeter is used in place of a special comparator voltmeter.

Report. Give the detailed diagrams of connections used and the original readings. Compute the ratio error and the phase angle at different loads, and compare their relative magnitude with that which might be expected from the general vector diagram of a transformer (§ 401). Indicate the sources of inaccuracy in the methods of measurement employed and in the apparatus used. In each case give the order of magnitude of the error.

421. Absolute Methods of Calibrating Potential Transformers. — Such methods require special electrical measuring devices of high accuracy, and for this reason the work of standardization of potential transformers is usually undertaken only in a few well-equipped laboratories. Standardized transformers are then used for the determination of errors of commercial potential transformers, as described above. The principal methods of standardization are only briefly described below, and for the details the reader is referred to special monographs of the Bureau of Standards.

(1) *Alternating-current potentiometer.* The two kinds of alternating-current potentiometers are described in §§ 85 and 86. By the use of either kind, the primary and the secondary voltage of a potential transformer can be measured directly, as well as the phase angle between the two.

(2) *Differential dynamometer voltmeter.* This device is provided with two separate sets of coils arranged as in Fig. 60. The torque due to one set of coils is balanced against that due to the other. One set of coils is connected to the source of primary e.m.f., the other set is connected first to the primary and then to the secondary terminals of the transformer under test, through large non-inductive resistances. This process corresponds to the measurement with current I_1 in Fig. 260 and gives the ratio of transformation. To obtain the phase angle, the current in one of the moving coils is supplied from the primary terminals through a condenser. This gives a quadrature current and corresponds to the measurement with the current I_2.

(3) *The two-electro-dynamometer method.* A high non-inductive resistance is connected across the primary terminals of the transformer under test, and the nearest balance is sought between the voltage e'

across part of this resistance and the secondary voltage e of the transformer. Let these two voltages be OA and OB in Fig. 260. By means of a sensitive dynamometer-type wattmeter the point A is reduced to C, giving a zero reading between the current I_1 and the voltage n. By means of another similar dynamometer the value of n is read, with a quadrature current I_2 from a separate source.

(4) *The inductance method.* A large non-inductive resistance is connected across the primary terminals of the transformer under test, and the nearest point of balance is sought against the secondary voltage, as in the preceding method. Since the two voltages are slightly out of phase, some inductance and capacitance is added to the part of the resistance to be balanced, so as to change the phase of the voltage drop across it until a vibration galvanometer reads zero. When the values of the resistance, inductance, and capacitance are known, both the ratio and the phase angle may be readily computed.

(5) *The locus method* (H. S. Baker). A special testing transformer is used in which the number of turns in the secondary is known and can be varied, one by one. The voltage of this transformer is " bucked " against that of the transformer under test, and the difference AB (Fig. 260) between the two voltages is measured by means of a separately excited wattmeter, using in succession currents I_1 and I_2 (or I) as in § 417. Then the ratio of the testing transformer is changed in small steps and the test repeated at each step. The results are plotted in the form of a vector diagram, and give a locus of point A, that is, the line OA, in its true angular relationship to OB. The perpendicular BC then determines the true ratio and the phase angle of the transformer under test.

CURRENT TRANSFORMERS

422. On a high-tension alternating circuit, or with an alternating current exceeding a certain value, it is safer, more convenient, and more economical to connect an ammeter, a current relay, or the series winding of a wattmeter, to the line through a *current transformer* (Fig. 50). Such a transformer consists of a closed iron core, and two windings, one of which (the primary) is connected to the line and the other (the secondary) to the measuring instrument, relay, etc. For high voltages, such a transformer is usually immersed in oil, and in its essential details it is similar to a power transformer (Chapter XVI), except for its small size and a nominal rating (sometimes 10 to 25 volt-amperes).

In order that an ammeter connected through a current transformer may indicate the correct line current, either the instrument must be calibrated with the transformer, or else the current ratio of the transformer must be accurately known. In a good modern current-transformer the magnetizing current and the core loss are very small, so that the ratio of the currents is very nearly equal to the inverse ratio of the turns (§ 402), at least within a certain range. There are cases in which an accurate knowledge of the ratio of transformation is of importance; for example, when a current transformer is used with a watt-hour meter through which energy is supplied to a large industrial or municipal load. In such a case, even a small error in the ratio would lead to a considerable difference in the monthly bill.

Besides an error in the current ratio, a current transformer introduces also a small error in the phase angle between the primary and the secondary currents (Fig. 252). This angle modifies the reading of a watt-hour meter or a wattmeter, and therefore must be kept low. Its influence is much more pronounced on an inductive load, as is shown by a numerical example in § 415. The principal methods of experimental determination of the ratio error and of the phase angle of a current are given below. Such measurements usually require quite accurate instruments and a skilled observer, unless a rough check only is required.

423. Types of Current Transformers and Some Precautions in Use. — A current transformer which is provided with two windings, a primary and a secondary, may be called "self-contained." Each winding sometimes consists of sections which may be connected in series or in parallel, thus considerably increasing the range of the device. This is particularly convenient in portable transformers used for calibration and testing purposes. For heavy primary currents the primary winding is sometimes omitted altogether, and the line conductor or the bus-bar is put through the hole in the iron core (Fig. 263). Such a transformer is variously called the "through" type, the "loop-through" type, the hole or loop type, the stud or bus-bar type, the open primary type, etc. With the same transformer the primary conductor may be passed through the hole once for the high current range, twice for the intermediate range, and three or four times for the low range.

There is also a "split-type" current transformer in which one side of the core is detachable, so that the transformer can be slipped over a line conductor and then tightly clamped in place. Such transformers are used for current measurements in circuits which it would be inconvenient or impossible to open. They are apt to lead to quite a considerable and variable error, especially in the phase angle, and should not be used with a wattmeter. They are useful only for an approximate measurement of current, for finding a fault in a cable, etc.

A low exciting current is essential in a current transformer, since it insures a constant ratio and a small phase angle. For this reason the core must be of extremely low reluctance and the winding so proportioned as to require but a low flux density. An accurate current transformer should have no joints in the magnetic circuit. In a split-type transformer the two variable air-gaps increase the exciting current and therefore affect the accuracy of the device. If such a transformer is used for permanent work, the joints must be carefully overlapped and the laminations drawn tightly together with insulated bolts.

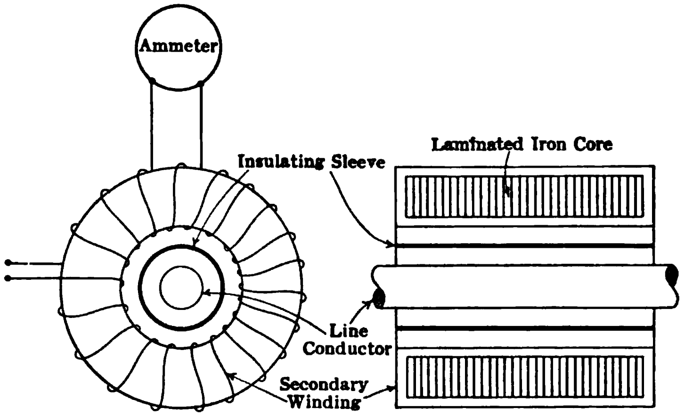


FIG. 263. A current transformer of the "through" or stud type.

Because of the unavoidable magnetizing current and internal losses, current transformers are properly *compensated* within their range with extra turns, to reduce the influence of these errors. For this reason, a transformer which gives a correct ten-to-one ratio will not give a correct one-to-ten ratio when the functions of its primary and secondary windings are reversed.

When using a current transformer with a wattmeter or watt-hour meter, it is of importance to observe the proper *polarity* of connections. Otherwise the meter may indicate backwards, or a polyphase meter (§ 121) may indicate the difference instead of the sum of the readings of the two elements.

The primary of a current transformer should never be left connected to the line with the secondary circuit open, as this will set up a heavy flux through the core, saturating the iron, and may induce a dangerous voltage in the secondary winding. Moreover, considerable residual magnetism is liable to remain in the core after the line circuit is opened, and the transformer ratio will be impaired. If for any reason it becomes necessary to disconnect the meter from the secondary winding, the latter should first be short-circuited by a piece of wire. Some portable transformers are provided with a special short-circuiting switch for this purpose. Should an open-circuit occur accidentally while the transformer is connected to the line, the residual magnetism can be removed by inserting a resistance between the secondary terminals and gradually reducing it to zero while a considerable alternating current is passing through the primary. If this is not done, the transformer may show quite an appreciable error. Such demagnetization should always precede a calibration.

Care must be taken to avoid placing two current transformers close together or having a heavy current lead adjacent to a transformer under test, as stray fields affect the ratio of transformation.

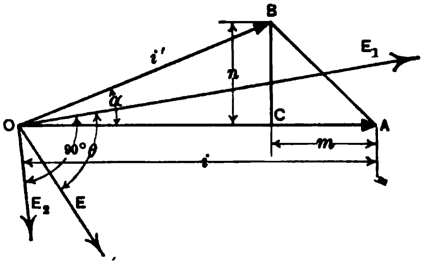


Fig. 264. The general principle of measurement of the ratio error and of the phase angle in a current transformer.

424. Errors in Ratio and in Phase Angle. — The various methods used for the determination of the ratio and of the phase angle in a current transformer may be best explained by reference to Fig. 264. $OA = i$ is the vector of the unknown secondary current in the transformer under test, and $OB = i'$ is a reference current. It is required to determine the magnitude of i and the phase angle α. Let BC be perpendicular to OA. Since angle α is always small, not over 2 degrees in the extreme, OC is nearly equal to OB, cos 2° being equal to 0.9994. Thus, the *algebraic* difference between OA and OB is practically equal to $AC = m$, or

$$i - i' = m \quad \dots\dots\dots\dots \quad (13)$$

AC may be considerably smaller than the *geometric* difference AB between the two vectors. If $BC = n$ is known then

$$\sin \alpha = n/i' \quad \dots\dots\dots\dots \quad (14)$$

Thus, if m and n are known, the unknown current i and its phase displacement α with respect to the reference current can be computed from eqs. (13) and (14).

In practice i' is usually the secondary current of another current transformer of the same nominal ratio as the transformer under test, the two transformers being connected in series on the primary side. This reference transformer must be carefully calibrated in an especially equipped laboratory, both as to its ratio and as to its phase angle. The ratio and the phase angle of the transformer under test can then be determined in their relation to those of the standard transformer, by comparatively simple means. Such methods are known as the *relative* methods of calibration (§§ 425 to 428). On the other hand, the *absolute methods* (§ 429) are those by means of which the ratio and the phase angle are determined directly by comparison of the primary with the secondary current in the transformer under test. See the last paragraph in § 416.

In the methods described below it is necessary to supply the potential circuits of the wattmeter or of the watt-hour meters with a source of alternating voltage differing by a given phase angle from that supplied to the current coils. A potential phase-shifter (§ 127) may be conveniently used for this purpose.

Compensated Current Transformer. The "two-stage" current transformer devised by Mr. H. B. Brooks (Fig. 264*a*) gives greater accuracy of ratio and freedom from phase-angle errors than can be attained by current transformers of ordinary construction. It consists of two iron cores linked with three secondary windings. The larger, or principal core, is wound with a principal secondary winding, s, of five ampere rating. The smaller core is wound with a winding a' of very much smaller rating, called the auxiliary secondary. This smaller core is also encircled by a five-ampere winding, s', of the exact number of turns which the principal secondary winding, s, on the left-hand core would have if the principal core had infinite permeablity and zero core loss. The winding s' is in series with the principal secondary winding, and in opposition to the primary winding p'. The terminals of s and s' are connected to the current winding of a wattmeter, watt-hour meter, etc. The auxiliary secondary coil is connected in the meter to an auxiliary winding of the same number of turns as the regular current winding.

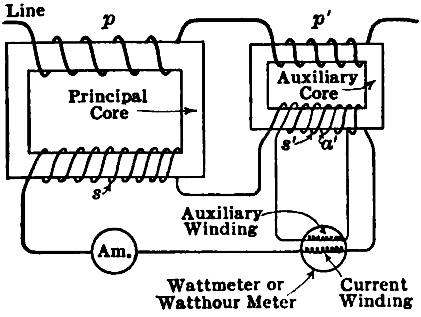


Fig. 264*a*. A two-stage current transformer which corrects errors in ratio and phase angle.

In operation, there will be no flux in the auxiliary core if the conditions are such that the principal secondary current is correct in ratio and phase, because then the m.m.f.'s of p' and s' are exactly equal and opposite. Any departure from the correct value sets up a magnetomotive force in the auxiliary core, and a current will flow in the auxiliary windings of both the transformer and the meter, and give resultant ampere-turns of nearly correct value in the meter. If the principal secondary current is too low, the auxiliary secondary current is automatically additive, and vice versa. If the principal secondary current has the right value but is out of phase, the auxiliary current is automatically of such phase as to correct it. In other words, the auxiliary transformer is so constructed and connected that the difference of the m.m.f.'s of p' and s' acts as its primary excitation, and the winding a' as its secondary. By a suitable modification in the secondary circuit, the auxiliary winding in the meter may be made unnecessary.

Instead of the two physically distinct transformers shown diagrammatically in Fig. 264*a*, it is more convenient to use a single primary winding and a single secondary winding encircling both cores, with the auxiliary secondary winding and a few turns of the main secondary winding surrounding the auxiliary core only. This method of construction produces a two-stage transformer which is physically a single compact unit.

425. The Wattmeter Method. — The transformer under test and the standard transformer (Fig. 265) are connected in series on both the primary and the secondary side. A suitable load is provided for each, equivalent to the impedance of the instrument or relay windings for which the transformers have been designed. The load on the primary circuit itself must be as nearly non-inductive as possible. The ratios of the two transformers or their phase angles being, in general, different from each other, a differential current will flow between them. This current passes also through the coil C_1 of a dynamometer-type wattmeter (§ 90) connected between the two secondary windings. This current corresponds to BA in Fig. 264, and its components m and n are measured by sending through the other winding, C_2, an alternating current of the same frequency and of a known phase and magnitude. Let C_1 be the current coil and C_2 the potential coil, and let the latter be connected across the same phase E_1 as the two transformer primaries are. Since the load is non-inductive, the voltage E_1 is nearly in phase

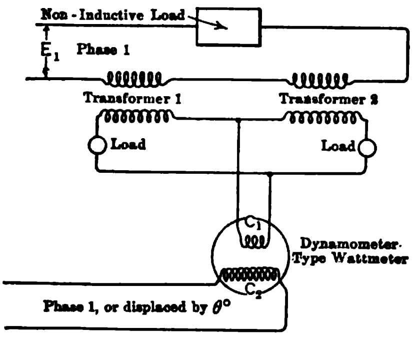


Fig. 265. The wattmeter method of calibration of a current transformer.

with both i and i'. Let the wattmeter reading be P_1 watts. We then have

$$m = P_1/E_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

After this, the potential winding is connected to a source of voltage E_2 in quadrature with E_1. In this case the wattmeter reading, P_2, is proportional to the component n of the differential current, in phase with E_2. Thus

$$n = P_2/E_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

If the second phase is displaced with respect to the first by an angle θ other than 90°, the computations are made according to formulæ (3) or (4) in § 417, with the self-evident interchange of the functions of the currents and the voltages. When m and n are known, the current ratio and the phase angle are computed as in § 424.

The winding C_1 may also be the potential winding of the wattmeter and C_2 its series winding, supplied with a current of known magnitude and phase from a separate source. In this case the resistance multiplier which is usually provided in series with the potential coil is omitted, in order not to introduce an unnecessarily high voltage. It is also advisable to set the moving coil at right angles to the stationary coil so as to avoid the electromotive force set up by mutual induction from the current coil. As a matter of fact, the use of a standard wattmeter is only a makeshift, and in order to get the best results it would be necessary to use a special dynamometer-type instrument similar to the comparator voltmeter described in § 417, and particularly adapted for the calibration of current transformers.

Signs of errors. The components m and n of the differential current may be either positive or negative, that is, the transformer under test may have either a greater or a smaller current ratio and phase angle than the one to which it is being compared. The signs of m and n must be very carefully considered, as otherwise the test would be misleading. The question of the signs has to be decided in each individual case, but the following suggestions will be found useful:

(1) Shunting one of the transformers will either reduce or increase the wattmeter deflection. Using an inductive and then a non-inductive shunt will help to decide the approximate phase position of the vector AB (Fig. 264) and hence the signs of its two components.

(2) The direction of deflection of the wattmeter pointer, when only one of the two transformers is connected to it, instead of both, will indicate the one whose action predominates. This can be tried with both voltages, E_1 and E_2.

(3) Adding a non-inductive resistance in the secondary of a current transformer tends to advance the phase of the secondary current.

426. The Watt-hour Meter Method. — The connections are shown in Fig. 266 and the principle is similar to that described in § 419. The primaries of the two current transformers to be compared are connected in series, and each secondary winding is connected to the current coil of a watt-hour meter, usually of the induction type (§ 120). The potential circuits of the two meters are connected in parallel and are supplied with current from the same phase as the current coils. A run is made over a known period of time and the meter readings are taken. Then the potential windings are connected to another phase displaced by an angle θ (say 90°) with respect to the first, and another run is made. Referring to Fig. 264, the speed of rotation of the meters in the first case is proportional to the components of the currents i and i' in phase with E_1, in the second case to those in phase with E_2 or E. In order to eliminate the meter constants, two more runs are made, with the meters interchanged. The computations are the same as in § 419, with the self-evident interchange of the functions of the voltages and currents. This method is very convenient for field testing of transformers, and possesses considerable accuracy even with a fluctuating voltage supply.

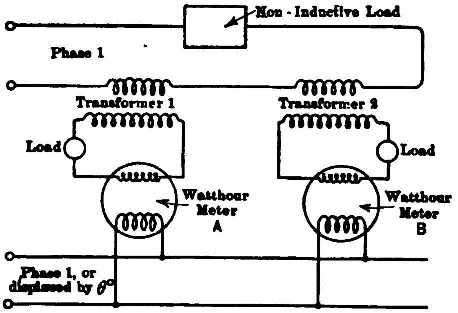


FIG. 266. The use of two watt-hour meters for calibration of current transformers.

A modification of this method, described at the end of § 428, consists in checking directly the primary of the transformer under test against its own secondary, without the use of a standardized transformer. This simplification is possible only at moderate values of primary current, because the whole primary current must flow through the series coils first of one and then of the other meter.

427. EXPERIMENT 18-B. — Calibration of a Current Transformer. — For directions and report requirements see Experiment 18-A, § 420.

428. Standardization of a Loop-Through Transformer. — The foregoing methods of calibration of current transformers necessitate the use of a standardized transformer of the same nominal ratio as the one under test. On a large electric supply system, current transformers of many different ratios are usually required, and the problem of reducing the number of standard transformers to a minimum is of considerable practical importance. Some portable transformers have the primary winding divided into sections so that by varying the connections between the sections, different current ratios may be obtained. Even then the number of ratios is usually limited to four.

A " loop-through " transformer (§ 423) may be conveniently provided with a primary winding of a variable number of turns and used as a universal standard. For example, let the highest desired primary current be 800 amperes and let the secondary winding consist of 160 turns designed to carry 5 amperes, this being the standard instrument current. The primary winding is then conveniently made of 16 turns of 10-strand cable, giving 160 turns in series for the one-to-one ratio. The same turns may also be used in several series-parallel combinations and as one straight parallel combination. One or two taps on the secondary winding will still further increase the possible number of current ratios.

Care must be taken to have both windings well distributed around the iron core, and always to use the same turns for a given ratio, because the calibration depends appreciably upon the position of the two windings on the core. Every lack of symmetry affects the ratio as well as the phase angle of the transformer. With a well-distributed secondary winding the difference in the ratio caused by changing from a distributed to a bunched primary winding may be less than 0.1 of one per cent, but if the secondary winding is bunched, then changes in the primary winding affect the ratio so much as to make such a transformer unsuitable for calibration purposes.

With proper precautions in the arrangement of the two windings, such a transformer may be calibrated at the one-to-one nominal ratio, and the same phase angle used with any other primary number of turns. Thus, a convenient universal standard is obtained with only one set of constants. If there are taps on the secondary winding, a separate calibration must be made for each.

Such a transformer may be calibrated and standardized without outside assistance, by means of two watt-hour meters (§ 426) and then used for checking other current transformers. With the one-to-one nominal ratio, one of the watt-hour meters is connected in series with the primary winding of the transformer and the other in series with its secondary winding. With a perfect transformer the two readings should be identical, and any discrepancy is due to an error in the ratio or to a phase angle. Referring to Fig. 266 and to the theory in § 426, the standard one-to-one transformer is here assumed to be perfect. Therefore it is omitted, and the corresponding watt-hour meter is connected directly in the primary circuit. Thus, the foregoing formulæ are directly applicable, if this particular condition is kept in mind.

429. Absolute Methods of Calibration of Current Transformers.— The methods of standardization of potential transformers described in § 421 are applicable, with suitable modifications, to the absolute calibration of current transformers. The problem is made more difficult because of the necessity of measuring large alternating currents with a high degree of precision. Accurately made non-inductive ammeter shunts are used in both circuits, and potential differences are measured across these shunts by means of precision instruments of electrodynamometer type. Otherwise, the problem and the methods are essentially the same as in the calibration of potential transformers.

CHAPTER XIX

THE ALTERNATOR — OPERATING FEATURES

430. An alternator is an electric generator for producing alternating current or currents by a relative motion of conductors and a magnetic field. More specifically, a synchronous alternator is one in which the magnetic field is excited with direct current. The essential parts of a synchronous machine are shown in Figs. 267 and 268. The stationary armature core consists of soft-steel laminations held together in a cast-iron frame. Slots are punched in these laminations and the armature coils placed in these slots are held in place by suitable wedges. In low-speed machines the revolving spider is made of cast-iron, and in high-speed machines of steel of great tensile strength. Individual pole-pieces are fastened to this spider and each is provided with a field coil. The field winding is excited with direct current and produces opposite magnetic polarity in adjacent pole-pieces. The exciting current is conducted into the field coils through two slip rings shown in the sketch.

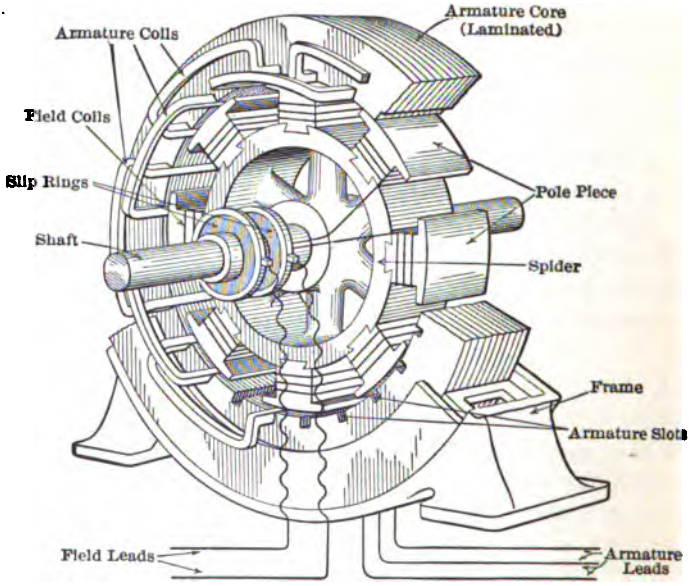
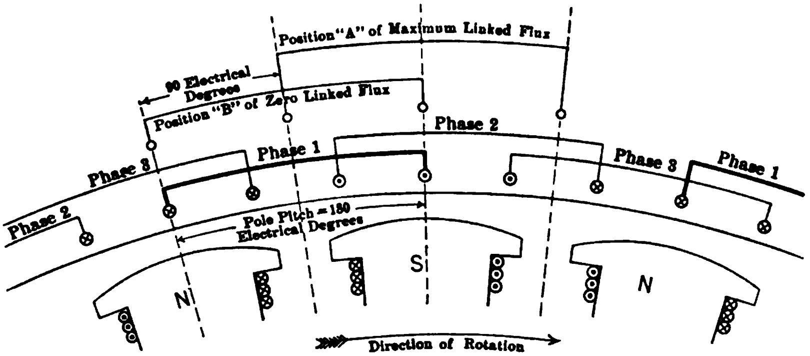


FIG. 267. The principal parts of an alternator or of a synchronous motor.

The particular alternator shown in Fig. 267 is an eight-pole, three-phase machine; it has four armature coils per phase, or one coil side per pole. The armature winding and the field poles are shown partly developed in Fig. 268. All the armature conductors which at the instant under consideration are under the centers of the poles belong to phase 1 and are electrically connected in series. At this instant the voltage induced in these conductors is a maximum, because they are cut by the magnetic flux at its maximum density. The directions of the exciting current and of the induced voltage are shown in the conventional manner by dots and crosses. As the poles move on, the voltage induced in this stage decreases to zero, then increases in the opposite direction, and reaches its negative maximum after the poles have moved by one *pole-pitch*, i.e., when the north poles come into the position occupied by the south poles, and vice versa. Thus, one alternation of induced voltage occurs during the interval of time within which the poles move over one pole-pitch.



Position "A" of Maximum Linked Flux
90 Electrical Degrees
Position "B" of Zero Linked Flux
Phase 2
Phase 1
Phase 3
Phase 2
Phase 3
Phase 1
Pole Pitch = 180 Electrical Degrees
N
S
N
Direction of Rotation

FIG. 268. A three-phase armature winding, one slot per pole per phase.

The coils marked " phase 2 " are identical with those in phase 1, but are displaced with respect to them by two-thirds of the pole-pitch, in the direction of rotation of the poles. The pole-pitch is said to correspond to 180 electrical degrees, and therefore phase 2 is displaced with respect to phase 1 by 120 electrical degrees. The same voltage which is induced in phase 1 is also induced in phase 2, one-third of a cycle later. In other words, the wave or the vector of the voltage induced in phase 2 lags behind that in phase 1 by 120 electrical degrees. Similarly the coils which belong to phase 3 are so placed that the voltage wave induced in them leads that in phase 1 by 120 degrees.

Such an arrangement of armature coils is called a *three-phase* winding, and the three induced voltages may be used either singly or in combination. Most alternators used in power and lighting systems are of the three-phase type. For the general properties and advantages of three-phase generation and transmission over single-phase, see Vol. II, under Polyphase Systems. For the elementary experiments described in this volume it is sufficient to become familiar with the simplest method of connecting the three phases, the so-called Y-connection described in § 447 below.

All the tests described in this chapter may be performed on either a polyphase or a single-phase machine. For the sake of experience the student should perform them on a single-phase machine first, because the connections and the computations are somewhat simpler. Later, the same experiments should be performed on a three-phase alternator, partly as an exercise in polyphase relations and measurements, and partly because the theory of voltage regulation, explained in Chapter XX, applies more correctly to polyphase machines.

The machine shown in Fig. 267 has a revolving field and a stationary armature. Most machines used for lighting and power are of this type, because the armature is usually wound for fairly high voltage, and a stationary structure can be better insulated and more easily supervised in operation. Moreover, in a three-phase machine a revolving armature would necessitate three slip rings instead of the two needed for the exciting current. There is, however, a type of alternator with a revolving armature and stationary poles, similar in its construction to the direct-current machine (Fig. 175). It is used mostly in small sizes for special purposes. There is also a so-called inductor type of alternator used mostly in high-frequency work, with both the exciting and the armature windings stationary. The pole projections alone are rotated, and the e.m.f. is induced in the armature coils by periodic changes in the reluctance of the magnetic circuit.

An alternator, like a direct-current machine, is not only capable of converting mechanical energy into electrical, but may also be operated as a motor, converting an electrical input into mechanical energy available on its shaft. It is then called a synchronous motor. In this chapter and in the two following chapters, simple experiments are described on the synchronous generator only. For more advanced and special tests on the synchronous generator, and for the theory of and experiments on the synchronous motor and rotary converter, see Vol. II.

431. Induced Voltage and Frequency. — According to the fundamental law of induction (§ 157), the electromotive force induced at any instant in a single turn of an armature coil is

$$e = -d\phi/dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where ϕ is the instantaneous value of the field flux linking with the turn under consideration, and t is time. Multiplying both sides of this expression by dt and integrating within an arbitrary finite interval of time Δt, we get the following expression for the *average voltage* induced during this interval:

$$e_{ave}\Delta t = \int_{t}^{t+\Delta t} e dt = -\Delta\phi \quad \ldots \ldots \quad (2)$$

or

$$e_{ave} = -\Delta\phi/\Delta t \quad \ldots \ldots \quad (3)$$

Here $\Delta\phi$ is the change in the flux linked with the turn during the interval of time Δt.

In a certain position A of an armature coil (Fig. 268), it embraces the whole flux per pole. In another position, B, distant from the first by 90 electrical degrees, the total flux linking with the turn is zero, half of it entering from one pole and the other half from a pole of the opposite polarity. These two characteristic relative positions of the field poles and armature winding are also shown in Figs. 269 and 270. The interval of time, within which the flux linked with a coil changes from its full value Φ to zero, is one-quarter of a cycle, or $\frac{1}{4}T$, where T is the duration of one cycle. Substituting in equation (3) $\frac{1}{4}T$ for Δt, we get, apart from the minus sign,

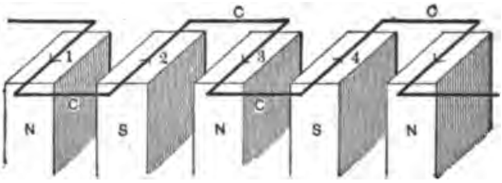


FIG. 269. Relative position of the poles and armature winding when the induced e.m.f. is a maximum.

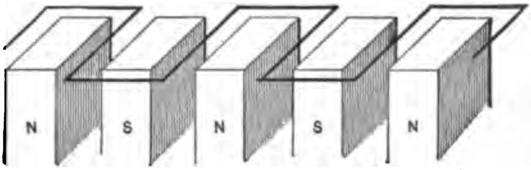


FIG. 270. Relative position of the poles and armature winding when the induced e.m.f. is zero.

$$e_{ave} = 4\,\Phi/T \quad \ldots \ldots \quad (4)$$

where Φ is the total useful flux per pole. In practice it is customary to characterize an alternating voltage or current not by the duration of a cycle, but by the frequency f which is defined as the number of cycles per second. For example, if the duration of one cycle, $T = \frac{1}{60}$ of a second, the frequency $f = 60$ cycles per second. Thus, in general terms

$$f = 1/T \quad \ldots \ldots \quad (5)$$

Substituting this value in eq. (4) we get

$$e_{ave} = 4\,f\,\Phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The e.m.f. wave of most alternators closely resembles a sine-wave (Fig. 109). This wave-form is usually required in practice, and the pole-pieces and winding are designed accordingly. For a sine-wave the ratio between the maximum and the average value is equal to $\pi/2$; the effective value (§§ 48 and 50) is obtained by dividing the maximum value by the square root of two ($\sqrt{2}$). Since all ordinary voltmeters and ammeters read effective values, and most practical computations, data, and ratings refer to effective values, it is convenient to change eq. (6) accordingly. We then get

$$e_{eff} = \tfrac{1}{2}\,\pi\,4\,f\,\Phi/\sqrt{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

or

$$e_{eff} = 4.44\,f\,\Phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Let the armature winding consist of N turns in series per phase, and let the voltages induced in all the turns be equal and in phase with one another. Then the preceding expression for the induced e.m.f. per turn must be multiplied by N to obtain the total induced voltage per phase. In reality the requirement for the best utilization of the winding space and other mechanical and electrical considerations make it necessary to place the individual coils in such positions that their induced e.m.f.'s are somewhat out of phase with one another. Sometimes it is also desirable to use coils of a width smaller than the pole-pitch. Therefore, the total induced e.m.f. is somewhat reduced, and this reduction is taken into consideration by the so-called *breadth factor* k_b smaller than unity. The foregoing equation then becomes

$$E_{eff} = 4.44\,k_b\,f\,N\,\Phi\,10^{-2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

In this expression E_{eff} is in volts and the flux Φ per pole is in megalines; this necessitates the factor 10^{-2}. For further details see the chapter on Armature Windings in Vol. II, and also the author's book entitled "The Magnetic Circuit," under "breadth factor."

The relationship between the number of poles p of the machine, its speed of rotation n, in r.p.m., and the frequency f of the induced voltage, in cycles per second, is as follows: At f cycles per second the induced voltage has a frequency of $60 \times 2\,f = 120\,f$ alternations per minute, because each cycle corresponds to two alternations. During one revolution of the machine p poles pass under each group of conductors, thus inducing p alternations. Consequently the number of alternations per minute is pn, and we have

$$120\,f = pn \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

When the number of poles or the speed of the machine is not known, the frequency can be measured directly with a suitable instrument. For a description of such *frequency meters* see §§ 443 to 446 below.

432. Corresponding Experiments on A.C. and D.C. Generators. — The fundamental experiments on alternators, such as the no-load test, voltage regulation under load, efficiency from losses, heat run, etc., are similar in their purpose and general method to those described for direct-current machinery in Chapters X to XV. Since the reader is expected to have performed at least the most elementary experiments on a direct-current generator before he begins a study of the alternator, it would be an unnecessary repetition to give here again all the minute instructions for the fundamental experiments. The student will do well to consult the corresponding sections on D.C. machinery and also his own laboratory reports, partly to get a clear idea of the points of similarity and the differences, and partly so as to be on his guard against the difficulties that may be encountered and the mistakes of judgment committed when performing an experiment for the first time.

The principal additional factors in testing an alternator, as compared to a direct-current generator, are as follows:

(1) A separate source of D.C. excitation is always required.

(2) The induced voltage is characterized not only by its magnitude but also by its frequency (§§ 430 and 431).

(3) The load current is characterized not only by its magnitude, but also by its phase angle with respect to the terminal voltage of the machine. Therefore a wattmeter or a power-factor meter is usually required in the circuit.

(4) An alternator is often a three-phase or a two-phase machine, and some knowledge of polyphase relations is necessary (§ 447).

433. EXPERIMENT 19-A. — No-Load Characteristics of an Alternator. — The purpose of the experiment is to investigate the influence of the exciting current and speed upon the induced e.m.f. at no load. The theoretical formula is given in § 431. The corresponding D.C. experiment is 10-A. It is preferable to perform this experiment on a single-phase machine, or to use one phase only of a polyphase winding, because the relationship is the same in each phase. The diagram of connections is shown in Fig. 271. A frequency meter (§ 443) may also be connected across the terminals.

(1) Have the alternator connected to an adjustable-speed motor, and run the set at the highest safe speed. Demagnetize the field and begin readings at a low value of the exciting current. Read the speed, the terminal voltage, the field current, and the frequency meter, if used.

Increase the field current in steps to a safe maximum and take similar readings at each step, keeping the speed as nearly constant as possible. Take a few readings with the field current reduced in steps to zero, in order to determine the influence of the residual magnetism (Fig. 177). Make one or two similar runs at lower speeds.

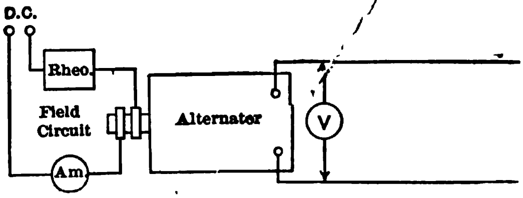


Fig. 271. Diagram of connections of a single-phase alternator at no load.

(2) Wire up the driving motor so as to be able to read its input and later to compute the corresponding output. The corresponding D.C. experiment is 13-B. Take a saturation curve of the alternator under test at the rated speed, as above, and at each step read the input into the driving motor, its field current, and speed. The last reading should be taken with the alternator field circuit open, to get its friction and windage. Disconnect the driving motor and run it alone to determine its losses. Measure its armature resistance.

(3) If the machine has a three-phase winding, take a few corresponding readings of phase voltages and line voltages (§ 447).

(4) Count the number of field poles and make a rough sketch of the armature winding.

Report. (1) Plot the observed saturation curves as in Fig. 177. (2) Show from the curves that with a given field current the induced voltage is proportional to the speed. Compute the speed at which the machine has to be run for a standard frequency, say sixty cycles. (3) From the input and losses of the driving motor compute its output, which is equal to the core loss, friction and windage of the alternator. Subtracting the friction and windage will give the core loss alone. Plot its values against field current as abscissæ. (4) Make a clear sketch of the winding showing the field poles in the proper relative position. (5) Check the ratio between the phase and line voltages and explain the departure from the theoretical ratio of $\sqrt{3}$, if any.

434. Short-Circuit Test. — This test consists in running the alternator at a constant speed with the armature winding short-circuited through a low-resistance ammeter. The field excitation is varied, but is kept quite low so as not to produce an unsafe armature current. The results are plotted as armature amperes vs. field amperes (Fig. 272). On the straight part of the saturation curve the voltage is proportional to the field current. Since the short-circuit armature current is proportional to the induced voltage it is also proportional to the field current.

The short-circuit test is used in practice for various purposes, the principal applications being as follows:

(a) *To check the assembly and the connections of the machine.* For a standard type machine, of which several have been built and thoroughly tested before, it is often unnecessary to perform a load test; in such cases a no-load run (§ 433) to check the magnetic circuit, and a short-circuit test to check the armature winding, will be sufficient. If the short-circuit curve checks closely with those taken on the preceding machines, it shows that the machine has been assembled and connected correctly, and that the soldered connections are good.

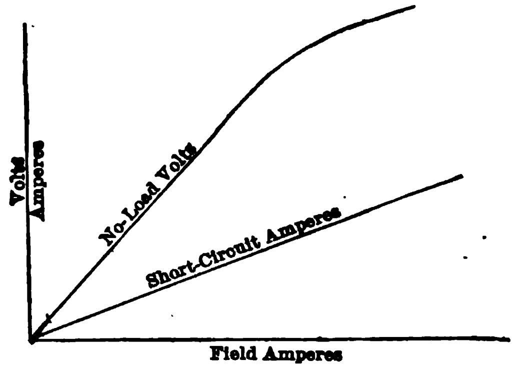


FIG. 272. No-load and short-circuit characteristics of an alternator.

(b) *Heat run and copper loss.* Since the output on short-circuit is zero, only a small motor is required to drive the alternator and to furnish the losses which consist mostly of friction and copper loss. The iron loss is quite small because the field flux is small. Thus, a heat run for the armature winding alone may be conveniently and economically performed, although its results must be used judiciously, because the iron core, being cooler, is helping to conduct the heat away. The input into the driving motor, corrected for the losses in the motor itself, may be taken for the value of the copper loss, friction and windage of the alternator, and used in efficiency computations (§ 436).

(c) *Voltage regulation.* Knowing the exciting current necessary to produce the rated current on short-circuit, one may determine by computation the voltage regulation of the loaded machine at zero power factor (or operating as a synchronous condenser). With certain assumptions, the performance at other values of power factor can also be estimated. This subject is treated more in detail in Chapter XX and in Vol. II.

The technique of the short-circuit test is quite simple, and it is thus desirable for the beginner to perform it, aside from the practical uses to which the results may be put.

435. EXPERIMENT 19-B. — Short-Circuit Test on an Alternator — The purpose of the experiment is to obtain the relationship between the exciting current and the armature current when the armature winding is short-circuited. The practical uses of such a test are explained in the preceding section. The experiment should be performed on the machine used in Exp. 19-A, in order that the efficiency may be computed from the losses, as explained in § 436. The alternator is connected as in Fig. 271, except that an ammeter is placed across the terminals instead of a voltmeter. On a three-phase machine an ammeter should be used in each phase, or else the same ammeter should be transferred from phase to phase (§ 58). In the latter case, impedances equal to that of the ammeter should be used in the other two phases to balance the circuit.

(1) Connect up the driving motor so as to be able to read its input and to compute the corresponding output (§ 319). Run the machine under test short-circuited and at the rated speed. Bring the field current up until the armature current reaches a safe maximum value. Read the A.C. armature current, the corresponding field current, the input into the driving motor, and its field current and speed. Reduce the alternator excitation in steps to zero and take similar readings at each step. The last readings should be taken with the alternator field circuit and armature circuit open. Disconnect the driving motor and run it alone to determine its own losses. Measure its armature resistance. The corresponding D.C. Experiment is 13-B.

(2) Increase the alternator speed to a safe upper limit and adjust the field current so as to maintain an overload armature current that the machine can carry for some time without dangerous overheating. Read the speed, the field current, and the armature current. Keep the field current constant and reduce the speed in steps to as low a value as possible. At each step read the speed and the armature current.

(3) Measure the resistance of the armature winding with and without that of the ammeter and connections.

Report. (1) Plot the short-circuit curve at the rated speed, as in Fig. 272. (2) Plot the readings taken at a constant field excitation and variable speed, and give a theoretical explanation for the general shape of the curve (see the end of § 454). (3) From the values of input into the driving motor compute its output, which represents the sum of the losses in the alternator. Subtract the I^2R loss in the ammeter and its leads, and plot the remainder against field current as abscissæ. The value at zero excitation gives the friction and windage loss, the remainder represents the armature copper loss and a small iron loss. For comparison, plot on the same sheet the computed I^2R loss in the armature. (4) Compute the synchronous reactance according to the definition given in § 454.

436. Efficiency from Losses. — The efficiency of an alternator or of a synchronous motor may be computed from its losses in a manner similar to that described in Chapter XIII for direct-current machines. The losses in a synchronous machine consist of the following: (a) Copper loss in the armature winding; (b) the amount of power necessary for excitation, including the loss in the field rheostat; (c) hysteresis and eddy currents in the armature core; (d) windage and friction; (e) eddy currents in the pole faces, end-shields, frame, etc.

The copper loss in the armature may be computed from the resistance of the winding, or it may be measured directly on short-circuit (§ 434). The latter method is used when the winding consists of heavy conductors and there is reason to suspect an appreciable eddy-current loss and skin effect, which will make the effective resistance higher than that measured with direct current.

The core loss and the friction and windage can be determined by one of the three methods described in Chapter XIII in application to direct-current machines, viz.: (a) The machine is driven mechanically at no load by a small auxiliary motor; (b) The machine is driven electrically as a synchronous motor; and (c) The retardation method is used. The reader should have no difficulty in devising similar experiments for a synchronous machine.

The excitation loss may or may not be taken into account in computing the efficiency. This depends upon the agreement between the parties concerned and upon local conditions, such as the source of the exciting current. When specifying the efficiency of a synchronous machine it is therefore necessary to state whether or not the excitation loss has been included, and, if included, how it was computed.

The losses in the pole faces cannot be measured in any simple way, and are usually disregarded, although under some circumstances they may reach considerable proportions. This is especially true in single-phase machines and in polyphase machines with an induced e.m.f. of poor wave-shape, that is, one departing considerably from a pure sine-wave. The pole-face losses are of the nature of additional load losses, which are discussed in § 316; the statements made there in regard to conventional efficiency apply to synchronous machines as well.

When the losses are known, the efficiency at a desired load is computed by means of one of the formulæ in § 312. An element of uncertainty is introduced by the difficulty of predetermining the exact value of the field current required for the desired load current at a given power factor and terminal voltage. The most accurate way is to make an actual load run under these conditions. If this is not feasible, the required field excitation is determined as explained in § 458, or by a more accurate method described in Vol. II. Another element of uncertainty is caused by the difference in the core losses at no load and in the loaded machine, at the same value of field current. Because of the impedance drop in the armature and of the armature reaction, the true field flux under load is different from that at no load, both in its magnitude and in its space distribution; hence the core loss is also different. This factor is difficult to take into account by computation; if the value of core loss used has been determined from a no-load test, the efficiency should be plainly marked "conventional."

The data obtained in Experiments 19-A and 19-B are sufficient for computing the conventional efficiency from the losses. The first experiment (run 2) gives the core loss and friction, the second experiment (run 1) gives the same friction and the armature copper loss. The latter is augmented, it is true, by some unavoidable core loss which has to be estimated. For a determination of the efficiency of a synchronous machine from an opposition run, see similar experiments on D.C. machines and on transformers (§§ 340 and 391).

437. EXPERIMENT 19-C. — Efficiency of a Synchronous Machine from its Losses. — The theory is explained in the preceding section, and the data available from the preceding two experiments may be used. The field current required for a given load and voltage is best determined by a direct test, or is computed according to § 458.

VOLTAGE REGULATION

438. When a synchronous alternator is loaded at a constant speed and constant field current, the terminal voltage is a function not only of the load current but also of its phase displacement with respect to the terminal voltage. In other words, the operating characteristics of an alternator at a constant speed comprise four factors:

A — terminal voltage
B — field current
C — armature current
D — power factor of the load.

It is impossible to represent the influence of all these factors by a single curve, or even by a single set of curves or a surface (§ 237). Several sets of curves are necessary. For example, in Fig. 273, *A* is plotted against *D* for various values of *C*, keeping *B* constant and equal say to B_1. Other similar sets of curves may be obtained for $B = B_2$, $B = B_3$, etc., covering the useful range of field current.

In Fig. 274, A is plotted against C for different values of D, keeping B constant. There are several different combinations of sets of performance curves, depending on which two quantities are used as coordinates, and which quantities are maintained constant. Of these combinations the following four are of particular practical importance:

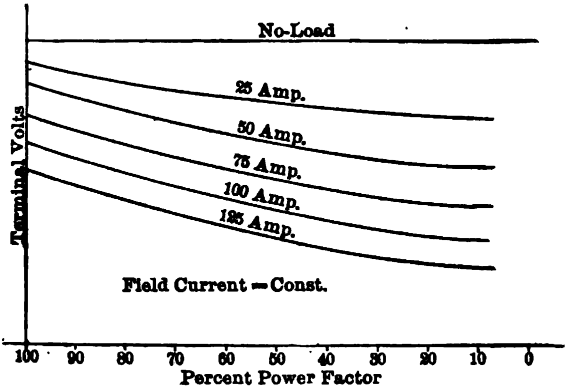


FIG. 273. Voltage characteristics of an alternator as a function of the power factor of the load.

(1) Voltage characteristics at a constant armature current and excitation (Fig. 273).

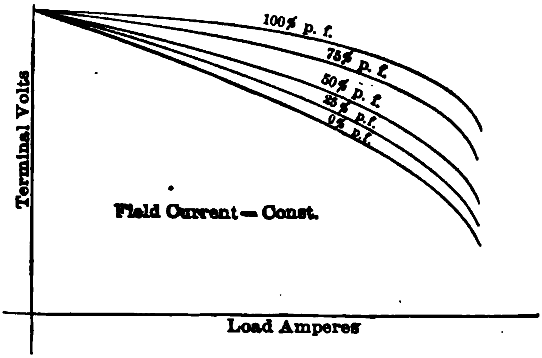


FIG. 274. Voltage characteristics of an alternator as a function of the load current.

(2) Voltage characteristics at a constant power factor and excitation (Fig. 274).

(3) Excitation characteristics at a constant load current and constant terminal voltage (Fig. 275).

(4) Load saturation curves at a constant armature current and power factor (Fig. 276).

One complete set of curves gives a sufficient number of points for determining the operating conditions at any desired values of A, B, C, and D. Therefore, from any one complete set, any of the remaining sets can be plotted by recombining the proper points on the curves of the first set.

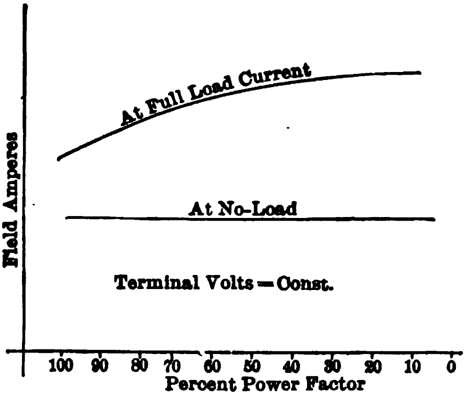


FIG. 275. Excitation characteristics of an alternator.

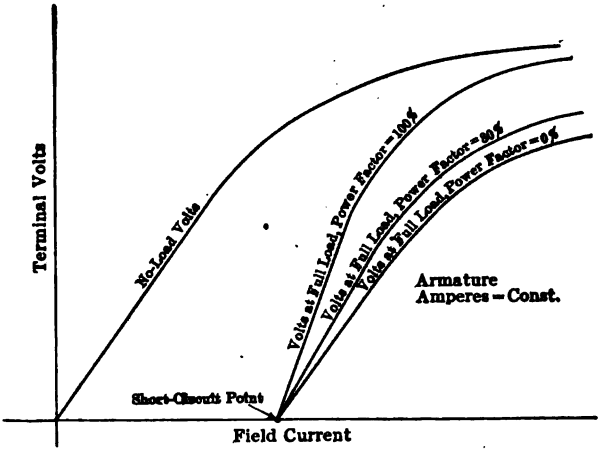


FIG. 276 Load saturation curves of an alternator.

More combinations may be obtained by imposing additional conditions upon the four variables. For example, curves may be plotted between B and C when the product CD is constant and A is constant. Each curve gives a relationship between the armature current and the field current at a constant terminal voltage, when the output of the machine remains constant. Such curves are known as *phase characteristics*, and correspond to the well-known V-curves of a synchronous motor. These phase characteristics are of interest in the operation of a synchronous generator when the prime mover (usually a water-wheel) has no governor and is made to deliver a definite amount of power. The input into the generator is constant, and consequently the output is also approximately constant.

An inspection of the curves in Figs. 273 to 276 shows that with a given lagging current the voltage drop increases as the power factor becomes lower, that is, as the phase displacement between the current and the voltage increases. When the armature current is leading, the opposite is the case. A leading current causes an internal voltage rise instead of a drop. With a sufficiently large leading current, the field excitation necessary for maintaining a given terminal voltage may be less than at no load.

A theoretical explanation of the fact that the terminal voltage is a function not only of the armature current, but of the power factor as well, is given in Chapter XX. It is sufficient here to take this statement as an experimentally observed fact.

439. Load Test on an Alternator. — In order to obtain experimentally the curves shown in Figs. 273 to 276, the alternator is loaded as is shown in Fig. 277. The diagram of connections is shown in application to a single-phase machine, or to one phase of a polyphase machine.

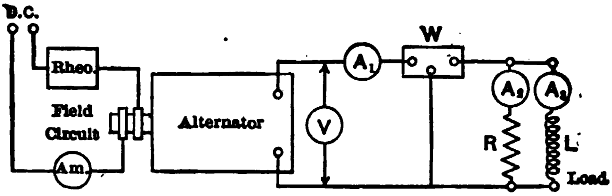


FIG. 277. Diagram of connections of a loaded single-phase alternator.

While a great majority of the alternators used for lighting and power purposes are three-phase machines, the readings, as well as the connections, are somewhat more involved. Moreover, it is difficult to keep the load in the three phases accurately balanced. For these reasons the beginner should first get his experience in loading a single-phase alternator.

The load is shown as consisting of a resistance and an inductance in parallel. This will be found to be a convenient arrangement for regulating both the current and the power factor, although the same result may be accomplished with a resistance and a reactance in series. When the resistance R and the inductance L are used in parallel, it is advisable to have an ammeter in each branch, as this facilitates current adjustment. When R and L are in series, voltage readings across each will make it easier to adjust the power factor to the desired value.

W is a wattmeter, and the power factor is computed as the ratio of watts to volt-amperes. The precautions mentioned in § 91 regarding the power consumption in the instrument itself must be carefully considered, although in many cases this amount may be negligible. An indicating power-factor meter (§ 104) is convenient for an approximate load adjustment, even though it may not be accurate enough for final readings.

When working up the data, it is convenient to draw the three ammeter readings as vectors (Fig. 278). *OE* is an arbitrary direction of the vector of terminal voltage, and the current A_2, through the non-inductive branch of the load, is in phase with it. On the base A_2, the triangle $A_2A_3A_1$ is constructed, the other two readings being used as the remaining sides. This is because the geometric sum of the branch currents A_2 and A_3 must equal the total line current A_1. The angle ϕ at O is equal to the phase displacement between the total current and the voltage, and the power factor is equal to the ratio of *OM* to *ON*. This furnishes a check on the reading of the power-factor meter and on the value of the power factor obtained as the ratio of watts to volt-amperes.

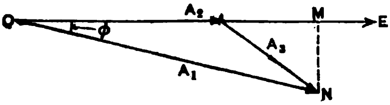


FIG. 278. Diagram of currents read on the three ammeters in Fig. 277.

If the main purpose of the experiment is to become familiar with the technique of testing, it may be sufficient and more instructive to take a few curves of each kind rather than a complete set. In performing the experiment always begin with the most difficult conditions, i.e., the highest field excitation, the lowest power factor, and the largest possible armature current.

However, when the particular purpose of the load test is to analyze the armature reaction (Chapter XX), the load saturation curves (Fig. 276) are of particular importance. Several of these should be taken, with the current both lagging and leading. To complete the set a curve should be taken at a power factor as near zero as possible. A synchronous motor constitutes a convenient load at very low values of power factor. For a leading current it is overexcited, for a lagging current underexcited, or even run on residual field only.

Sometimes it is impossible to run a synchronous motor at abnormally high or low values of field current, because the machine will not stay in step, especially when operated single phase. It can then be driven by a small direct-current motor, to enable it to operate in parallel with the machine under test. When the power factor of the load is below 20 per cent, the terminal voltage of an alternator, at a given armature current, remains practically constant and independent of the power factor (Fig. 273). This performance feature of the alternator makes it unnecessary to adjust the power factor of the load exactly to zero.

440. EXPERIMENT 19-D. — Load Test on an Alternator. — The purpose of the experiment is to determine the influence of the armature current, load power-factor, and field current upon the terminal voltage of the machine. The general procedure is discussed in § 439, and sample curves are shown in Figs. 273 to 276. According to the specific purpose of the test and the available experimental facilities any two factors may be kept constant and the other two varied. The readings may be conveniently recorded on a data sheet such as is shown below.

	Amperes			Volts	Watts	Power Factor	Field Amps.	Speed
	A_2 Non-ind.	A_3 Ind.	A_1 Total					
Inst. No.								
Constant								

If the no-load and short-circuit curves of the machine have not been obtained before (§§ 433 and 435) they should be taken in connection with this experiment. The resistance of the armature should also be measured.

Report. (1) From the wattmeter readings compute the values of the power factor and check them with those read on the power-factor meter and with those obtained from the three ammeter readings (Fig. 278). (2) Plot the test data in some systematic way, so as to bring out clearly the characteristics of the machine and the influence of the desired factor. Show on one or two examples how to obtain "cross-curves," that is, curves for which some other quantity is kept constant. For example, if the terminal voltage is variable for all the curves that have been taken, show how to combine certain points on different curves so as to obtain a relationship between the field current and the armature current for a constant terminal voltage and some constant power factor.

(3) If familiar with portions of the theory given in Chapter XX, use the data obtained to illustrate and to check the effect of the armature reaction and reactance upon the voltage regulation of the machine.

441. Tirrill Regulator. — Because of the armature reaction and leakage reactance, the terminal voltage of an alternator varies within wide limits with changes in load; this characteristic makes an automatic voltage regulation highly desirable. Numerous attempts at compounding alternators have so far proved to be too complicated. Another solution, namely, the use of an automatic regulator *outside* the machine, is quite satisfactory. The Tirrill regulator, described in § 245 in application to direct-current generators, is used with some modifications for maintaining constant terminal voltage of alternators.

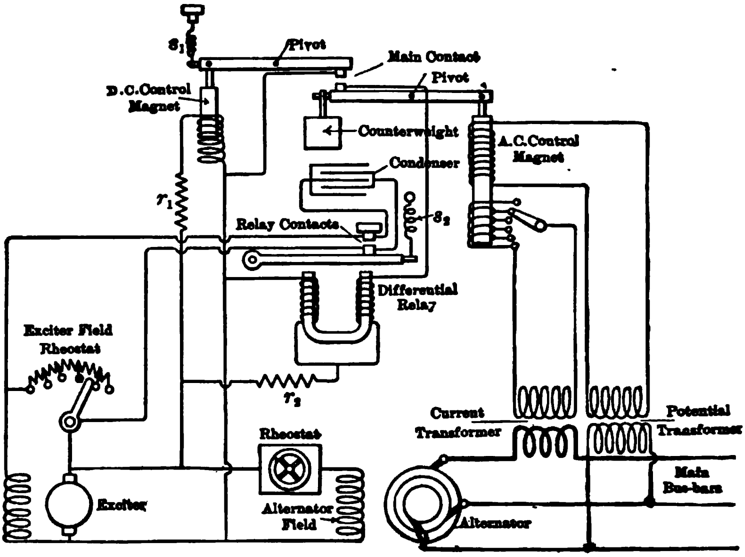


Fig. 279. Diagram of connections of the Tirrill voltage-regulator used with an alternator.

The regulator is shown diagrammatically in Fig. 279. Its function consists in periodically short-circuiting the field rheostat of the exciter. The relative duration of short-circuit and open circuit of the contacts is made to depend on the terminal voltage of the alternator. The exciter rheostat is short-circuited between two relay contacts, when the differential relay under the contact arm is deënergized. This relay is controlled by the main contacts, which are closed and opened by the direct-current electromagnet shown at the left. The duration of the contact is determined by the position of the right-hand lever actuated by the alternating-current control electromagnet.

Assume the main contacts to be open, so that only the left-hand side of the differential relay is energized and the relay contacts are opened. This brings the exciter voltage down, and the spring s_1 closes the main contacts. A current flows through the right-hand leg of the differential relay, destroying its magnetism. Thereupon the relay contacts are closed by the spring s_2, and the exciter voltage increased. The D.C. control magnet overcomes the action of the spring s_1 and again opens the main contacts, etc. Thus, both contacts are vibrating all the time, maintaining a certain exciter voltage. A condenser is placed across the relay contacts to reduce sparking.

Since the lower main contact is attached to a movable arm, the interval of time during which this contact is closed depends upon the position of the arm. Should the terminal voltage of the alternator rise above normal, the core of the A.C. control magnet is pulled upward. This moves the main contacts farther apart, and the time of short-circuit of the exciter rheostat is reduced. The alternator voltage is thus brought back to normal. Should the A.C. voltage be low, the same solenoid brings the main contacts closer together.

Sometimes it is desired to have the alternator over-compounded, that is, to give a higher voltage with increasing load. This is sometimes advisable in order to compensate for the impedance drop in a long transmission line. For this purpose the A.C. control electromagnet is provided with a second winding connected to the main line through a current transformer, as shown in the sketch. When the line current increases, the series winding pulls the core of the A.C. control magnet down, thus forcing the main contacts to operate on a higher plane. This increases the tension of the spring s_1 and a higher exciter voltage is required to balance it. The increased exciter voltage gives the correct A.C. voltage under the changed load conditions.

The current and potential transformers are so connected to the busbars that at unity power factor the current in the series winding is practically in quadrature with that in the potential winding. Referring to Fig. 284, it will be seen that the current I_1 is in quadrature with the voltage E_{23} when $\phi = 0$. At zero power-factor of the load, the currents in the two windings are in phase opposition with each other. Thus the controlling action of the series winding is a minimum at 100 per cent power factor and reaches its maximum at zero power-factor when more excitation is needed. The number of turns in the compensating winding is adjustable for different degrees of over-compounding. More accurate voltage regulation at the distant end of the line is obtained by means of a line-drop compensator such as are used with voltmeters.

When two or more alternators in parallel (Chapter XXI) are controlled by Tirrill regulators, it is necessary to check reactive circulating currents between the individual machines by automatically adjusting their excitation. In this case the A.C. control electromagnet of each regulator, in addition to a potential winding, is provided with a compensating winding of the type already described. When the power factor is unity and the load current is properly balanced between the different machines, the field produced by the current coil is 90 degrees out of phase with the field produced by the potential coil, and has but a slight effect. Should a circulating current tend to flow between the generators, the regulator will be instantly affected and will raise or lower the generator excitation, as required, to eliminate the circulating current. This is because the latter is nearly 90 degrees out of phase with the load current, and therefore in phase with, but directly opposed to, the current in the potential coil, thereby changing the pull of the potential winding until balanced conditions have been restored. Should it be necessary to compensate also for the voltage drop in the line, additional current transformers and line-drop compensators are required.

442. EXPERIMENT 19-E. — Study of the A.-C. Tirrill Regulator. — The device is described in the preceding section. An experimental investigation of a similar D.C. regulator is outlined in Exp. 11-B. An additional study should be made of the effect of the power factor of the load and of the adjustment of the counter-weight placed on the arm controlled by the A.C. electromagnet.

FREQUENCY METERS

443. The frequency, or the number of cycles per second of an alternating-power supply, can be measured directly by means of instruments called frequency meters. There are three types of such instruments in general use, which may be designated as

(1) The vibrating-reed type;
(2) The differential-voltmeter type;
(3) The iron-needle type.

These three types are described below in more detail. For frequency measurements by means of A.C. bridges see Vol. II.

444. The Vibrating-Reed Frequency Meter. — This meter, also known as the Frahm frequency meter, is based upon the principle of mechanical resonance, the property by virtue of which an elastic body vibrates vigorously when subjected to rhythmic impulses of the same frequency as the natural period of vibration of the body itself.

How this well-known law has been utilized for the measurement of frequency may be gathered from Fig. 280. A number of reeds *R* (usually 3 mm. wide) consisting of special spring steel, carefully tempered, are screwed in a row to a bridge-piece *B*, to which is attached the armature *A* of a small electromagnet *M* mounted close to it. When the instrument is connected across the circuit whose frequency is to be measured, the current, after passing through a series resistance *G*, excites the magnet, which thus imparts to the armature *A* an impulse for each alternation of the current. The vibration set up in *A* is transmitted to the reeds *R* and just as one tuning fork in operation will excite another of the same period of vibration, so will that reed which is in tune with the frequency of the current, at once respond vigorously. In order to give a clear indication, the reeds have a small portion of their upper ends bent over at right angles and enameled white, so as to make them conspicuous against the black interior of the instrument.

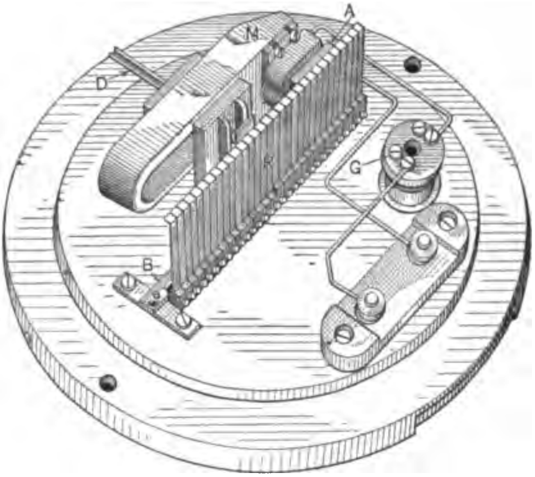


Fig. 280. Vibrating-reed frequency meter.

On first consideration it may seem that this method of indication is only capable of giving results in an intermittent, step-by-step fashion. The number of reeds in one meter is naturally limited, and it would therefore appear that indications would depend on the chance of the frequency to be measured corresponding exactly to that of one of the reeds in the instrument. In practice this is not the case. The vibrations of each reed are not confined to one mathematically exact value, but extend over a certain range of frequencies. They commence about 2 per cent below full value, reach their greatest amplitude at the exact frequency for which they are tuned, and at 2 per cent above this still show clearly. Actually, therefore, more than one reed is usually in motion, and from the relative lengths of the vertical bands formed by them the exact value of the frequency may be estimated to a fraction of a unit.

The instrument is fitted with an adjusting screw (shown at *D*) by means of which the distance between the armature and the magnet may be varied. This enables the user to secure full amplitude of the reeds for voltages varying within 20 per cent of that for which the instrument was calibrated. This is particularly desirable with portable meters, where the voltage to be employed is frequently not known beforehand.

The fact that in Frahm instruments the reeds are excited mechanically through the bridge-piece, affords means for providing any meter with a second scale range (double that of the first) by a very simple device. For this purpose a second electromagnet, having a permanent magnet for its core instead of one of soft iron, is connected in. The current then merely weakens and strengthens the flux of the permanent magnet, thus giving to the armature but one impulse for each complete cycle (2 alternations). Consequently, double the frequency is required with this magnet to cause any particular reed to vibrate in tune. A special felt-lined damping bar is fixed behind the scale, just out of reach of the swinging reeds under normal conditions. In cases of sudden overload, the reeds touch this bar and are thereby prevented from swinging through so great an arc as to suffer permanent deflection or to become liable to fracture. These instruments have no pivoted moving parts or jeweled bearings, are accurate in any position of the meter, and their indications are independent of the voltage or the wave-form of the supply. They are not affected by external magnetic fields and their energy consumption is very small.

445. The Differential-Voltmeter Type Frequency Meter. — The frequency meter shown in Fig. 281 is essentially a combination of two induction voltmeters described in § 52. Two split-phase electromagnets, M_1 and M_2, act in opposite directions on the aluminum disk D, thus constituting a differential voltmeter. To make the instrument respond to changes in frequency, the winding of one of the electromagnets is connected in series with an inductance L, and the other winding with a resistance R. The current in the branch M_1, containing the resistance, is practically independent of the frequency; the current in the branch M_2 decreases as the frequency increases, thus giving

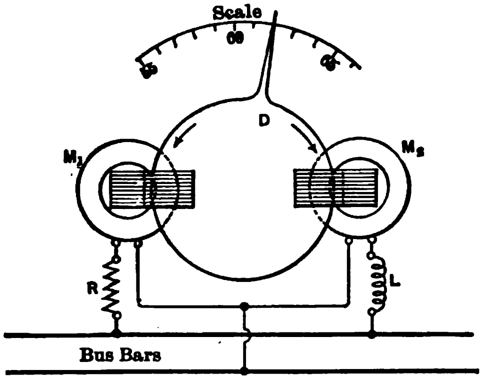


FIG. 281. A frequency meter of the differential-voltmeter type.

preponderance to the electromagnet M_1. For each frequency there is a definite stable position of the aluminum disk, and the instrument is calibrated in cycles per second by connecting it to an alternator whose speed is measured directly with a speed counter.

The aluminum disk, acted upon by the magnets, is so arranged that when the shaft turns in one direction the torque of the magnet tending to rotate it decreases, while the torque of the other magnet increases. The pointer therefore comes to rest where the torques of the two magnets are equal. This arrangement insures freedom from error due to varying voltage.

446. The Iron-Needle Type Frequency Meter. — This meter (Fig. 282) has two sets of stationary field coils 1 and 2, mounted with their axes perpendicular to each other. An iron needle is pivoted and free to rotate within these coils. A reactance, x_1, is connected in series with coil 1, and a resistance, r_2, in series with coil 2. The first combination is shunted by a resistance r_1, the second by a reactance x_2. The whole is connected across the source of supply, in series with the reactance x.

At a low frequency, a larger current flows through coil 1 and a smaller one through coil 2. The magnetic field of coil 1 predominates, and the iron needle takes the corresponding position of the resultant field, indicating the frequency on the scale. At a high frequency, the field due to coil 2 predominates. Thus, for every frequency within the limits of the instrument range there is a definite position of the iron needle.

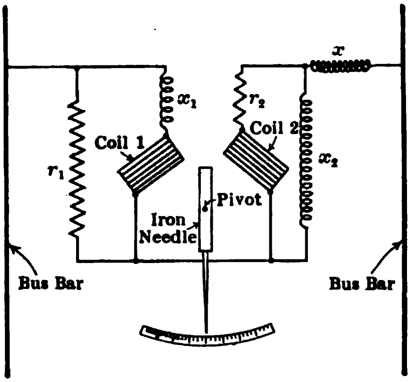


Fig. 282. The iron-needle type frequency meter.

The reactor x serves to cut down higher harmonics in the supply voltage. By a proper design of the reactors, the instrument has been made practically independent of fluctuations in the applied voltage.

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447. Three-Phase Y-Connection. — The three separate armature windings, shown in Figs. 267 and 268, are usually connected according to Fig. 283. The vectorial relations of currents and voltages are shown in Fig. 284. Each winding is connected on one side to a common point O, and on the other side to a line conductor. The load is similarly connected in Y to the line and to the neutral point O'. The two neutral points may or may not be connected together. This neutral connection, or the fourth conductor, is shown by a dotted line and is often omitted. Sometimes one or both neutral points are simply connected to the ground.

An ammeter is shown in each phase, and the load can be so balanced that the three instruments read exactly the same. In this case the three currents differ from each other in time phase by exactly 120 electrical degrees, as shown in Fig. 284. According to the first Kirchhoff law, the current in the neutral connection is equal to the sum of the three line currents. But the geometric sum of three equal vectors displaced by 120 degrees is equal to zero. *In a balanced three-phase system with strictly sinusoidal currents, the common return current is equal to zero.* For this reason the return conductor is often omitted when the load can be kept fairly well balanced. With non-sinusoidal currents there is some current in the neutral even on balanced load. This current consists of the third, ninth, fifteenth, etc. harmonics.

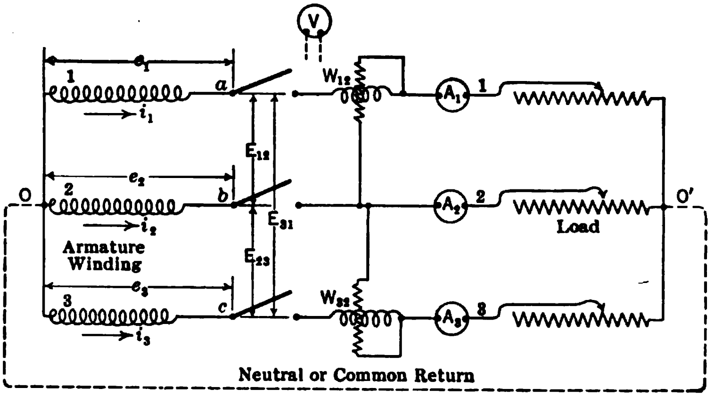


Fig. 283. A three-phase Y-connected system.

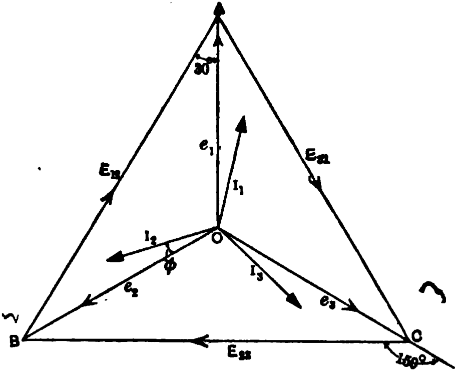


Fig. 284. Current and voltage relations in a Y-connected three-phase system, at a balanced partly inductive load.

Two kinds of voltages are distinguished in a Y-connection, viz., the *phase voltages*, e_1, e_2, e_3, and the *line voltages*, E_{12}, E_{23}, E_{31}. At any particular instant the voltage e_1 indicates the amount by which the potential of point a is above that of point O. Similarly, e_2 gives the potential of point b above that at O. Therefore, the instantaneous value of the voltage E_{12} between a and b is equal to the difference between the instantaneous values of e_1 and e_2. But a relationship which holds algebraically for instantaneous values of currents or voltages, holds true geometrically for their vectors. For this reason, in Fig. 284 each E is shown as the geometric difference between the two corresponding e's. The three e's are displaced in phase by 120 degrees because of the arrangement of the armature windings. As a consequence, the three E's are also displaced by 120 degrees, but are by 30 degrees out of phase with the e's.

From the geometry of the figure it will be found that *in a balanced three-phase system with pure sinusoidal e.m.f.'s the ratio between the line voltages and the phase voltages is equal to the square root of three;* in other words,

$$E = e\sqrt{3} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (11)$$

With a balanced system the points O and O' are at the same potential, and the phase voltages may be measured with respect to either. When the induced voltages are not sinusoidal, the ratio of E to e differs from $\sqrt{3}$.

With a non-inductive load the three currents are in phase with the corresponding voltages aO', bO', cO'. On a balanced load, these voltages are equal to the phase voltages e_1, e_2, e_3, so that the currents I are in phase with the voltages e. When the load is partly inductive, the currents I lag behind the voltages e by the corresponding angle ϕ, as shown in Fig. 284.

The power delivered to the load can be measured either in each phase separately, or by the two-wattmeter method (§ 100), using the wattmeter movements W_{12} and W_{32} (Fig. 283). See also Figs. 87 and 104. A more detailed treatment of polyphase systems will be found in Vol. II.

CHAPTER XX

THE ALTERNATOR — ARMATURE REACTION

448. THE brief outline of theory given below is intended as a supplement to the preceding chapter, in order that the reader may account at least qualitatively for the observed changes in the terminal voltage of a loaded synchronous alternator. Some statements are given without a rigid proof, and for such proofs the reader is referred to special works on the subject, among others to the author's "Magnetic Circuit."

449. Causes of Voltage Drop in an Alternator. — There are three component causes of the difference between the no-load voltage and the terminal voltage under load, with the same field current and speed, viz.:

(1) *Armature Resistance.* The resistance drop is in phase with the armature current I and is equal to IR. When there are considerable eddy currents in armature conductors, the effective resistance (§§ 149 and 385) should be used, and not the value measured with direct current.

(2) *Leakage Reactance of Armature Winding.* Alternating currents in the armature winding excite magnetic leakage fluxes, linking partly with one phase only, partly with two or more phases. These fluxes act like those of self- and mutual inductance (Chapter VI). With a symmetrical polyphase winding the effect of the mutual inductance can be combined with that of the self-inductance of each phase, giving the so-called equivalent inductance per phase. Multiplying this inductance by $2\pi f$, where f is the frequency of the armature currents, we get the so-called equivalent reactance x per phase. The voltage drop Ix, due to this reactance, is in leading quadrature with the current I.

(3) *Armature Reaction.* When a current is flowing through the armature winding, it becomes a source of magnetomotive force which is combined with that of the field winding, weakening (or strengthening) and distorting the field flux. The flux having thus been modified, the e.m.f. induced by it in the armature winding is different from that induced by the original flux at no load. The actual effect of the armature reaction is quite complicated; but for practical purposes the

magnetomotive force of the armature may be resolved into two components:

(a) Direct armature reaction, whose ampere-turns can be directly subtracted from (or added to) those of the field winding.

(b) Transverse armature reaction, whose effect in a generator is to shift the field flux against the direction of rotation of the poles, and in a synchronous motor in the direction of rotation of the poles.

The armature reaction is simpler in a polyphase machine than in a single-phase one, because in a polyphase machine the magnetomotive forces of the individual phases are combined into one resultant magnetomotive force which glides along the air-gap at the same angular velocity as the field poles (§ 472). Therefore the relative position of the field and armature m.m.f.'s remains unchanged with the time.

In a single-phase machine the armature reaction is pulsating, while the field magnetomotive force is rotating. A pulsating magnetomotive force may be resolved into two magnetomotive forces gliding in the opposite directions, each component being of one-half the amplitude of the original m.m.f. The m.m.f. gliding in the direction of the poles is combined with that of the field winding, as in a polyphase machine. The other component excites a flux which is partly wiped out by eddy currents which it induces in the pole faces, and partly induces an additional double-frequency voltage in the armature conductors. This subject is outside the scope of this work.

450. Armature Reactance. — A current in the armature winding of an electrical machine produces leakage fluxes linking with the armature conductors. Three kinds of paths of such leakage fluxes are usually distinguished, *viz.*, those in the slots, those between the tooth tips, and those around the end connections. When the current is alternating, these fluxes vary with the current and induce in the armature winding e.m.f.'s in lagging quadrature with the current. In other words, the armature winding of an a.c. machine, with its leakage fluxes, acts as a reactive coil interposed between the machine and the external circuit. The voltage induced by these fluxes, divided by the corresponding armature current, is called the armature reactance. In order to be able to predetermine the performance characteristics of an alternator or synchronous motor, its armature reactance must be known.

In a polyphase machine, the leakage fluxes due to one of the phases induce voltages not only in that phase, but in the other phases as well. However, with a symmetrical winding and at a balanced load, the mutual effect of all the phases is the same, and the mutual inductance is usually taken into account by a suitable modification of the self-inductance of each phase. Therefore, in a polyphase machine (with the above limitations) it is possible to speak of the equivalent armature reactance per phase, as if there was no mutual inductance.

The leakage field linking with the armature conductors depends not only upon the currents in these conductors, but upon the field excitation as well, because the latter limits the leakage fluxes to certain paths. Therefore, it would be wrong to measure the armature reactance on a machine standing still, by simply sending an alternating current (or currents, in a polyphase machine) through the armature winding from an external source, as in the case of an ordinary impedance coil (§ 138). The fluxes excited by such external currents are entirely different from the leakage fluxes produced by identical currents with the machine excited and running at synchronous speed.

No simple direct method of measurement of armature reactance being available, the following indirect methods are used:

(*a*) From a short-circuit test (§ 434). In this case the direct armature reaction is practically the only one present, and it can be computed knowing the number of turns and the arrangement of the armature winding. The effect of the armature reaction having been eliminated, the remaining voltage drop is due to the armature impedance. For details see § 452.

(*b*) From a load saturation curve at zero power-factor (Fig. 276). In this case also the direct armature reaction can be eliminated, as explained in § 455, and the reactive voltage drop in the armature computed.

(*c*) From estimated permeances of leakage paths per unit length of coils. The values of unit permeances are based upon previous tests on machines with similar windings. For details see the author's "Magnetic Circuit," p. 229.

(*d*) From an air characteristic, as described below.

The Air Characteristic. It has been found by numerous experiments that the true leakage reactance of the armature of a polyphase synchronous machine may be computed from an impedance test on the same armature supplied with alternating currents from an external source, *the field structure being completely removed.* The straight line plotted between volts and amperes from such a test is known as the air characteristic of the machine. From the total measured voltage drop, the resistance drop is subtracted vectorially, and the remainder is equal to the voltage induced in the armature by the magnetic fluxes.

These fluxes may be roughly divided into (*a*) the true leakage fluxes, the same that exist in normal operation, and (*b*) those which extend over a pole pitch and which correspond to the useful fluxes in a loaded machine. Subtracting the induced voltage due to the latter fluxes, gives the voltage induced by the true leakage fluxes. This voltage, divided by the corresponding current, gives the desired armature reactance.

In evaluating the voltage due to the (*b*) fluxes we shall use the following notation and units:

E the e.m.f. per phase, induced by the (*b*) fluxes, in volts.
f the frequency, in cycles per second.
i the effective value of the armature current, in amperes.
k_b the breadth factor of the armature winding for the fundamental of the revolving m.m.f.
L the gross axial length of the stator.
m the number of phases.
M the amplitude of the fundamental of the revolving m.m.f. of the armature, in ampere-turns.
n the number of armature turns, per pole per phase.
p the number of poles of the machine.
Φ the flux described under (*b*) above, in maxwells per pole.

It has been proved by M. Schenkel (*Elektrotechnik und Maschinenbau*, Vol. 27, 1909, p. 207) that

$$\Phi = 0.8\pi\, ML \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

With the approximate assumptions made in the deduction of this formula, the flux Φ comes out independent of the number of poles and of the bore of the machine. In this expression

$$M = 0.9\, k_b\, mni \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

(see the author's "Magnetic Circuit," p. 130). The flux Φ induces in the armature winding the following voltage per phase:

$$E = 4.44\, k_b\, f\, n\, p\, \Phi\, 10^{-8} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

Substituting the value of Φ from eq. (1) and using in it expression (2) for M, we get

$$E = f(k_b n)^2\, miLp\; 10^{-7} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

By means of this formula the voltage due to the fluxes extending from pole to pole may be evaluated, provided that n and k_b can be ascertained. Subtracting E arithmetically from the total reactive voltage computed from the air characteristic, the armature reactance can be determined as is explained above.

While this method involves several assumptions, it gives results which are reasonably close for practical purposes. In good synchronous machines the purely reactive drop at the rated current is less than 20 per cent of the rated voltage; moreover this drop is mostly added to the induced or terminal voltage geometrically, at a considerable angle. Consequently, even a considerable error in the determination of the armature reactance will affect the resultant calculated voltage comparatively little. On the other hand, with modern highly saturated machines even a small difference in the induced voltage means a considerable increase in the existing ampere-turns, and for this reason the armature reaction should be known as accurately as possible.

451. Experiment 20-A. — Determination of the Armature Reactance of a Synchronous Machine. — The available methods are described in the preceding section. This is a somewhat advanced experiment which ought to be performed quite accurately, and the results computed judiciously in order to get consistent values. The reactance should be determined by two or three independent methods in order to have a basis for comparison and a mutual check.

452. Direct Armature Reaction on Short-Circuit. — The voltage induced by the field flux in a short-circuited armature (§ 434) is used up in overcoming its impedance drop. In large and medium-sized alternators of the usual proportions, the reactive drop is several times larger than the resistance drop, so that for many practical purposes the latter may be neglected. Therefore, *in a short-circuited armature the current is approximately in quadrature with the induced voltage*. This means that while the induced voltage in an armature conductor reaches its maximum when the conductor is opposite the center of a pole, the current in the same conductor reaches its maximum when the conductor is midway between two poles. By actually considering instantaneous values of currents and positions of the poles, the following general proposition may be proved:

In a polyphase synchronous machine in which armature currents reach their maximum values when the corresponding conductors are midway between the poles, the direct armature reaction is the only one present. It opposes the field m.m.f. when the armature current is lagging behind the induced e.m.f., and strengthens it when the current is leading. By direct armature reaction is meant a magnetomotive force with its maxima opposite the center lines of the field poles and moving synchronously with the poles.

Neglecting the armature resistance, the short-circuit current satisfies the foregoing condition, and hence on a short-circuit the direct armature reaction is practically the only one present. This makes the short-circuit test and the corresponding curve (Fig. 272) particularly valuable in the study of alternator characteristics.

Let the field ampere-turns (Fig. 285) be Oa at a certain armature current I on short-circuit, and let the demagnetizing action of the armature be equivalent to $ab = In$ ampere-turns, where n is the number of *effective* armature turns per pole. Therefore, the *net* excitation is equal to Ob ampere-turns, and the induced voltage is equal to bc. This voltage causes the current I to circulate through the armature winding, and therefore bc is equal to Ix, since the ohmic drop IR is neglected.

Thus, for a machine whose design constants are known, the field excitation necessary for a given short-circuit current I can be computed

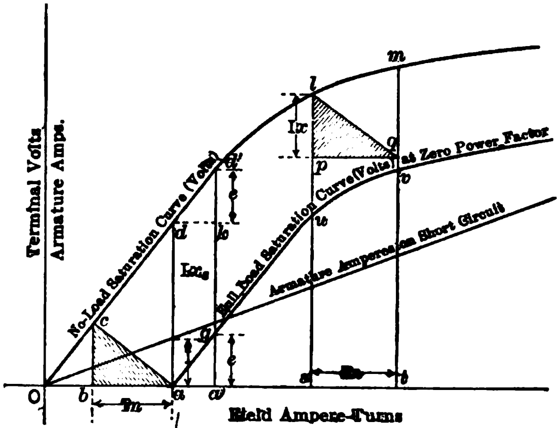

Terminal Volts
Armature Amps.
No-Load Saturation Curve (Volts)
Full Load Saturation Curve (Volts) at Zero Power Factor
Armature Amperes on Short Circuit
Field Ampere-Turns
m
l
Ix
q
p
v
u
a'
d
b
e
Ix_s
c
g
e
a
b
O
b
In
a
a'

FIG. 285. Direct armature reaction and synchronous reactance in a synchronous machine.

as follows: Compute the value Ix and find the corresponding abscissa Ob on the no-load saturation curve. To this value add the ampere-turns In of the direct armature reaction. This will give the required field ampere-turns Oa.

The problem is different when the machine constants, x and n, are not known, because the short-circuit test gives only the value of Oa, in amperes, for a given value of armature current I. The equivalent number of armature turns has to be determined either by some special measurement, by computation, or indirectly from the known value of $Ix = bc$. But the armature reactance x cannot be easily and accurately determined either by test or by computation (§ 450). For these reasons, the armature reactance and the direct armature reaction are sometimes combined into one fictitious quantity known as the *synchronous reactance* (§ 454).

453. EXPERIMENT 20-B. — A Study of the Phase Position of Short-Circuit Current in a Synchronous Machine. — While it is fairly evident from the foregoing discussion that the short-circuit current must be nearly in quadrature with the induced voltage, unless the ohmic resistance of the armature is unusually high, some simple experimental proof may be desirable. The following three experimental demonstrations on polyphase alternators may be of interest:

(1) Two identical alternators are coupled mechanically in such a way that their no-load e.m.f.'s are in phase with each other. One machine is short-circuited through the series winding of an indicating wattmeter and an ammeter. The other machine is run at no load, and its terminals are connected to the potential winding of the same wattmeter, and to a voltmeter in parallel with it. The wattmeter reading will be small, and the phase displacement ϕ between the current and the voltage may be computed from the three instrument readings. The two machines can also be coupled so that their poles are in space quadrature. Then the wattmeter reading will be large and the ratio of watts to volt-amperes will give the reactive factor, Sin ϕ (§ 102).

(2) An alternator is run short-circuited at a certain field excitation Oa (Fig. 285) and the armature current I. Then the armature is loaded on a highly inductive load (of nearly zero power-factor) and the field excitation is increased to such a value Oa' that the armature current is again equal to I. Let the measured terminal voltage be e. Plotting Oa' we obtain the corresponding no-load voltage $a'd'$, and since the amount ad is lost in the armature as before, the difference $a'd' - ad = kd'$ must be equal to the observed voltage e. This is true only if the point d' lies on the straight part of the saturation curve, and the experiment must be planned accordingly. The underlying idea of this proof is that the addition of an external reactance does not change the large phase displacement between the induced voltage and the current, so that the internal reactions in the machine remain the same. A similar experiment should be tried with a non-inductive load; the difference $d'k$ will not be found equal to the observed value of e.

(3) Arrange an electrical contact to be closed or opened every time the center of a pole is opposite the center of a group of adjacent conductors belonging to one phase. Connect a separate source of direct current (say a dry cell) through this contact and through one of the elements of a double oscillograph (Vol. II). Connect the other oscillograph element across a separate ammeter shunt in series with the ammeter which short-circuits the armature winding. The oscillograph record will consist of an alternating-current wave and of a series of marks indicating the instants when the centers of the poles passed by the groups of conductors. A study of this record, extended to the three phases, will demonstrate that the resultant m.m.f. of the armature is practically in space opposition with the field m.m.f., and that the short-circuit current exerts only a direct reaction.

454. Synchronous Reactance on Short-Circuit. — The difficulties connected with an experimental separation of the armature reaction from the armature reactance (end of § 452) have led to the adoption of a fictitious variable quantity known as the synchronous reactance of the machine. While the author is not in favor of this term or of the quantity itself, it has been used in the past and is occasionally mentioned in the literature of the subject. The student should therefore become familiar with the definition, uses, and limitations of this quantity.

By definition the synchronous reactance of an alternator armature on short-circuit, x_s, is a fictitious reactance which would cause the same voltage drop on short-circuit as the combined action of the true armature reactance x and of the armature reaction In. In other words, the drop $Ix_s = ad$ (Fig. 285) is taken to correspond to the induced voltage at no load for the field excitation Oa, as if the armature reaction did not exist. This quantity, x_s, remains constant only at zero power-factor, and then only when the magnetic saturation is low. It must be used with great discretion, and is liable to give wrong results.

Similarly, the *synchronous armature turns*, n_s, may be defined as a fictitious number of armature turns, such that $In_s = Oa$. In this case the armature is assumed to be devoid of reactance and to oppose the full field ampere-turns by an equivalent m.m.f. alone. The same warning applies to this concept as to that of synchronous reactance. Both the true armature reaction In and the true reactive drop Ix are unavoidably present, and while for certain practical purposes they may be combined into one fictitious quantity, the physical side of the phenomenon must not be lost sight of.

The synchronous reactance and synchronous turns are safe concepts to use as long as the operation is considered at very low values of power factor and on the straight part of the saturation curve. When these two conditions are fulfilled simultaneously, x_s and n_s remain constant and equal to their values as determined from a short-circuit test. As examples of the use of synchronous reactance, the following two practical applications will be considered:

(a) *Transient short-circuit current.* The value of the steady armature current obtained from the short-circuit curve (Fig. 285) is considerably below that which flows at the first instant after a *sudden* short-circuit. Let the machine be running at no load and then be suddenly short-circuited. The main magnetic flux, which was excited by the field winding alone, cannot be affected instantly by the armature reaction, because the exciting current rises, on account of the transformer action, and opposes the armature reaction. Thus, for the first few cycles the short-circuit current is practically limited by the true armature reactance x alone. Consequently, the transient short-circuit current is greater than the stationary short-circuit current in the ratio of x_s to x. In large low-frequency turbo-alternators this ratio may be quite large, and a momentary short-circuit current is sometimes destructive to the machine.

(b) *Variations of short-circuit current with speed.* Within wide limits of speed, and at a given field excitation, the short-circuit current increases very slowly with the speed. This is because both the synchronous reactance and the induced voltage are proportional to the speed. Neglecting the resistance of the armature winding, the short-circuit current is equal to the ratio of the induced voltage to the synchronous reactance, and is therefore independent of the speed. At low frequencies the effect of the resistance becomes appreciable, and the short-circuit current rapidly decreases to zero.

455. Voltage Drop with Load of Zero Power-Factor. — In Fig. 285 the curve *auv* represents the terminal voltage with a purely inductive load, when the armature current is kept at a constant value I, and the field excitation varied from the value *Oa* on short-circuit to a higher value. If the armature reactance x and the leakage factor of the machine (§ 169) did not depend upon saturation, a point could be located on this curve by taking an arbitrary point l on the no-load saturation curve and laying off *lq* equal and parallel to *ca*. It could then be argued that, the armature current being the same, the values of In and Ix must be the same on short-circuit as under load, and that the triangle *abc* could be transferred into position *qpl*. In other words, curve *auv* could be obtained from *Olm* by shifting the latter parallel to itself by the amount and in the direction of *ca*.

In reality, point v lies somewhat below q, and the shifted no-load saturation curve needs a further empirical correction in its upper part, to make it coincide with the actual load saturation curve at zero power-factor. In the case of many turbo-alternators and high-speed water-wheel generators the discrepancy is small; but it may be quite pronounced in machines having high reactance, high saturation, and large magnetic leakage.

The synchronous reactance at point v can be computed from the equation $vm = Ix_s'$. It will be seen from the figure that $vm < ul < ad$, so that the synchronous reactance decreases considerably as the magnetic circuit of the machine becomes saturated. It would be wrong, therefore, to use the value of x_s, computed from the short-circuit test, for a predetermination of voltage drop and regulation within the normal operating range of the machine, even at zero power-factor.

The relationship between the terminal voltage E and the induced e.m.f., E_0, at zero power-factor, with the current lagging, is also shown vectorially in Fig. 286. The armature reaction exerts a direct demagnetizing action, and the induced voltage is reduced from OD_0 at no load to the value OD. Now, taking OD as the true induced e.m.f. and neglecting the ohmic drop, we obtain the terminal voltage by subtracting the true reactive drop Ix. The same difference between E_0 and E is also shown to be due to a fictitious reactance drop Ix_s. With the current I in leading quadrature, both BD and DD_0 are added to OD_0, making the terminal voltage E higher than the induced e.m.f., E_0.

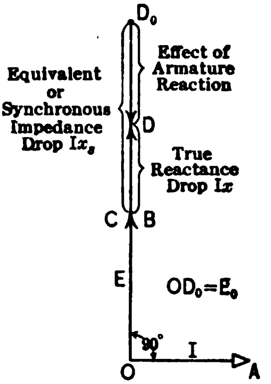

D₀
Equivalent or Synchronous Impedance Drop Ix_s
Effect of Armature Reaction
D
True Reactance Drop Ix
C
B
E
OD₀=E₀
90°
I
O
A

FIG. 286. Voltage drop in a synchronous machine at zero power-factor, with a lagging current.

The diagram shown in Fig. 286 also explains why an over-excited synchronous motor at no load draws a leading current from the line. The machine is shown over-excited because the induced voltage E_0 is greater than the terminal voltage E. The current I is lagging with respect to E, but is leading the line voltage, $-E$ (equal and opposite to E), by 90 degrees. In this case both the armature reaction and the reactive drop tend to reduce the induced voltage, and the current I automatically assumes such a magnitude that the reduced voltage is equal and opposite to the given line voltage.

456. EXPERIMENT 20-C. — Alternator Characteristics at Zero Power-Factor. — The purpose of the experiment is to obtain the lower saturation curve *auw* shown in Fig. 285, and to apply to it the theory given in the preceding section. The experiment should be performed on a three-phase or two-phase machine on which the no-load and short-circuit characteristics have been previously taken (§§ 433 and 435), or else these curves must be included as part of this experiment. A load is provided, and the readings at zero power-factor are taken, as explained in § 440. As a matter of fact, the load saturation curve at zero power-factor is included in Exp. 19-D, but it is desired here to obtain this curve and the other two shown in Fig. 285 with particular accuracy, for the purpose of applying the theory. The analysis of the data can be carried much further if the results of Exp. 20-A on the same machine are available.

Report. (1) Plot the curves shown in Fig. 285. (2) Cut out a piece of paper to correspond to the shape of the no-load saturation curve and find by trials the direction in which it must be shifted to fit the lower part and the knee of the other curve. This will give *ac* in magnitude and direction, and consequently the values of *In* and *Ix*. (3) Check the value of x thus obtained with that found by direct measurement in § 451. (4) Plot the theoretical saturation curve as the locus of points q. (5) On the same curve sheet, plot a curve of values of synchronous reactance.

457. Voltage Regulation at Values of Power Factor other than Zero. — This subject cannot be fully treated here because of the complexity of the numerical relations. The brief description given below is intended merely to impart to the reader a correct qualitative idea of the factors involved. While both the armature reaction and reactance act simultaneously, it is advisable to consider their effect separately.

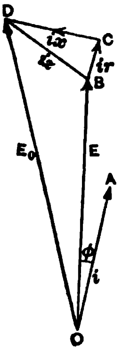


FIG. 287. Voltage drop in an alternator, at a high power factor.

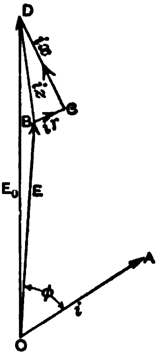


FIG. 288. Voltage drop in an alternator, at a low power factor.

(a) *Armature reactance.* The vectorial relations at a high and at a low value of power factor are shown in Figs. 287 and 288 respectively. The terminal voltage is denoted by E and the phase angle between it and the load current by ϕ. The net induced voltage, E_0, is obtained by adding to E the ohmic drop ir in the armature, in phase with the current, and the reactive drop ix in leading quadrature with i.

It will be seen from the geometry of the figures that, with the same value of the induced voltage E_0, the higher the power factor of the load, the higher will be the terminal voltage E. By drawing a similar diagram for a leading armature current, the reader can satisfy himself that the terminal voltage may be even higher than the induced e.m.f. In other words, the armature reactance may cause a rise instead of a drop in voltage.

The practical significance of this relationship is as follows: Let the machine run at a constant load current i and at a constant terminal voltage E, the field excitation being varied in accordance with the power factor of the load. It will be seen from Figs. 287 and 288 that the necessary induced e.m.f., E_0, becomes smaller and smaller as the load becomes less and less inductive. Disregarding, for the present, the influence of the armature reaction (which only accentuates the relationship), the field excitation has to be reduced as the current becomes less and less lagging. Conversely, if the field current is kept constant the terminal voltage increases as the load becomes less inductive, and the voltage rises still higher with a leading current. In other words, the voltage regulation of the machine is improved as the current vector moves further and further in the counter-clockwise direction. This is in accordance with the actual performance curves shown in Figs. 273 to 276.

(b) *Direct armature reaction.* It has been shown in § 452 that a lagging armature current, which reaches its maximum when the corresponding conductors are midway between the poles, exerts a demagnetizing action upon the field flux. Such a current is displaced in time phase by 90 degrees with respect to the no-load voltage, because the latter reaches its maximum when the groups of conductors are opposite the centers of the poles. This current is practically a reactive current, although its phase displacement with respect to the terminal voltage is not quite equal to 90 degrees.

When the power factor of the load is not zero, the total armature current, i, may be resolved into two components, one of which reaches its maximum midway between the poles and the other when the conductors are opposite the centers of the poles. The statements made in §§ 452 and 455 in regard to the direct armature reaction apply to the first component of the current. The practical difficulty consists in finding the value of this component.

The larger the phase angle ϕ of the load current, the greater is this component of the current and the more pronounced is the demagnetizing or direct armature reaction. But the influence of the armature reactance is also more pronounced at higher values of ϕ (Fig. 288), so that the two factors are cumulative in lowering the terminal voltage of the machine.

(c) *Transverse armature reaction.* The m.m.f. of the other component of the current, viz., that which reaches its maximum when the conductors are opposite the centers of the poles, should also be considered in detail from instant to instant. It can be shown that its average action consists in shifting the main flux towards the trailing pole tip. The effect is qualitatively the same as if fictitious exciting coils were placed between the main poles in lagging space quadrature with the real exciting coils. For this reason the armature reaction due to this component of the current is called the transverse armature reaction. It does not directly weaken the flux, but indirectly reduces its value by saturating the trailing pole tip. Moreover, it modifies the distribution of flux density on the pole face, and thereby also distorts the wave-shape of the induced e.m.f. The actual influence of the transverse armature reaction upon the induced voltage of the machine is rather difficult to take into account quantitatively.

Thus, in estimating the full-load voltage of an alternator, the correct procedure (for which credit is due to A. Blondel) requires that the armature current be resolved into two components, one of which is in quadrature and the other in phase with the induced e.m.f. at no load. The first component (if lagging) exerts a demagnetizing action on the field poles, the other, a cross-magnetizing or transverse action, as described above. After each action has been taken properly into account, and the field flux has been corrected for the effect of both, the net induced e.m.f. E_0 may be found from the no-load saturation curve of the machine. After this, the diagram shown in Figs. 287 and 288 may be constructed, and the terminal voltage E determined. It is also possible to begin with a given terminal voltage and current, and to determine the necessary field excitation.

While this correct method of predetermination of the voltage characteristics of an alternator is used to some extent in designing a new machine or in checking its performance, it can hardly be applied to testing a machine whose constants are not known, because of the difficulty of obtaining correctly the two components of the armature reaction. For this reason, various approximate and empirical methods for predetermining the voltage regulation of an alternator have been proposed and used. Of these methods the one recommended by the American Institute of Electrical Engineers is described below. For a more detailed treatment of the subject see Vol. II and the author's book entitled "The Magnetic Circuit."

458. The Synchronous-Reactance Method of Predetermination of Voltage Regulation at any Value of Power Factor. — This method is empirical and has been devised to do away with the difficulties of the more correct method described in the preceding section. This empirical method is based upon the following assumption: *The synchronous reactance* (§ 455), *computed for certain values of armature and field currents at zero power-factor, holds true for the same values of currents at any other value of power factor.* The armature reaction, as such, is entirely disregarded, being included in the synchronous reactance.

Let the curves in Fig. 285 be given and let it be required to draw the diagram shown in Fig. 287, where x now stands for the synchronous reactance x_s. Because the true armature reaction is disregarded, E_0 is the induced voltage due to the field flux which would exist with the same field current at no load. It will be remembered that in § 457 E_0 stands for the e.m.f. induced by the flux modified by the armature reaction. In other words, we now use a larger E_0, but also subtract from it a larger ix, so as to get an E which is approximately the same as before. Let Ot in Fig. 285 be the field excitation corresponding to the voltage E_0 so that $tm = E_0$. The synchronous reactance drop at zero power-factor is equal to mv, and this value is *empirically* taken to be equal to CD in Fig. 287. Knowing E_0, ix, ir, and the angle ϕ between ir and E, one can determine the latter quantity either graphically or analytically. The usual way is to construct the triangle BCD and to draw the direction of BO at an angle ϕ to CB. An arc of radius E_0 is described with D as a center, and its intersection with BO determines point O and the length of the vector E. Kapp's diagram (§ 405) can also be used for a constant value of field current.

Such a construction, performed for different values of power factor and field current, will give a set of curves like those in Figs. 273 to 276, without the necessity of obtaining them by an actual load test. When applying this method to a new or somewhat unusual type of machine, it is well to check the computations by a few load readings to satisfy oneself as to their accuracy.

459. EXPERIMENT 20-D. — Load Saturation Curves of an Alternator at Different Values of Power Factor. — The purpose of the experiment is to check the actual test curves shown in Fig. 276, with those computed according to the method given in the preceding section. The results of Experiments 19-D and 20-C may be used, if the data are sufficiently accurate. In such a case no actual new experiment is necessary, but only additional computations for the report. It is advisable, however, to take a special set of curves with very careful settings, and with all the readings at exactly the value of armature current for which the computations are to be performed. The method of loading the machine and the data sheet are the same as in § 440.

Report. (1) Plot the experimental curves as in Fig. 276. (2) For different values of field current measure and plot the values of synchronous reactance drop, using the load saturation curve at zero power-factor. (3) Construct diagrams like those in Figs. 287 and 255 for the values of power factor for which experimental curves are available; choose points of low, medium, and high saturation. (4) Compare the computed values of terminal voltage with those measured on test and find out whether the method of synchronous reactance gives *optimistic* or *pessimistic* curves, that is, whether the computed values lie above or below those observed on test. (5) In case of a serious discrepancy between the observed and computed values, suggest a further empirical correction in order to bring the computed values nearer the test data.

CHAPTER XXI

ALTERNATORS IN PARALLEL

460. Two direct-current machines can be put in parallel as soon as their voltages are approximately equal, provided that the correct polarity is observed in connecting them (§ 248). But before two alternators are switched in parallel, three conditions must be fulfilled: their terminal voltages must be

(1) of the same magnitude;

(2) of the same frequency;

(3) in phase with each other.

Unless all these requirements are met, there will be a disturbing circulating current between the machines. Only when the terminal voltages of the alternators are equal and opposite *at all moments* — which requires the fulfilment of the above conditions — will their actions be properly combined, and each machine can send its share of the total current into the line.

The process of adjusting the magnitude, frequency, and phase of the voltage of an alternator or synchronous motor to those of another machine or source of supply, is called *synchronizing* the machine. The word means " bringing into the same rhythm."

461. The Three Adjustments before Switching in Parallel. — Let us investigate what would happen if two alternators should be connected to the same bus-bars without having all the above-stated conditions fulfilled.

(1) If the two machines are so excited as to give *different voltages*, the other two conditions being fulfilled, a reactive current will circulate between the two machines, leading in the machine excited lower, and lagging in the machine excited higher. This has the effect of strengthening the field of the first machine and weakening the field of the second machine; the resultant voltage at the bus-bars will be somewhere between the voltages of the two machines. The set may still work satisfactorily, provided that this reactive current is not too heavy. However, such a circulating current does not help the operation and gives an unnecessary I^2R loss in the armatures of both machines. Should this current become sufficiently large, it may cause a dangerous

temperature rise in the armature windings or open one of the circuit-breakers, even though the external load current may be quite small.

(2) If the voltages of the two machines are *not in phase* with each other, in other words, if the maxima of their e.m.f. waves do not occur at the same moments, even though the two e.m.f.'s are of the same frequency and magnitude, an energy current will circulate between the machines. This current, sometimes called the synchronizing current, tends to bring the machines into phase and it may be quite large, if the difference in phase is considerable. If the torque due to this synchronizing current is large enough to pull the lagging machine into phase promptly, the current itself will be of short duration and will not damage the machine. Otherwise, the armature winding of the machine may be overheated and the insulation damaged.

(3) If coupling without synchronizing is attempted, when the two frequencies differ from each other by a few per cent, the conditions are approximately those described under (2) above, with the phase angle continually changing. To show this clearly, two sine-waves of nearly the same frequency should be drawn. It will be found that at some instants the waves are in phase with each other, then they are in phase quadrature, in phase opposition, and, after a number of cycles, in phase again. Unless the energy component of the circulating current is sufficient to pull the lagging machine into synchronism promptly, and to hold it there, one of the machines may be injured by the inrush of current. The theoretical side of the foregoing statements is elaborated somewhat more fully in §§ 466 and 467 below.

An ordinary carbon or oil-type circuit-breaker is much too slow to protect an alternator against an excessive transient current due to a short-circuit or to faulty synchronizing. Therefore, one has to be very careful when synchronizing a large machine for the first time.

Knowing the frequency of the supply and the number of poles of the machine to be synchronized, it is easy to determine the speed necessary for the desired frequency. If, for example, the machine has 30 poles there will be 30 alternations of the e.m.f. during one complete revolution. If the frequency of the supply is 7200 alternations per minute, the machine must revolve 7200/30 = 240 times per minute in order to give this number of alternations. The simple rule is: the speed of the alternator in r.p.m. is found by dividing the frequency of the supply (in alternations per minute) by the number of poles of the machine (§ 431). For a description of direct-reading frequency indicators see §§ 443 to 446.

After the correct speed has been approximately obtained, the field current of the alternator is so adjusted as to give about the same voltage as that across the bus-bars to which the machine is to be connected It only remains then to bring the machine *into phase* with the bus-ba voltage. This is done either by means of properly connected synchronizing lamps, or by special instruments, called synchroscopes o synchronism indicators.

462. Synchronizing Lamps. — Synchronizing lamps are usually connected as shown in Fig. 289, around the main switch which connect the machine to the bus-bars. As long as the alternator voltage is not i phase opposition with that o the line, the lamps will b alternately lighted and extinguished with a frequenc equal to the difference of th frequencies of the two voltages, that is, with the frequency of the beats of the circulating current. By varying the alternator speed it is possible to extinguish the lamps; this will show that the machine is in perfect synchronism, and the switch can be closed. The sketch illustrates the case of a three-phase machine; with a single-phase machine one of the lines, say CC_1, is omitted.

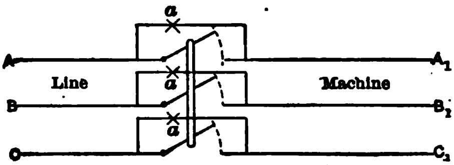


FIG. 289. Synchronizing lamps.

The reason for current beats in the synchronizing lamps is as follows: Let the two alternators be running at slightly different frequencies, e.g. 60 and 59.5 cycles per second. The terminal voltages cannot be made equal and opposite at all instants. Let the voltages be approximately equal and opposite at a certain instant, so that there will be no circulating current due to voltage difference when the machines are connected in parallel. A second later the machines will be out of phase by one-half of a cycle, so that their induced e.m.f.'s will be added together, and a circulating current will flow through the lamps. Another second later, the voltages will be again in opposition and the lamps will be extinguished. The student should plot two such sine-waves of voltage and add the ordinates point by point in order to get a better idea of the shape and the frequency of the resultant voltage. He will find that the frequency of the so-called "beats," that is, the frequency at which the synchronizing lamps are lighting up, is not the true frequency of the resultant current but that of its "envelope." Each flash of the lamps corresponds to several cycles of the circulating current, but the eye receives only one continuous impression.

Some operators prefer to have synchronizing lamps crossed, as shown in Fig. 290. The machine is in synchronism when the lamps glow the brightest. As an advantage of this arrangement, it is claimed that should a lamp burn out during the process of synchronizing, the opertor would immediately notice it, while with the first arrangement e might assume the machines to be in perfect synchronism and close he switch with disastrous results. However, the possibility of a synhronizing lamp burning out is rather remote, and with two- or threehase machines the burning out f one set of lamps would not ffect the others. On the other and, it is generally considered asier to observe moments of total extinguishing than moments of naximum brilliancy of the lamps.

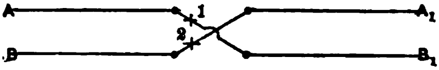


Fig. 290. Crossed synchronizing lamps.

With three-phase machines, crossed synchronizing lamps in two hases are very convenient, especially when the lamps are arranged in a ircle, as in Fig. 291. In this case, maximum brightness occurs in the hree sets of lamps in rotation, so that the light appears to be traveling long the circle. The direction in which the light rotates depends on whether the speed of the incoming machine is low or high. When the machine is in synchronism, the rotation of the light ceases; the lamps marked "3" are dark, while the lamps "2" and "1" glow brightly.

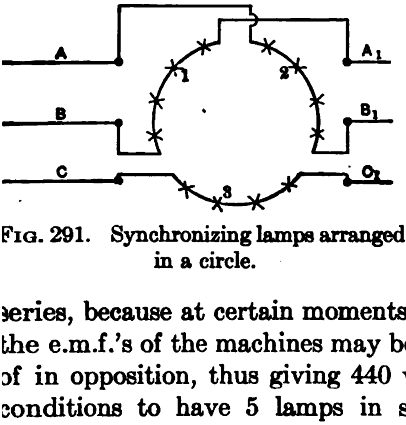


Fig. 291. Synchronizing lamps arranged in a circle.

When two 220-volt single-phase machines are to be put in parallel, at least four ordinary 110-volt synchronizing lamps should be used in eries, because at certain moments during the process of synchronizing, the e.m.f.'s of the machines may be acting in the same direction instead of in opposition, thus giving 440 volts. It is even better under these conditions to have 5 lamps in series, so as not to let them glow too brightly; it is then easier to observe the periods of extinguished lights. With two three-phase machines of the same voltage as above, three 110-volt lamps in series in each phase are sufficient, although four lamps may give better service. For higher voltages synchronizing lamps are usually connected through small transformers.

An ordinary indicating *voltmeter can be used for synchronizing*, instead of lamps. It is connected between the machines, across the switch, either directly or through a potential transformer. When the machines are in synchronism, the voltmeter pointer comes to zero; otherwise it swings to and fro.

When providing a synchronizing connection between two polyphase machines, it is necessary to find out their *phase sequence*. Otherwise, when A is connected to A_1 (Fig. 289), B may be connected to C_1, and

C to B_1. If synchronizing lamps were placed only in phase A and t switch closed in accordance with their indication, there would be a parti short-circuit between the machines. Once the proper phases hav been found, however, it is safe to synchronize afterwards in one pha only, provided that neither the direction of rotation nor the connectio are changed.

463. Synchronism Indicators. — In larger installations, synchr nizing lamps have gradually given place to the more accurate instr ments, so-called synchroscopes or synchronism indicators. Such device has the appearance of an ordinary switchboard instrument (Fig 292), except that the pointer has no retaining spring or weight, and i free to revolve through 360 degrees. When the speed of the alternat to be synchronized is low, the pointer revolves in one direction; wh it is high, the pointer rotates in the opposite direction. When the spe is correct, the pointer stands still; and when the machine is "in phase the pointer shows zero, indicating that the main switch may be close

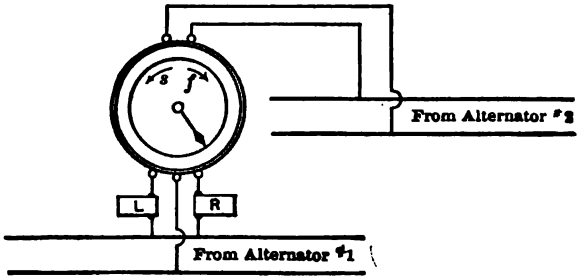


FIG. 292. Connections to a single-phase synchronism indicator.

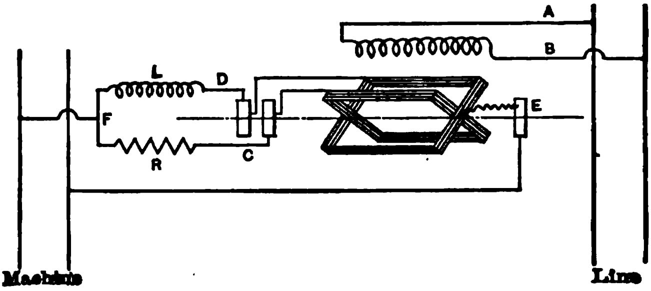


FIG. 293. The diagram of connections of a single-phase synchronism indicator.

The usual synchronism indicator resembles a small alternating-cur rent motor. The field winding is connected to one of the machine or to the line, and the armature to the other machine (Fig. 293). Whe the frequencies of the two machines are different, the resultant field in the synchronism indicator constantly changes its position, making the armature revolve in one or the other direction. When the frequency is the same, the field is stationary in space; when finally the machines are in phase with each other, the field occupies such a position that the pointer, connected to the armature, shows zero.

The synchronism indicator, illustrated in Figs. 292, 293, and 294, is a single-phase device. The stationary field winding, *AB*, is connected to the bus-bars (line); the revolving armature is connected to the machine to be "thrown in," in series with an inductance *L* and resistance *R*, usually an incandescent lamp. The armature is of the drum type; it has two coils rigidly fastened at right angles to each other and connected in series. Their junction is connected through the collector ring *E* to the binding post marked *E* in Fig. 294. The other two terminals are brought out through collector rings *C* and *D* to the binding posts marked with the same letters in Fig. 294.

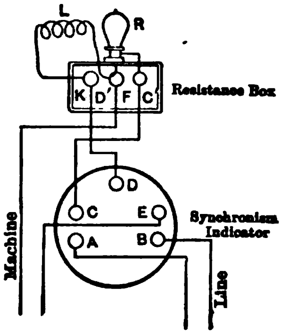


Fig. 294. The actual connections between the split-phase box and the synchronism indicator.

R and *L* are used for splitting the phase of the armature current, so that the current in the coil which is in series with *L* lags behind that in the other coil. Each armature coil is subjected to a torque from the stationary field. When the frequencies of the two machines are equal, the armature of the synchronism indicator assumes the position in which the two torques are equal and opposite. When the frequencies are slightly different, the position of equilibrium varies with the time and the armature revolves at a speed equal to the difference of the two frequencies.

In another make of synchronism indicator the split-phase arrangement shown in Fig. 293 is stationary, so that no slip rings are necessary. A rotating magnetic field is produced around this winding by currents from the bus-bars passing through it. In this rotating field is placed a movable iron vane (Fig. 90) or armature magnetized by a stationary coil connected across the terminals of the incoming machine and corresponding to the winding *AB* in Fig. 293. The iron vane takes a position where zero of the rotating field occurs at the same instant as zero of the stationary field. Thus its position at every instant indicates the phase angle between the voltage of the incoming machine and that of the bus-bars. As this angle changes, on account of the varying frequency, the iron vane with the pointer attached to it rotates, and whe synchronism has been reached it remains stationary.

For one or two other types of synchronism indicators, see a handbook for electrical engineers. When using a single-phase synchroscope with polyphase machines, the same precaution should be observed i regard to the phase sequence as in the case of synchronizing lamps (§ 462).

With the development of automatic power plants and substations operated without attendants, a need arose for an automatic synchronizer. Such a device is essentially a relay which closes the operating circuit of the main switch of the incoming machine when the latter is in synchronism. A dash-pot on one of the parts prevents the contact from being closed too soon and requires the existence of a condition of synchronism for a certain period of time before the circuit is closed.

464. EXPERIMENT 21-A. — Exercises in Synchronizing an Alternator. — The purpose of the experiment is to acquire skill in connecting a synchronous machine in parallel with a source of supply. This source should preferably be a regular A.C. line used in the laboratory, so as to concentrate the student's attention on one machine only, with its driving motor. If such an outside source is not available, another alternator, driven by a separate motor, may be used. The alternator under test should be driven by a shunt-wound direct-current motor, with a fine speed adjustment. The wiring diagram is the same as in Fig. 277, except that the load may be omitted if the machine is to be synchronized with an A.C. system which carries a sufficient load.

(1) Connect synchronizing lamps " dark " or " straight," as in Fig. 289; at first use the machine single-phase and synchronize two leads only. After this has been successfully accomplished, find the phase sequence and synchronize all three phases. Be careful not to close the switch too soon; let the lamps remain extinguished for several seconds in order to make sure that the machine is in synchronism and is running at the right speed.

(2) Practice synchronizing with the lamps crossed (Figs. 290 and 291) and make clear to yourself the advantages and disadvantages of both methods. Vary the number of lamps in series and form an opinion as to whether a bright or a dull glow gives the best results.

(3) Try synchronizing with D.C. and A.C. voltmeters built on different principles (Chapter II) and select the one that gives the best results.

(4) Connect a synchronism indicator and practice synchronizing with it. Find out if the use of lamps at the same time is of assistance.

Repeat each experiment several times and then make a final trial to find how many seconds it should take under ordinary practical conditions to start the set, to connect it in parellel, and to make the final adjustments until it is carrying its correct share of the load. The ammeter and the wattmeter need not be read accurately, but approximate readings will help in writing the report.

(5) Try connecting the machine in parallel without having one of the three necessary conditions fulfilled (§§ 460 and 461). Since this may cause an excessive transient current, such a test must be performed with great caution. Nevertheless it is desired that the student form an idea as to the ability of the machine "to pull itself into step."

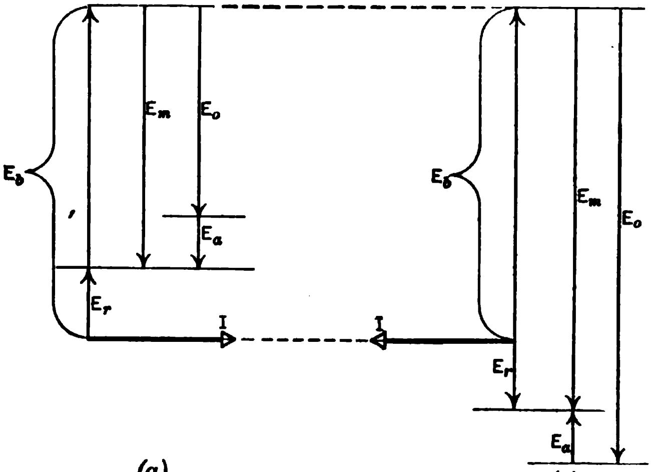
(6) Connect a choke coil without iron core in each lead between the alternator and the line. This choke coil will limit the transient current when the machine is switched in without accurate synchronizing. Bring the machine to a speed slightly above synchronism and close the switch. In slowing down, the machine will synchronize itself. Then short-circuit the choke coils. Experiment with different values of reactance and find the best amount.

(7) Repeat the preceding test with series resistance in place of reactance.

Report. (1) State your findings as to the amount of time that it should take an experienced operator to synchronize the alternator with the different methods tried. (2) Describe the phenomena observed when the main switch was closed without having the machine properly synchronized. (3) Give the data and your opinion as to the usefulness and practicability of using choke coils or resistors in the main leads for the first closing of the switch. (4) Show that the results obtained with the choke coils and resistors are in accord with the theory given in §§ 466 and 467.

465. Operation of Synchronous Machines in Parallel. — Let it be assumed for the sake of simplicity that a comparatively small alternator under test is operated in parallel with a source of power supply of practically unlimited capacity, so that fluctuations in the power output or the circulating current of the small machine do not appreciably affect the operation of the other machines. The bus-bar voltage and its frequency are therefore supposed to be absolutely constant under all possible conditions of operation of the machine under test. The magnitude and the phase angle of the current which the alternator under test is sending into the network, with its very large connected load, is a function of two independent variables, viz., (1) of the speed setting of the prime mover or electric motor which drives the alternator, and (2) of the field excitation of the alternator. The influence of these two factors will be considered separately.

466. Field Excitation. — Let the prime mover or the driving motor of the machine under test be so adjusted that the alternator sends practically no power into the network, and furthermore let the excitation of the alternator be so adjusted that the induced e.m.f. is equal to the bus-bar voltage, so that there is no appreciable reactive circulating current. Now, let the field current of the alternator be somewhat reduced, the new conditions being shown in Fig. 295*a*. In this vector diagram E_b is the bus-bar voltage and E_m is the machine voltage, or



(a) (b)

FIG. 295. Reactive circulating current between a large source of power supply and a synchronous machine (a) under-excited, (b) over-excited.

more exactly, the induced e.m.f. corresponding to the new excitation. E_r is the difference of the two or the resultant voltage acting upon the armature of the machine. In most alternators the armature resistance may be neglected as compared to its reactance, so that E_r causes a lagging quadrature current, I, between the machine and the rest of the system.

With respect to the induced voltage, E_m, of the machine this current I is in leading quadrature; hence it exerts a direct armature reaction which strengthens the field excitation and increases the induced e.m.f. (§ 455). Thus, the induced e.m.f. E_m may be thought of as consisting of two parts: E_0, which is the e.m.f. induced with the same excitation at no load, and E_a, due to the effect of the armature reaction. The conditions with the field over-excited are shown in Fig. 295*b*. Here the armature current lags behind the induced e.m.f., the armature reaction demagnetizes the field, and the reactive voltage drop E_r is again equal to the difference between the induced voltage and that at the bus-bars.

Thus, by varying the field excitation it is possible to cause an alternator operating in parallel with others to deliver a lagging or leading current of any safe desired magnitude, without appreciably changing the working component of the current. The total reactive current in the connected system depends upon the character and magnitude of the load, and is fixed for a given load. Thus, when the reactive current delivered by one of the alternators is reduced, the total reactive component supplied by the other machines is increased by the same amount. With several interconnected power stations or substations, it is of advantage to furnish as much reactive current as possible from the station nearest to the load, so as to reduce the needless I^2R loss in the transmission lines.

467. Speed Setting. — With two or more direct-current generators operating in parallel (§ 248) the amount of power delivered by one of them can be increased by increasing its field excitation. Even though the prime mover driving that particular machine should slow down somewhat, the increased induced e.m.f. would cause the machine to deliver a greater current. A direct-current generator can also be made to deliver a higher current by increasing the speed of its prime mover or driving motor. On the other hand, the power output of an alternator working in parallel with others can be increased by the second expedient only, that is, by changing the speed setting of its prime mover. As is shown above, an increase in the field excitation merely produces a reactive current which demagnetizes the field.

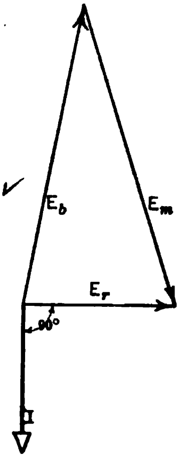


Fig. 296. A diagram showing that the phase of the induced e.m.f. has to be advanced to make an alternator deliver an energy current.

Let an alternator working in parallel with a large system be so excited that its induced e.m.f. is practically equal to the bus-bar voltage, and let the governor of the prime mover be adjusted for a very small output, so that the alternator delivers practically no current to the network. Now let the setting of the governor be changed to that for a slightly higher speed. The new conditions are shown in the vector diagram in Fig. 296. E_b is again the bus-bar voltage and E_m is the induced e.m.f. of the machine. Because of the higher setting of the governor, the reversed vector E_m leads E_b slightly. The difference between the two, E_r, will cause the machine to deliver a current. Again neglecting the armature resistance as compared to its reactance, we may say that this current I is in lagging phase quadrature with E_r. This current is almost in phase with E_m and therefore is an energy current which produces a useful counter-torque.

The alternator under consideration experiences but a momentary acceleration, after which it continues to run in synchronism, delivering an energy current to the line and absorbing an increased amount of power because of a higher setting of the governor. The same consideration applies to the case in which an alternator is driven by a shunt-wound direct-current motor. Weakening the field of the latter acts in the same way as setting a centrifugal governor for a higher speed.

Let two identical alternators be driven side by side unloaded and connected in parallel to the same bus-bars. Let the center lines of the shafts of the two machines lie on the same straight line. It is assumed that other alternators are furnishing all the required power. Now let the governor setting of machine No. 1 be changed to make the machine deliver some power to the line, while machine No. 2 still continues to run idle. Originally the revolving parts of the two machines were running in agreement, that is, for an observer viewing along the shaft line, the field poles of one machine would seem to cover the corresponding poles of the other. When machine No. 1 is loaded, its revolving part is shifted forward with respect to that of machine No. 2. This phenomenon may be made visible by drawing a radial line in an identical position on the revolving part of each machine and illuminating both lines with an arc lamp fed from the bus-bars. The stroboscopic effect will cause these two lines to appear stationary even when the machines are running; the shifting forward of the line on the loaded machine can be readily observed. Two contacts and a telephone receiver can also be used, as in Fig. 218.

As the prime mover is set for higher and higher input, the revolving structure of the alternator is shifted forward accordingly, thus generating a larger energy current and developing a higher opposing torque. When the angle of shift has reached a certain limit (about 90 electrical degrees) a further increase in the angle will give a smaller torque. Thus, if the governor is set for an output higher than this maximum output of the alternator, the latter becomes electrically "uncoupled" from the rest of the system and the set will run away.

468. Operation as a Synchronous Motor. — Like a direct-current generator, an alternator is convertible in its action and may be made to operate as a so-called synchronous motor. To show this operation experimentally, it is convenient to drive the machine by a direct-current shunt-wound motor whose field may be adjusted within wide limits. The alternator is synchronized in the usual way and the field currents of both machines are so adjusted that the alternator delivers practically no current to the line. If the field current of the driving motor be reduced, so that the set tends to speed up, the synchronous machine will operate as a generator delivering power to the A.C. line. However, if the field excitation of the driving motor be increased, so that the set tends to slow down, an energy current will flow into the alternator armature from the bus-bars, driving the A.C. machine as a motor. This will convert the D.C. motor into a generator and will cause it to send electric power back into its source of D.C. supply.

In this case there is also a definite overload limit for the synchronous machine, similar to that which exists when the machine is acting as a generator (see the end of the preceding section). With motor operation, the field structure is shifted back as the load increases; when the angle of shift has exceeded that corresponding to the maximum torque, the machine drops out of step and comes to rest.

For further details regarding the synchronous motor and hunting of synchronous machines when operated in parallel, see Vol. II. Hunting is also discussed more in detail in the author's article on " Hunting and Parallel Operation of Synchronous Machines," *Sibley Journal of Engineering*, March 1920.

469. EXPERIMENT 21-B. — Parallel Operation of Alternators. — The purpose of the experiment is to study the division of the load between two or more synchronous alternators in parallel, and the influence of the field excitation upon the reactive component of the current. The theory is given in §§ 465 to 468 above. The machine under test should be driven by a shunt-wound direct-current motor, in parallel with the laboratory power supply or with another alternator. The speed of the driving motor should be adjustable in fine steps near the synchronous speed of the alternator. Instructions for synchronizing are given in §§ 462 to 464. A suitable load should be provided, consisting of a combination of adjustable resistances and reactances (Fig. 277), unless the power from the alternator under test can be conveniently used in the system with which it is working in parallel. The following instruments should be provided for both the alternator and the driving motor: a voltmeter, an ammeter in the armature circuit, and a field ammeter. A wattmeter should be connected for measuring the A.C. output.

(1) Having synchronized and " switched in " the machine under test, adjust the field currents of both the A.C. and the D.C. machine

to such values that the alternator sends practically no current into the line. Keep the motor excitation constant and increase the excitation of the alternator in steps until the machine delivers its maximum safe current. At each step read all the instruments. Devise some simple arrangement to show that the current delivered by the machine is of low power-factor and is lagging and not leading. A power-factor meter (§ 104) may be used for this purpose. If the machine under test is working in parallel with another machine of limited capacity and not with a large network, it may be necessary to adjust the load and the field excitation of the other machine in order to keep the terminal voltage of the machine under test constant.

(2) Bring the excitation of the alternator again to the value that it had at the beginning of the preceding run. Reduce the field current in steps and take similar readings. Show that the current in this case is leading.

(3) Reestablish the normal conditions as at the beginning of the first run. Now keep the alternator field current constant and decrease the motor excitation in steps to the lowest possible value. At each step read all the instruments. Devise some simple method by which the forward shift of the alternator field with the load may be demonstrated, and if possible measured. Determine the limits within which the alternator excitation has to be varied in order to keep the current strictly non-inductive between no load and full load.

(4) Having brought the conditions back to normal, open the main switch on the D.C. side to show that the alternator can operate as a *synchronous motor*. Take a few readings of the instruments on the A.C. side with different values of field current, above and below normal. In each case, be sure to determine whether the armature current is leading or lagging.

(5) With the conditions as in (1) above, increase the field current of the driving motor above normal, in steps, and observe the reversal of the flow of power in both machines. The D.C. motor will become a generator and will send power into the D.C. line. If necessary, reverse the instrument connections and take several sets of readings as above. Show that varying the field excitation of the loaded synchronous motor merely changes the reactive component of its armature current, but does not affect the wattmeter reading very much.

Report. (1) Show from your readings that varying the field excitation of a synchronous generator or motor mainly influences the reactive component of the armature current. Describe how the sign of the reactive component (leading or lagging) was determined experimentally and check the results with the theory of direct armature reaction. (2)

Explain how, by varying the field excitation of the driving motor, the alternator is changed into a synchronous motor. Substantiate the explanation with the data taken in the laboratory. (3) Compute a few values of the efficiency of the set, with the power flow in both directions. (4) Describe the demonstration of the mechanical shifting of the field structure with the load, both forward and backward. (5) Predict theoretically the combined effect of field current regulation of both the driving motor and the synchronous machine. Assume the set to constitute one of the several power plants on a large inter-connected system, with a central load dispatcher to whom the local conditions as to the load and power factor are reported from each generating station at frequent intervals. Under what conditions would he direct the operator to regulate the prime mover (in this case the driving motor), and under what other conditions would he direct him to adjust the alternator field? How could this regulation be accomplished automatically?

CHAPTER XXII

THE POLYPHASE INDUCTION MOTOR

470. The construction and principle of operation of the polyphase induction motor are shown in Figs. 297 to 299. The stationary part, or the *stator*, consists of steel laminations with slots along the inner edge. A winding is placed in these slots and connected to a two-phase or three-phase source of supply (§§ 430 and 447). The winding is similar to that shown in Figs. 300 and 301, and the combination of alternating currents in it produces a *revolving magnetic field* in the air-gap, as is explained below. An instantaneous distribution of this flux in a six-pole machine is shown in Fig. 299. The poles are alternately marked N and S, and the flux density is assumed to be distributed in space approximately according to the sine law. This distribution of flux density moves uniformly along the air-gap, so that at any other instant the distribution is the same as in the sketch, except that the

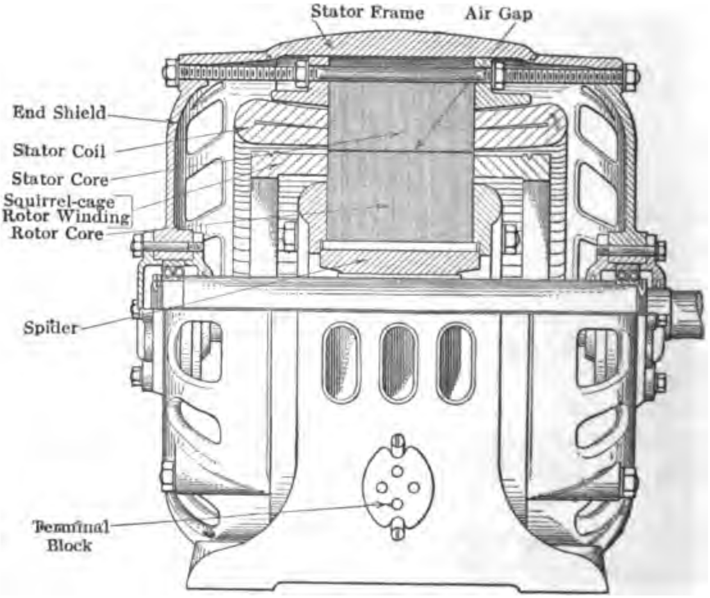


Fig. 297. An induction motor.

points of maximum and zero flux density have been shifted. The motion of the air-gap flux takes place at the *synchronous speed;* that is, the flux glides by a pair of poles during the interval of time corresponding to one complete cycle of the exciting currents. The flux revolves to the left or to the right, depending upon the connections between the stator winding and the source of power.

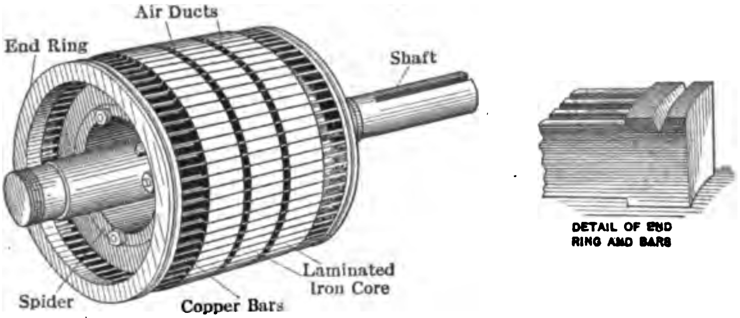


Fig. 298. A squirrel-cage rotor.

The *rotor* also consists of steel laminations assembled on a spider or directly on the shaft, and provided with slots on the periphery. A winding is placed in these slots and is short-circuited upon itself. This winding usually consists of copper bars (Fig. 298) slightly insulated from the iron core and connected at both ends to rings made of an alloy of comparatively high resistivity. Such a winding is known as a *squirrel-cage winding.* It is the simplest and most reliable type of winding known.

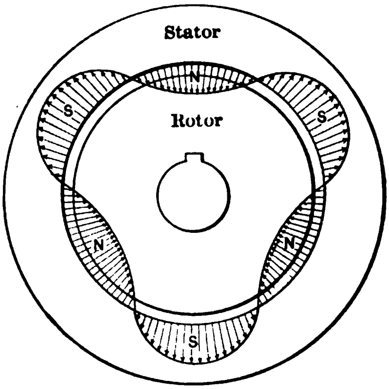


Fig. 299. The field in the air-gap of a six-pole induction motor.

As a direct-current motor requires a starting resistance in series with the armature, so an induction motor should have a starting resistance in series with its rotor winding, in order to reduce the starting current and to increase the starting torque per ampere of line current. This resistance should be cut out gradually or in steps as the rotor speeds up, and under the normal operating conditions the rotor resistance should be as low as possible, in order to reduce the secondary copper loss to a minimum and to increase the efficiency of the motor. Unfortunately, there are no simple means known for connecting a starting resistance in series with a squirrel-cage winding, and therefore such a winding is usually designed as a compromise between the best starting and operating conditions. The resistance of the end rings, as a rule, is not high enough for the best starting torque and current, and is too high for the best possible efficiency at full speed.

Thus, while squirrel-cage rotors are largely used for ordinary driving purposes, because of their simplicity and lower cost, rotors which must start at a heavy torque, as in crane service, or where the starting current of a large motor must be limited on account of its effect upon the line, are provided with a *three-phase winding* similar to the stator winding. This winding has three terminals to which resistances may be connected temporarily while the motor is being started. These resistances are usually stationary, and the connection with the rotor is established through three slip rings and brushes. Sometimes the resistances are mounted within the rotor and are connected to the winding either manually or automatically, by a centrifugal device. The resistances must be used in the rotor only during the starting period, and then gradually cut out as the motor speeds up.

ELEMENTARY THEORY

471. The Principle of Operation. — The principle upon which a polyphase induction motor operates is similar to that in the familiar experiment in which a permanent magnet is rotated in front of a copper disk and sets the disk in rotation by means of eddy currents induced in it. If the disk carries no mechanical load, its tendency is to revolve at the same speed as the magnet, because then the eddy currents are reduced to zero and there is no electromagnetic torque, since none is needed. When a reasonable mechanical load is applied to the disk it slows down to such a speed that the induced currents form with the magnet a torque equal and opposite to the load torque.

The difference between this arrangement and the induction motor is that in the latter the revolving field is produced by means of stationary windings, and the power for driving the rotor is furnished to these windings from the line electrically. The secondary winding consists of parallel bars, instead of a solid copper disk, in order to force the secondary currents to flow in directions parallel to the shaft, and to suppress harmful eddy currents in other directions. The electromagnetic force is perpendicular to the directions of both the flux and the current. Thus, with the magnetic flux perpendicular to the surface of the rotor and the currents parallel to the shaft, the driving force is tangential to the rotor surface in planes perpendicular to the shaft.

472. The Air-Gap Flux. — The key to an understanding of the performance of the polyphase induction motor lies in a clear conception of its revolving field. A theory of this field is given in Vol. II, and the practical results are as follows:

(1) When two stator windings (Fig. 300) are displaced *in space* by 90 electrical degrees and are supplied with two-phase currents, that is, with currents displaced by 90 electrical degrees *in time*, a magnetic field is produced which moves synchronously along the air-gap, covering a space equal to two pole-pitches, or 360 electrical degrees, during each cycle of the alternating currents.

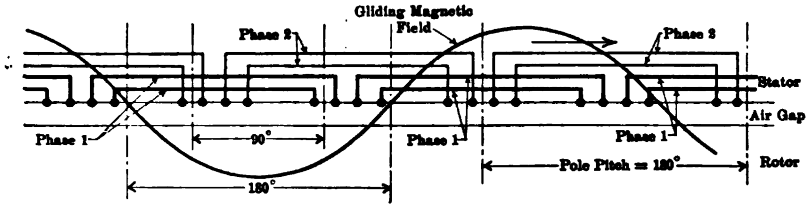


FIG. 300. A two-phase winding and a gliding magnetic flux produced by it.

(2) When three stator windings (Fig. 301) are displaced *in space* by 120 electrical degrees and are supplied with three-phase currents, that is, with currents displaced by 120 electrical degrees *in time*, a magnetic field is produced which moves synchronously along the air-gap, covering a space equal to two pole-pitches, or 360 electrical degrees, during each cycle of the alternating currents.

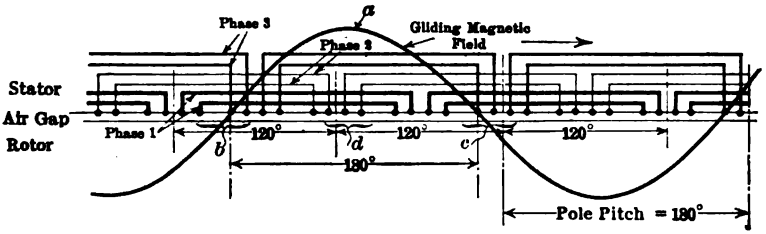


FIG. 301. A three-phase winding and a gliding magnetic flux produced by it.

(3) The flux glides in the direction from the phase in which the current is leading to the one in which it is lagging. To reverse the direction of rotation of the flux in a two-phase machine, the terminals of one of the phases are reversed. To reverse the flux in a three-phase machine, two out of three terminals are reversed.

At no load the revolving flux is excited by the stator winding only, and the current drawn from the line consists of the magnetizing current in quadrature with the terminal voltage and of a small energy component which supplies the core loss, the copper loss, and the friction and windage.

When the machine is loaded, the air-gap flux is due to the combined action of the primary and secondary m.m.f.'s, as in a transformer (§ 387). Since the secondary currents oppose the primary, larger currents are drawn from the line to compensate for the secondary magnetomotive force, and to leave a sufficient m.m.f. for the production of the air-gap flux.

An induction motor is usually called upon to operate at a constant primary voltage. Under these conditions the air-gap flux is nearly constant and independent of the load. This property of the polyphase induction motor greatly simplifies its theory. The applied voltage is distributed in the stator windings approximately in the same manner as in a transformer, viz.

(1) The greater part of it is balanced by the useful counter-e.m.f. induced in the windings by the air-gap flux.

(2) A small part is balanced by the e.m.f.'s induced in the stator windings by the leakage fluxes in the slots and around the end connections. This is called the primary reactance drop of the machine.

(3) A small part goes to overcome the ohmic drop of the primary current in the resistance of the stator windings.

In a well-designed machine, parts (2) and (3) are small, so that the counter-e.m.f. (1) is nearly equal to the applied voltage. If the latter is constant then the counter-e.m.f. is also nearly constant. But this counter-e.m.f. is induced by a flux revolving at a constant speed independent of the load. Consequently, the air-gap flux is also nearly constant, that is, independent of the load. In reality the flux decreases by a small amount as the load increases, on account of the voltage lost in (2) and (3).

In a machine designed for a special use, or in a very small motor, the primary impedance drop may be quite large, and the air-gap flux may decrease considerably with the increase in the load. Also when a motor is just starting, at several times its rated current, the air-gap flux may drop to about one-half its normal value, because of a large primary impedance drop. Therefore, the statement that the air-gap flux is nearly constant applies only to the working range of a standard induction motor of moderate or large size, designed for continuous operation.

473. Slip and Torque. — The speed of the revolving magnetic field depends upon the frequency of the line currents and upon the number of poles of the stator. During one alternation of the supply current the field travels from one pole to the next pole; therefore the number of revolutions of the rotating field per minute is equal to the number of alternations per minute divided by the number of poles of the stator winding. For example, on an ordinary 7200-alternation lighting circuit the revolving field of a 6-pole induction motor makes 7200/6 = 1200 revolutions per minute (§ 431).

The speed of the rotor is lower than that of the revolving field, because the secondary conductors must cut the lines of force in such a direction that the induced currents tend to pull the rotor into synchronism. In practice, the speed of the armature at full load is a few per cent less than that of the revolving field. For example, the actual speed of the above-mentioned motor is somewhere near 1140 r.p.m. This difference between the speed of the revolving field and the actual speed of the rotor is called *the slip* of the induction motor, and is usually measured in per cent of the speed of the revolving field. This latter speed is also called the *synchronous speed*, and is the speed of a synchronous alternator with the same number of poles. It is also the actual speed of an ideal rotor which has no ohmic resistance. The synchronous speed of the above motor is 1200 r.p.m., and the slip at full load is

$$\frac{1200 - 1140}{1200} = 5 \text{ per cent.}$$

The slip depends on the load and increases with it. At no load the motor runs almost synchronously, because a very small difference in speed between the revolving field and the rotor is sufficient to induce secondary currents which, with the air-gap flux, form a small driving torque equal to that of the friction and windage. As the load increases the rotor slows down somewhat, while the speed of the revolving field remains the same. Therefore the rotor conductors cut the air-gap flux at a higher and higher relative speed. As a result, larger and larger secondary currents are induced which, with a nearly constant magnetic field, exert a higher and higher tangential force upon the rotor. For each load there is a speed at which the driving electromagnetic torque is equal to the load torque plus that due to friction and windage.

Low-resistance rotor. Experiment and theory show the general character of the speed-torque curves of an induction motor to be that shown in Fig. 302. Consider first the case of a low-resistance rotor. The working part of the torque curve is *hf*. At no load and zero torque the machine runs at practically the synchronous speed *Oh*, where the origin *O* is at the stand-still point. As the load increases the operating point moves from *h* towards *f*. At the rated torque, corresponding to the ordinate *kg*, the machine runs at the speed *Ok*, the slip being equal to *hk*. As the torque is further increased, the point f is reached at which the motor "pulls out" and quickly comes to a stop. The part ff_0 of the curve corresponds to an unstable operation and Of_0 is the starting torque. If the actual torque applied to the motor at start is less than Of_0, the machine will start, run up the part f_0f of the curve and will operate in a stable manner somewhere between f and h. If the applied load torque at start is greater than Of_0 the motor will not start at all.

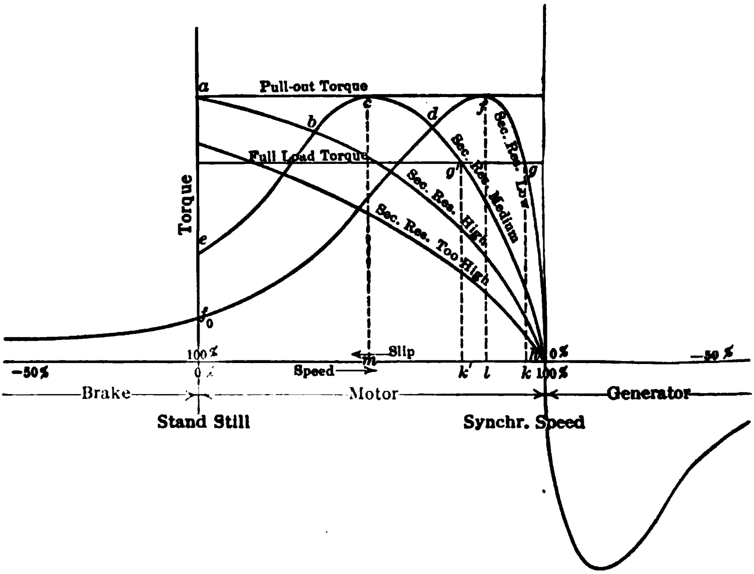


FIG. 302. Speed-torque curves of a polyphase induction motor.

The existence of the critical point f is due to the leakage inductance, L_2, of the rotor winding. When currents flow through the secondary winding they cause magnetic fluxes around the conductors, in slots and at the end connections. These fluxes cause the currents to lag behind the induced voltages. The tangent of the angle of lag, ϕ_2, is equal to the ratio of the reactance $2\pi f_2 L_2$ to the resistance r_2 (see Chapter VI) or $\tan \phi_2 = 2\pi f_2 L_2 / r_2$. Both L_2 and r_2 are constant, but the frequency of the secondary currents increases with the slip, so that as the load increases these currents get more and more out of phase with the induced voltages. The electromagnetic torque is proportional not to the whole secondary current I_2, but only to that part of it which is in phase with the flux. This is illustrated in Fig. 303. At a small load the secondary current is I_2', and as the load increases the current becomes equal to I_2''. The component Oa'' of I_2'', in phase with the flux Φ, is greater than the corresponding component Oa' of I_2', and the motor is capable of carrying the increased load. This corresponds to the stable portion *hf* of the curve. As the load further increases, the slip increases, and while the secondary current also increases, its vector assumes such a phase position, I_2''', that its component, Oa''', in phase with Φ is smaller than before, and the motor pulls out and stops.

Behavior at low and high values of slip. The difference in the rotor reaction at low and high values of slip may be further elucidated with reference to Fig. 301. Instead of considering the gliding magnetic field as being produced by stationary polyphase currents, imagine a fictitious winding concentrated at places marked *b* and *c*, 180 degrees apart, excited with direct current, and moving synchronously along the air-gap. Such a winding can be made to produce the same gliding magnetic flux as the actual polyphase winding, and the secondary currents may be described with reference to this fictitious winding.

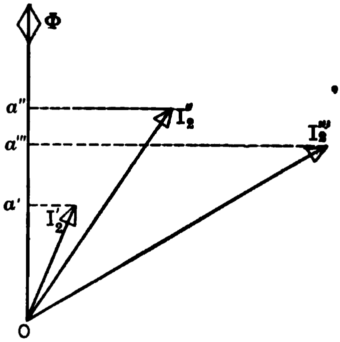


Fig. 303. A vector diagram showing that a larger secondary current may have a smaller torque component than a smaller current.

At low values of slip the induced secondary currents are of a low frequency, and therefore their value is mainly determined by the rotor resistance and not by its inductance. These currents are essentially in phase with the induced voltage, and when the air-gap flux is a maximum at point *a*, the secondary currents are at a maximum in the rotor region marked *d*. Such currents, being in phase with the flux, exert a considerable mechanical torque, but they do not tend to wipe out the flux, because *d* is in space quadrature with the primary exciting regions *b* and *c*. The flux persists because such primary A.C. currents are drawn from the line that their m.m.f. is equal and opposite to that of the secondary currents, leaving the exciting current practically intact.

At very high values of slip, for example, when the rotor is driven mechanically against the direction of rotation of the flux, the induced secondary currents are of a high frequency, and their value is mainly determined by the rotor inductance. These currents are practically in time phase quadrature with the induced voltage and in space quadrature with the air-gap flux. Thus, the currents follow the gliding flux mainly in the regions *b* and *c*, instead of *d*. Their mechanical torque is very small because the flux is small in the regions *b* and *c*. Being directly opposite the fictitious primary exciting coil *bc*, these secondary currents tend to wipe out the flux; or, if the flux value is fixed, an extra magnetizing current is drawn from the line to counteract their m.m.f. These results may be briefly summarized as follows:

At low values of slip, a low-resistance rotor of a polyphase induction motor behaves like a friction coupling, transmitting mechanical power and drawing an energy current from the line. At very high values of slip, the rotor behaves like the secondary of a transformer carrying a purely inductive load. At intermediate values of slip both characteristics are present to a varying degree.

The effect of increased rotor resistance. Should the rotor resistance of an induction motor be increased, without any other change in its dimensions, the speed-torque curve assumes the shape *hce*, instead of hff_0. Theoretically the value of the pull-out torque, *cm*, remains equal to *fl*, but it takes place at a lower speed *Om*, or at a higher per cent slip *hm*. For the same useful torque as before, say $k'g' = kg$, the slip *hk'* is greater, because a higher induced e.m.f. is required in the rotor to establish the same current through a higher impedance. The starting torque is now equal to *Oe* and is greater than before. By suitably increasing the rotor resistance still further, the speed-torque curve can be made to assume the shape *hba*, with the maximum torque at start. Such a motor would have a very low efficiency at running speeds and would show great variations in speed with the load.

The advantages of the curves *abh* and f_0fh can be readily combined by using an additional resistance in the rotor, for starting only. The machine is started on the curve *ab* at the highest obtainable torque, then part of the resistance is cut out and the performance continues on the curve *bcd*. Finally, when the machine has reached a considerable speed, all of the external resistance is cut out, and the rotor reaches its normal speed on the curve *dfg*. The resistance may be cut out in any number of steps. A liquid rheostat, in which the number of steps is infinite, is sometimes used.

STARTING CONDITIONS

474. The Starting Torque. — The starting torque of a polyphase induction motor is approximately proportional to the square of the applied voltage, for the following reason: When standing still, the machine is identical with a transformer (§§ 401 and 402) and may be replaced by a combination of resistances and reactances. But in such a combination all the currents are proportional to the applied voltage, the equivalent impedance being constant. Therefore the net magnetomotive force and the air-gap flux are proportional to the applied voltage. But the torque is proportional to the product of the air-gap flux and the secondary current, and as each of these is proportional to the applied voltage, their product, or the torque, is proportional to the square of the voltage.

At a given voltage the starting torque is a function of the line frequency. While a motor designed for a certain frequency could not be expected to operate satisfactorily at a widely different frequency, small variations in frequency are unavoidable. Sometimes a high starting current of the motor may pull down the alternator speed a few per cent. Theory shows that the starting torque at a given voltage is inversely proportional to the expression $[(r_{eq}^2 + (2\pi f L_{eq})^2]f$, where r_{eq} is the total or equivalent resistance of the motor, L_{eq} is the equivalent leakage inductance, and f is the frequency (See the author's "Electric Circuit," p. 129). Thus, the starting torque increases when the frequency decreases, and the increase is more rapid than the corresponding decrease in the frequency.

475. Starting of Induction Motors. — A very small induction motor, or one provided with a high-resistance rotor, can be started at full voltage by simply connecting the stator to the line. This is usually permissible only when the machine starts at a light load, as in the case of a motor connected to a centrifugal blower, and when the first rush of current would not melt the fuses or be otherwise objectionable. A starter with primary resistance is also sometimes used.

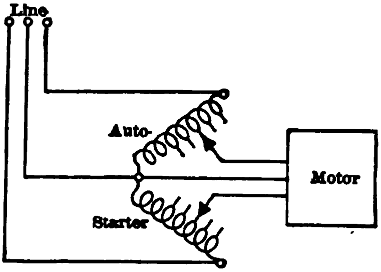


Fig. 304. Starting an induction motor by means of two V-connected auto-transformers.

A medium-sized motor, provided with a squirrel-cage rotor, is usually started at a reduced voltage (Fig. 304) with two V-connected auto-transformers (Vol. II). Taps are provided in the transformer windings at different fractions of the total number of turns, so that a starting voltage may be chosen to suit the required starting torque. In order to reduce the starting current, as low a starting voltage as possible should be selected, because the starting current is proportional to it. On the other hand, the starting torque decreases as the square of the applied voltage (§ 473) and this fact determines the lowest limit of the starting voltage. As the rotor speeds up the stator is connected directly to the line and the auto-transformers are cut out. For details of induction motor starters see Vol. II.

The foregoing method is not well suited for a large induction motor, a motor intended for starting at a high torque, a motor connected to a line on which the starting current is strictly limited, or one which has to be started and stopped at frequent intervals. In such cases a three-phase rotor is provided with three slip rings (Fig. 305) which at the start are connected to three external resistances, and later short-circuited upon themselves. Internal resistances which revolve with the rotor are but seldom used because an arrangement whereby such resistances can be regulated and short-circuited, automatically or manually, is usually more complicated and less reliable than slip rings with stationary resistances. The relationship between the starting torque and the resistance of the rotor circuit is explained in § 473 and illustrated in Fig. 302. It is important to connect all the resistance " in " again as soon as the motor is stopped, to be sure that the machine is ready for the next start.

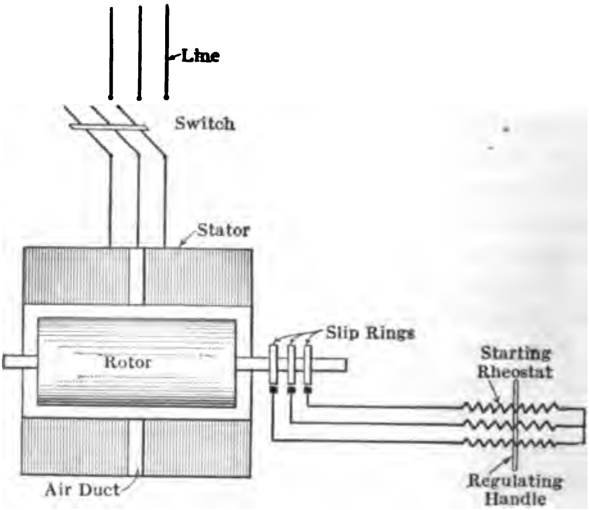


FIG. 305. Starting an induction motor by means of secondary resistances.

There are some special methods of starting induction motors, devised to do away with the complication of the two standard methods described above. Such special arrangements are seldom used and need be mentioned here only briefly.

(1) Starting with the stator windings Y-connected and running with the same windings delta-connected. This is equivalent to applying a starting voltage equal to $1/\sqrt{3} = 57.7$ per cent of the line voltage.

(2) Starting at no load, with a centrifugal clutch between the motor and its load.

(3) Starting simultaneously with the generator which supplies the motor with power.

(4) In a plant equipped with several motors, using one or more auxiliary lines from sources of lower voltage or frequency, for starting only.

(5) The rotor is provided with two separate windings, one of high resistance, active at start only, the other of low resistance which automatically comes into play as the motor speeds up, and then carries the bulk of the secondary currents. Several arrangements are in existence for accomplishing this result.

(6) Rotor windings are divided into two unequal parts which are connected in opposition for starting and are cumulative in their action in normal operation.

(7) The use of inductances has been suggested in place of resistances in the secondary circuit (Fig. 305) to limit the secondary currents at the start. The disadvantage of this method is that the starting torque is greatly reduced, and the power factor of the machine is lowered unless the inductances are later cut out.

(8) Choke coils of high reactance, but of low ohmic resistance, are connected across the slip rings in parallel with comparatively high non-inductive resistances. The frequency of secondary currents at start being high, a large portion of them flows through the resistances. As the motor speeds up the secondary frequency decreases and a larger and larger fraction of the currents flows through the reactances. At full speed the whole starting arrangement may be short-circuited, or left in, if so desired for the sake of simplicity.

476. Speed Control of Induction Motors. — An induction motor is not well adapted for speed control because of the constant speed of its revolving flux. The electrical energy input from the line is transmitted through the air-gap into the rotor, with the exception of a few per cent lost in the primary copper loss and stator core loss. Let the tangential effort between the stator and the rotor be F kg., the linear speed of the revolving field v_1 meters per second, and that of the rotor conductors v_2 meters per second. Then the electrical input into the rotor is Fv_1 kg.-meters, while the mechanical output of the rotor, corresponding to the electromagnetic torque, is only Fv_2 kg.-meters. The remainder, $Fv_1 - Fv_2$, is used up in the rotor, mainly as the secondary copper loss. Thus, of the power transmitted into the secondary, the fraction v_2/v_1 is converted into mechanical power on the shaft (including friction and windage), and the remainder, $(v_1 - v_2)/v_1$, is the fraction of the electric power lost in the rotor winding. This relationship is usually expressed in the words: *Per cent secondary copper loss is equal to per cent slip.* Let, for example, 100 kw. be transmitted inductively from the stator into the rotor, and let the machine be operating at 5 per cent slip. Then the Joulean loss in the rotor is 5 kw., and 95 kw. is used to drive the mechanical load and to overcome the torque due to the friction, windage, and hysteresis in the rotor core.

A clear understanding of this relationship is necessary for the study of different methods of speed control. If all but a small percentage of the input into the secondary is to be converted into mechanical power, then the difference between the speed of the air-gap flux and that of the rotor must be small. If the rotor is to run at various operating speeds, the speed of the stator flux must be adjustable. But this speed is a function (a) of the number of poles, and (b) of the frequency of the supply, so that at least one of these factors has to be varied.

On the other hand, if the speed of the rotor is to be varied without modifying the speed of the revolving flux, there are the following two possibilities: (c) A large secondary i^2r loss is permitted to occur; or (d) The excess electrical power delivered to the rotor is returned into a network or converted into useful mechanical power in another machine. Of these four possibilities (a) and (c) are used regularly, while (b) and (d) are used in some large drives. The practical means for accomplishing these results are as follows:

(a) *Change in the number of poles.* Either the same winding is used and properly re-connected, or two separate windings are placed in the same slots and used one at a time. This method allows only two or three speeds and is used, among other applications, in marine propulsion and in some electric locomotives.

(b) *The frequency of the supply* can be varied at will if the generating set furnishes power only to the motors whose speed is to be adjusted. Thus, in an electrically propelled ship, the speed of the main turbo-alternators can be varied within certain limits for the purpose of regulating the speed of the induction motors which drive the propeller shafts.

(c) *The use of secondary resistances* for speed control is the standard method for all intermittent work, on cranes, hoists, switching locomotives, etc. For lack of a better and simpler method, a considerable loss of power in external resistances is accepted as being inevitable. For details of rheostats and controllers see Vol. II. Speed-torque curves for different values of secondary resistance are shown in Fig. 302.

(d) *The regenerative control* in the secondary circuit may be of two kinds: (1) The excess secondary electric power is converted into electric power of the primary frequency and is returned to the primary circuit, or it is converted into direct current and used in another circuit; (2) The excess secondary electric power is sent through another motor and converted into mechanical power which is used on the same shaft with the main motor, or separately. All such methods of regenerative control require at least one and often two or three additional machines, and are therefore used only in very large drives, such as rolling mills and mine hoists, where the advantages of speed control within wide limits outweigh the additional complication and expense. It is in such drives that polyphase commutator motors find their useful place as adjuncts to the induction motor. Several descriptions of such systems will be found in the volumes of *General Electric Review* and *Electric Journal*.

477. EXPERIMENT 22-A. — The Starting Torque and Current of a Squirrel-Cage Induction Motor. — The purpose of the experiment is to investigate the value of the starting current and starting torque as functions of the applied voltage, frequency, and rotor resistance. The necessary theory is explained in §§ 474 and 475. A suitable Prony brake must be provided (§ 303), preferably with a spring balance, so arranged that the lever can be moved up and down while the motor is starting, in order to eliminate the bearing friction of the machine. Auto-transformers with numerous taps can be used for applying different voltages (Fig. 304) although it may be simpler to use directly a separate alternator whose voltage and frequency can be varied at will. It is sufficient to have an ammeter and a voltmeter in one phase only, if a preliminary experiment shows that the three phases are fairly well balanced. An indicating tachometer should be connected to the motor shaft. The frequency of the supply must be kept constant except where its influence upon the starting torque is to be determined. Copy the name plate of the motor under test.

(1) Find the lowest possible voltage at which the motor will just start at no load. Read volts and amperes at the instant of starting, find the number of seconds which it takes to attain the final speed and read the speed, the final volts, and amperes.

(2) Repeat the test at several higher values of the voltage, taking care not to overheat the motor. Should the windings become dangerously hot, run the motor for a time at no load and at the rated voltage, to allow the windings to cool off by the windage.

(3) Make a few runs similar to (1) and (2) at frequencies up to 10 per cent higher and lower than the rated frequency of the motor.

(4) Apply a brake to the motor and gradually raise the terminal voltage until the current reaches the highest safe value (it may be twice the rated current or even higher). Read the current, the voltage, and the brake setting at which the motor will just start and run up to speed. Read the final volts, amperes, speed, and starting time. The test

should be made as quickly as possible so as not to overheat the machine.

(5) Repeat similar runs at a few lower values of starting current.

(6) If another squirrel-cage rotor of different resistance is available, take a few readings with it so as to have a basis for comparison with the first machine.

Report. (1) Tabulate the data of runs (1) and (2) in such a way as to bring out clearly the effect of the starting voltage upon the current inrush and upon the time of running up to speed. (2) Show that the effect of the frequency upon the starting torque and current is such as would be expected theoretically. (3) Show from the readings that the starting current under maximum torque conditions is proportional to the terminal voltage; extrapolate the straight-line relationship to the full rated voltage. On the same curve sheet, mark the rated current of the machine. (4) Show from the readings that the maximum starting torque is nearly proportional to the square of the terminal voltage. Extrapolate the parabolic curve to the rated voltage. From the rated output and speed of the motor, compute the full-load torque (§ 267) and mark it on the same curve sheet. (5) From the curves of current and torque, suggest a value of starting voltage suitable for a kind of service in which the motor starts practically at no load. Also suggest a starting voltage for another kind of service where the motor has to overcome a starting torque of the same order of magnitude as the full-load torque. (6) If tests have been made with a rotor of different resistance, show that with the same starting voltage the starting torque is higher for the rotor of higher resistance.

478. EXPERIMENT 22-B. — The Starting Torque and Current of an Induction Motor with a Phase-Wound Secondary. — The purpose of the experiment is to investigate the effect of a variable rotor resistance upon the starting torque and current. The necessary theory is explained in §§ 474 and 475. It is advisable to use the same stator as in the preceding experiment. A suitable Prony brake must be provided (§ 303), preferably of the self-recording type, because the starting torque of a phase-wound rotor varies considerably in its different positions with respect to the stator. It is essential therefore to note several values of the torque with the same setting of the secondary rheostat, moving the rotor by a small angle at a time. A self-recording brake, or "stationary torque recorder," can be improvised out of an ordinary Prony brake and a steam-engine indicator. An ammeter should be provided in each stator phase and in each rotor phase, so that the resistances in the three phases can be balanced. A voltmeter should be provided to measure the line voltage and the voltage between the slip rings. When balancing the resistances it may be convenient to connect the voltmeter in succession between each of the slip rings and the neutral point. It must be remembered that the frequency of the secondary currents and voltages is equal to that of the line only when the rotor is stationary. The secondary frequency decreases to zero as the rotor approaches synchronism. Therefore it is essential to have in the secondary circuit instruments that are not affected by the frequency, or else not to read the secondary quantities except when the rotor is standing still. A reliable indicating tachometer should be connected to the shaft. All the tests specified below are to be performed at the rated stator voltage.

(1) Find the lowest safe value of the secondary starting resistance, that is, the value at which either the stator current or the rotor current has its highest safe value as regards overheating. Read the maximum starting torque on the brake; also the primary and the secondary volts and amperes.

(2) Repeat the test at several higher values of the starting resistance. Be sure to go beyond the value at which the starting torque reaches its maximum.

(3) Having found the best value of the resistance, that is, the one at which the torque is near its maximum, take an ampere-speed curve when the resistance is cut out in equal steps at equal intervals of time. Keep a constant brake torque and read instantaneous speeds and currents in at least one stator phase, every few seconds, while cutting the resistance out. Take a few complete curves, using a different time interval, a different resistance step, or a different load for each curve.

(4) Repeat the preceding test, using varying intervals of time and variable step resistances, the object being to accelerate the motor in the shortest possible time, without exceeding a safe maximum current. The motor is started at this maximum current and the resistances are cut down in steps, as soon as the current drops appreciably below this value. After a few trials, find and record the recommended values of resistances, the intervals of time, and the current values which you consider to be best for the motor starting at a certain load and accelerating in a minimum time. These values are of importance in specifying an automatic starter for the motor and in estimating the performance of the motor for an intermittent service, with frequent starting and stopping.

(5) Try starting the motor on a single-phase line (Fig. 315) and make clear to yourself the limitations of this method.

(6) Try some special methods of starting described in § 475.

Report. (1) Describe the preliminary experiments on fluctuations of the starting torque with different positions of the stator, and explain whether the average, the minimum, or the maximum torque has been used in the computations and curves specified below. (2) Against primary starting current as abscissæ, plot the observed values of starting torque, secondary current, and secondary voltage. If the primary applied voltage was incorrect, change the currents and the secondary voltage readings directly as the voltage, and the torque as the square of the voltage. (3) From the preceding curves compute the value of the starting resistance for which the starting torque is a maximum. This resistance, per phase of Y, is equal to the ratio of the secondary Y voltage to the secondary current. (4) Draw ampere-speed curves for the various rates of cutting out the starting resistance. Specify the best resistance steps to be used in a permanent starting rheostat, and the approximate interval of time within which the motor could be guaranteed to reach its full speed under certain specified torque conditions. (5) Explain in detail how the modified starting conditions may be estimated when the motor also has to accelerate considerable revolving masses. (6) Make a comparison between the starting characteristics of a squirrel-cage motor and one with a phase-wound secondary. (7) Describe the tests with special starting arrangements and state the advantages and disadvantages of each.

PERFORMANCE TESTS

479. The Performance Curves. — The operating characteristics of a polyphase induction motor are shown in Fig. 306, plotted to kilowatts output as abscissæ. While the rating of electric motors in horse-power was formerly common, the more recent international usage is to express both the input and the output in kilowatts, no matter whether electrical, mechanical, or thermal power is meant, and to avoid the horse-power unit altogether. The performance characteristics may also be plotted against kilowatts input, against primary amperes, against torque, slip, etc., depending upon the purpose for which the curves are to be used.

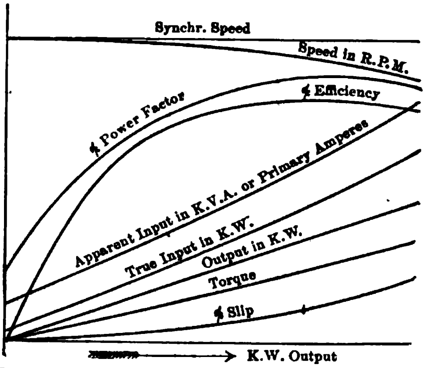


Fig. 306. Performance curves of a polyphase induction motor.

The curves are self-explanatory, except that a few words in regard to the curve of "apparent input" may not be amiss. If the effective value of the primary current per phase is I_1 and the primary Y voltage is E_1, then the apparent input per phase, in volt-amperes, is defined as E_1I_1. The true or real power input per phase is less, and is equal to $E_1I_1 \operatorname{Cos} \phi_1$ watts, where ϕ_1 is the phase-displacement angle between E_1 and I_1. The total apparent input is equal to $3\ E_1I_1$, or, using the line voltage (§ 447) and kilovolt-amperes,

$$P_{app} = E_{\Delta} I_1 \sqrt{3}/1000 \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (1)$$

where $E_{\Delta} = E_1 \sqrt{3}$.

Similarly, the total true input, in kilowatts, as measured by a wattmeter, is

$$P_{true} = E_{\Delta} I_1 \operatorname{Cos} \phi_1 \sqrt{3}/1000 \quad . \ . \ . \ . \ . \ . \ . \quad (2)$$

In addition to the curves of true and apparent power, a power-factor curve is shown on the same sheet; from any two of these curves the third one may be readily plotted. The ordinates of the curve of apparent power are proportional to the values of primary current, and for this reason the current curve is not shown separately. In some cases the current curve is of more interest than that of apparent power and is plotted in its place.

The general shape of the curves shown in Fig. 306 follows from the theory of the motor given in §§ 471 to 473. At no load, or at zero output, the true input is small and covers only the losses in the motor. The power factor is low because the greater part of the primary current is a reactive component which excites the main flux. The efficiency is zero, and the rotor runs practically at synchronous speed because the torque is almost zero. As the load is increased the primary current becomes larger, because in addition to a magnetizing component it contains a power component corresponding to the increasing rotor current. Thus the power factor and the efficiency are increased and the slip also becomes greater, to enable larger currents to be induced in the secondary. When the motor is overloaded, the slip and the frequency of the secondary currents increase so much that the effect of the secondary leakage inductance causes a low secondary power factor. This lowers the primary power factor, the effect being similar to that of an inductive load in a transformer (§ 390). For this reason the power-factor curve is shown slightly drooping at heavy loads. The copper loss increases rapidly with the current, and therefore the efficiency also passes through a maximum and decreases on an overload.

480. The Load Test. — The foregoing curves can be computed either from an actual load test, or from a no-load and short-circuit test, as in the case of a transformer. The first or direct method is described below and the second or indirect one is treated in Vol. II.

A load test may be performed in two different ways, according to whether the mechanical output is measured directly or is computed from the input and the losses. The output may be measured directly by means of a Prony brake (§ 303), by means of a dynamometer (§§ 304 and 305), or by using a calibrated generator as a load. The general arrangement is nearly the same as in the load test of a direct-current motor (Chapter XII).

The principal difficulty in performing an accurate brake test on an induction motor is to hold the load constant while the speed and the slip are measured. Moreover, the load must be exactly the same when watts and amperes are read in the three phases. For these reasons it is preferable to use a steadier load than is possible with a Prony brake. An electric generator, a blower, or a centrifugal pump, will be found particularly convenient for the purpose. Provision must be made for varying this load at will. The test is conducted similarly to a brake test, except that the output is not determined experimentally, but is calculated afterwards from the input and the losses. The latter can be partly measured and partly computed, as is explained below.

Experience shows that with this method more accurate results are obtained than with an ordinary brake test. One reason is that the speed and the load are easily maintained constant for any length of time, so that the input can be read quite accurately. Another reason is that all the readings and calculations are purely electrical, the mechanical torque being calculated from the input, instead of being read on the brake.

With any load test on an induction motor it is of importance to measure the speed and the slip accurately. While the speed may differ only by a small percentage from its synchronous value, there is no provision in a squirrel-cage motor for adjusting the speed to a desired value, as one can do in a direct-current motor by means of a field rheostat. For this reason the exact speed and its variations with the load must be accurately known in order to provide a proper gear ratio or pulley for a particular drive. When the output is computed from the losses the slip must be determined with particular accuracy, because the secondary copper loss is directly proportional to it (§ 476). The speed and slip measurements are described in §§ 481 to 484.

The electrical input into the stator is usually measured with a polyphase wattmeter (§ 97) so that the whole power is obtained in one reading. If a polyphase wattmeter is not available, the two-wattmeter method is used (§ 100), a single-phase wattmeter being read in succession in two current phases. It would hardly be practicable to provide a separate ammeter and voltmeter for each phase; a special "polyphase board" is used, by means of which the same instruments may be connected in succession in the three phases. Any of the polyphase boards described in § 58 can be used for the purpose.

481. Measurement of Frequency and Speed. — The speed measurement of an induction motor deserves particular attention. The speed depends on the load and on the frequency of the supply since the latter determines the speed of the revolving field. If the power for testing is taken from an industrial circuit, the frequency may at times be a few per cent above or below normal, and unless the exact frequency is known at the time when the speed is taken, the speed determination is of little value.

When the generator is accessible, its speed may be measured simultaneously with that of the motor, so that the two speeds refer to the same frequency. If the generator is not accessible, a small synchronous motor may be run from the same line to which the induction motor is connected. As the speed of a synchronous motor is always equal to that of a generator with the same number of poles, and automatically follows speed variations of the latter, the synchronous motor will at any moment give the actual speed of the generator. A frequency meter (§ 443) may also be used for measuring the frequency of the supply.

Instead of measuring the speed of the motor and the frequency of the supply, it is sometimes preferable to measure simultaneously the speed and the slip of the induction motor. Their sum gives the synchronous speed, from which the frequency of the supplied currents can be computed. If, for example, the motor speed is 702 r.p.m., and the slip is 22 r.p.m., the synchronous speed is 724 r.p.m. If the motor is a 10-pole machine, the frequency of the supply at that particular moment is 7240 alternations per minute, instead of the standard 7200. When plotting a speed curve, the corresponding correction must be made.

The slip usually constitutes but a few per cent of the speed; at the same time an accurate knowledge of its value is of considerable importance to the designer as well as to the user of the motor. When the slip is determined as a difference between the synchronous speed and the actual speed of the motor, a small error in the determination of either may lead to a considerable error in the value of the slip figured out as the difference of the two. For this reason it is preferable to measure the slip and the speed directly and independently of each other, and to check the frequency of the supply as the sum of the two.

482. Stroboscopic Slipmeters. — The term "stroboscopic" is usually applied to devices and methods based upon the fact that the eye receives a continuous impression when the frequency of flickering o the light exceeds a certain limit, a moving body being visible only a certain points on its path.

(*a*) *Sectored-Disk Slipmeter.* A simple slipmeter based on this principle is shown in Fig. 307: A pasteboard or sheet-metal disk, with white sectors painted on it, is mounted on the shaft of the motor. The number of sectors should be equal to the number of poles of the motor. An alternating-current arc lamp, fed from the same supply as the motor, is placed before the disk. If the motor were revolving synchronously the white sectors would appear stationary; since, however, the speed of the rotor is a few per cent below synchronism, the sectors appear to the eye to be slowly rotating in the direction opposite to that of the shaft.

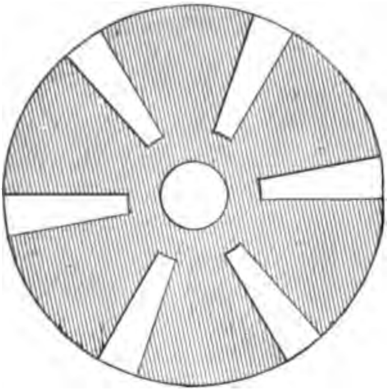


FIG. 307. Disk with painted white sectors, for measuring the slip of induction motors.

The reason for this phenomenon is that an alternating-current arc is actually extinguished once during each alternation of the current. With a synchronous rotation of the disk, each sector has just enough time during one alternation to move into the position of the preceding sector. Thus, when the lamp relights, the observer's eye finds the sectors in seemingly the same position as before. Since the flickerings of the light occur several thousand times per minute, the phenomenon appears continuous. When the rotor lags behind synchronism, the sectors do not have enough time to move through one polar division during one alternation of the current, so that each time the arc relights the eye finds the sectors in a slightly different position. This lagging behind goes on continually, and the sectors appear to the eye to be slowly moving backward, while in reality they are revolving at a high speed in the direction of rotation of the shaft.

The higher the slip of the motor, the faster are the sectors moving backward through the field of vision; the number of *apparent* revolutions of the disk is a direct measure of the slip. In order to count the sectors more conveniently, it is advisable to limit the field of vision so as to see only one sector at a time. Let us assume, for example, that 120 sectors have passed through the field of vision in one minute, while the speed of the motor, as measured by an ordinary speed-counter, was

1765 r.p.m.; let the motor be a four-pole, 60-cycle machine. With these data the slip of the motor and the actual frequency of the supply are figured out as follows:

With 120 sectors passing through the field of vision during one minute, the rotor has skipped 120 alternations of the supply. As the motor has four poles it took 120/4 = 30 slip revolutions to lose 120 alternations. Hence, the synchronous speed of the motor was 1765 + 30 = 1795 r.p.m., and the slip amounts to

$$\frac{30}{1795} = 1.67 \text{ per cent.}$$

When the speed was measured, the frequency of the supply was equal to 1795 × 4 = 7180 alternations per minute, instead of the standard frequency of 7200.

It will be seen from the above example that the stroboscopic slipmeter is more suitable for 25-cycle motors and for light loads than for high-frequency motors and for heavy loads, because in the latter case the sectors pass quite rapidly through the field of vision, and it is difficult to count them.

(*b*) *Vibrating-Reed Slipmeter.* Another stroboscopic slipmeter, which does not require an arc lamp, consists of an alternating-current electromagnet (Fig. 308) connected to the source of supply and provided with a steel reed near one of its ends. Alternating current flowing through the electromagnet sets the reed into synchronous vibrations. The reed is loaded at its extremity with the weight *w*, so as to make its natural period of vibration correspond to that of the supply. A disk with a slot in it is mounted on the shaft of the motor, and the vibrating reed is viewed through this slot. If the rotor were revolving synchronously, the reed would appear to the eye to be stationary, because

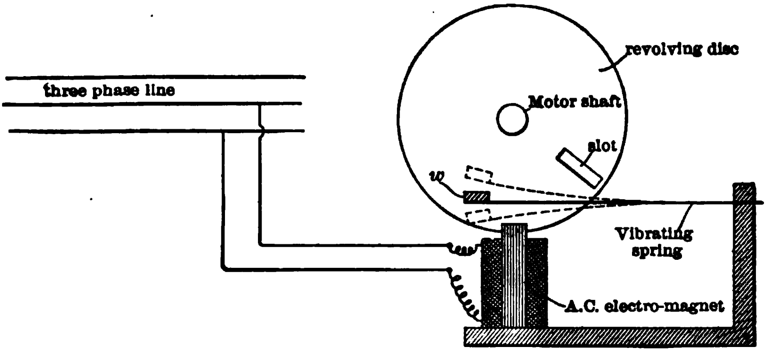


FIG. 308. A vibrating-reed slipmeter.

it would always be viewed by the observer during the same part of its vibration. Since the rotor lags behind synchronism the reed appears slowly moving up and down. The number of strokes per minute is proportional to the slip, as in the case of the arc-lamp slipmeter.

The reed may vibrate at a frequency which is a multiple of that of the supply, and it is safer to calibrate this instrument experimentally, by measuring the slip of a motor by some other method. This will give a constant by which to multiply the number of strokes in order to find the number of alternations skipped per minute.

483. Commutator Slipmeters. — In places where many induction motors are tested regularly, it is desirable to have a more convenient device for measuring slip than the above-described stroboscopic slipmeters. A device of this kind is shown in Figs. 309 and 310. It is a commutator with as many segments as the motor has poles, and is pressed by hand against the end of the motor shaft, as an ordinary speed-counter. The device is connected through a resistance to the power supply and to a sensitive D.C. ammeter. If the speed of the motor were exactly synchronous, the impulses of the current sent through the commutator into the ammeter would be always at the same point on the alternating e.m.f. wave, and its indication would be steady. However, since the rotor *lags* behind the revolving field, these impulses occur at different intermediate values of the current, and the ammeter needle swings at a frequency equal to the *difference* of the speeds of the revolving field and the rotor, and hence proportional to the slip of the motor. If the slip is not too high, the number of swings per minute can be easily counted and the slip calculated. A polarized bell (Fig. 370) can be used instead of an ammeter, and the number of strokes per minute counted.

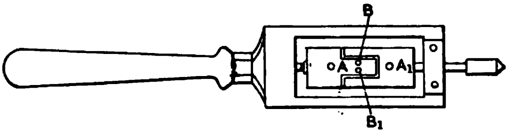


FIG. 309. A commutator-type slipmeter.

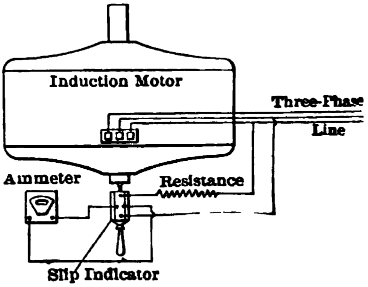


FIG. 310. Electrical connections to the slipmeter shown in Fig. 309.

As with a stroboscopic slipmeter, it is difficult to count the number of impulses when the frequency is high and the slip is considerable. The Bianchi slipmeter (Fig. 311), while similar in its principle to that in Fig. 309, has an attachment for automatically registering the number of revolutions of slip. The impulses produced by the revolving commutator D, instead of being counted by the observer, are sent through the electromagnet M which actuates a ratchet-and-pawl recording mechanism q, through the permanent magnet c. The number of revolutions of slip is thus recorded on the dial Z_2; at the same time the number of revolutions of the rotor is shown on the dial Z_1, connected to an ordinary speed-counter. The operation of this device is entirely automatic; it is pressed against the shaft of the motor at p, as an ordinary speed-counter, and is held for, say, one minute. The reading on one scale Z_1 gives the actual speed of the motor; the reading on the other scale gives the number of revolutions of slip; adding the two gives the synchronous speed of the motor, and consequently the frequency of the supply.

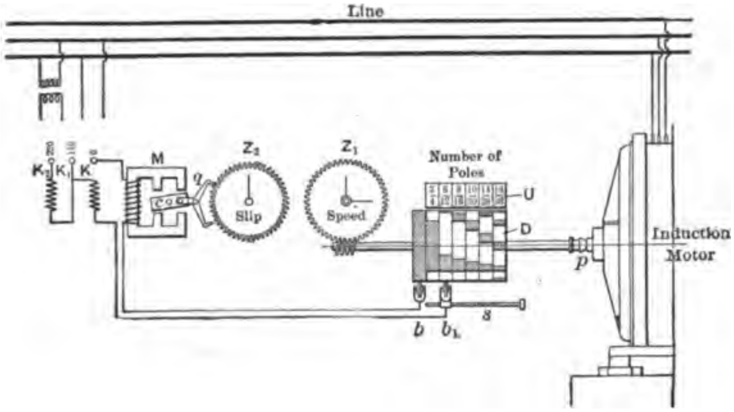


FIG. 311. The Bianchi automatic slipmeter.

The slipmeter is connected to the line either between the terminals K and K_1, or K and K_2, according to the voltage of the motor. For higher voltages a potential transformer is used. The drum D is provided with different combinations of contacts, for motors with various numbers of poles. The setting is done with the screw s which moves the roller brush b_1. The number of poles is indicated on the scale U.

484. A Differential-Gear Slipmeter. — The slipmeter shown in Fig. 312 is a convenient arrangement for mechanically comparing the angular velocities of two shafts. One shaft is driven at a constant speed by the synchronous motor of the slip indicator and the other at the speed of the motor under test by means of a flexible coupling between them.

The indicator is mounted on an iron base, fitted with a handle at each end, for carrying or handling it. The synchronous motor has a bipolar stator and a four-pole rotor without winding. The shaft carries a fly-wheel on one end and on the other a 32-tooth brass gear wheel, which may be made to mesh with any one of a nest of seven gears mounted on a parallel shaft, by shifting the motor on the brass plate. The various gears are provided to enable the synchronous motor to drive the parallel shaft at different speeds, thus adapting the instrument for testing motors with various numbers of poles.

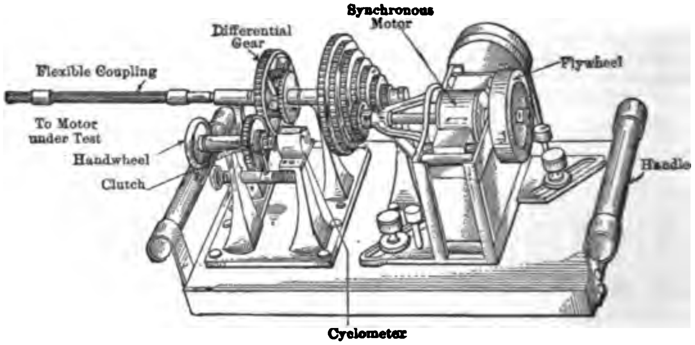


FIG. 312. A differential-gear slipmeter.

The parallel shaft is equipped at its other end with a bevel-gear wheel meshing with two pinions of a differential gear. Meshing with these pinions on the other side is another bevel gear, carried on the shaft connected to the machine under test. A short auxiliary shaft shown in front of the differential gear has on one end a gear wheel meshing with the large wheel of the differential gear, and on the other end a small handwheel by which to hold this shaft. A clutch, operated by a lever, connects the auxiliary shaft to a cyclometer which registers the number of revolutions of the auxiliary shaft. The gear ratio is such that one revolution of the auxiliary shaft corresponds to one-half revolution of the differential gear.

The indicator is used as follows: With the speed of the alternator held constant at the normal frequency of the motor, the indicator is connected to the induction motor shaft by means of a "split tip." The bevel gear on the shaft carrying the split tip is then driven at the speed of the induction motor. By grasping the hand-wheel on the auxiliary shaft, and thus holding the large gear of the differential stationary, the synchronous motor is mechanically brought up to the speed of the induction motor by power transmitted through the other side of the differential and the nest of gears. When the line switch of the synchronous motor is closed it will immediately fall into step with the alternator and run at the synchronous speed. Should it fail to do so on the first trial, a second or third trial in the same way will usually be successful.

With the synchronous motor running and driving one bevel gear of the differential at the synchronous speed, and the induction motor driving the other bevel gear at the speed of the induction motor, the large gear of the differential will rotate and drive the auxiliary shaft and cyclometer. In this way, the difference between the synchronous speed and that of the induction motor is mechanically recorded.

485. EXPERIMENT 22-C. — Brake Test of an Induction Motor. — The purpose of the experiment is to obtain the performance curves of the motor (Fig. 306). The theory of the method and the various measurements are explained in §§ 480 and 481. For wattmeter connections see § 100; for a polyphase board for transferring an ammeter and a voltmeter from phase to phase, see § 58. To obtain accurate curves it is necessary to keep the terminal voltage and the frequency of the supply constant throughout the test; suitable provisions should be made before beginning the readings. The data sheet shown on the opposite page will be found convenient for recording the readings. The motor should first be run light for about thirty minutes to warm the bearings. Begin the test with an overload of about 25 per cent above the rating of the motor. Keep the brake load as steady as possible, read volts, amperes, watts, brake load, speed, and slip. Reduce the load to zero in approximately equal steps, and take similar readings at each step. Finally take readings at no load.

Report. Give the actual diagram of electrical connections and describe the apparatus used, viz., the kind of brake or dynamometer, frequency meter, slipmeter, etc. Compute the data for the curves and give sample computations. Plot the curves and mark the points of full-load, ¾, ½, and ¼ load, as well as 25 per cent overload.

486. Losses in the Induction Motor. — As is explained in § 480, it is difficult to hold the brake load constant while the readings are being taken. A convenient modification of the brake test consists in connecting the motor to a steady load, such as an electric generator, and omitting the output readings from those specified in the preceding experiment. The output is computed from the input and the losses in the machine. These losses are as follows:

(a) The primary copper loss;
(b) The secondary copper loss;
(c) The core loss in the stator;
(d) The core loss in the rotor;
(e) Bearing friction, slip-ring friction, and windage.

The primary copper loss may be computed from the resistance of the winding at the proper operating temperature. The actual loss may be

Date.................................Section.....

Observers:

....

....

....

....

Length of Brake Arm..................................

Tare or Zero Brake Reading..........................

Make of Motor...

Type — Open, semi-enclosed, enclosed, ventilated frame { Yes. No.

Volts.................................Amps. per terminal ....

H.P.......................Mfrs' No..............No. of Poles ....

Speed { At full load.................. At no load .... Alts............Cycles........

Phase.......................................Connections Y or Δ.

Type of Rotor { Phase wound. Squirrel Cage. Fans { Yes. No.

Method of Starting ....

	Amperes.			Volts.			Watts.		Speed.		Slip indic.	Torque.
	A	B	C	AB	BC	CA	A — AB	C — CB	Motor.	Synchr.		Lbs.
Inst. No.												
Const.												

considerably higher because of eddy currents and an unequal distribution of the current in the conductors. The same applies to the secondary copper loss. For this reason, the "effective" A.C. resistances are sometimes determined from the wattmeter readings on the so-called short-circuit test similar to that on a transformer (§ 386). It is difficult, however, to eliminate the core loss from such readings.

The stator core loss depends upon the stator flux. With a constant applied voltage, the stator flux is practically constant (§ 472) and therefore the primary core loss is usually considered constant and equal to that at no load. If greater accuracy is required, a slight reduction in the core loss with an increasing load may be taken into account. The rotor core loss is zero at synchronism, and is quite small at usual values of slip, because the frequency of the secondary magnetization is low. As the load increases, the secondary core loss increases while the primary core loss slightly decreases. For most practical purposes it is sufficient to disregard the secondary core loss altogether and to consider the primary core loss as having its maximum value (the no-load value) at all loads.

When the core loss and the mechanical losses are known separately, the *correct* procedure in computing the output of an induction motor, for a given input and slip, is as follows:

(1) From the measured primary current and resistance, the primary copper loss is computed and added to the primary core loss. The sum of these losses is subtracted from the input, and the result represents the stator output, or the input into the rotor.

(2) According to the theory given in § 476, the percentage of secondary copper loss is equal to the per cent slip. Therefore, the input into the secondary, after being reduced by the percentage equal to that of slip, gives the gross mechanical output.

(3) This total mechanical output is further reduced by the amount of friction and windage, and the result represents the useful net output available on the shaft.

In practice this procedure is sometimes difficult to follow, because there is no simple method of measuring the core loss and the mechanical losses separately. With the motor running at no load, the total input is equal to the core loss, plus the friction and windage, plus the primary copper loss. The secondary copper loss is negligible because the slip is practically zero. Correcting the wattmeter reading for the computed primary copper loss, gives the total no-load loss, that is, friction, windage, and core loss. The experimental separation of the core loss is explained in § 489 below, and if it is performed on the motor under test, the correct procedure outlined above can be followed. If, however, only the total no-load loss is known, an *approximate* procedure is followed, viz.:

(1) The primary copper loss and the total no-load losses are subtracted from the input, and the result is taken to be equal to the input into the rotor.

(2) The input into the secondary is reduced by the percentage of slip and the result is taken to represent the useful output available on the shaft.

This approximate procedure is accurate enough for large and medium-sized machines in which the losses and the slip are comparatively small. But with small motors and with special low-efficiency motors, the accurate procedure should be followed.

487. An Example of Computation of Output from Input and Losses. — Let a 10-kw., 440-volt, three-phase induction motor be found to take 800 watts at no load; let the no-load current be 5.5 amperes per phase and the primary resistance 0.75 ohm per phase. The iron loss and friction is equal to 800 watts less a correction for the copper losses in the primary and the secondary windings. But the secondary copper loss is negligible at no load, because the slip is very small. Thus the correction amounts to $3 \times \overline{5.5^2} \times 0.75 = 68$ watts, and the losses in question are equal to $800 - 68 = 732$ watts.

Now take a point on the current curve, for example that corresponding to an input of 15 amperes per phase. Let the measured power input at this current be 9920 watts and the slip 5.4 per cent. The primary copper loss is

$$3 \times \overline{15^2} \times 0.75 = 506 \text{ watts;}$$

therefore $9920 - 506 = 9414$ watts represents the output plus all the losses except the primary copper loss.

From now on, the procedure differs according to whether the correct or the approximate method is followed, as outlined in the preceding section.

(a) *The approximate method.* The input into the secondary is $9414 - 732 = 8682$ watts. The secondary copper loss is $0.054 \times 8682 = 469$ watts. The net output is $8682 - 469 = 8213$ watts. After the output corresponding to a given input has thus been determined, the efficiency, the torque, etc. may be computed as in an actual brake test.

(b) *The exact method.* Let the friction and windage be determined separately (§ 489), and let them be equal to 415 watts at the synchronous speed. Then the iron loss is $732 - 415 = 317$ watts. The input into the secondary is $9414 - 317 = 9097$ watts, the secondary copper loss is 0.054 × 9097 = 491 watts, and the total power output, including that converted into friction and windage, is 9097 − 491 = 8606 watts. At 5.4 per cent slip, the friction and windage are less than at the synchronous speed. Let their estimated value, or one taken from an available curve, be 370 watts. Then the net useful output is 8606 − 370 = 8236 watts, which is only 23 watts more than the value determined above by the approximate method.

488. EXPERIMENT 22-D. — Performance of an Induction Motor from Input and Losses. — The purpose of the experiment is to obtain the performance curves shown in Fig. 306 without actually measuring the output with a brake or dynamometer (§§ 486 and 487). The experiment should preferably be performed on the same motor as experiment 22-C, in order to check the results. The motor is belted to a generator or a blower, serving as a load, and readings are taken as explained in § 485, except that the brake readings are omitted. At the end of the load test, careful readings should be taken of amperes and watts at no load, and the resistance of the primary winding determined at the proper operating temperature. When a resistance is measured between two terminals of a Y-connected winding, the result represents the resistance of two phases in series. Therefore, in order to obtain the *average* resistance per phase, take the resistances between the terminals $A - B$, $A - C$, and $B - C$, add them together, and divide the result by six. If possible, separate the core loss from the windage and friction by one of the methods described in the following section.

Report. The same curves should be plotted as in § 485, the results to be derived by the method described in §§ 486 and 487. In figuring out the primary copper loss, use the value of the primary resistance at the highest temperature specified for that particular class of insulation in the Standards of the American Institute of Electrical Engineers. Compare the performance curves with those obtained on the same motor by a brake test, and explain the discrepancies, if any.

On a separate sheet plot curves of total loss, primary and secondary I^2R losses, iron loss, and friction, against the output as abscissæ, so as to get a clear idea of the relative importance of these losses at various loads.

489. Separation of Iron Loss from Friction. — The same three methods may be used for the separation of the core loss from the mechanical losses in a polyphase induction motor as in a direct-current machine (Chapter XIII), viz., (a) the machine may be driven mechanically, (b) the machine may be driven electrically, (c) the retardation method may be used

(a) *The machine driven mechanically.* The induction motor under test is belted or otherwise mechanically connected to a small shunt-wound direct-current motor and is driven at different speeds near synchronism, both below and above the synchronous speed. The stator circuit is left open so that there are no losses, except the friction and windage. The input into the driving motor and its losses being known, the friction and windage loss of the motor under test can be determined. The driving motor should be sufficiently small in order to run at a point of high efficiency; then a small error in the estimation of its losses will not appreciably affect the results. From the wattmeter readings taken on a no-load run on the same induction motor, the sum of iron loss and friction can be determined. Subtracting the friction loss from this sum, the value of the iron loss alone becomes known.

It is also of interest to have the stator connected to a source of A.C. supply while the machine is being driven by the auxiliary motor. An ammeter and a polyphase wattmeter should be connected in the A.C. circuit. When the field of the D.C. motor is strong, its armature tends to run rather slowly and the induction motor drives it as a generator. As the D.C. field is weakened and the motor speeds up, it begins to drive the induction motor, overcoming its friction and windage and the secondary core loss. This is shown by a reduction in the wattmeter indication. When the synchronous speed is reached, the A.C. circuit furnishes only the primary core loss and the primary copper loss. The driving motor supplies the power for overcoming the rotor friction and windage.

If the field of the D.C. motor be further reduced, the motor will drive the induction machine as a generator, " pumping " power back into the A.C. line. The wattmeter indication will be reversed and the readings of both the A.C. ammeter and wattmeter will increase with the speed. Just at synchronism the secondary hysteresis loss disappears and there may be a noticeable decrease in the current input into the driving motor. For the operation of the induction machine as a generator, see Vol. II.

(b) *The machine driven electrically.* The induction motor under test is connected in the usual way to a source of A.C. power, with a voltmeter, an ammeter, and a wattmeter in the primary circuit. A provision should be made for varying the applied voltage within wide limits. The first run is made at a reasonable over-voltage, and then the voltage is reduced in numerous steps to the lowest value at which the motor can run at a nearly synchronous speed. The current, the voltage, the watts, and the speed are read at each step. The watt readings are corrected for the primary copper loss and a curve is plotted between the watts and the volts. This curve gives the core loss plus friction, as a function of the applied voltage. Being extrapolated to zero voltage, the curve gives the friction and windage alone, because the core loss would be zero if the motor could be run at zero voltage.

There is a small inaccuracy involved, because the speed of the machine goes down as the applied voltage is reduced. This can be corrected by increasing the generator speed to keep the motor speed constant. By taking several series of runs, at speeds corresponding to zero slip, 0.5 per cent slip, 1.0 per cent slip, etc., a curve of friction and windage can be plotted as a function of slip. This curve can then be used in computing the output from the input (§ 486). For an accurate extrapolation of a curve see § 336.

(c) *The retardation method.* The machine is brought up to a speed well above synchronous, either by an auxiliary motor or by increasing the generator speed. Then the source of power is cut off and the machine is allowed to slow down. The friction and windage are computed from the shape of the speed-time or retardation curve. For details see §§ 329 to 333.

490. EXPERIMENT 22-E. — Separation of Friction from the Core Loss in an Induction Motor. — The three available methods and the necessary computations are explained in the preceding section. This is a somewhat advanced experiment which requires considerable skill and precision if it is to give satisfactory results. The friction loss should be determined by all three methods on the same motor, so as to obtain a mutual check and to form an opinion as to the relative convenience and accuracy of each method.

NOTE. For some advanced tests on the induction motor, see Vol. II.

CHAPTER XXIII

THE SINGLE-PHASE INDUCTION MOTOR

491. A SINGLE-PHASE induction motor differs from the polyphase motor shown in Figs. 297 and 298 in that its stator is provided with a single-phase winding similar to that shown in Fig. 269, for connection to a single-phase supply. The rotor is usually provided with a squirrel-cage winding, although it may also have a three-phase winding and slip rings. At standstill such a motor has only an alternating and not a gliding magnetic field, and will not start without some auxiliary means, since there is no reason why there should be more torque in one direction than in the other. Electromagnetically, such a motor at standstill is similar to a stationary transformer.

To make the motor self-starting, an auxiliary winding is usually provided on the stator, in space quadrature with the main winding (§ 492). Another method consists in starting the machine as a repulsion motor (§ 496) and then short-circuiting all the individual rotor coils.

Once brought up to a considerable speed, the motor develops a torque and without load runs almost up to synchronism. After this, when loaded, it behaves essentially as a polyphase induction motor treated in the preceding chapter. Its performance characteristics, while inferior to those of a polyphase motor, are essentially those shown in Fig. 302, and its speed-torque curve (Fig. 302) has the shape *hfd*; beyond point *f* the torque rapidly drops to zero.

There are two different ways of explaining the fact that, once brought up to speed, a single-phase induction motor possesses a torque and can be operated under load near the synchronous speed. Both theories are given in Vol. II, and the fact itself may be accepted here as a result of electromagnetic reactions between the stator and the rotor. A simple experiment is as follows: bring a two-phase or three-phase induction motor up to speed, apply a moderate load, and then break the connection between one of the phases and the line. The motor will continue to operate single-phase, although at a lower power factor and at a much lower overload capacity.

An important characteristic of the single-phase induction motor is that at synchronous speed and within a few per cent slip below and above synchronism it has a practically perfect revolving air-gap flux. The net magnetizing m.m.f. of the stator (stator ampere-turns minus the opposing rotor ampere-turns along the same axis) furnishes only an alternating flux. Hence the rotor, when in motion, must develop an alternating m.m.f. stationary in space, and in time and space quadrature with the net stator m.m.f. This is because two such m.m.f.'s are necessary to produce a revolving flux (§ 472). At lower speeds this quadrature m.m.f. is smaller than the net magnetizing m.m.f. along the stator axis, and the revolving field becomes irregular, or, as it is called, elliptical. At standstill the quadrature field disappears, and for starting purposes must be excited by means of an auxiliary winding on the stator.

Single-phase induction motors are used in small sizes only, for the following reasons: (a) Their weight and cost per kilowatt of rating are considerably higher than those of two-phase or three-phase motors. (b) They either have to be started and brought up to speed at no load, or else the starting current reaches several times the rated current, even for a moderate torque. This current is objectionable from the point of view of the line service to other users of electric power. (c) The power factor and the overload capacity are lower than those of polyphase motors. In small sizes, and for certain classes of service, single-phase induction motors are very useful, because often they can be connected to an ordinary single-phase lighting circuit and do not require a special three-phase wiring with more expensive transformers, switches, etc.

492. Starting with an Auxiliary Phase. — A revolving magnetic field, however imperfect, is necessary in order to start an induction motor. In a single-phase motor such a field is usually produced by adding an auxiliary stator winding in space quadrature with the main winding (Fig. 313). If the current in this auxiliary phase could be made to lag behind or to lead the current in the main winding by exactly 90 degrees and be equal to it, a perfect revolving field and a high starting torque could be obtained.

In reality the phase displacement between the two currents is always considerably less than 90 degrees, because both windings are connected to the same single-phase source of supply, and the current is made to lag or to lead in one of them by means of a series resistance or reactance. (In rare cases a condenser has been used.) Thus, a fluctuating or elliptical revolving field is produced, one that reaches higher values opposite the centers of the main coils than opposite the centers of the auxiliary coils. Nevertheless, such a field can be made of sufficient magnitude to start the rotor at no load and to bring it up to a considerable speed beyond which the rotor itself furnishes a quadrature field necessary for running up to full speed.

The auxiliary phase is opened at a proper speed, either by hand or automatically by a centrifugal device. The rotor is usually connected to its load through a friction clutch operated by a centrifugal governor, so that the rotor comes up to speed light, and is not engaged until it is capable of developing the required torque. To reverse the direction of rotation, the electrical connections either to the main phase or to the auxiliary winding are reversed.

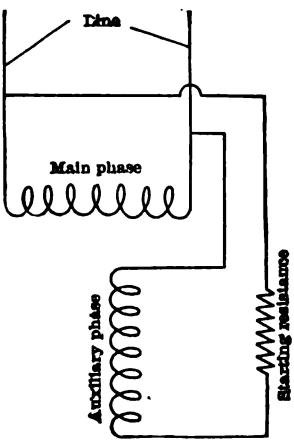


FIG. 313. Connections for starting a single-phase induction motor by means of an auxiliary phase.

Cases in which the auxiliary winding is left permanently in the circuit. In watt-hour meters, in indicating instruments, in phase-shifting transformers, etc., based upon the principle of the single-phase induction motor (Chapters II to V), the auxiliary phase winding has to be left in the circuit permanently. In all such devices the part corresponding to the rotor either is stationary or moves at such a low speed that the induced secondary currents do not excite a sufficient quadrature flux, and the latter has to be produced by an auxiliary primary current.

The shading coil. A special case of an auxiliary phase is the so-called shading coil S (Fig. 314) placed unsymmetrically upon a stator

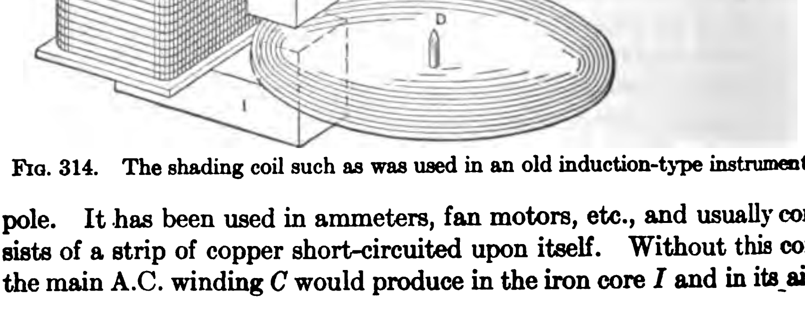


FIG. 314. The shading coil such as was used in an old induction-type instrument.

pole. It has been used in ammeters, fan motors, etc., and usually consists of a strip of copper short-circuited upon itself. Without this coil, the main A.C. winding C would produce in the iron core I and in its air-gap a uniform alternating flux, causing symmetrical eddy currents in the pivoted aluminum disk *D*. There would be no tangential force tending to turn the disk. The shading coil acts as a short-circuited secondary of a transformer, weakening the flux within itself and causing it to lag with respect to the flux outside the coil. Thus, two sets of unsymmetrical eddy currents are produced in the aluminum disk; each set forms a torque with the other flux and tends to rotate the disk. The electromagnetic reactions are similar to those shown in Fig. 103.

493. EXPERIMENT 23-A. — Adjustment of the Starting Phase in a Single-Phase Induction Motor. — The purpose of the experiment is to obtain the best starting conditions for a single-phase motor in which an auxiliary stator phase is used for starting (§ 492). It is desired to produce a torque sufficient for a positive start at no load, with as small an initial current as possible. Wire up the machine as in Fig. 313, with an ammeter and a wattmeter in the line and in both phases. A voltmeter should be connected to measure at will the line voltage, that across the starting phase, and across the resistance or impedance in series with it.

(1) Find out by preliminary trials whether a resistance or a reactance in series with the auxiliary phase gives better starting conditions. Increase and decrease this resistance or reactance until the smallest initial line current has been obtained at which the motor is sure to start. Since numerous trials at a full voltage would overheat the motor, conduct these tests and adjustments at a reduced voltage, with the rotor stationary. The aim should be to obtain, in the current that flows through the auxiliary winding, as high a component in time quadrature with the main current as possible. This can be determined by a properly connected wattmeter or power-factor meter.

(2) Having obtained the best conditions, check the results by actually letting the rotor start and come up to speed. Measure the initial line current and watts, the currents and the watts in the two phases, and the voltages across both phase windings and across the starting resistance or reactance. Determine whether the current in the auxiliary phase lags behind or leads that in the main phase. Estimate the number of seconds which it takes to reach the full speed, and find the best intermediate value of speed at which the auxiliary phase should be disconnected. Measure the line current when the motor is running at full speed, with the auxiliary phase " in " and " out."

(3) Adjust the starting resistance or reactance to bring the motor up to speed within the shortest possible time, even though at

the expense of a much larger starting current. Make the same measurements as before.

(4) Put a Prony brake on the motor, and by adjusting the starting resistance determine the largest practicable starting torque that the motor can develop without an excessive current. Make the same measurements as before.

(5) To investigate the air-gap flux, place in the air-gap a flat exploring coil of a width equal to the pole-pitch and connect the coil to a voltmeter. Block the rotor, set the starting resistance or reactance for the best conditions found under (1) above, and connect the stator to a reduced line voltage so as not to overheat the motor. Move the exploring coil in steps over a pole-pitch, and at each step measure the induced voltage in magnitude and in phase position. The latter can be determined with a wattmeter, or by balancing the induced voltage against that of a potential phase-shifter (§ 127) or an A.C. potentiometer (§ 84). Repeat the same measurements with the main phase disconnected, and then with the auxiliary phase disconnected. For comparison take a few readings with a wrong value of starting resistance or reactance, that is, with one which gives a poor starting torque.

Report. (1) Draw vector diagrams of starting currents and voltages in their proper magnitude and relative phase position, under the different conditions investigated. (2) Tabulate the test data to show how different values of starting resistance or reactance affect the total starting current, the time of coming up to speed, and the starting torque. (3) With the assistance of the test data explain the effect of disconnecting the auxiliary phase too early or leaving it " in " too long. (4) Give the results of the test with the exploring coil. From these results deduce the space and time distribution of the actual flux in the air-gap, especially the magnitude and the time phase of the quadrature flux with respect to the main flux.

494. A Three-Phase Motor on a Single-Phase Circuit. — There are cases in which it becomes desirable or necessary to operate a three-phase induction motor on a single-phase circuit, for example when one of the line wires or a transformer is out of order. A satisfactory starting arrangement is shown in Fig. 315. Two of the phase windings, OA and OC, are connected in series across the line, as the " running " or main phase. The third winding, OB, is used for starting only, and is connected at b, between a resistance r and an impedance z (as nearly as possible a pure inductance). After the motor has reached a considerable speed, the circuits of the starting device and of the phase OB are opened, and the machine operates as a single-phase motor.

The voltages, with OB open, are shown in Fig. 316. OE is the line voltage, *Oi* is the current in the main phase *AC*, lagging behind *OE* because of the inductance of the winding. The voltages across *r* and *z* are represented by *OF* and *FE* respectively. The current i_1 through *r* and *z* is in phase with *OF* and lags considerably behind *FE*. The voltage across the starting phase, *OB*, is represented by the vector *DF*, since this phase is connected between the middle point of *AC* and the junction of *r* and *z*. It will be seen from the sketch that the voltage *DF* may be made to lag considerably behind *OE*, so as to imitate a two-phase combination. However, when a current flows through *OB*, the phase relations become less favorable and the phase angle between *DF* and *OE* decreases. Nevertheless, by suitably selecting *r* and *z*, a torque can be produced sufficient for starting the motor.

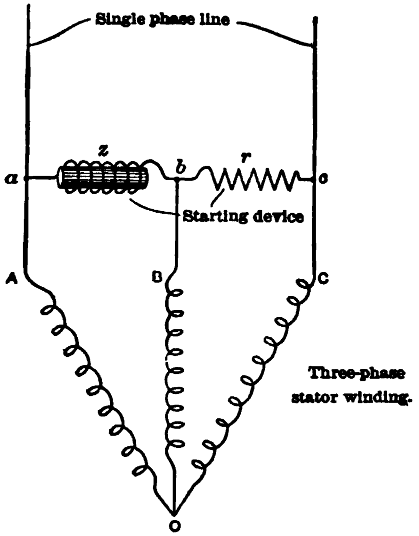


FIG. 315. Starting a three-phase induction motor on a single-phase circuit.

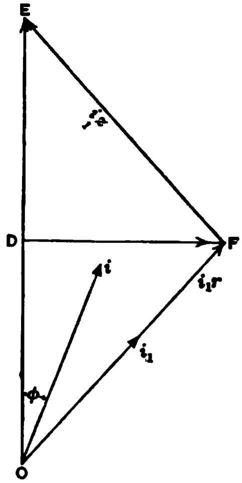


FIG. 316. Current and voltage relations with the connections according to Fig. 315.

495. EXPERIMENT 23-B. — Exercises in Starting a Three-Phase Induction Motor on a Single-Phase Circuit. — The method is explained in the preceding section. (1) First of all satisfy yourself that a three-phase motor can be operated on a single-phase circuit. For this purpose connect it to a three-phase circuit in the usual way, bring it up to speed at no load, and read volts, amperes, watts, speed, and slip (Chapter XXII). Disconnect one of the stator phases and take similar readings. (2) Stop the motor and adjust the resistance *r* and the impedance *z*

(Fig. 315) so as to get any starting torque whatever. Then by varying their values improve the starting conditions as far as possible. After this, either r or z should be kept constant, and curves taken showing the influence of the other factor. Read the currents, voltages, and watts in the several branches of the circuit, so as to permit the construction of diagrams, like the one shown in Fig. 316. Readings should also be taken with the phase OB closed and open, in order to see the influence of the current in this phase upon the current and voltage relations in the circuit abc. If a starting torque can be obtained, its value should also be investigated with a Prony brake. (3) For the rest of the experiment follow the instructions for the preceding experiment, with suitable modifications. For a companion experiment see § 499.

Report. (1) Give the no-load data for the motor operated single-phase and three-phase. Explain theoretically the relative values of exciting current and watts input under these two conditions. (2) Draw vector diagrams of starting currents and voltages under the best obtainable conditions, with the phase OB open and closed. (3) See the report requirements of the preceding experiment for the remainder of the work on the data taken.

496. Starting as a Repulsion Motor. — There is a class of single-phase motors (Fig. 317) which possess quite a considerable starting torque; this torque is obtained, however, at the expense of a commutator and brushes. The particular motor, shown in the sketch, has a stationary field winding, FF, connected across the line and producing an alternating field indicated by the arrows. The armature A is similar to that of a direct-current machine, and is provided with a commutator. The brushes bb are placed at an angle with respect to the direction of the field, and are short-circuited upon themselves, thus forming a closed armature circuit. The alternating field induces secondary currents in the armature, and the latter tends to move into such a position as to be subjected to a minimum of induction. This is impossible because the direction of the magnetic axis of the armature is deter-

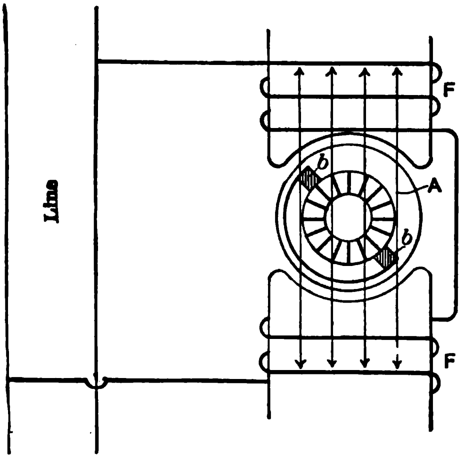


Fig. 317. Connections of a repulsion motor.

mined by the position of the brushes, so that the armature revolves continuously, trying to come into a stable position.

At most instants when the upper field pole is a north pole, the currents induced in the armature are such that its north pole is also at the upper brush *b*. Hence, the upper part of the armature is repelled to the left. It is probably for this reason that such a motor is called a " repulsion " motor. Motors of this type are seldom used at present on account of difficulties in commutation, and because of their so-called *series* characteristics. The speed of the motor greatly increases as the load decreases, the repulsion motor being in this respect similar to a direct-current series-wound motor (Fig. 215).

The desirable starting characteristics of a repulsion motor and the desirable running characteristics of an induction motor can be combined by starting the machine as a repulsion motor and then short-circuiting all the individual commutator segments when a sufficiently high speed has been reached. The machine then operates as an ordinary squirrel-cage single-phase induction motor. In practice the commutator is automatically short-circuited by a centrifugal device which at the same time lifts the brushes in order to do away with unnecessary friction. Such motors are useful for drives in which the load cannot be conveniently disconnected at start, and where a large starting current is permissible. For a more detailed theory of the repulsion motor at start see Vol. II.

497. EXPERIMENT 23-C. — Study of a Repulsion-Start Single-Phase Induction Motor. — The purpose of the experiment is to investigate the effect of various factors which affect the starting characteristics of the motor described in the preceding section. The factors to be varied are the brush shift, the ratio of the primary and secondary turns, and a resistance or reactance in place of the short-circuiting conductor between the brushes. The desired characteristics to be investigated are the starting torque and current, the time of coming up to speed, sparking at the brushes, etc. On the basis of the instructions for the experiment 23-A, the student should write similar instructions for this test, never varying more than one factor at a time. It is of particular importance to bring the experimental results regarding the starting torque and current in accord with the theory of the brush shift given in Vol. II.

498. EXPERIMENT 23-D. — Brake Test on a Single-Phase Induction Motor. — The purpose and the method of the experiment, the curves to be obtained, etc., are similar to those described in §§ 479 to

485, except that with but one stator phase, the wiring, the readings, and the computations are much simpler. It is of importance to load the motor, if possible, up to its pull-out torque, because a comparatively low ratio of the pull-out torque to the full-load torque is a weak point of the single-phase induction motor, and should be known when choosing such a motor.

If the stator has an auxiliary phase winding, in space quadrature with the main winding, it should be connected to an A.C. voltmeter and the induced e.m.f. measured at each load. The quadrature flux at no load is practically equal to the main flux, so that the voltage induced in the auxiliary winding at no load is a measure of both the main flux and the quadrature flux. The observed decrease in the quadrature flux with the load should be shown to be at least approximately in accord with the theory given in Vol. II.

499. EXPERIMENT 23-E. — Comparison of Characteristics of an Induction Motor Operating Three-Phase and Single-Phase. — The method of operation of a three-phase motor on a single-phase circuit is explained in §§ 494 and 495. The purpose of this experiment is to obtain complete performance characteristics (Figs. 302 and 306) of a three-phase motor from a brake test, and then to take similar characteristics of the same motor operated on a single-phase circuit. All the necessary information in regard to the test itself will be found in §§ 480 to 485.

Report. Plot on the same curve-sheet the performance curves of the motor operated three-phase and single-phase, so that a direct comparison may be made. Where possible, explain the observed differences theoretically.

500. EXPERIMENT 23-F. — Performance of a Single-Phase Induction Motor from Losses. — The reasons for determining the performance of a single-phase motor from the losses, instead of by direct brake test, are the same as those given in § 486 for polyphase induction motors. The experiment is conducted as explained in § 488. See also § 498 in regard to the pull-out torque and the measurement of the quadrature flux. The only difference in figuring out the results is in the expression for the secondary copper loss. It is shown in § 476 that in a polyphase induction motor the per cent secondary copper loss is equal to per cent slip (the input into the secondary being taken as 100 per cent). In a single-phase motor, at small values of slip, the per cent secondary copper loss is very nearly *twice* as large as per cent slip. For example, if the slip of the motor at a certain load is 4 per cent, the secondary copper loss amounts to 8 per cent. In other words, 8 per cent of the input into the secondary is converted into heat in the rotor windings. This added copper loss is due to a much more unfavorable distribution of the secondary currents, as a result of the rotor providing an m.m.f. for the quadrature flux (§ 491). The exact relationship between the slip and the secondary copper loss is much more involved, and the reader is referred to special treatises on the subject.

501. The Phase Converter. — A single-phase induction motor can be used to convert single-phase energy into two-phase or three-phase energy, and is then called a phase converter. When a single-phase motor is running at no load, the air-gap flux is practically a uniformly rotating one, as in a polyphase machine. Hence, the e.m.f. induced in the auxiliary winding (Fig. 313) is in time quadrature with the counter-e.m.f. induced in the main winding, and therefore is approximately in time quadrature with the voltage applied to the main phase. Thus the single-phase source of supply and the auxiliary phase, together, form a source of two-phase energy. By properly choosing the number of turns in the auxiliary winding, the two voltages can be made of nearly the same magnitude.

If the auxiliary phase is electrically loaded, the phase relations become more complicated, but as long as the load does not exceed a certain moderate limit, the machine continues to run at nearly synchronous speed, taking energy from the main phase and delivering it to the auxiliary phase. Some of the transferred energy is stored, for a fraction of a cycle, in the mechanical inertia of the revolving part and in the magnetic fluxes, since the instantaneous amounts taken from the main phase and delivered to the auxiliary phase are not equal to each other.

Such a phase converter can be used for supplying energy not only to a two-phase circuit, but to a three-phase circuit as well. In this case the auxiliary winding has fewer turns than the main phase, and the two windings are T-connected (Vol. II), forming a source of three-phase supply. Such converters have been successfully used on some electric locomotives operated from a single-phase trolley circuit, and they can also be used for other purposes where polyphase currents are needed and only a single-phase source of supply is available.

502. EXPERIMENT 23-G. — Load Test on a Phase Converter. — The machine is described in the preceding section. An ammeter, a voltmeter, and a wattmeter should be provided in each circuit and in the line, or else the same instruments transferred from circuit to circuit by means of a suitable polyphase board (§ 58).

(1) Start the motor in the usual way, with an auxiliary phase (§§ 492 and 493), and then disconnect one of the terminals of the auxiliary winding from the source of supply. With the machine running at full speed and at no load, measure the voltages across the main phase, across the auxiliary phase, and across the two together. This will give a triangle of vectors so that the voltages can be drawn in their proper relative phase positions.

(2) Connect the auxiliary phase to a small non-inductive load and measure the currents, the voltages, and the watts in both phases; also the speed, the slip, and the frequency of the supply. One point of the two windings should be left in common as before, so that the three voltages may be read in their proper phase position. Increase the load in steps until the temperature limit is reached or until the machine stops. Read the instruments at each step, as before.

(3) Perform similar runs with inductive loads of different power factor. If possible, also try a leading load current.

(4) Provide a balanced two-phase load, one phase connected to the auxiliary winding, the other across the main line. In this case, currents and watts should be read in four places, viz., in both load phases, in the main winding, and in the line. Make a regular load run as before, and observe the unbalancing in the voltages and currents as the load is increased.

(5) Adjust the number of turns in the auxiliary winding so as to have a no-load voltage equal to $1/\sqrt{3} = 0.866$ of that in the main winding. If this is not feasible, use a transformer across the auxiliary phase to bring the voltage to the desired value. Connect the middle point of the main winding to one of the terminals of the auxiliary phase (or of the secondary winding of the transformer), to form a T-connection (Vol. II). Provide a balanced three-phase load, for example a three-phase motor, and make a regular load test as above. Be sure that measuring instruments are read in the proper places so that vector diagrams can be constructed later.

(6) Invert the machine by connecting it to a source of three-phase power and load it single-phase across the main winding. Show that with such an arrangement there is less unbalancing in the polyphase supply than if the same single-phase load were taken directly across one of the phases, without the converter.

Report. (1) Plot all the observed quantities against suitable abscissæ, such as the load current, load watts, or the input, whichever quantity seems to be the most suitable from an inspection of the data. (2) On the same curve sheet plot efficiency and power-factor curves. (3) Draw a few characteristic vector diagrams showing all the currents and voltages, to illustrate the conditions on a single-phase, two-phase, and three-phase load, at a high and at a low power factor. (4) Show that the observed variations in the terminal voltages are at least qualitatively in accord with the theory of the machine. (5) Give the data on the operation of the inverted machine; compute the percentages of power delivered to the load directly and through the auxiliary phase. Connect the observed facts with the theory.

CHAPTER XXIV

PRIMARY CELLS

503. An electric cell is a device for generating electric current by means of chemical reactions. In more scientific language, it is a device "for the conversion of the potential energy of chemical separation into the energy of an electric current" (Carhart). A single cell consists of two electrodes of different substances immersed in a liquid (or a paste, as in dry cells). With proper materials an e.m.f. is produced between the electrodes, and if the circuit outside the cell is closed, the cell becomes a source of power. When two or more cells are connected together to increase the voltage, the current, or both, the combination is called an *electric battery.*

The production of current is accompanied by chemical reactions in the cell itself: The electrodes and the solution (electrolyte) are gradually changed, and finally become *neutral* with respect to each other. Here the reaction stops, and the cell is said to be completely discharged or exhausted.

Some kinds of cells require a renewal of one of the electrodes and of the solution in order to be able again to generate current. Others need only recharging by an external electric current sent through the cell in the opposite direction; this current restores the elements of the cell to their original state. Cells of the first type are called *primary cells;* those of the second type, *secondary or storage cells.* The reason for these names is that in the first type of cells electric current is obtained directly from the chemicals constituting the cell; whereas a storage cell merely returns the electric energy previously put into it.

Primary batteries are used only where comparatively small amounts of energy are required and where the service is intermittent; or else, if the service is continuous, the current must be very small. They are used in bell and telephone work; in telegraph, signal, and alarm service; for ignition purposes in some gas engines, etc. Storage batteries (Chapter XXV) are also used for these classes of service, but their principal fields of application are (a) in gasoline automobiles for starting, lighting, and ignition; and (b) in light and power stations, in electric railway substations, etc., where it is of advantage to store large amounts of electrical energy during periods of small demand for use during periods of large demand. Moreover, a storage battery is an excellent reserve in case of an accident to the generating machinery.

The Principal Types of Primary Cells. According to the physical state of the electrolyte, the primary cells are broadly divided into *dry cells* and *wet cells*. The name " dry cell " is somewhat of a misnomer, since the electrolyte is a moist paste, and the name simply implies that the cell may be operated in any position without spilling the electrolyte. The important types of wet cells are as follows:

(1) Gravity cell;
(2) Leclanché cell;
(3) Lalande cell.

The dry cell is the most commonly used primary cell. The gravity cell has been largely replaced by the Lalande cell and its more recent modifications. The Leclanché wet cell also has been supplanted in most services by the dry cell.

504. The Anode and the Cathode. — In the following description of primary cells the terms " anode " and " cathode " are used to indicate the electrochemical functions of the electrodes, instead of the expressions " positive electrode " and " negative electrode." This is done because the reader is sometimes confused by the fact that the positive electrode inside a cell becomes the negative pole at its external terminal. In accordance with a ruling of the American Electrochemical Society, battery electrodes inside a cell should always be named in accordance with their polarity in the external circuit.

A solution which can be decomposed by an electric current is called an electrolyte. When a direct-current is passed through such an electrolyte, two kinds of products of decomposition are formed. The anode is the electrode at which the non-metallic or acid product of reaction is liberated. The cathode is the electrode at which the metallic or basic product is set free.

In a primary cell the anode goes into solution, forming a salt with the adjacent product of decomposition. On the cathode side, either a metal is precipitated out of the solution or hydrogen is liberated. The latter is objectionable in that it increases the internal resistance of the cell and causes a counter-e.m.f. which reduces the useful voltage of the cell.

A cell in which the cathode is covered with hydrogen bubbles is said to be polarized. In most cells a *depolarizer* is used near the cathode, that is, an oxide which readily combines with hydrogen and prevents its liberation on the cathode.

Zinc, being decidedly electro-positive with respect to many substances, is used as the anode in all the cells described below. This means that zinc becomes positively charged and tends to send (within the cell) a positive current to the cathode. Hence, in the external circuit the same current flows from the cathode to the zinc. Thus, zinc is the anode and at the same time the negative terminal of a cell. Carbon, copper, or copper oxide are used as the cathode.

The words anode and cathode are derived from Greek and are used in the sense of " upper " and " lower " electrode, the electro-positive electrode (such as zinc) being called the upper electrode.

DRY CELLS

505. Construction of Dry Cells. — The outer electrode (Fig. 318), which is the anode and also serves as the container, is a zinc cup to which the negative terminal of the cell is attached. The zinc cup is protected on the outside by a cardboard tube or jacket. The positive terminal is attached to a vertical carbon rod placed in the center of the cell. The intervening space is filled with a mixture consisting of manganese dioxide (MnO_2), sal ammoniac (NH_4Cl), a solution of zinc chloride ($ZnCl_2$), and granulated carbon or graphite. The latter is added to increase the electrical conductivity of the cell.

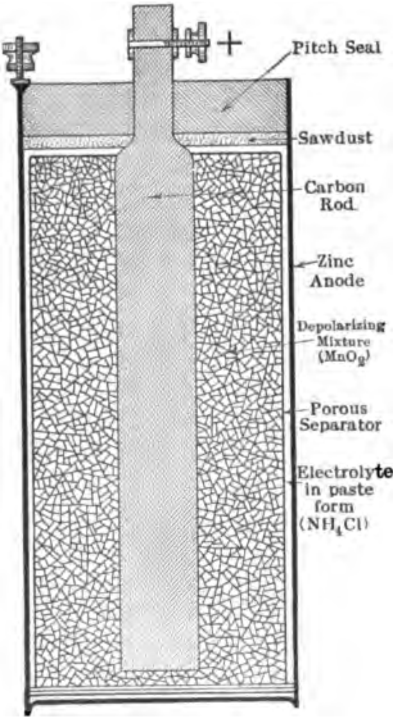

+
Pitch Seal
Sawdust
Carbon Rod
Zinc Anode
Depolarizing Mixture (MnO_2)
Porous Separator
Electrolyte in paste form (NH_4Cl)

FIG. 318. The cross-section of a dry cell.

This " depolarizing " mixture, which together with the carbon rod forms the cathode of the cell, is separated from the zinc anode by a layer of porous paper or cloth, which also acts as an absorbent for the electrolyte. Between this porous separator and the zinc is a paste layer which serves as the electrolyte. It consists of a solution of ammonium chloride (NH_4Cl) to which zinc chloride ($ZnCl_2$) is added to reduce the corrosion of the zinc when the cell is not in action. The solution is thickened with flour or some such material. The rest of the electrolyte is held in the depolarizing mixture of ground carbon and manganese dioxide. The whole is sealed on top with some bituminous pitch or rosin sealing wax. A layer of moisture-retaining material, such as sawdust, may be used under the seal.

506. Chemical Reactions in a Dry Cell. — The principal chemical reactions in a dry cell, when the external circuit is closed, are as follows:

$$Zn + 2NH_4Cl \rightarrow ZnCl_2 + 2NH_3 + H_2 \quad . \; . \; . \; . \; . \quad (1)$$

$$H_2 + 2MnO_2 \rightarrow Mn_2O_3 + H_2O \quad . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

In other words, zinc displaces ammonium (NH_4) in the electrolyte, orming zinc chloride, ammonia gas (NH_3), and free hydrogen. The atter rapidly reduces manganese dioxide to a lower state of oxidation, probably Mn_2O_3, while the ammonia gas combines with the water, or escapes around the seal.

The ammonia gas which remains dissolved sets itself into equilibrium with zinc chloride, forming a complex zinc-ammonium chloride $Zn(NH_3)_2\,Cl_2$. Thus, the foregoing two equations may be combined nto one, of the form

$$Zn + 2NH_4Cl + 2MnO_2 \rightarrow Zn(NH_3)_2\,Cl_2 + Mn_2O_3 + H_2O. \quad (3)$$

Equation (2) shows the function of manganese dioxide as a depolarizer § 504). Without it, bubbles of liberated hydrogen would soon accumulate around the carbon rod and would polarize the cell. Manganese dioxide, being rich in oxygen, readily converts hydrogen into water and thus helps to keep the internal resistance of the cell low.

The depolarizer, being solid, does not act promptly when the cell is made to deliver a considerable current. For this reason dry cells are better suited for intermittent work which allows the depolarizer to oxidize the liberated hydrogen during periods of rest. For continuous ervice the current density should not exceed 2 milli-amperes per square nch of active zinc area. This density corresponds to about 0.1 ampere or a standard dry cell 6 inches high and 2.5 inches in diameter.

507. Electrical Characteristics of Dry Cells. — In view of the importnce of dry cells in the electrical industry, the following brief account f their properties and characteristics may be of interest, especially when specifying tests for some particular purpose.

(1) *Open-circuit e.m.f.* The open-circuit voltage of an unused dry ell should be between 1.5 and 1.65 volt. Lower voltages than 1.45 olt may indicate deterioration due to age, an internal short-circuit, tc. The value of the open-circuit e.m.f. alone gives very little indiation of the service capacity of the cell, since this electromotive force hanges by a relatively small amount during the life of the cell. When a cell is nearly deteriorated, its internal resistance becomes so high that ven a small current taken by the voltmeter appreciably reduces the

terminal voltage. In such a case it is necessary to measure the voltage without drawing any current, for example with a potentiometer or with a condenser (§ 514).

(2) *Short-circuit current.* It is customary to test a dry cell by short-circuiting it for an instant through a low-resistance dead-beat ammeter and reading the maximum swing of the needle. The temperature of the cell is an important factor and must always be measured. Such a short-circuit test is easily made, but its value lies only in the relative indication which it gives of the condition of the cell as compared to normal cells of the same manufacture, type, and size. For example, if other new and unused cells of the same type are known to give a short-circuit current of about 30 amperes, then any cell which gives a current considerably below this value, is either partly deteriorated, shelf-worn, previously used, or has some defect. On the other hand, another type of cell may regularly test at 18 amperes when new, and may in no way be inferior to the first cell, but only have a different internal resistance, being intended for a different service. This point will become clearer after the factors which determine the internal resistance of a cell have been described.

(3) *Internal resistance.* The internal resistance of a new cell is rather low, the manganese dioxide being quite near the zinc anode. As the cell is discharged and the depolarizer used up, the effective distance between the zinc and the nearest active layer of manganese dioxide is increased, with a consequent increase in the internal resistance of the cell. This resistance is also increased because of some secondary reactions in the cell which probably take up water and result in the formation of products which clog the pores of the paper lining and paste, and incrust the surface of the zinc. As a result, the available current paths are reduced and the internal resistance of the cell is increased.

The electrical conductance of a cell depends essentially upon the granulated carbon or graphite in its depolarizer mixture. By increasing the amount of carbon, the conductance and the short-circuit current can be increased, but with a given available space the amount of manganese dioxide must be reduced, and this means a lower service capacity or a shorter life of the cell. Thus, unless the conductance is increased by using carbon or graphite of lower resistivity, a high short-circuit current may mean an inferior cell. In types of cells intended for large intermittent currents, such as ignition of internal-combustion engines, the internal resistance is kept low, while in so-called telephone cells, intended for small currents, the internal resistance may be higher, and the service capacity greater.

(4) *Effect of temperature.* Temperatures above 25° C. are detri-

mental. Heat tends to produce leakage and increases the rate of the chemical reactions within the cell. Therefore cells in storage or being shipped should be kept as cool as possible. Similarly, cells used for an intermittent service, at a small current, will have the longest life at a temperature just above the freezing point of water.

On the other hand, the short-circuit current increases appreciably with the temperature, and this fact should be kept in mind when testing cells. Consequently, when cells are used for heavy service with a low external resistance, and when a long life is of secondary importance, a larger current may be obtained by keeping the cells warm, since heat stimulates the chemical reactions. Of course, under such conditions the cells will deteriorate much more rapidly than when kept cool.

508. Capacity of Dry Cells for Service. — It is impossible to state the capacity (in ampere-hours) of a given type of dry cell with anything like the same degree of precision as for a storage cell. The useful output depends essentially upon the current at which the cell discharges, upon the relative and absolute duration of intervals of service and rest, and upon the end current or voltage at which the cell is to be discarded (§ 515). Considerable information on the performance of dry cells will be found in the Transactions of the American Electrochemical Society, and in the corresponding foreign publications, mostly in the form of tables and curves connecting various factors which affect the useful life of a cell. The following data, among others, have been found useful in the past:

(1) Hours of continuous service to various final values of voltage, for various discharge rates, or for various values of external resistance.

(2) Same for intermittent service, with different lengths of time of discharge and rest.

(3) Service efficiency expressed in " hours per ohm of external resistance," at various values of this resistance and with a continuous or intermittent discharge.

(4) Useful life as a function of " severity of service," the latter being somewhat arbitrarily defined as the percentage of the total time during which the cell is actually discharging, divided by the resistance of the external circuit.

(5) Hours of service with a given type of miniature incandescent lamp, the discharge taking place neither at a constant current nor at a constant resistance, since both vary with the temperature of the filament.

Dry cells gradually deteriorate even on open circuit, and the effect of this " shelf life " upon intermittent service must be considered. A cell may give a smaller total useful output on a lighter service than an identical cell on a heavier service, because in the first cell there is more opportunity for shelf-life corrosion of zinc and for a corresponding consumption of ammonium chloride.

While it is customary to estimate the deterioration of a cell by the percentage decrease in short-circuit current, there is no direct relationship between the two. For example, a decrease of 40 per cent in the short-circuit current does not mean that 60 per cent of the service capacity of the cell still remains. This fact becomes obvious when one considers that the end voltage at which the cell is discarded depends entirely upon the particular application of the cell.

509. Grouping of Cells. — When two or more cells are combined into a battery, careful attention should be given to the method of grouping. Thus, six cells may be arranged all in series, all in parallel, two series groups in parallel, or three series groups in parallel. The best grouping has to be decided in each individual case according to the characteristics of the cells and the particular purpose in view.

If the cells are to give the longest possible service, the connections must be such that the average current per cell will be of about the value at which the cell has its maximum efficiency. If, on the other hand, the largest possible current is desired, the cells should be so grouped that their combined internal resistance will be nearly equal to the resistance of the external circuit. The latter rule may be proved as follows:

Let the internal resistance of each cell be r and its e.m.f. e. Let the total number of cells be n and the load resistance R. If the unknown number of cells in series be x, then n/x cells are connected in parallel. The current in the external circuit is

$$i = \frac{xe}{R + \dfrac{xr}{(n/x)}} = \frac{e}{R/x + rx/n} \quad \ldots\ldots \quad (4)$$

If the current is to be a maximum, the denominator of the foregoing expression must be a minimum. Taking the derivative of the denominator with respect to x and equating it to zero, we find

$$-R/x^2 + r/n = 0,$$

or

$$R = \frac{xr}{(n/x)} \quad \ldots\ldots\ldots\ldots \quad (5)$$

Comparing this expression with the denominator of eq. (4), we find that the external resistance must be equal to the combined internal resistance of the battery. This proves the above proposition.

Solving eq. (5) for x, we get

$$x = \sqrt{nR/r} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

In this expression R and r are usually given, and the value of n must be so chosen as to make x and n/x integers.

WET CELLS

510. Gravity Cell. — This cell has a zinc anode on top of the cell and a copper cathode on the bottom of the jar. The copper electrode is surrounded with crystals of copper sulphate ($CuSO_4$) commonly known as blue vitriol or bluestone, and the jar is filled with water. A little sulphuric acid and a weak solution of zinc sulphate ($ZnSO_4$) are added on top of the water.

This solution must be poured in carefully so as not to bring the copper sulphate solution in contact with the zinc, as copper would be immediately deposited on the zinc. The name "gravity cell" is due to the fact that the copper sulphate solution is kept below the zinc sulphate solution, without mixing with it, merely because of the difference in the specific gravities of the two. The separating "blue line" must be kept about midway between the electrodes. Evaporation is reduced by a thin layer of mineral oil on top.

The chemical reactions in the cell are of the following general form:

$$Zn + H_2SO_4 \rightarrow H_2 + ZnSO_4 \quad . \; . \; . \; . \; . \; . \; . \quad (7)$$
$$H_2 + CuSO_4 \rightarrow Cu + H_2SO_4 \quad . \; . \; . \; . \; . \; . \; . \quad (8)$$
$$Zn + CuSO_4 \rightarrow ZnSO_4 + Cu \quad . \; . \; . \; . \; . \; . \; . \quad (9)$$

In other words, zinc is dissolved, and both zinc and copper ions travel with the electric current towards the bottom of the cell, zinc gradually displacing copper in the copper sulphate and copper being deposited on the positive electrode.

The e.m.f. of the cell on open circuit is about 1.07 volts. The positive terminal is attached to the copper electrode and the negative to the zinc. The cell is not well adapted to stand on open circuit, and a small current should be allowed to circulate through a high external resistance, in order to prevent copper ions from diffusing upward and precipitating copper on zinc.

511. Leclanché Cell. — Zinc is used as anode, and the cathode consists of a hollow carbon cylinder filled with crystals of manganese dioxide (MnO_2) which acts as a depolarizer (§ 504). A solution of sal ammoniac (NH_4Cl) is used as electrolyte. The e.m.f. of the cell is about 1.5 volts. The Leclanché cell is the precurser of the modern dry cell, and both are based on the same chemical reactions; see § 506.

512. Lalande Cell. — The elements employed in this cell, commonly called a "soda cell," are as follows: The anode is of zinc (Zn), the cathode of black oxide of copper (CuO), and the electrolyte is caustic soda solution (NaOH). The cathode may consist of molded plates or cylinders of copper oxide, or of perforated containers filled with granular oxide. The assembled cell must have a layer of mineral oil floating on the electrolyte. Its purpose is (a) to prevent evaporation of the electrolyte, (b) to prevent the salts from creeping up the terminals or the jar sides, and (c) to preclude an action of the atmospheric carbon dioxide upon the alkaline solution. In unassembled cells this oil is carried either in a bottle, or in the interstices of the cathode if it contains granular oxide.

The chemical process broadly consists in the oxidation of zinc and the de-oxidation of copper oxide. When the circuit is closed, the water of the electrolyte is decomposed into nascent oxygen and hydrogen. The oxygen goes to the zinc plate and is united with it, forming zinc hydroxide. This in turn is dissolved in the caustic soda solution, forming zincate of soda (Na_2ZnO_2). The hydrogen unites with the oxygen contained in the copper oxide cathode, forming water and leaving behind metallic copper. As the cathode is porous throughout, this action goes on, when the cell is in service, until the oxide is completely reduced to metallic copper. Thus, the principal chemical reaction is

$$Zn + 2NaOH + CuO \rightarrow Na_2ZnO_2 + H_2O + Cu \quad . \; . \quad (10)$$

The porosity of the copper oxide permits rapid depolarization; so the Lalande cell is well suited for closed-circuit work, and is capable of delivering large intermittent currents. It is most economical in services where the continuous current discharge does not exceed 0.02 ampere per square inch of active zinc area (about 1 ampere for a cell of 500 ampere-hour capacity). In intermittent service it will deliver five to ten times as much current for short periods of time. The e.m.f. of the cell is 0.95 volt.

High-Voltage National cell. This cell, which has been in use only since 1916, is a modification of the Lalande cell and contains the same elements. The improvement consists in the use of granular copper oxide to which a catalyst is added. This special depolarizer causes the working voltage to be about 20 per cent higher than that of the Lalande cell, permitting the use of fewer cells for a given circuit.

TESTING OF PRIMARY CELLS

513. The principal characteristics for which primary cells are tested, are as follows:

(1) Open-circuit voltage.
(2) Short-circuit current (§ 507).
(3) Useful life in continuous service.
(4) Useful life with intermittent service.
(5) Deterioration on open circuit (shelf test).
(6) Internal resistance.
(7) Polarization and promptness of recovery.
(8) Effects of shocks, heat, etc.
(9) Chemical examination.

All these tests are comparatively simple and most of them are described below. In an important case, such as a decision among several competitive types of cells on a large contract, the details of such tests must be carefully worked out. Publications of the Bureau of Standards on the subject and some papers and committee reports of the American Electrochemical Society will be found useful.

514. Measurement of Electromotive Force. — The electromotive force on open circuit and the terminal voltage on closed circuit are measured with an ordinary voltmeter (Fig. 319), unless even a small voltmeter current appreciably alters the condition of the cell. A good modern voltmeter takes but 0.01 ampere for a full-scale deflection, and this current is negligible under ordinary circumstances (except in a standard cell, § 72).

In the case of worn-out dry cells, even a small voltmeter current causes an appreciable voltage drop in the cell. In such a case the terminal voltage may be measured as shown in Fig. 320. A condenser C is connected across the terminals of the cell under test, and is charged at the voltage of the cell, by closing the two-way key to the right. Then the condenser is discharged through the ballistic galvanometer Ga (§ 158) by closing the key to the left, and the deflection is noted. Then the same condenser is charged by means of a cell of known e.m.f. and again discharged through the galvanometer. The ratio of the deflections gives the ratio of the voltages under comparison. Since the voltage of the standard cell is known, the voltage of the cell under test may be calculated.

If very accurate results are required, the voltage of the cell under test is compared to that of a standard cell by means of a potentiometer (§ 73).

515. Service and Life Tests. — Identical primary cells yield widely different amounts of service under varying conditions, such as different values of external resistance, different periods of rest and discharge, etc. (§ 508). No way has been found as yet to make accelerated tests from which the performance and useful life of a cell for a given service could be predicted to a fair degree of accuracy. Hence, a life test has to be practically a duplicate of the actual proposed service and takes a considerable time to perform, especially for an intermittent discharge.

(a) *Continuous tests.* One or more cells in series are connected to a known fixed resistance (Fig. 319) and allowed to discharge until the voltage falls to a prescribed low value depending upon the purpose of the test. Readings of current, voltage, and temperature are taken periodically and plotted to time as abscissæ. The instantaneous short-circuit current is also read from time to time (§ 507).

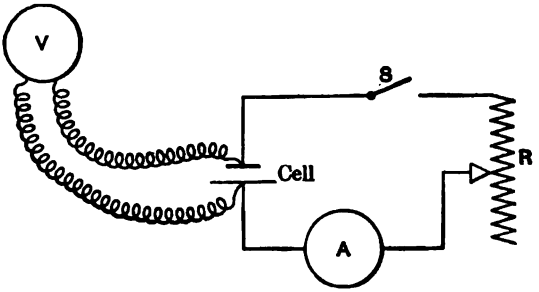


FIG. 319. Electrical connections for testing a cell.

(b) *Intermittent tests.* Such tests have been standardized for dry cells, for the principal classes of service. The table below gives four such test specifications. The first line gives the so-called ignition test. The circuit is closed for one hour in every twelve hours, the periods of rest being eleven hours each. The test is considered complete when the impulse current through a 0.5-ohm resistance at the end of a discharge period falls below 4 amperes. The second line gives an older telephone test and the third line its more recent modification, used by the American Telegraph and Telephone Co.

For wet cells, and for dry cells intended for classes of service not included in the table, similar test specifications may be written, keeping in mind the actual current demand and the general characteristics of the cells.

For intermittent tests an automatic apparatus controlled by a pendulum clock has been built by the Bureau of Standards. It consists of two selective relays having three rotating shafts which make complete revolutions in one hour, one day, and one week respectively. The coils of the first relay are energized by the clock once every minute. The coils of the second relay are energized once each hour by the first relay. In this way, rapidly moving parts are eliminated and the accuracy of the time intervals is increased. Any periodic test may be controlled by this apparatus by means of suitably designed commutators

STANDARD INTERMITTENT TESTS
For Dry Cells

	No. of cells in series	External resistance in ohms	Days per week	Hours per day	Duration of a cycle in hours	Duration of a discharge, in minutes	Final conditions	Results expressed as
Ignition Test........	6	16	7	24	12	60	4 amperes through 0.5 ohm. (***)	No. of hrs. of actual discharge
Telephone Test......	3	20	7	24	1	2	2.8 volts on closed circuit	No. of days the test lasted
Telephone Test...... (A. T. & T.)......	3	20	6; 1 (**)	10	1; 2	4	2.8 volts on closed circuit	No. of days the test lasted
Flashlight Test......	(*)	4 per cell	7	24	24	5	0.5 volt per cell	No. of minutes of actual discharge

(*) The number of cells in series depends upon the lamp voltage, although in making the test it is necessary to use a fixed resistance, since the resistance of a lamp varies with the impressed voltage.

(**) The circuit is closed for four minutes every hour during ten consecutive hours for six consecutive days. On the seventh day every other period is omitted.

(***) Impulse current only.

which are placed on the rotating shafts. The contacts on the commutators are platinum against platinum-iridium. Test boards are provided for various sizes of cells, with multiple switches to close the individual circuits as desired.

(c) *Shelf test.* This test consists in storage of the cells on open circuit at room temperature over a considerable period of time, during which the changing condition of the cells is ascertained by open-circuit voltage and short-circuit current readings. No definite end point is taken, but the results are expressed as per cent drop in amperage for certain periods of time. This is an important test, and should be made at as nearly a constant temperature as possible.

516. Measurement of Internal Resistance. — Let the e.m.f. of a cell on open circuit be E, and let the terminal voltage be e when the cell is delivering a current i. Assuming no polarization, and taking the whole voltage drop, $E - e$, to be due to the internal resistance r of the cell, we have

$$r = (E - e)/i \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (11)$$

In reality, the drop $E - e$ is partly due to the counter-e.m.f. of polarization (§ 506) which, on closed circuit, increases with the time (Fig. 321). Therefore, it is essential *to measure the voltage e immediately upon closing the circuit,* before any appreciable polarization has set in. Moreover, the cell should have had a sufficient period of rest, in order that all previous polarization may have been removed. The measurements may be conveniently performed with the connections shown in Fig. 320. The no-load voltage is read by charging and discharging the condenser with the two-way key, leaving the switch S open. Then the two-way key is closed to the right, the switch S is closed, and immediately afterwards the key is again pressed to the left. The galvanometer deflection gives the terminal voltage e. The galvanometer is supposed to be calibrated with a cell of known e.m.f.

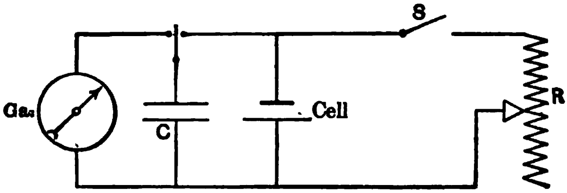


Fig. 320. Measuring the voltage of a cell by means of a condenser and a ballistic galvanometer.

The internal resistance of a cell cannot be measured with an ordinary Wheatstone bridge, using direct current, because the e.m.f. of the cell would entirely vitiate the result. One way out of this difficulty is to measure the resistance of two identical cells connected in opposition, so that their e.m.f.'s cancel each other. Another method is to use a

Wheatstone bridge with alternating currents (§ 19). For variations of the internal resistance of a dry cell with the age, see § 507.

517. Polarization and Recovery Tests. — The connections are made as in Fig. 319, or as in Fig. 320, according to whether or not it is permissible to use a voltmeter. A potentiometer (§ 73) may also be used in the latter case. The cell is closed on a certain resistance *R*, and the terminal voltage is measured at regular intervals, with the switch *S* closed and open. The results are plotted as in Fig. 321.

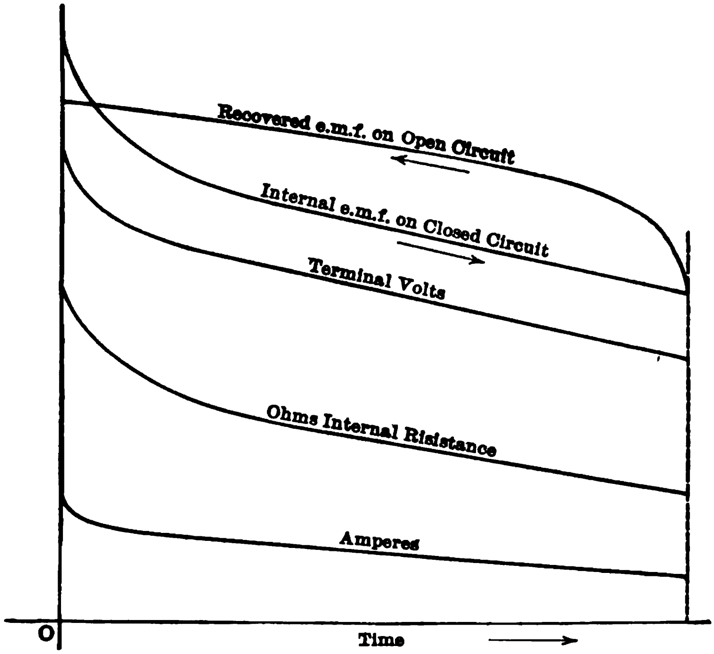


Fig. 321. Performance curves of a primary cell.

The curve marked "Terminal Volts" refers to the readings with the circuit closed; the curve marked "Internal e.m.f. on Closed Circuit" gives voltmeter readings immediately after the switch *S* has been opened. The current is read directly on an ammeter; the internal resistance of the battery (§ 516) is calculated by dividing the difference of the ordinates of the two voltage curves by the corresponding current. The circuit must be kept open just long enough to read the voltmeter, and then closed again.

At the end of a certain period of time, say one or two hours, the circuit is permanently opened, and the cell is allowed to recover from polarization. The voltage gradually rises, as shown by the upper curve.

The test, in order to be conclusive, must be repeated on several cells of the same type, and with various rates of current; also with cells closed for different lengths of time.

518. EXPERIMENT 24-A. — Setting Up and Study of a Gravity Cell. — The cell is described in § 510, and should be set up by the student himself, using fresh materials. An old cell should also be available for measurements and inspection, in order that the deterioration in service may be seen. Investigate the result of a careless or wrong setting up, such as an excess or deficiency of one of the chemicals, stirring up the copper sulphate solution, etc. Any of the tests mentioned in § 513 may be performed, especially (1), (2), (6), and (7). The internal resistance should be measured by the three methods described in § 516, the results carefully compared, and the discrepancies explained.

519. EXPERIMENT 24-B — Setting Up and Study of a Leclanché Cell. — The cell is described in § 511, and the experiment is similar to the preceding one. A careful study of the action of the depolarizer is particularly recommended, since dry cells are based on the same principle as the Leclanché cell, but cannot be studied as conveniently as a wet cell.

520. EXPERIMENT 24-C. — Setting Up and Study of a Lalande Cell. — See §§ 512 and 518.

521. EXPERIMENT 24-D. — Characteristics of Dry Cells. — Dry cells are described in §§ 505 to 507. The possible tests are enumerated in § 513. Of these it is desired in this experiment to become familiar with the tests described in §§ 507, 514, 516, and 517. Several cells, of the same make and type and at different stages of exhaustion, should be provided. In this way the characteristics of new and old cells may be compared without making a long life-test.

522. EXPERIMENT 24-E. — Service and Life Tests on Primary Cells. — Such tests are described in §§ 508 and 515. Since it is not feasible to perform a test of this kind in a student laboratory in one period, with reasonable rates of discharge, the experiment may be performed in one of the following two ways: (a) Cells are subjected to excessive rates of discharge so as to complete their useful life in one laboratory period. (b) Cells are set up for a reasonable rate of discharge, to continue for several days or weeks, and each student in turn takes a set of measurements of current, voltage, internal resistance, etc., and enters his results in a continuous log or chart. In this way each student will get experience in making life tests and can see the general character of the changes in the cells day by day, by looking over the finished set of readings.

523. EXPERIMENT 24-F. — Grouping of Primary Cells. — The purpose of the experiment is to learn to select the best number of cells for a given service, and their proper connection in series or in parallel. The principles of grouping are explained in § 509. A sufficient number of identical cells of comparatively high internal resistance should be provided; the value of this resistance should be approximately known. Exercises and problems can be readily devised with different values of current and external resistance and with different groupings of cells. In each case the purpose should be (a) to find the best grouping for a long life; (b) to find the best grouping for a maximum current; and (c) to check the theoretical conclusions with the experimental data.

CHAPTER XXV

STORAGE BATTERIES

524. The difference between a primary cell and a storage cell is explained in § 503. A storage cell or electric accumulator is a device in which electric energy can be stored in chemical form and then delivered again to an external circuit as electric energy. Two or more cells connected together form an *electric storage battery.*

525. Function of a Storage Battery in Steadying Generator Current and Load Voltage. — A storage battery (Fig. 322) is usually connected in parallel with one or more D.C. generators (or rotary converters) for the purpose of steadying the load of the latter or acting as a reserve. When the load is light (Fig. 323) some current is sent into the battery, charging it. During peaks of the load the battery discharges in parallel with the generators, relieving them of injurious heavy overloads.

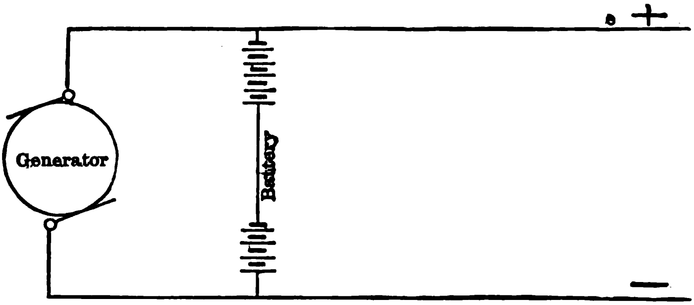


Fig. 322. A generator and a storage battery supplying a line in parallel.

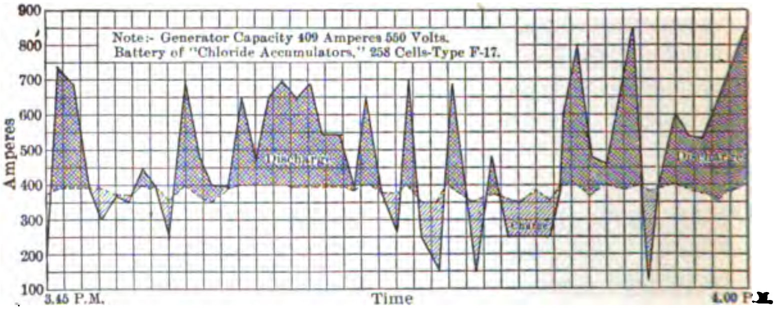

Note:- Generator Capacity 409 Amperes 550 Volts.
Battery of "Chloride Accumulators," 258 Cells-Type F-17.

Fig. 323. Effect of a battery in steadying generator load.

In an emergency or during the hours of very light load, say at night, the generators can be shut down, and the battery made to supply the entire load until discharged. The advantages of such a use of a battery are as follows:

(a) The size of the prime movers and generators can be somewhat reduced, since the battery carries the peaks of the load over and above the average value.

(b) The prime movers and the generators can be made to operate at a more nearly constant load, within the range of their best efficiency.

(c) Mechanical stresses in rotating machinery, due to sudden changes in the load, are reduced.

(d) The power-generating machinery may be shut down during certain hours of the day, with the resulting economies in operation.

(e) The battery constitutes an emergency source of power which is available to tide over short interruptions in the regular sources of supply (stand-by service).

On the other hand, the first cost of a storage battery and of its accessories is quite high, and there is a considerable additional expense for its upkeep and depreciation. Nevertheless, stand-by storage batteries are used in many large electric power and railway substations, especially in cities where it is of particular importance to avoid interruptions in service. A battery placed at the receiver end of a long feeder helps to maintain a more nearly constant load voltage (Fig. 324) by reducing

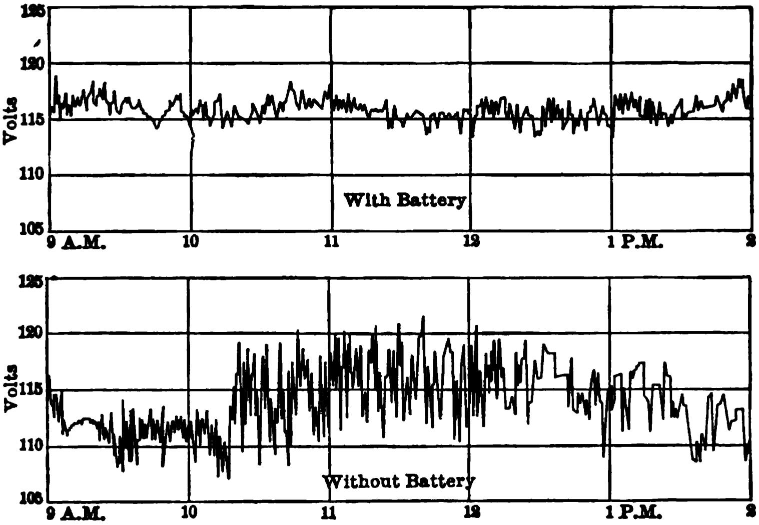


Fig. 324. Effect of a battery in steadying the line voltage.

fluctuations in the line current and therefore causing a more uniform line voltage drop.

526. A Hydraulic Analogue. — The part which a storage battery plays in the distribution of electrical energy is somewhat the same as that of a storage tank in a water supply system (Fig. 325). Without the tank, the pumps would have to supply a variable demand and their capacity would have to be sufficient for the *maximum* demand. Moreover they would have to be operated twenty-four hours a day, the power consumption would be much increased, and the efficiency reduced, to say nothing of the severe mechanical strains imposed by sudden variations of the load. Besides, there would be no stored water in reserve for short interruptions in the pumping plant.

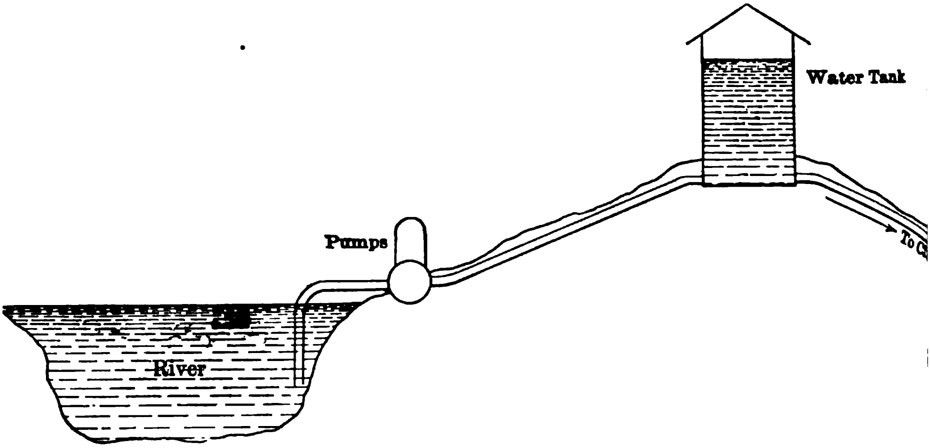


FIG. 325. A water-supply plant, as an analogue to an electric plant with a storage battery.

A water tank of sufficient capacity remedies all these drawbacks. The capacity of the pumps needs to be sufficient for the *average* demand only, and they may be operated at practically full load. When the demand is below the average, the excess water is pumped to raise the level in the tank. When the demand is above the average, the tank supplies the necessary excess of water into the city mains. With a variable load, the tank allows a more constant pressure to be maintained in the mains, and makes it possible to shut down the pumps during the hours of light demand.

527. Storage Batteries for Starting and Lighting Automobiles and for Electric Vehicles. — An important use of storage batteries is for starting internal-combustion engines on passenger automobiles, trucks, tractors, motor boats, etc. The same battery is also used to operate the lights, the signal horn, special heating appliances, etc. The general arrangement is shown in Fig. 200. The battery must be light and durable, must repeatedly stand an excessive rate of discharge during the period of starting, and must not be damaged by long overcharging with a moderate current. Batteries of similar type are used for the propulsion of electric passenger vehicles, trucks, some street cars, mining and industrial locomotives, boats, etc.

Among other uses of storage batteries may be mentioned house and farm lighting, radio, miscellaneous uses on board ship, telephone, telegraph and signal field, operation of large electric switches, relays, etc. in power plants, portable mine lamps, laboratory service, etc.

528. Types of Storage Cells. — Two types of storage cells are in practical use at the present time, although some other chemical combinations are theoretically possible. The most generally used type is the so-called lead-acid cell. A more recent cell, which has a rather limited field of application, is the nickel-iron cell, also known as the alkaline or Edison cell. A brief description of these cells is given below; for a more detailed description and for instructions as to their care and management, troubles and their remedies, the reader is referred to numerous special works on the subject.

THE LEAD-ACID CELL

529. Chemical and Physical Features. — The positive active material is peroxide of lead (PbO_2), and the negative active material is spongy lead (Pb). Dilute sulphuric acid (H_2SO_4) is used as electrolyte. When the cell discharges, both active materials are partially converted into lead sulphate ($PbSO_4$), and the acid thus becomes more dilute. On charging, a reverse action takes place, the plates being again restored to lead peroxide (positive plate) and spongy lead (negative plate). The specific gravity of the electrolyte increases to its maximum value, and the cell is again ready for discharge. These chemical changes may be represented by the following formula:

$$PbO_2 + Pb + 2H_2SO_4 \underset{\xrightarrow{\text{discharge}}}{\overset{\xleftarrow{\text{charge}}}{=}} 2PbSO_4 + 2H_2O \quad . \; . \; . \; . \quad (1)$$

In reality the chemical reactions are more complicated and are not known at present in all their details. The above fundamental equation is, however, sufficient for a general understanding of the operation of lead storage batteries.

Certain impurities in the electrolyte, or in the water added to it, are injurious to the plates and may ruin them in a short time. Therefore, pure sulphuric acid and distilled water should be used whenever possible. If there is any doubt, the proposed acid and water should be submitted to the manufacturer of the battery for analysis and approval.

The density of the electrolyte in a cell, when in a fully charged condition, varies over quite a wide range, depending on the design of the cell and the service for which it is adapted. Where cells are designed to go into a very limited space and where minimum weight is essential, there is very little space for electrolyte, and in order to provide sufficient quantity of sulphuric acid to give full capacity, it is necessary to make the electrolyte somewhat more dense. In such cases the specific gravity on full charge runs as high as 1.275 to 1.300. However, where the space is not so restricted, a lower density is used, down to about 1.200 to 1.210. From the standpoint of the life of the plates and wood separators, the lower density electrolyte is preferable.

The foregoing values of density refer to 25° C. Tables and charts are available which give the correct density at other temperatures and also proportions of water and concentrated acid by weight and by volume for an electrolyte of a desired density.

As the cell is being discharged, the specific gravity of the electrolyte decreases by a definite amount, on account of the abstraction of some acid which forms lead sulphate on the plates. Thus, if the gravity of a fully charged cell is 1.275, at the end of discharge it will be between 1.175 and 1.150. The battery should not be discharged below this point. The limits of density variations are usually within 10 per cent, and therefore an accurate hydrometer should be used for tests, especially when the state of charge or discharge is to be judged by the specific gravity of the acid.

When a cell is fully charged, the specific gravity of the electrolyte reaches its maximum and remains constant if the battery receives an overcharge. With reference to the fundamental chemical equation given above, this means that all sulphate has been converted into lead and lead peroxide. This is the best indication of a complete charge of a cell. Similarly, other states of charge and discharge can be judged by the specific gravity of the electrolyte, since changes in its density are approximately proportional to the ampere-hours discharged. This procedure is fairly safe with a familiar battery whose performance has been observed during some preceding cycles of charge and discharge.

Acid should never be added to the electrolyte except when all the plates in the cell are fully charged and the density is still below a prescribed minimum. If there is any doubt about this point, it is safer to operate the cell with a weak electrolyte than to impair its life by one that is too strong.

Sulphation. Whenever a battery is discharging, lead sulphate ($PbSO_4$) is formed on both the positive and the negative plates, as a part of the process of producing current. After a normal discharge the sulphate is finely crystalline and of such a nature that it is easily reduced by the current flowing through the battery on charge. If charging is neglected and the battery allowed to stand in a discharged state, the sulphate gradually ceases to be crystalline; it fills the pores of the plates and eventually makes the active material dense and hard, thus prolonging the time required for a charge. In this latter condition the battery is spoken of as being " sulphated."

If sulphation is allowed to go too far, it will cause buckling of plates, loss of active material, a much decreased capacity and efficiency of the cell, and an increased internal resistance. A prolonged charging at a low rate of current will restore a moderately sulphated cell or battery. In more aggravated cases, several cycles of charge and discharge, or even a special treatment, may be necessary. For such special treatments see some practical book of instructions for the care and handling of storage batteries. By keeping the battery well charged and filled with proper electrolyte, and by avoiding excessive rates of discharge, serious sulphation can be avoided.

530. Mechanical Construction of Plates. — Both active materials, viz., spongy lead and peroxide of lead, are mechanically weak and must be properly supported. According to the method of support and manufacture, two types of plates are distinguished, the Planté plate, and the Faure or pasted plate.

A *Planté plate* consists originally of pure lead with its surface area greatly increased by cutting a large number of grooves. The plate is then used as an anode (§ 504) in an electrolyte of dilute sulphuric acid containing a small amount of some other acid, such as nitric, which will attack lead. When a direct current is passed from an outside source, an external layer of lead is oxidized into lead peroxide. The plate is then ready to be used as a positive plate in a storage cell. If it is to be used as a negative plate the active layer is " reversed," or reduced, into spongy lead. This is done by using the plate as a negative in a cell and charging it.

A widely used positive plate of Planté type is shown in Figs. 326 and 327. The grid consists of cast lead-antimony alloy, designed to resist the " forming " effect of electrolytic action of the cell, and of sufficient rigidity to prevent troublesome growth and buckling. The grid is provided with circular openings slightly tapering toward the center.

Into these openings are forced by hydraulic pressure rosettes or buttons of soft lead which constitute the active portion of the plate. These rosettes are formed of strips of pure lead, corrugated transversely and rolled into a spiral. After being forced into place in the grid they are subjected to the "forming" process, whereby the active material or lead peroxide is developed electrochemically on the transverse surfaces. The expansive action of this forming process, combined with the "hour-glass" shape of the openings, securely locks the buttons in place.

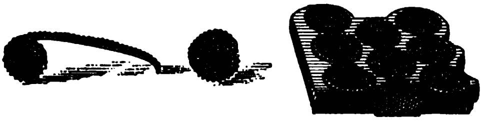


FIG. 326. Lead buttons used in the positive plates shown in Fig. 327.

A *Faure or pasted plate* generally consists of a flat frame or grid of lead-antimony alloy into which the active materials, in the form of a paste, are pressed. The structure of the grid is such that the active materials are held securely in place. There are many forms of grids; the one shown in Fig. 327 for the positive plate gives a general idea of their structure. In the "box type" negative pasted plate, shown in the same sketch to the left, the grid is made of horizontal and vertical ribs spaced about 1 inch apart. These ribs form pockets closed on both sides with perforated sheet lead, and in these pockets the active material is permanently held in place. Since this active material need not be self-supporting, it may be of such composition as is best suited to maintain the desired porosity and permanent capacity in service.

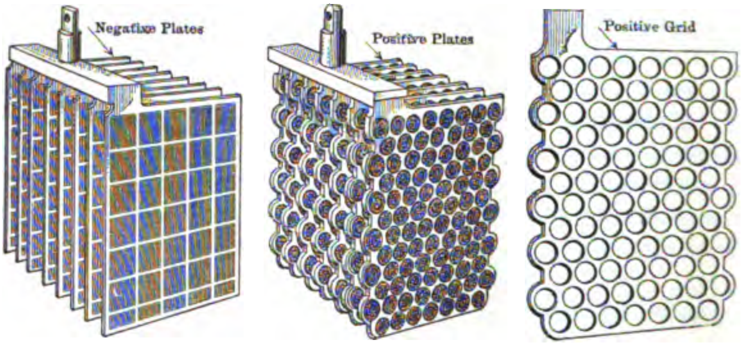


FIG. 327. An example of Planté positive and pasted negative plates.

A positive plate with lead peroxide held in compartments is shown in Fig. 328. It has a grid composed of a number of parallel vertical lead-alloy rods united integrally to horizontal top and bottom frames, the former being provided with the usual conducting lug. Each vertical rod forms a core which is surrounded by a cylindrical pencil of peroxide of lead, the active material. This, in turn, is enclosed by a hard-rubber tube having a large number of horizontal slits. These slits serve to provide access for the electrolyte to the active material, and yet are so fine as to practically eliminate the washing out of the material. The outside tubes are reinforced by leaving the exposed edge solid, that is, without slits.

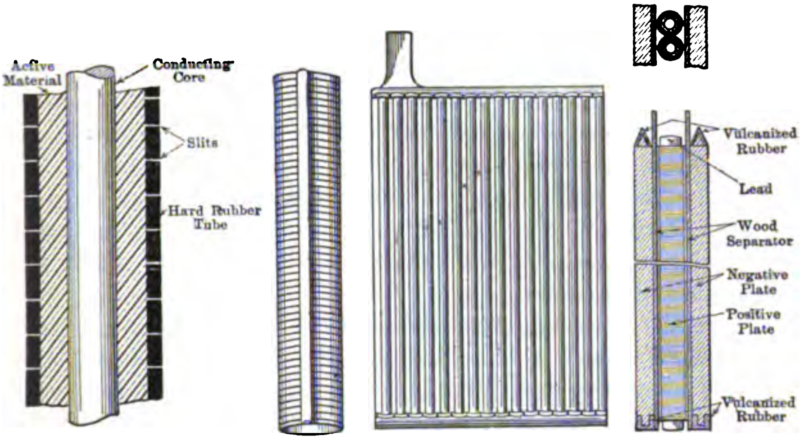


FIG. 328. A positive plate with the active material in hard-rubber tubes.

Each tube is provided with two parallel vertical ribs projecting on opposite sides, at right angles to the face of the plate. These ribs not only serve to stiffen the tubes, but also act as insulating spacers, allowing the use of plain wood-veneer separators. The rubber tubes have a certain amount of elasticity, allowing them to compensate for changes in volume of the active material, due to expansion and contraction during charge and discharge.

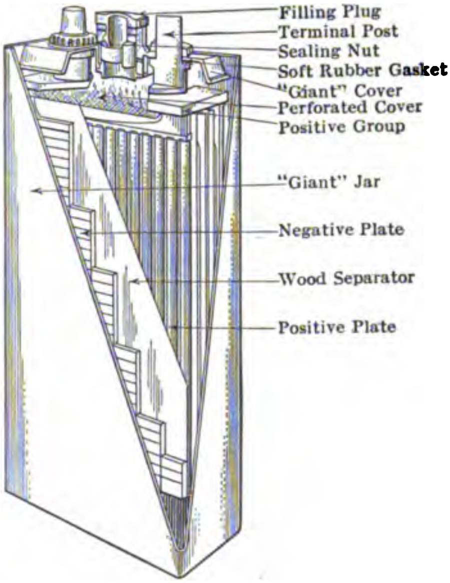


FIG. 329. A cut-away view of a complete lead-acid storage cell.

A complete cell embodying this plate is shown in Fig. 329, with part of the jar cut away to show the construction. There is one more negative plate than there are positive plates, the two outside plates being negative. The same excess of negative plates is shown in Fig. 327.

Separators. The active plates are generally separated from each other by thin sheets of wood veneer known as separators (Figs. 328 and 329). The separators are porous to permit electrolytic conduction, but they prevent metallic conduction which would short-circuit the cell. The separators are usually corrugated on one side and smooth on the other. The wood must be either naturally free from injurious substances or treated to remove them. The wood separators in tractor batteries are generally kept away from the positive plates by means of thin sheets of perforated hard rubber.

In one make of storage cells, threaded-rubber separators are used in place of wood. Such a separator consists of a sheet of soft-rubber compound containing a great many cotton threads running transversely through the sheet. Each thread acts as a wick, and the threads take the place of pores in a wooden separator.

531. Voltages during Charge and Discharge. — A fully charged storage cell has an e.m.f. of a little over 2 volts on open circuit. If it is allowed to be discharged indefinitely, the voltage will at first remain practically constant at about 2 volts, then will gradually fall off, at first slowly, then more and more rapidly, down to zero (Fig. 330). The voltages given in the curve are supposed to be measured while a normal discharge current is passing through the cell. The voltage drop in the cell is caused by the internal resistance of the cell (§ 535) and by some polarization (§ 504) due to a weakening of the acid in the pores of the active material.

A complete discharge down to zero voltage would be impracticable, because for all ordinary purposes the terminal voltage of the battery must be constant within rather narrow limits. Moreover, such a complete discharge, if frequently repeated, would in time sulphate the plates (§ 529) and shorten their life. In practice, a cell is considered to require a new charge when the voltage has dropped to 1.75 volts (or better, 1.8 volts). With stationary batteries this voltage is measured while the cells are supplying a current which corresponds to the eight-hour rate of discharge.

When a cell is being charged, the external voltage applied at its terminals must be high enough to overcome the counter-e.m.f. of the cell and to force the charging current through its ohmic resistance. At the beginning of a charge the charging voltage is a little above 2 volts per cell. As the battery becomes recuperated, this voltage must be gradually increased; until at the end of the charge it is necessary to apply about 2.6 volts in order to get full charging current through the cell. The end of the charge is also recognized by an excessive liberation of gases (gassing) due to a decomposition of water in the solution.

One of the best indications of a complete charge is given by the specific gravity of the electrolyte, measured with a hydrometer (§ 529).

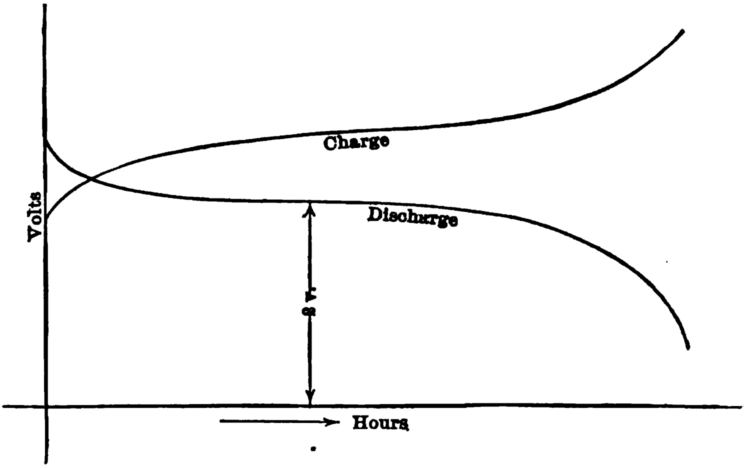


FIG. 330. Curves of charge and discharge of a lead storage cell.

The open-circuit voltage of a cell does not indicate its state of charge, but does indicate internal troubles, such as sulphation, short-circuited plates, etc. In all such cases the voltage will be below normal. The voltage will vary with changes in strength of electrolyte, temperature, and age of the cell.

532. Cadmium Tester. — In order to ascertain separately the state of charge on each plate, a cadmium tester or electrode is sometimes used. It consists of a stick of pure cadmium placed in the acid of the cell under test. It is well to have the cadmium protected by a hard-rubber tube with perforations for the circulation of the acid. At the end of the charge the voltmeter shows about 2.45 volts between lead peroxide and cadmium, and about 0.10 volt between the lead plate and cadmium. This is a more or less positive indication of the end of the charge. The voltage between the two plates is equal to the sum of the two readings:

$$2.45 + 0.10 = 2.55 \text{ volts.}$$

The same tester can be used to ascertain the end of discharge. In this case the voltages are approximately + 2.05 and − 0.25 volts respectively; the cell voltage is

$$2.05 - 0.25 = 1.80 \text{ volts.}$$

Voltages on charge and on discharge should be read while a normal current is passing through the cell. On charge the cadmium tester is electro-positive to the sponge lead plate and is negative to the peroxide plate. On discharge the cadmium is negative to both plates. This explains the plus and minus signs in the foregoing equations. Different authorities give somewhat different numerical data for the potentials to cadmium, and these values depend somewhat upon the conditions of the test and upon the cadmium electrode itself.

A high-resistance voltmeter, or a potentiometer, is essential in this test, as otherwise there will be an appreciable polarization about the cadmium electrode that will vitiate the results. For details of the use of the cadmium electrode and for the necessary precautions, see the Technological Paper of the Bureau of Standards, No. 146.

In case it is found that one of the plates is not fully charged, the charge must be continued until the cadmium electrode shows the required voltage. If it is feared that an excessive charge may damage the other plate, the plate which requires additional charging may be charged in a separate cell. A cadmium tester gives reliable indications in the hands of an experienced observer, especially when many tests are made on batteries of the same type. Otherwise, it is safer to judge about the state of the charge from the acid density and the voltage.

533. Capacity of Storage Cells. — The *capacity* of a cell, or the amount of electricity which it can give on discharge, is measured in *ampere-hours.* A cell which can supply 25 amperes for 8 hours, before the lower limit of the e.m.f., say 1.8 volt, is reached, is said to have a capacity of 25 × 8 = 200 ampere-hours. Experience shows that the capacity of a lead cell depends essentially on the rate of discharge. The more rapid the discharge the less is the capacity; thus the above cell, if discharged at a rate of 100 amperes, would be completely discharged in one hour instead of two hours. Therefore, *when speaking of the capacity of a storage battery it is always necessary to mention the number of hours in which the battery is supposed to be discharged.* It is customary to rate stationary batteries on the basis of an eight-hour discharge, and batteries used in electric automobiles on the basis of a four-hour discharge. Storage batteries used in electric railway substations, for taking up fluctuations of the load, are usually rated on the arbitrary basis of one-hour discharge, or even less.

If a battery is intended to be discharged within a shorter period of time than normal, its rated capacity must be reduced in a ratio usually given by the manufacturer. In the table below the data refer to a particular type of cell only, but they give a general idea of the order of magnitude of changes in capacity. The second column contains the terminal voltages at which the discharge should be stopped. In each case the voltage is supposed to be measured with a discharge current flowing at the corresponding rate.

Hours Discharge.	Final Voltage.	Relative Value of Current.	Relative Capacity in Amp. Hours.
8	1.75	1	8 (100%)
3	1.70	2	6 (75%)
1	1.60	4	4 (50%)
⅓	1.40	8	2⅔ (33⅓%)

When a battery is discharged at the one-hour rate for one hour and is completely discharged *at that rate,* it still has additional capacity which is available at lower rates of discharge. Additional capacity may also be taken out at the one-hour rate if the battery is allowed to stand and recuperate. In fact, the full eight-hour capacity of a battery may be taken out at the one-hour rate, provided the discharge is intermittent in character and is distributed over a total elapsed time of eight hours. Higher rates than the one-hour rate are utilized in many applications, such as momentary load regulation and standby service. In the latter service, rates as high as four times the one-hour rate are occasionally used.

The above-described behavior on discharge and the reduction in capacity at higher rates is principally due to a reduced density of the electrolyte in the pores of the plates. When the cell is allowed to recuperate, fresh acid gradually finds its way into the pores, an additional quantity of lead sulphate can be formed, and more ampere-hours taken out of the cell.

For starting and lighting batteries of gasoline automobiles, the American Society of Automotive Engineers prescribes a double rating in the following rule:

"Batteries for combined lighting and starting service shall have two ratings, of which the first shall indicate the lighting ability and be the capacity in ampere-hours of the battery when discharged continuously at a 5-ampere rate to a final voltage of 1.8 per cell, the temperature of the battery beginning such discharge being 80 degrees F. The second rating shall indicate starting ability and shall be the rate in amperes at which the battery will discharge for twenty minutes continuously to a final voltage of not less than 1.65 per cell. The temperature of the battery beginning such discharge to be 80 degrees F. (27 degrees C.)."

Effect of Temperature. The capacity of a lead-acid storage cell decreases as its temperature is lowered. This is particularly unfortunate in the case of starting batteries of automobile engines and of electric vehicle batteries in extremely cold weather, when a maximum of energy is usually required and when the available capacity of the battery may be only 50 per cent of what it is in the summer.

One would naturally suppose from the high rates of current demanded from the automobile storage battery that failures in service would be due principally to this reason. As a matter of fact, the deterioration of the battery plate is due largely to overheating in the summer time. This overheating acts in two ways. It tends to loosen the active material in the battery plates and also carbonizes the wood insulation between the plates. The charging rate should, therefore, not be excessive, especially during warm weather.

534. Electrical Efficiency. — Two kinds of efficiency of a storage cell or battery are distinguished, viz., the ampere-hour efficiency and the watt-hour efficiency. Let an ampere-hour meter (§ 119) and a watt-hour meter (§ 111) be connected in series with a cell which has been fully charged, and let the cell be discharged to a low point at a predetermined constant or variable rate. Then let the cell be again fully charged at a desired rate. Let the two instruments be read both on charge and on discharge. The ratio of the total ampere-hours delivered by the cell to those put into it gives its ampere-hour efficiency. A similar ratio of the readings of the watt-hour meter gives the watt-hour efficiency of the cell. The second efficiency is lower than the first because the average voltage on discharge is lower than the average charging voltage. The numerical values of efficiency refer to the particular rates of charge and discharge used and to the chosen end-voltage.

535. Internal Resistance. — The resistance of a storage cell is due to a number of factors, including the resistance of the electrolyte, the resistance of the active material which varies during charge and discharge, and that of the grids and terminals. The resistance of a storage cell rises toward the end of discharge to more than double its resistance when fully charged. When a discharged cell is charged, the internal resistance falls again to its original value, corrected for temperature.

Although there is a considerable change in the density of the electrolyte, the change in its conductivity is small. The change in resistance which takes place at the plates of a lead-acid cell is largely explained by the variable amount of diluted electrolyte in the pores of the active materials. The more the acid in these pores is diluted, through an excessive discharge, the higher is the internal resistance.

Another cause of variations in the external resistance is the presence of lead sulphate (§ 529). During discharge the lead and peroxide particles become more or less densely covered with a layer of non-conducting lead sulphate which is reduced on the subsequent charge. If this layer of sulphate is allowed to become hard and dense, the internal resistance of the cell may rise to a high value.

The determination of the *true* ohmic resistance of a storage cell is rather difficult, because this resistance is very small and variable, and is to some extent masked by the effect of polarization. Moreover, it is not the true resistance, but rather the *virtual* resistance of the cell that is of interest to the user. This virtual or equivalent resistance represents the total drop of voltage in the cell, due to whatever causes. If it were not for a variable amount of polarization, the simplest method of determining the internal resistance R of a cell would be to observe its voltage E_0 on open circuit, and then immediately note the voltage E with a certain charging current I flowing through the battery. Then, evidently, we should have

$$R = \frac{E - E_0}{I}. \quad \ldots\ldots\ldots\ldots \quad (2)$$

Because of polarization, a better method is to measure two terminal voltages, E_1 and E_2, corresponding to two different values, I_1 and I_2, of charging current. We then have

$$\left.\begin{aligned} E_1 - p - E_0 &= RI_1 \\ E_2 - p - E_0 &= RI_2 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (3)$$

where p is the counter-e.m.f. of polarization. Eliminating p and solving for R, we obtain

$$R = \frac{E_1 - E_2}{I_1 - I_2}. \quad \ldots\ldots\ldots\ldots \quad (4)$$

An objection to this method is that the e.m.f., p, of polarization is not quite constant with various rates of charge. Another objection is that the difference $E_1 - E_2$ is rather small, and this impairs the accuracy of the result. It is advisable to perform a large number of tests with various values of I_1 and I_2 and to take an average of the calculated values of R. Experience shows that more consistent results are obtained on discharge than on charge; the same formulæ (2) and (4) are used as given above.

With the so-called " break " method of measuring the resistance of a cell on discharge, the external load resistance is adjusted to a certain value, and when the conditions become steady, the circuit is suddenly opened. The e.m.f., read on a voltmeter, rises instantly by a certain amount and then continues to rise *gradually* as stronger electrolyte is diffused into the pores of the plates, taking the place of the electrolyte weakened by the discharge. From this sudden rise in voltage and a knowledge of the current that was flowing through the cell, the internal resistance can be calculated. Let, for example, the voltage rise from 1.8 volts to 1.9 volts when the switch is opened, and let the current be 100 amperes. The resistance of the cell is 0.1 ÷ 100 = 0.001 ohm.

Instead of opening the circuit, the observer may suddenly change the current by a definite amount and read the corresponding instantaneous rise in voltage. The virtual resistance can be computed from eq. (4). See also § 516 on the internal resistance of primary cells and § 19 on the resistance of electrolytes.

536. Charge and Discharge Characteristics. — Supplementing the curves shown in Fig. 330, a more complete set of characteristic curves for a lead-acid storage cell is shown in Fig. 331.

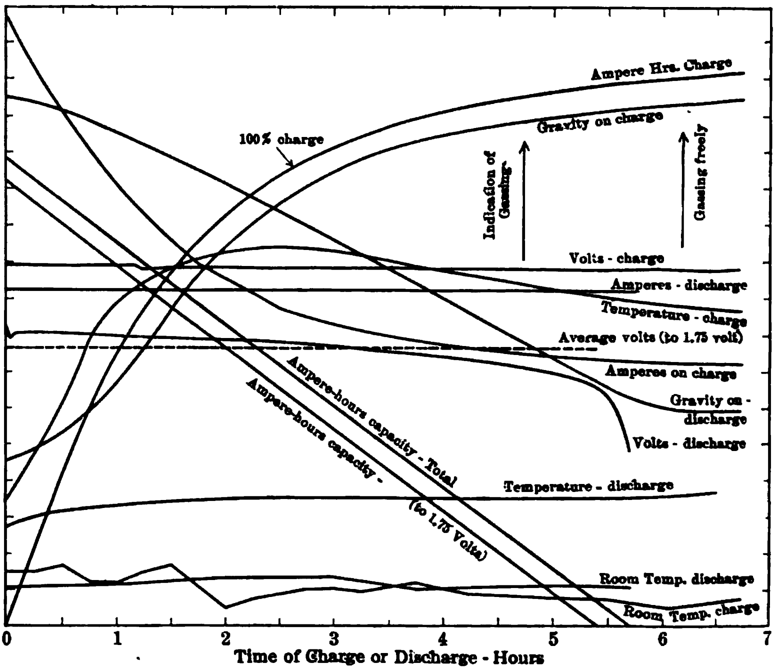


FIG. 331. Charge characteristics of a lead-acid cell at a constant voltage, and its discharge characteristics.

In Fig. 330 the cell is supposed to be charged at a nearly *constant current*, except that at the end of the charge the current is reduced to the "finishing rate," to avoid violent gassing which loosens the active material and shortens the life of the cell. This gassing is due to a decomposition of the water in the electrolyte into hydrogen and oxygen. At the end of the charge, when the remaining amount of lead sulphate is small, the chemical reaction in the pores can go on only at a moderate speed, and if there is an excess current it is wasted in decomposing water. When charging at a constant current, the required voltage increases as the charge proceeds.

The curves shown in Fig. 331 refer to charging at a *constant voltage* of about 2.3 volts per cell. At the beginning of the charge the difference between 2.3 volts and the counter-e.m.f. of the cell is rather large, and therefore the initial charging current is also large. With the cell nearly discharged, a large charging current causes no gassing and simply reduces the total time required for charging. As the charge process advances and the counter-e.m.f. of the cell increases, the current is automatically reduced, and with a terminal voltage of 2.3 volts the final or finishing rate of current is nearly correct, that is, it permits the final reduction of lead sulphate without excessive gassing.

Thus, the advantages of the constant-potential system of charging, over the older constant-current system, are (a) that no current regulation is required, and (b) that the total time of charging is considerably reduced. In practice, a modified constant-potential system of charging is often used (§ 544). When only one cell is being tested, the current usually has to be adjusted to a desired value by means of a rheostat as the charge proceeds.

It has been found that if the charging rate is kept below a value equal to the number of ampere-hours out of the cell, excessive gassing and high temperature will ordinarily be avoided. As an illustration, if a cell having a capacity of 200 ampere-hours is completely discharged, the initial rate of charge may be 200 amperes, but must be continually reduced, so that when the cell is half charged the rate will be 100 amperes, and when it is three-quarters full, the rate will be 50 amperes. Toward the end of the charge, when the rate has been finally reduced to the normal or finishing rate, it need not be further reduced, and the charge may be completed at this rate. Some gassing will result at the end, but the amount will not be excessive.

537. EXPERIMENT 25-A. — Test of a Lead Storage Cell. — A complete test on an unknown cell should comprise the following points: (a) performance curves, such as shown in Figs. 330 and 331, at various rates of discharge; (b) variations of voltage and capacity with temperature; (c) charge and discharge rates and capacity per unit weight of the cell and per unit area of both plates; (d) the efficiency of the cell under different conditions; (e) the internal resistance of the cell and its variations; (f) the relationship between the maximum density of the electrolyte and the discharge characteristics; (g) loss of charge by leakage and local action; (h) durability of the cell and its adaptability for a proposed service.

All these tests are either described in the preceding sections or follow directly from their purpose and from the general characteristics of the lead storage cell. While each test in itself is rather simple, a complete series, such as outlined above, would take many weeks to perform. If it is desired merely to become acquainted with the technique of the principal tests, one partial discharge and a following charge will be sufficient. If it is not convenient to charge and to discharge a single cell, two or more cells in series may be tested, but all voltage and density readings should be taken on one and the same cell.

The cell or cells (Fig. 332) are connected to a double-throw switch, by means of which they can be connected at will either to a source of direct current for charging or to a load resistance for discharge. An ammeter and an ampere-hour meter are provided in the battery circuit. A watt-hour meter may also be used as a check, if the battery voltage is sufficiently high for its potential circuit.

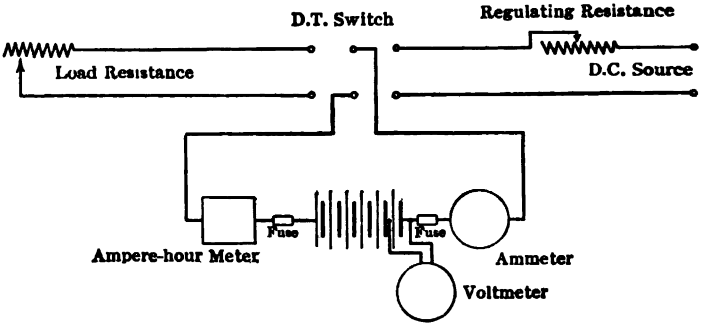


FIG. 332. Connections for testing a storage battery.

Assuming the cell to be fully charged, take the initial readings of voltage and specific gravity on open circuit, and measure the internal resistance and the temperature of the cell and of the room. Connect the cell to the load rheostat, adjust the current to a desired value and keep it constant throughout the test.

At first read all the instruments and thermometers at frequent intervals; later, when the conditions become steady, less frequent readings are sufficient. Use a cadmium electrode as a check on voltage readings. From time to time measure the internal resistance. Always stir the liquid before reading the hydrometer so as to measure the true average density.

To save time, two or more independent cells or batteries can be tested simultaneously, some charging, others discharging. One voltmeter with a good multipoint switch is sufficient for all. An ammeter shunt may be used in each circuit, with a common milli-voltmeter for reading the currents. In a student laboratory, tests at low rates of discharge, sulphation tests, investigation of local action, durability, service tests, etc., may be continued day after day by different observers, and the results entered in a common log to be analyzed at the end of the complete test.

At the end of the experiment, measure all the dimensions of the cell and of its elements, so as to be able to make a drawing to scale. Determine the weight of the plates, of the electrolyte, and of the complete cell. Do not keep the negative plates out of the liquid longer than is necessary; they may be damaged by the action of the atmosphere.

Report. Plot the observed performance curves and give answers to the items (a) to (h) enumerated at the beginning of the experiment. Where the laboratory data are insufficient, specify in detail the necessary additional tests and measurements.

THE NICKEL-IRON CELL

538. Chemical and Physical Features. — The secondary cell, known as the Edison or alkaline cell, is shown in Fig. 333. In a fully charged state the active material of the positive plates is nickelic hydroxide, $Ni(OH)_3$, and that of the negative plates is metallic iron (Fe). The electrolyte is a 21 per cent solution of caustic potash (potassium hydroxide, KOH). When the circuit is closed and the cell allowed to discharge, iron is oxidized to ferrous hydroxide, $Fe(OH)_2$, while the high nickel oxide is reduced to lower oxides, and finally to nickelous hydroxide, $Ni(OH)_2$. On charging, the iron oxide is again reduced to iron and the nickelous oxide is oxidized to the higher oxide, $Ni(OH)_3$. These reactions can be expressed by the following reversible equation:

$$\underset{\xrightarrow{\textit{discharge}}}{\mathrm{Fe} + 2\mathrm{Ni(OH)_3}} = \overset{\xleftarrow{\textit{charge}}}{\mathrm{Fe(OH)_2} + 2\mathrm{Ni(OH)_2}} \quad . \; . \; . \; . \quad (5)$$

Because of this reaction the cell may be termed a "hydroxyl lift" cell.

The function of the electrolyte is to furnish hydroxyl ions (HO) at the anode and to absorb them at the cathode. During the discharge, potassium ions tend to migrate from iron to nickel, leaving hydroxyl ions to combine with the iron. Potassium ions recombine with hydroxyl ions moving from nickel. Thus the electrolyte becomes concentrated in the pores of the positive plate and weakened in the pores of the negative plate. On charge, an opposite migration takes place. Diffusion ultimately destroys these differences in concentration and leaves the electrolyte in the original condition, since at any instant the total quantity of water and potassium hydroxide (including that in the pores of both plates) remains constant.

For this reason, the specific gravity of the electrolyte, at a given temperature, remains practically constant both on charge and on discharge, and a hydrometer cannot be used to ascertain the state of charge of a cell, as in a lead-acid accumulator (§ 529). Distilled water has to be added frequently to the alkaline electrolyte to make up for the losses due to gassing on charge, and the whole electrolyte has to be completely renewed about once a year.

The fundamental principle of the cell is, in short, the oxidation and reduction of two metals in an electrolyte which does not dissolve and will not combine with either the metals or their oxides. Sodium hydroxide may be used in place of potassium hydroxide.

Under normal conditions the gravity of electrolyte should be from 1.200 to 1.220. When potassium electrolyte has fallen to 1.160 it should be renewed. Sodium electrolyte may be used down to 1.130.

A small amount of lithium hydroxide (LiOH) is always added to the electrolyte. It is said that lithium causes the active material within the nickel electrode to swell and to make a better contact with the containing tubes and with the nickel flakes added to nickelic hydroxide for better electric conductivity. Whatever the explanation may be, the fact has been established that lithium hydroxide appreciably increases the capacity of the cell.

539. Mechanical Construction of the Plates. — The active material of the positive plates (nickelic hydroxide) is packed under high pressure in small reinforced steel tubes, with alternate layers of nickel flakes (Fig. 333). This is necessary because nickel hydroxide itself is a poor conductor of electricity. The tubes are provided with numerous lateral perforations to allow the electrolyte to permeate the active material. Packed tubes are solidly mounted in a steel grid so that a finished plate somewhat resembles that shown in Fig. 328, although of course the materials are entirely different.

Negative plates consist of thin rectangular pockets of perforated sheet steel mounted in steel frames or grids. The active material, iron oxide and metallic iron, is contained in the pockets. The grids, tubes, and pockets are nickel plated. The general type of construction is therefore similar to that of the negative plates shown in Fig. 327 to the left. Mercuric oxide, subsequently reduced to mercury, is added to increase the electric conductivity of the active material.

There is always one more negative plate in the cell than there are positive plates. All like plates are connected together by nickeled steel rods, and the whole assembly is placed in a container of nickeled steel. Hard-rubber separators are placed between adjacent plates of opposite polarity, and hard-rubber bushings, rods, and supports are used throughout to insulate the plates from each other and from the container. The whole mechanical construction has been devised so as to secure the rigidity, strength, and durability necessary in traction work and for uses as portable batteries.

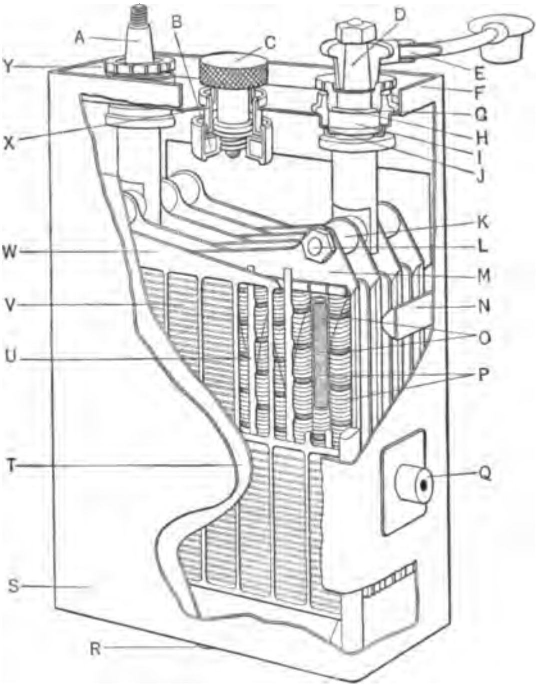


FIG. 333. **Details of construction of Edison's alkaline or nickel-iron secondary cell.**

A — Negative pole.
B — Valve.
C — Filler cap.
D — Positive pole.
E — Copper wire swedged into steel lug.
F — Cell cover welded to container.
G — Stuffing box.
H — Gland ring.
I — Stuffing box gasket.
J — Weld to cover.
K — Spacing washer.
L — Connecting rod.
M — Positive grid.
N — Grid separator.
O — Seamless steel ring.
P — Positive tubes (nickel hydrate and nickel in layers).
Q — Suspension boss.
R — Cell bottom (welded to sides).
S — Solid steel container.
T — Side insulator.
U — Pin insulator.
V — Negative pocket (iron oxide).
W — Negative grid.
X — Cell cover.
Y — Hard rubber gland cap.

540. Electrical Characteristics. — The normal open-circuit e.m.f. of an alkaline cell is about 1.34 volts. When the cell is fully charged, the e.m.f. rises to a somewhat higher value for a short time. The open-circuit voltage should never be used to determine the state of charge or the extent of discharge. Neither can the specific gravity of the electrolyte of an Edison cell be used as a guide, since the density remains practically constant (§ 538).

The proper method of determining the state of charge is to read the cell voltage while the battery is on charge or discharge. On charge, if the voltage remains constant at 1.8 volts per cell or above this for thirty minutes, with normal charge current flowing through the battery, then the battery can be considered fully charged. A very good check on this is to observe the extent of gassing. If a cell is gassing freely with normal current flowing, it is a pretty good indication that a condition of full charge has been reached. In this connection, a distinction must be made between gassing and frothing.

Frothing at the filler openings of a battery indicates either too rapid charging, too high a level of electrolyte, or impurities in the cells. If frothing takes place while charging at normal rate with the level of the electrolyte at the proper height, it is a sure indication that some form of animal fat or oil has gotten inside the cell. If the frothing continues for any length of time, the cell should be emptied, rinsed out, and new electrolyte put in.

If a cell is discharged immediately after charging, the initial voltage, with the rated current, is 1.45. The voltage rapidly falls to 1.3 volts and then drops slowly to 1.15 volts. At this point it begins to fall more rapidly, and the discharge is usually stopped at 0.9 volt. The average voltage while discharging to the foregoing point is about 1.2 volts. When discharging at five times the normal rate, the final voltage can be as low as 0.5 volt. The advisable final voltage between these limits may be taken to vary according to a straight-line law.

The normal charging and discharging rate for an Edison battery is seven hours. Gas is evolved throughout the charge, and as this gas contains considerable hydrogen, a lighted match or an open flame should not be brought near. The temperature rises constantly when the battery is discharging, but does not increase much on charge.

The capacity of an Edison cell and its efficiency increase with the temperature, but temperatures above 50° C. will shorten the life of the cell. At low temperatures the capacity and efficiency decrease rapidly, and there is a critical temperature (which varies with the rate of discharge) below which the output is very small. For a general discussion of the voltage, capacity, efficiency, and internal resistance of storage cells see §§ 531 to 536.

541. A Comparison of the Lead-Acid and Nickel-Iron Cells. — The Edison cell has not superseded the lead-sulphuric acid cell, and cannot even compete with it, except in some classes of service, such as electric trucks, tractors and automobiles, portable batteries for radio work, submarine uses, etc. The lead battery has a field of its own in power plants and for starting automobile engines. A comparatively high internal resistance and low e.m.f. of the nickel-iron cell are the principal inherent obstacles to its universal use in power work. On the other hand, an alkaline battery is somewhat lighter and takes less space than a lead battery of the same capacity and voltage. The alkaline battery is also much more robust and fool-proof, and can stand an amount of hard usage, abuse, and neglect that would ruin a lead battery.

The following other disadvantages of the nickel-iron cell may be mentioned as limiting its application in various fields: (a) Its first cost is higher. (b) The electrolyte is quite expensive and must be renewed annually. (c) The energy and current efficiencies are lower than those of a lead cell. (d) The voltage varies considerably during a discharge. (e) The cell is very inefficient at low temperatures. (f) After a full charge it loses 10 per cent of its energy in twenty-four hours; if an Edison battery has to stand idle it is better to charge it only to 80 per cent of its capacity. (g) Gases are evolved continuously during the charge, and under certain conditions may be of an explosive nature.

On the other hand, the nickel-iron cell has the following advantages over the lead-acid cell:

(a) *On charge:* An Edison battery may be forced to full charge at almost any rate of current, without injury. A short boosting charge at a high rate is particularly desirable in electric-vehicle service. The battery may be charged in the wrong direction without permanent injury. It emits no corrosive fumes and there is no acid to handle.

(b) *On discharge:* A repeated excessive discharge will not result in a permanent injury. Even an occasional short-circuit is said to have a beneficial effect, if any. The ampere-hour capacity is more independent of the rate of discharge than with a lead cell.

(c) *Mechanical strength:* A nickel-iron cell is not subject to buckling or sulphation, and there is no shedding of active material which ultimately may cause a short-circuit due to the sediment. Continued vibration has practically no effect on the cell.

(d) *When standing idle:* An Edison battery may stand indefinitely in any state of discharge, without damage. The electrolyte may be drained or partially removed without injury to the plates. The cell will retain over half its charge at the end of six months or more of idleness. Cases are quoted of a prompt recovery of an alkali battery after remaining unused for three years.

542. EXPERIMENT 25-B. — Test of a Nickel-Iron Storage Cell. — The general scope of the test is the same as in § 537, but the differences in the chemical reactions and in the physical characteristics of the two cells must be kept in mind.

OPERATION AND CONTROL OF STORAGE BATTERIES

543. Desirable Tapering of Charging Current with the Time. — In order to charge a battery fully after discharge it is necessary to pass through the cells in the proper direction (opposite to that of discharge) an amount of electricity equal, in ampere-hours, to that taken out on discharge plus some excess to make up for the losses.

If the charging rate in amperes is not too high, practically all the current is useful for charging the plates. If the charging rate is increased, a point will be reached where "gassing" begins — due to bubbles of oxygen and hydrogen formed at the surface of the plates by decomposition of the water in the electrolyte; this gassing increases with further increase in the charging rate. This decomposition absorbs an amount of electricity proportional to the amount of gas produced, and this portion of the current is wasted, as it produces absolutely no useful effect in charging the plates. Charging rates sufficiently high to produce violent gassing not only are wasteful of electric energy, but tend to dislodge the active material from the plates and produce excessive temperature rise, thus very materially shortening the life of the plates; and it cannot be too strongly emphasized that *the excess which produces gassing does not charge the plates.*

In general, any charging rate is permissible which does not produce excessive gassing or a cell temperature exceeding 43° C. with lead-acid cells and 50° C. with nickel-iron cells.

The value of the charging current at which appreciable gassing begins depends upon several factors, such as state of charge, temperature, specific gravity of electrolyte, type of plate, etc., but the principal factor is the state of charge of the battery. When a battery is fully charged, any rate of charge, however small, will produce gassing, but the rate may be reduced to such a low value that, unless abnormally prolonged, the small amount of gassing which results is practically harmless. This safe rate is called the "finishing" rate (§ 536). During the earlier stages of the charge, the charging rate may be several times the finishing rate, without producing violent gassing, and the more completely the battery is discharged, the higher the charging rate may be without causing excessive gassing.

From the above it will be observed that there is a wide range of satisfactory charging methods, from that employing a constant current at the finishing rate (the " Constant Current " method) requiring with a lead battery about sixteen hours for a full charge, to that involving a comparatively high rate at the beginning, tapering off to the finishing rate at the end, under which conditions a battery may be charged in five hours or even less.

For each stage of charge there is a maximum value of current which should not be exceeded, and when the charging current can be automatically tapered in accordance with this rate, the charging is completed within the least possible time. It is often of importance to accomplish this aim at least approximately, for example in the case of batteries used on locomotives, trucks, etc., where the time taken for charging may reduce the amount of service of which the vehicle is capable, unless spare batteries are available.

An *ampere-hour meter* (§ 119) can be used for terminating the charge automatically. The ampere-hour meter must be connected in the battery circuit at all times during discharge as well as charge, in order that it may register the state of charge of the battery when the battery goes on charge. It should run slower on charge than on discharge, in order to take into account the losses in the battery and to give it a proper overcharge before the charging circuit is opened. Under these conditions an ampere-hour meter designed to open the charging circuit when the indicator hand returns to zero will give satisfactory results, provided a suitable instrument is employed and is maintained in operating condition.

In important installations of lead batteries a *recording and signaling hydrometer* is sometimes used. As is stated in § 529, the specific gravity of acid electrolyte is the best indicator of the state of charge. However, it is often troublesome or impossible for an attendant to take a sufficient number of readings and to determine accurately when the gravity has reached a predetermined point, either on charge or discharge. Moreover, the readings taken with an ordinary hydrometer require a correction for temperature in order to be directly comparable, and this the attendant frequently has not the opportunity to compute.

The recording hydrometer is placed on a pilot cell, that is, a cell whose condition is representative of the normal cells of the battery. The instrument traces a continuous record of density on a sheet of coördinate paper, the ordinates being automatically corrected for the temperature. When the density reaches its maximum, a contact is automatically closed and a visual or audible signal is given to warn the operator that the charge should be ended. Similarly, a signal is given to end the discharge when the specific gravity has reached its permissible minimum.

544. The Modified Constant-Potential System of Charging. — A simple method of obtaining an automatically tapering rate of charge consists in connecting the battery, in series with a comparatively low resistance, across a constant-potential source of direct current. This is known as the *modified constant-potential system,* to distinguish it from the older " straight " constant-potential system with which no current-limiting resistance is used (§ 536).

Let the finishing value of current be i_f, that is, the value usually prescribed by the manufacturer at which the charge should be completed when gassing begins. Let an applied voltage of e_f volts per cell (about 2.5 volts for a lead cell) be required to maintain this finishing rate of current at the end of a charge. If the bus voltage is E and the number of cells in series is n, the excess voltage is $E - ne_f$, and the required protective resistance R is determined by the condition

$$R = (E - ne_f)/i_f \quad \ldots\ldots\ldots\ldots \quad (6)$$

At the beginning of a charge, the current, i_b, will be

$$i_b = (E - ne_b)/R \quad \ldots\ldots\ldots\ldots \quad (7)$$

where e_b is the applied voltage per cell necessary to cause the charging current i_b to flow at the beginning of a charge. For a given type of cell the relationship between i_b and e_b is approximately known.

The protective resistance R, computed from eq. (6), must give a value of current i_b not to exceed a safe limit for the cells and for the charging equipment. If the initial current is excessive, several remedies are possible, for example:

(a) A higher resistance R may be used, thus lengthening the total duration of charge.

(b) A generator, a rectifier, or some other source of current with a distinctly drooping voltage characteristic may be used, to limit the initial charging current.

(c) The protective resistance R may be reduced in one or two steps, after the initial period of charging is over.

The foregoing difficulty arises when the available source of voltage is not high enough for the battery, leaving no margin at the end of the charge. On the contrary, when the available source is too high, the protective resistance, adjusted for the proper finishing rate, will unnecessarily cut down the initial rate of current, thus lengthening the total time of charging. Moreover, the efficiency of the whole arrangement is reduced when a considerable part of the applied voltage is absorbed in a resistance. In such a case the following remedies are possible:

(d) A motor-generator set is installed to reduce the bus voltage to the best value.

(e) Two or more batteries are charged in series.

(f) The protective resistance *R* is increased in one or two steps after the initial period of charging.

(g) Counter-e.m.f. cells may be used (§ 550), if the required reduction in voltage is not too great.

By the use of counter-e.m.f. cells the characteristics of the modified constant-voltage method may be secured with a bus voltage somewhat higher than normal, and without manual adjustment during charge, whereas, if an ordinary rheostat is used, it must be adjusted from time to time to secure the same results.

If the line voltage varies from time to time, the number of counter-e.m.f. cells in the circuit may be adjusted by means of a cell switch (§ 552) or a series of knife switches, to maintain a constant voltage on the charging bus. These cells may also be used to provide a second constant-voltage bus at a lower voltage than the main bus, for charging batteries of fewer cells.

When the value of the protective resistance has to be changed automatically during the charge, this can be done by using an additional relay contact on the ampere-hour meter. When a certain prescribed number of ampere-hours has passed through the battery this contact is closed, and a circuit-breaker is tripped or an electrically-operated switch is closed in the resistor circuit, thus increasing or reducing the resistance by the desired amount.

545. EXPERIMENT 25-C. — Constant-Potential System of Charging a Storage Battery. — The method is described in §§ 536 and 544. The purpose of the experiment is to study the effect of different values of bus voltage and protective resistance upon the magnitude of charging current at different stages of charge. To save time, it is advisable to have three or more identical batteries, one fully charged, one discharged to the lowest allowable limit, and the others at known intermediate stages of discharge. By closing the charging circuit for a short time only, their state of charge is not materially altered during the experiment. In this way it is possible to duplicate the conditions which would obtain with one and the same battery at different stages of charge, without waiting for a battery to become charged to each particular stage.

A suitable source of direct current should be provided, such as a generator whose voltage can be varied within wide limits. An ammeter, a voltmeter, and an adjustable series rheostat are connected in the generator circuit. Each battery is provided with a separate switch by means of which it can be connected to the generator.

(1) Measure the voltage of the fully charged battery, adjust the generator voltage to be slightly above it, and connect the battery so that it is charging, with the series rheostat short-circuited. Raise the generator voltage to charge the battery at the finishing rate with the electrolyte gassing freely. Read volts and amperes. Substitute the other batteries in turn, keeping the generator voltage constant, and read the charging current. For a fully discharged battery it may be necessary to reduce the generator voltage or to introduce some protective resistance. This test is intended to illustrate the straight constant-potential method of charging.

(2) Measure the voltage of the fully discharged battery, adjust the generator voltage to be slightly above it, short-circuit the rheostat and connect the battery on charge. Raise the generator voltage to produce as high a charging current as the equipment will stand. Keeping the voltage at this value, substitute the fully charged battery and read the current. This test will show that with the straight constant-potential system of charging, the end of the charge is needlessly prolonged, because of a low final current.

(3) Adjust the generator voltage and the series resistance by trials until the charging current has the correct "finishing" value for the fully charged battery and does not exceed a safe value for the discharged battery. Read the currents, the voltages across the different batteries, and the voltage drop in the rheostat. This illustrates the modified constant-potential system.

(4) Select a generator voltage somewhat below that in the preceding test and take readings with the different batteries, using first a value of resistance which gives the correct finishing rate and then one which gives a proper initial current. Try the remedies (a), (b), and (c) mentioned in the preceding section, or some other remedy that the facilities on hand will permit.

Select a generator voltage somewhat above that found in test (3) and repeat test (4). Use remedies (e), (f), and (g). Take a generator voltage much above that required for the last period of charge, and repeat the test. Try a remedy that seems to be the most suitable for such a case.

Report. (1) Give the data on the straight-potential charging and explain its disadvantages and limitations. (2) Give the data on the modified constant-potential system, and estimate the saving in time as compared with charging the same battery at a constant current. (3) Apply eqs. (6) and (7) to the values of currents, voltages, and resistances found by experiment. (4) Give the data on the tests with the generator voltage below and above normal and explain the remedies which have been applied in order to reduce the total time of charging to a minimum.

546. Control of Battery Charge and Discharge. — The exact method by which the discharge rate of a battery is controlled, or by which the total output is divided between the battery and the generators, depends upon the function of the battery, its size, and the local conditions. The methods of controlling the charge also differ widely, from a simple knife switch to a sensitive automatic booster. In the following experiments the student is given an opportunity to become familiar with a few typical systems of battery control. With these cases as a guide he will be able to understand and to choose among many other intermediate methods of battery control met with in practice.

The various cases are treated more or less in the order of their complexity, and boosters are described separately at the end. The automatic control of an automobile battery used for starting and lighting is described in § 260.

547. A Low-Voltage Battery Charged from a High-Voltage Circuit. — There are cases of small batteries which have to be charged from sources of much higher voltage. For example, a 125-volt battery used for operating switches in an electric railway substation may have to be charged from an available 600-volt trolley circuit. A 40-volt battery used for some special purpose may have to be charged from an available 110-volt supply, etc. If power economy is not important, and a constant voltage between the low-voltage buses is not essential, the battery is simply charged in series with a resistance which absorbs the excess voltage.

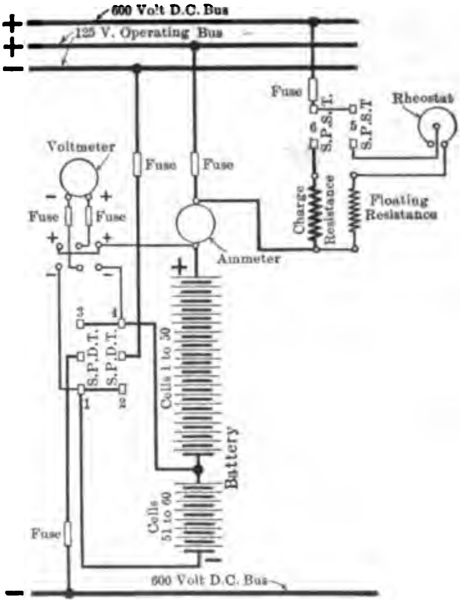


FIG. 334. A low-voltage battery connected to a high-voltage circuit through a resistance.

A typical installation is shown in Fig. 334. Two resistances, a "floating" resistance and a "charging" resistance, are shown between the 600-volt bus and the 125-volt operating bus on the positive side, with a knife switch in circuit with each. Ordinarily the charging resistance is disconnected, while the floating resistance allows an amount of current to pass from the 600-volt bus equal to that required by the 125-volt load, plus a small "trickling" charge into the battery. It is important that this "floating" resistance be of suitable value to effect these results, and for convenient adjustment a small variable rheostat is shown in series with the fixed resistance.

When the battery is to be charged, the charging resistance is connected in parallel with the floating resistance. A group of 10 extra cells is provided, which may be used at will or cut out when fully charged. Normally the switches 1, 2, and 5 are closed. At the beginning of the charge, switches 1, 4, 5, and 6 are closed, and at the end of the charge the switches 3, 4, 5, and 6. The voltmeter reads the total battery voltage when its switch is closed to the left, and that of the cells 1 to 50 when the switch is closed to the right.

548. EXPERIMENT 25-D. — Charging a Storage Battery Through a Resistance. — The purpose of the experiment is to learn the characteristics of the circuit shown in Fig. 334. The voltages need not be the same as shown in the sketch. A 110-volt laboratory circuit can be used for the higher voltage, and a battery of comparatively few cells charged from it. A suitable battery load should be provided, adjustable within wide limits. It is advisable to have at least three identical batteries, one fully discharged, one fully charged, and one about half discharged. In this manner, the current and the voltage relations may be studied under different operating conditions, without waiting for a battery to undergo a complete charge and discharge. If the number of cells is sufficiently large, the same result can be accomplished by varying the number of cells in the circuit. For example, 20 nearly discharged cells in series may approximately represent about 16 charged cells, in so far as the total voltage is concerned. An increase in the internal resistance of nearly discharged cells may be imitated by adding a corresponding resistance in series with the battery.

(a) Select and adjust the values of the three resistances shown in Fig. 334, to cover operation under the extreme conditions of load and charge. (b) Measure the operating bus voltage under various conditions of charge and discharge that may arise. (c) Try different numbers of extra cells (marked cells 51 to 60 in the diagram) and make clear to yourself their function in steadying the bus voltage, and the amount of charge which they receive as compared to the main cells. (d) Measure voltage fluctuations between the low-voltage buses when the voltage between the high-tension buses varies within reasonable limits.

Report. (1) Explain how the values of the three resistances were selected, and why an adjustable rheostat is needed in series with the floating resistance. (2) Give the limits of unavoidable fluctuations of the voltage between the battery buses, and explain the conditions under which each value occurs. (3) Explain the amount of charging current which the battery receives under different load conditions and at different stages of charge. (4) Give data to show the function of the extra cells, and explain which number should be recommended in the particular installation tested. (5) Give data on the effect of fluctuating voltage supply, and explain the results theoretically.

549. Charging Battery Sections in Parallel.—A simple method of charging a storage battery when the generator voltage is not sufficiently high, is shown in Fig. 335. The battery is divided into two halves which are connected in series when discharging, and in parallel for charging. This is done in order to obtain a sufficient voltage for charging, without raising the line voltage maintained by the generator. An example will make this clearer. Consider a battery intended for an ordinary 110-volt lighting circuit. The voltage of each cell at the end of discharge is about 1.8 volts; therefore the required number of cells is 110 ÷ 1.8 = 62. But the voltage necessary with this number of cells at the end of a charge is 2.6 × 62 = 161 volts, which is far above the line voltage. With the battery divided into two halves in parallel, only 80.5 volts are required for charge; the excess line voltage is absorbed in the rheostat *R*. The battery output on discharge is also regulated by the same rheostat. This method, although very simple, is used only in small installations, where the loss of power in the rheostat is not objectionable.

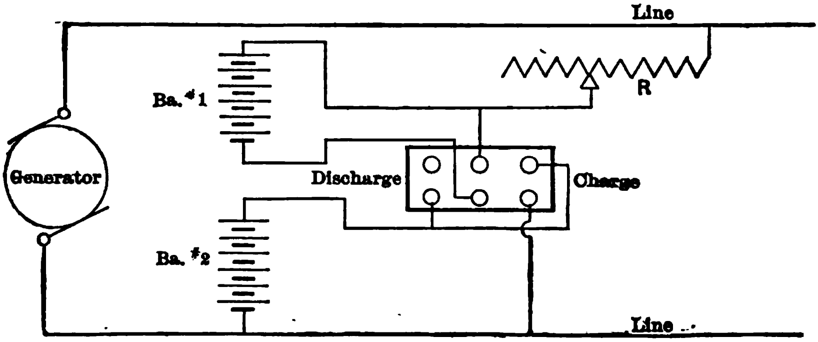


FIG. 335. Charging two halves of a storage battery in parallel.

A more economical method is to divide the battery into three equal parts; let them be denoted by the letters *A*, *B*, and *C*. First the parts *A* and *B* are charged in series, up to one-half of the total required ampere-hours; then the parts *B* and *C* are charged with an equal amount of electricity. Finally the parts *A* and *C* are connected in series and the charge is completed. Less energy is wasted in the resistances with this arrangement, although it may take longer to charge the battery. The voltage at the end of the charge is $\frac{2}{3} \times 161 = 107$ volts.

Other combinations are also possible; for example, *A* and *B* may be connected in parallel with each other and in series with *C*. The battery is charged at as high a rate as possible until *C* is completely charged. Then *C* is disconnected, *A* and *B* are connected in series, and the charge is completed.

550. Counter-E.M.F. Cells. — In place of, or in addition to, a rheostat in series with a storage battery, counter-e.m.f. cells may be used on both charge and discharge. These cells consist of unformed lead plates in dilute sulphuric acid. Because they have no active materials, they possess no capacity for discharge. When current is passed through such a cell it offers a counter-e.m.f. of about 2.3 volt. The difference between a resistance and a counter-e.m.f. cell is that in the former the voltage drop is proportional to the current, while in the latter it is nearly independent of the current.

Counter-e.m.f. cells must be so connected that current always passes through them in the same direction. Otherwise, plates would be gradually "formed," like Planté plates (§ 530), and the cells would attain a considerable ampere-hour capacity.

On charge, the counter-e.m.f. cells absorb the excess generator voltage. At the beginning of the discharge the same cells cut down the battery voltage while it is too high. The counter-e.m.f. cells can be connected and disconnected as a unit, or, for a closer regulation, one by one, by means of an end-cell switch (Fig. 336).

551. EXPERIMENT 25-E. — Charging a Battery in Sections. — Wire up the two halves of a battery as in Fig. 335, and make connections to a suitable generator. Ascertain the state of charge of the battery with a hydrometer, or by some other method. Provide the necessary ammeters and voltmeters, and a load in the form of adjustable resistances. Operate the installation under the following conditions:

(a) The battery and the generator supplying power to the line in parallel.

(b) The battery being charged, the generator at the same time supplying power to the line.

(c) The battery alone supplying power, the generator being shut down.

(d) The generator working alone, the battery being disconnected for inspection or repairs.

For each of these conditions select a few characteristic loads (light load, medium load, full load, and overload) and take all the necessary ammeter, voltmeter, and hydrometer readings, so as to have a complete record of the electrical relations in the circuit, with special reference to the performance of the battery. Observe voltage and current fluctuations, when the load is varied, first gradually and then suddenly. Perform all these tests both with a resistance and with counter-e.m.f. cells, and observe the difference in the behavior of the two.

Devise a convenient arrangement of switches for charging the battery in three parts, as explained in § 549. Connect the battery accordingly and observe the process of charging. Try a division of the battery into different sections which in your judgment will permit charging it with the least loss in the rheostat, or in a minimum time, or will have some other advantage. With the various arrangements tried, first use a regulating rheostat and then counter-e.m.f. cells; make clear to yourself the advantages and the disadvantages of both.

Report. (1) Draw the exact diagram of connections used, with all the instruments, fuses, etc. (2) Give the data on the division of the load between the battery and the generator. Estimate what this division would have been, were the battery fully charged or almost completely discharged. (3) Describe the effect of the battery when the load is changed suddenly. (4) Give the voltage drop in the rheostat when charging the battery in two and three sections, and estimate the total loss of energy in the rheostat, or in the counter-e.m.f. cells, as compared to the energy actually put into the battery. (5) Describe a better division of the battery into sections if one has been tried or suggested itself. (6) Give your opinion, substantiated by test data, as to the relative advantages of a resistance *vs.* counter-e.m.f. cells. (7) Assume the battery and the generator to represent an isolated plant; select a type of prime mover and a variable load curve of reasonable character. Write instructions as to the hours of day and night when the generator should be running, when the battery should be charging, and when it should be supplying the load alone. Also give brief instructions as to the care which the battery should receive.

552. End-Cell Switches. — In many small installations there is no demand for current during certain hours of the day. In such cases battery connections shown in Fig. 336 may be used. The battery is charged during the hours when the main switch S can be left open, the generator voltage being raised to the required value (say 161 volts) for charging the battery. During a discharge the battery voltage and output are regulated by the so-called *end-cell switch E*, by means of which more cells may be connected in the circuit as the voltage of each cell becomes lower.

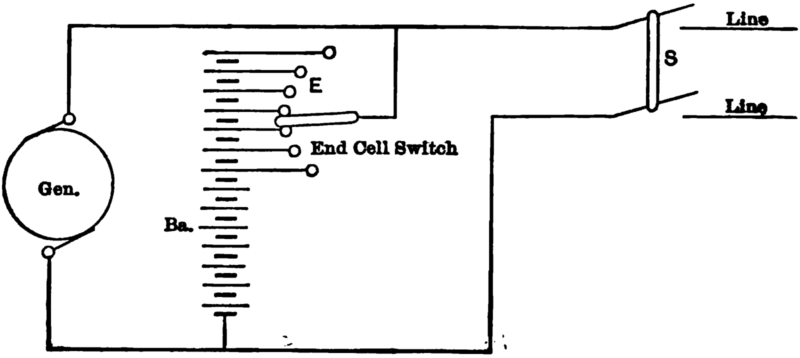


Fig. 336. Use of a single end-cell switch.

End-cell switches are also sometimes used in installations where charging is done by means of special machines, so-called " boosters " (Fig. 340). Thus, storage batteries in stations and substations of electric lighting companies in large cities are often regulated by end-cell switches and charged by boosters. Large end-cell switches are sometimes operated by small auxiliary electric motors, which are started and stopped either by the switchboard attendant, or automatically by a contact voltmeter.

The movable contact piece of an end-cell switch must be wide enough not to open the battery circuit while the arm is moved from one stationary segment of the switch to the next. On the other hand, if the contact were allowed to bridge two adjacent segments, it would short-circuit the cell connected to these two segments, which is not permissible. Therefore, the contact piece is made in two parts, with a *protective* resistance between, this resistance limiting the current in the short-circuited cell during the instant when the arm is moved from one contact to the next.

In some cases it is not practicable to have the main switch opened while the generator voltage is being raised for the charge; at the same time the size of the installation may not warrant the complication of a booster set. Two end-cell switches may be used in such cases, as shown in Fig. 337. By means of the end-cell switch E_1 the required voltage is maintained on the line, while the charging current and voltage are regulated by the generator field rheostat and the end-cell switch E_2.

With this arrangement the end cells are carrying the sum of the charging current and the line current, and are therefore charged faster than the rest of the battery. Actual practice does not show, however, any serious disadvantage of such an arrangement, provided that charging is done judiciously and during the hours of small demand. By actually computing the limits of voltages it will be found that more contact points are necessary with a double end-cell switch than with a single end-cell switch.

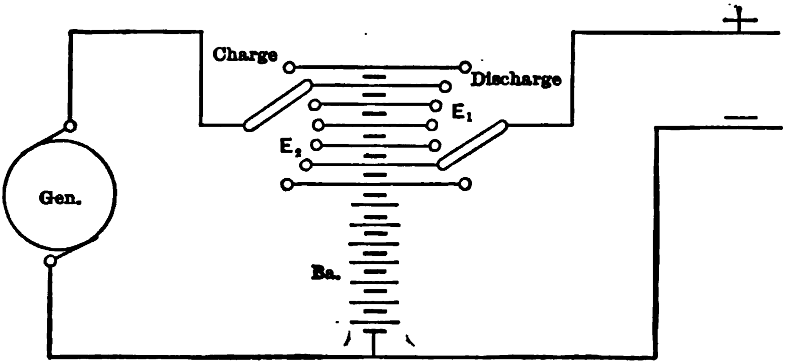


FIG. 337. Use of a double end-cell switch.

Counter-e.m.f. cells (§ 550) may be used as a modification of, or in addition to the connections shown in Figs. 336 and 337. When the battery is placed on charge, the generator current is divided; part of it flows through the main battery and the rest passes through the counter-e.m.f. cells to the line. In this way the generator voltage may be raised sufficiently to charge the whole battery, and at the same time part of the generator current supplies the load at the normal voltage. As the charge progresses and the generator voltage is increased, more counter-e.m.f. cells are connected in series with the line, by means of an end-cell switch.

Fig. 338 shows a complete diagram of connections for the arrangement according to Fig. 337. The switch S_1 controls the generator circuit, S_2 the battery circuit; S_3 is a double-throw switch. For charging the battery, S_3 is thrown to the right, and the switches S_1 and S_2 are closed. The charging current flows from the positive terminal of the generator, through S_3, to the charging end-cell switch E_2; thence through the battery, ammeter A_2, overload circuit-breaker CB_2, underload circuit-breaker CB_1, and switch S_2 to the negative terminal of the generator. At the same time, the line is supplied with current through the discharge end-cell switch E_1 and the ammeter A_1. The underload circuit-breaker prevents the battery from sending current back into the generator; it can be used only in battery plants in which the load is varying slowly, so that there are distinct periods of charge and discharge.

For discharging the battery into the line, in parallel with the generator, the switch S_3 is thrown to the left; this cuts the charging end-cell switch E_2 out of the circuit, while the underload circuit-breaker CB is short-circuited by the lower blade of the switch S_3. The discharge is regulated by the end-cell switch E_1 and by the field rheostat of the generator. On the other hand, if it is desired to shut down the generator, the switch S_1 is opened, and the battery continues to supply current alone. This is usually done when the demand for current is quite small. If the switch S_2 is opened, instead of S_1, the battery is disconnected from the line, while the generator continues to supply power to the load.

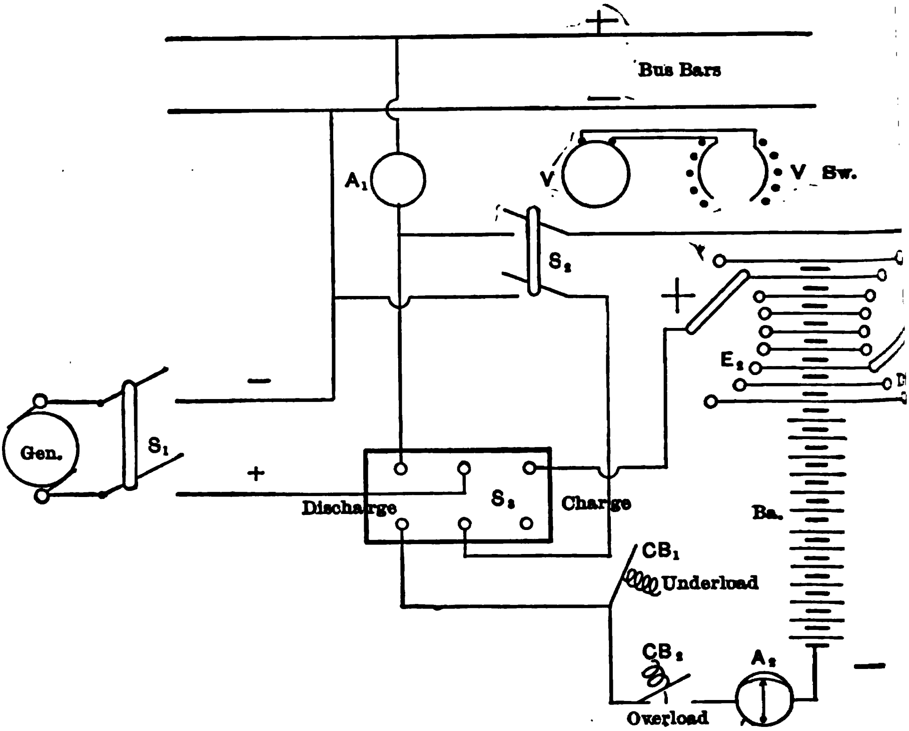


Fig. 338. Details of connections of a battery controlled by a double end-cell switch, as in Fig. 337.

V is a voltmeter which, by means of the switch $V. Sw.$, can be connected at will to show the generator voltage, the battery voltage, or the line voltage.

553. EXPERIMENT 25-F. — Control of a Storage Battery by an End-Cell Switch. — The connections with a single end-cell switch are shown in Fig. 336 and with a double end-cell switch in Fig. 337, or more in detail in Fig. 338. Try both systems in actual operation, under the conditions specified in § 551. Devise a diagram of connections with which counter-e.m.f. cells can be used, and the generator made to supply the load while charging the battery at an increased voltage.

Report the actual diagrams of connections and the numerical results of the test. State voltage fluctuations with sudden variations of the load, and give the relative fluctuations of the current in the generator and in the battery. Figure out the number of points actually necessary on the single end-cell switch and on the double end-cell switch in a practical installation.

544. Battery Control when a Variable Bus Voltage Is Permissible. — There are special cases in which a generator and a storage battery are connected in parallel to a load consisting of devices which can be operated satisfactorily within rather wide limits of voltage. In this case, end cells can be used without an end-cell switch. A typical installation of this kind is shown in Fig. 339, in the form in which it is used in power stations and substations for operating remote-control oil switches, disconnecting switches, etc. The difference between this case and that shown in Fig. 334 is that here a motor-generator set is available for charging the battery.

In order to permit giving the battery a charge to the maximum voltage by raising the voltage of the generator, and without subjecting the connected apparatus to this high voltage, a tap is taken from the battery to the switchboard by means of which a group of 10 cells may be cut out. At the beginning of charge, the

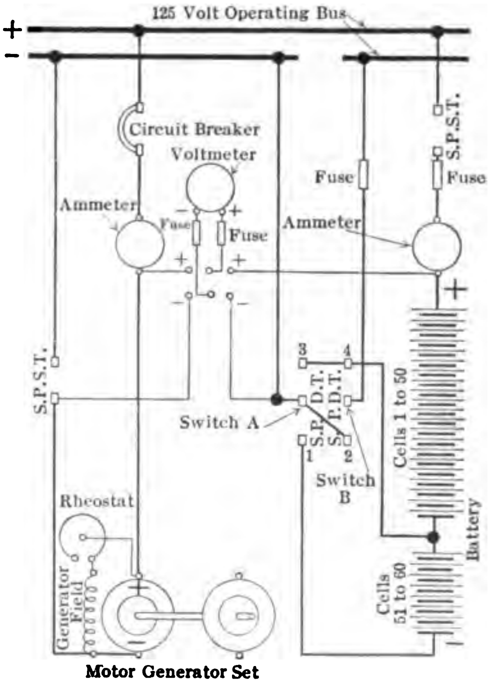


Fig. 339. Use of end cells to limit voltage variations.

entire 60 cells are connected to the generator whose voltage is raised sufficiently to deliver the charging current, while 50 cells are connected across the load circuit. Under these conditions the load current passes through the end-cell group, in addition to the charging current of the main battery, and the charging of the end cells is, therefore, completed before that of the main battery. The end-cell group is then cut out and the charging of the remaining 50 cells is completed. The maximum voltage of these 50 cells at the end of charge will be nearly 140 volts. The signal lamps used with oil switches are designed to stand this voltage for a short time, and the standard remote-control apparatus will operate satisfactorily at this voltage.

The negative bus is divided into two sections, and two single-pole, double-throw knife switches, *A* and *B*, are provided, one connected to each section of the negative bus. When A is thrown down, the 60 cells of battery are connected across the generator terminals, and when *B* is thrown down, the two sections of the bus are connected together. Current is then furnished to the load circuit by the dynamo with the battery floating in parallel. This is the normal position of these switches.

At the beginning of the charge, switch *B* is thrown up, connecting the load circuit across 50 cells, and the voltage of the generator, which is still connected across 60 cells, is raised until the desired charging current is obtained. When the end group is fully charged, as indicated by free gassing and maximum specific gravity, switch *A* is thrown up, cutting out the end-cell group, and the charging of the main battery is completed.

555. EXPERIMENT 25-G. — Storage Battery with End-Cells, Operating in Parallel with a Generator. — The connections and the method of operation are described in the preceding section. The order of the experiment and the requirements for the report are similar to those in §§ 548, 551, and 553, with such modifications as follow directly from the differences in the electrical connections.

556. Floating Batteries. — In power plants in which considerable voltage and load fluctuations on generators are not objectionable, or are unavoidable, storage batteries have been often used without any means for adjusting instant charge and discharge, simply as shown in Fig. 336. This allows the battery to be freely charged or discharged with the fluctuations of the bus voltage. Such "floating" batteries are used in some electric-railway substations, and also in plants containing cranes and elevators. The number of cells is so selected that when the generators give approximately their full output, the bus-bar voltage is equal to the battery e.m.f. Under such conditions no current flows into or out of the battery. The end-cell switch is adjusted from time to time to correct for the state of charge of the battery.

The battery current at different loads depends upon the voltage characteristics of the generators. Shunt-wound generators with drooping characteristics should be used in this case. Then, with a load below the rated load, the bus voltage is higher than the battery e.m.f., and a charging current flows into the battery. When the generators are carrying an overload the bus voltage is reduced and the battery discharges into the line, helping the generators (or rotary converters). With such an arrangement, the generator load is more nearly constant than without the battery. The battery is never entirely discharged or fully charged, but is maintained in a medium condition. Once every week or two it is necessary to give the battery a thorough charging, and even an overcharge, in order to prevent sulphation (§ 529).

Such a floating battery is more effective the wider the voltage fluctuations. The voltage at the end of a feeder varies much more than in the substation, on account of the ohmic drop in the feeders; therefore, it is better to have a floating battery at the end of the line. Further advantages are as follows: The average voltage being lower than in the substation, fewer cells are required; voltage fluctuations being more pronounced, the battery is charged and discharged within wider limits; the load on the generators or rotary converters is steadier; there is a considerable saving in line copper, since the feeders have to carry more nearly an average current, instead of the maximum current. The chief disadvantage of placing the battery at the farther end of a feeder is the additional space and attention required outside the substation.

Standby service. Owing to the desire always to have the battery fully charged for an emergency, and because of a comparatively rapid depreciation of the cells on load-peak service, the foregoing arrangement of allowing the battery to discharge freely during overloads has been gradually abandoned. It has become a standard practice for the lighting companies in the larger cities to provide storage batteries which are kept fully charged and connected to the bus-bars at all times, in order to insure against an interruption of service in the congested business districts. The batteries are used for no other purpose than insurance of continuity of service. In case of failure of the customary sources of energy, the stored energy of the battery is instantly available to supply the deficiency, without even the necessity of throwing any switches either by hand or automatically. The batteries are capable of discharging at very high rates and therefore are not protected by any form of overload circuit-breaker, nor is any device provided for breaking the battery circuit under load. In case of any interruption of power the batteries must meet the full demand until power is restored, or until the charge in the batteries is exhausted.

Such a "standby" battery is usually provided with an end-cell switch, and is charged at regular intervals with a plain non-automatic booster (§ 558).

557. EXPERIMENT 25-H. — Performance of a Floating Battery. — Connect the battery in parallel with a shunt-wound generator, as in Fig. 336, and select such a number of cells that at a desired load the battery neither charges nor discharges. Provide an artificial feeder of considerable resistance so as to represent the actual conditions of railway service; the resistance being such as to give from 10 to 15 per cent voltage drop at full load. The load, representing street cars, is connected at the further end of the line. Connect an ammeter in the generator circuit, one in series with the battery, and one in the main line. Connect a voltmeter so as to measure, at will, the voltage across either end of the line.

(a) First disconnect the battery and use the generator alone, increasing the load from zero to a heavy overload, without regulating the field rheostat. Measure the line amperes and the volts at each end of the line.

(b) Apply the same values of load with the battery and the generator working in parallel. Observe the proportion of the load taken by the battery, and the improved conditions of voltage regulation. If the results show that the battery is charged more than it is discharged, add one or more cells, and vice versa. Repeat the experiment until the battery gives the best performance, in other words, keeps the generator load as steady as possible.

(c) Add a few turns of series winding on the generator field, but not enough to make the machine flat-compounded. Observe the difference in the performance of the battery, and the new voltage regulation.

(d) Connect the battery at the farther end of the line and repeat the same runs. Investigate the influence of the resistance of the line, of the position of the load on the line, and of compounding the generator.

Report. Give the results in such a form as to show the relative usefulness of the floating battery under the various conditions investigated. Point out the limitations in its use.

BATTERY BOOSTERS

558. Non-Automatic Shunt-Excited Booster. — With the exception of smaller plants in which storage batteries are charged as described in the preceding sections, most of the important battery plants are provided with so-called *boosters*, or special generators for charging batteries. The same boosters are sometimes used for regulating the discharge.

A simple non-automatic booster is shown in Fig. 340. Its armature *B* is in series with the battery and is direct-driven by a motor *M*. The fields *F* are separately excited across the bus-bars; *E* is an end-cell switch (§ 552).

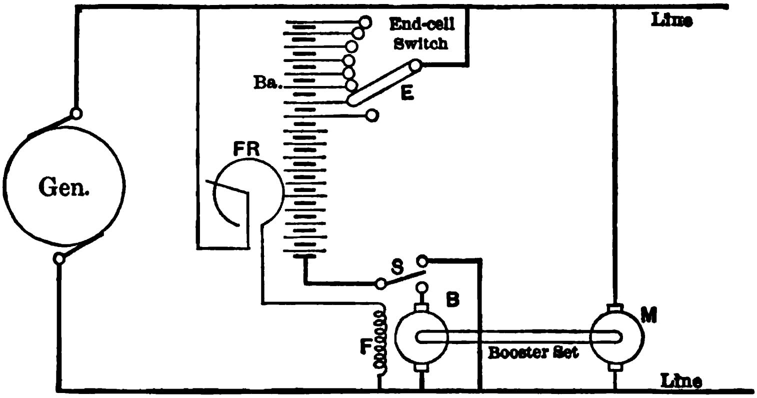


FIG. 340. Battery charged by a non-automatic booster and discharged through an end-cell switch.

When the battery is discharging, the switch *S* is thrown up so that the booster is cut out of the circuit; the discharge is regulated by the end-cell switch *E*. For charging the battery, the switch *S* is thrown down and the booster e.m.f. raised by means of the field rheostat *FR*, thus giving, together with the generator pressure, a voltage sufficient for charging. As the charge progresses, this voltage is regulated by means of the same rheostat *FR*.

Instead of using an end-cell switch, the same booster may also be used for regulating the discharge of the battery. In this case a reversing switch is connected in its field circuit, so that the direction of the induced e.m.f. may be changed. The booster is then called *reversible*. Sometimes the booster field is connected across the battery and not across the line; there is not much difference between the two methods.

559. EXPERIMENT 25-I. — Storage Battery Control by a Shunt-Wound Non-Automatic Booster. — The connections are shown in Fig. 340. Ammeters should be inserted to read the generator current, the line current, the battery current, and the booster field current. A voltmeter with a suitable switch should be provided to read the line voltage, the battery voltage, and the booster voltage. To represent changes in the battery e.m.f. that depend on the state of its charge, one or more cells may be disconnected, or more cells added to the battery. Some resistance may also be used in series with the battery, to imitate an increase in its internal resistance with discharge (§ 535).

(1) Assume the battery to be fully charged and the booster disconnected. Connect the battery to discharge in parallel with the generator, and set the end-cell switch to give a normal division of the load between the two. Read all the instruments and note the position of the end-cell switch. Repeat the readings under conditions corresponding to the battery half discharged and fully discharged.

(2) Connect the booster to charge the battery and again take a few sets of readings to correspond to the various stages of charge. See §§ 536 and 543 in regard to the desirable tapering of the charging current. Decide for yourself what amount of attention and manual regulation of the booster will be necessary during a charge under ordinary operating conditions.

(3) Add a reversing switch in the booster field, disconnect the end-cell switch, and investigate the limits, the advantages, and the disadvantages of regulating the battery discharge by means of the booster field, especially with a rapidly fluctuating load.

Report. (1) Give numerical data to illustrate the performance of the plant at different stages of battery charge and discharge. (2) Give data on the performance of the booster, both on charge and on discharge. (3) Describe the advantages and the disadvantages of using the booster, in place of the end-cell switch, for regulating the battery discharge.

560. Automatic Boosters. — The above described manually controlled booster is used in plants in which there is a distinct period of time during which the battery can be charged. The rest of the time the battery is either discharging or is kept in reserve (standby service, § 556). There are other cases, however, in which a battery is used for taking up instantaneous load fluctuations, with intervals of charge and discharge rapidly and irregularly following each other. Such is the case in some railway installations, in elevator service, in buildings with large printing presses, etc. In such plants it would be out of the question to regulate the booster field by hand in order to keep the generator output even approximately constant. It then becomes necessary to vary the field strength of the booster automatically, by means of a series winding in the load circuit (Fig. 341), or in the generator circuit (Figs. 344 and 345). Such boosters are generally known as automatic boosters. By properly proportioning and connecting the field windings, a great variety of performance characteristics of a battery plant can be obtained.

Of late years the tendency has been away from using storage batteries with automatic boosters for load regulation, batteries being used more as a reserve, and being charged by plain non-automatic boosters at regular intervals (§ 558). Nevertheless, automatic boosters deserve the student's attention in view of the fundamental principles involved and because of some of their ingenious features. Moreover, there may be a return to automatic boosters in proportion as storage cells are improved and made to stand numerous charges and discharges with less deterioration.

The following typical automatic boosters are described below:

(a) *Differentially compounded booster* (Fig. 341) in which one or two series windings are placed on the booster field poles.

(b) *Constant-current booster* (Fig. 342) which differs from the preceding one in that its armature is in series with the main generators and not with the battery.

(c) *Shunt-wound booster* with a separate series coil actuating a *carbon-pile regulator* (Fig. 344).

(d) *Shunt-wound booster* whose field is controlled by a separate *counter-e.m.f. machine* (Fig. 345) excited by the main generator current.

A series winding on the booster itself is objectionable because it necessitates a much larger machine, and because the booster field flux does not follow rapid fluctuations of the current in the series coils promptly enough. For these reasons, *externally controlled* or *relay boosters* have been developed, of which those mentioned under (c) and (d) above are the best-known examples.

With a clear understanding of the four above-mentioned typical kinds of connections, the reader should have no difficulty in comprehending other booster systems not covered in this book.

561. Differential Booster. — A booster of this type is shown in Fig. 341 and differs from the non-automatic booster (Fig. 340) in being provided with an additional series-field winding, F_2, which opposes the action of the shunt winding F_1. At a certain average load, F_1 and F_2 neutralize each other, and the booster e.m.f. is $= 0$. At this load the generator voltage must be made equal to that of the battery, so that the battery neither charges nor discharges. At a heavier load the action of F_2 is stronger than that of F_1, and the booster e.m.f. is added to that of the battery, causing it to discharge. On light loads the action of F_1 is stronger than that of F_2, and the booster sends a charging current into the battery. This action is entirely automatic, and the booster tends to maintain a nearly constant load on the generators, the battery taking up load fluctuations.

Experience shows that the action of the booster is improved and a closer regulation is obtained by adding a third field winding F_3, shown by the dotted line and acting in the same direction as F_2. This " inside " series field is particularly useful in correcting for the effect of polarization which occurs during short charges and discharges of the battery. It also corrects for variations in internal resistance, due to changes of temperature as well as to changes in the state of charge of the battery.

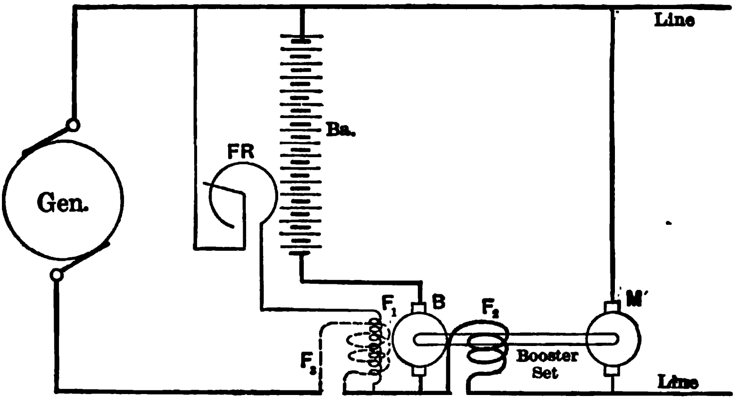


FIG. 341. Automatic differential booster.

In order to understand the action of this additional winding, first assume the battery to be very little polarized, and the current in the shunt-field winding to be so adjusted that the battery takes its proper share of the load. This means that when the generator is delivering its assigned current, the m.m.f.'s of the three field windings of the booster are such that the battery current is zero. Now let the battery be badly polarized due to a preceding heavy discharge. Then, with the same external load as before, but without F_3, the battery would take less than its share of current, thereby overloading the generator. With the winding F_3, a heavier current delivered by the generator automatically raises the booster e.m.f. and thus causes the battery to deliver a larger current, relieving the generator. Similarly the winding F_3 allows the booster voltage to be raised in the direction of charge when the generator output is below normal.

562. Constant-Current Booster. — A booster connected as shown in Fig. 342 is used in plants in which it is desired to protect a constant-voltage load, such as the lamps L, from elevator motors X and Y, electric welders Z, etc., whose current demand is subject to violent rapid fluctuations. The voltage across the lighting load must be practically constant, while a considerable drop in line voltage when a large motor is being started, is not only harmless but may even be desirable.

The booster is provided with a shunt-field winding and a series winding. The shunt field causes an e.m.f. to be induced in the same direction as that of the generator. The series field is connected to oppose it. At first, assume the load XYZ to be steady, and let the field rheostat FR of the booster be so adjusted that the generator furnishes a desired current while the battery supplies the rest. Now let the load be suddenly increased and let the first tendency in the generator be to deliver a larger current. This current, acting in the winding F_2, will lower the booster e.m.f. in the positive direction or increase it in the negative direction. The net result will be that the difference of potential between the points p and q will be reduced, and the battery will therefore be forced to take a larger share of the load.

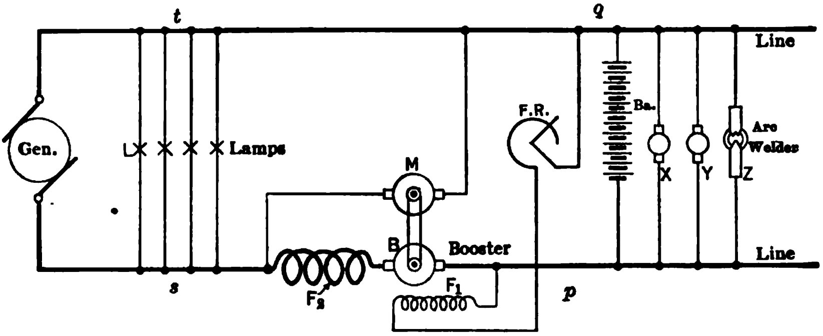


FIG. 342. A constant-current booster.

Should the load XYZ be reduced or disconnected, the booster voltage would rise and the voltage between the points p and q would be increased. The battery would be made to deliver a smaller current or even to charge. Thus, the generator is made to deliver a substantially constant current to a fluctuating load, and therefore it is possible to maintain a constant voltage at the lamps, between the points s and t. An end-cell switch may be used with this booster, if desired, for manual adjustment of the battery from time to time.

563. EXPERIMENT 25-J. — Operation of a Compound-Wound Booster. — The purpose of the experiment is to study the performance characteristics of the automatic boosters connected as shown in Figs. 341 and 342.

(1) Study the performance of the installation with a steady load. The booster voltage, the generator voltage, and the battery voltage are first adjusted to give the desired division of the current at some one particular load. Then this load is varied in steps from zero to a maximum, and all the currents and voltages are read at each step. This gives an idea of the closeness of the automatic regulation of the system.

(2) Change the characteristics of the battery to correspond to a different state of charge, to a different polarization, different internal resistance, etc. Take similar readings with different load currents.

(3) Change the load suddenly and measure the promptness or slowness of adjustment to the new conditions. Note the flickering, if any, of the lamps.

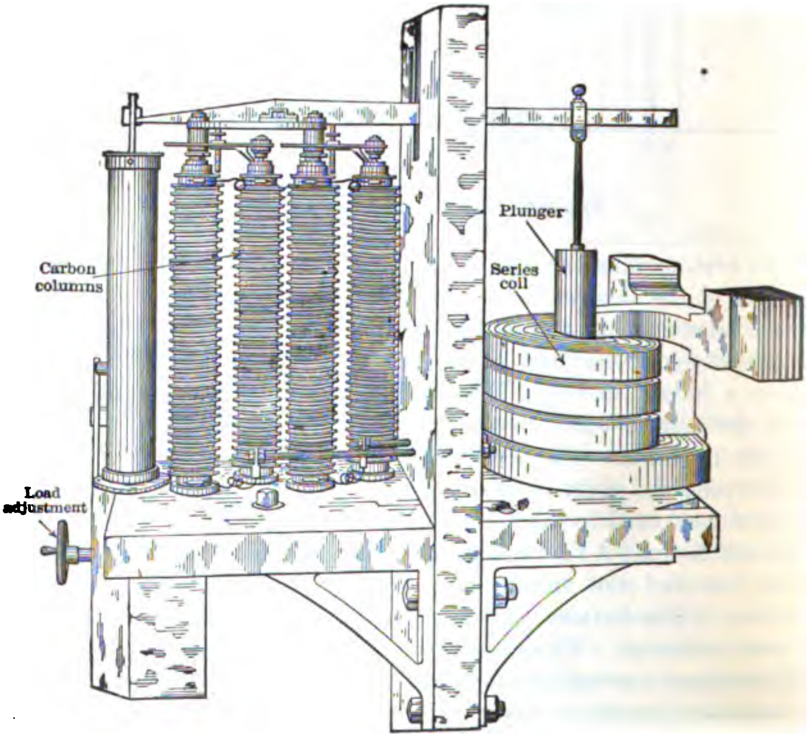


FIG. 343. Carbon-pile battery regulator.

(4) Adjust the field strength of the booster, the number of cells of the battery, and any other variables, to obtain the best all-round average performance of the system within some assumed limits of the load and its fluctuations. Take performance data within as wide limits as possible.

Report. Give the data on the best possible performance of the booster with a variable load, and describe whatever additional adjustment by hand is necessary or desirable. State the advantages and the disadvantages of such a booster as compared to a plain non-automatic shunt-wound booster (§ 557), and give concrete examples of practical installations in which one or the other kind is preferable.

564. Carbon-Pile Booster Regulator. — In order to avoid placing a bulky series winding on the field poles of a booster, several external regulators have been developed, one of which is shown in Figs. 343 and 344.

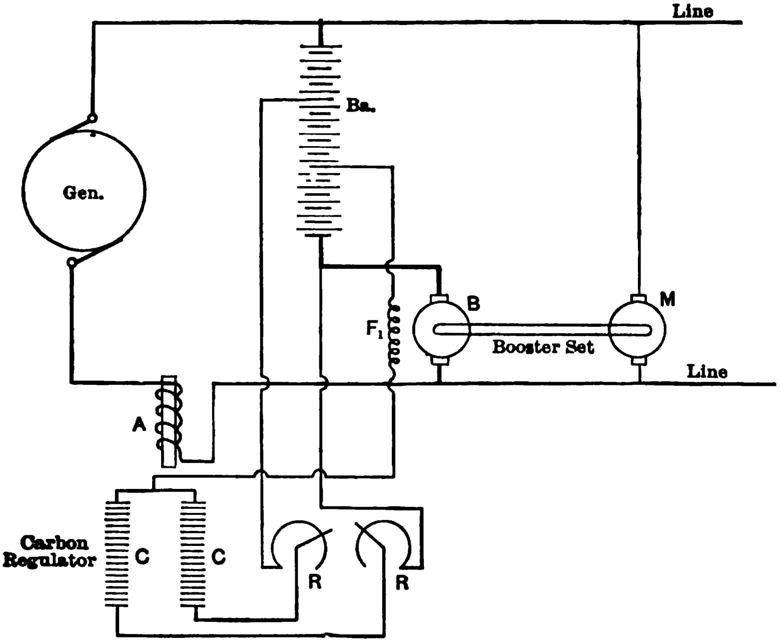


FIG. 344. Electrical connections of a booster controlled by a carbon-pile regulator.

The generator current, instead of passing through a series winding placed on the booster, passes through the solenoid A of the regulator. An iron core, actuated by this solenoid, compresses through a suitable leverage two sets of columns, CC, consisting of carbon disks. These carbon piles are connected to the shunt field of the booster and to the battery, as shown in Fig. 344. The electrical resistance of the piles consists chiefly of the contact resistance between the disks, and therefore varies within wide limits depending upon the pressure exerted by the core of the solenoid A.

With the normal value of the generator current, the pressure on both piles is adjusted to be the same. Their resistances being then equal, no current flows through the booster field F_1. The whole arrangement somewhat resembles the familiar Wheatstone bridge. When the generator current is below or above normal, one column is compressed more than the other, the bridge is not balanced, and a current flows through the booster field in one or the other direction, causing the battery either to charge or to discharge. The device is entirely automatic in its action, and takes the place of one or more series-field windings (§ 561).

In reality the arrangement is more complicated than that shown in Fig. 344. It would be too wasteful to circulate through the carbon columns a current sufficient to energize the booster field through a mere unbalancing of the resistances. Therefore, except in very small installations, *the carbon regulator actuates the field of a separate exciter, which in turn supplies current to the booster field.* The exciter being a much smaller machine than the booster, considerably less energy is lost in the regulator. The booster and the exciter are usually driven by the same motor.

Adjuster of generator output. The handwheel shown in Fig. 343 serves to regulate the spring tension and thus to control the relative output of the generator and the battery. For best results the spring should be so adjusted that at any time the average load is thrown on the generator and the battery is allowed to take up only very short fluctuations and momentary peaks. Since the average load itself varies with the time, the spring tension should be adjusted at frequent intervals. In most plants it is not feasible to make such an adjustment by hand, and therefore a small electric motor is provided which automatically varies the tension of the spring. This motor is electrically interconnected with the booster armature in such a way that the generator current is made to correspond to the load current averaged over comparatively long intervals of time, while the battery is allowed to absorb momentary fluctuations. A load-limiting device is also available which prevents more load from being transferred to the generator after its safe output has been reached. Above this limit the battery is made to carry sustained peaks as well as rapid fluctuations of the load.

565. Booster Regulation by Counter-E.M.F. — In the arrangement shown in Fig. 345 the booster shunt field F_1 is automatically regulated by a small counter-e.m.f. machine C. The field F_2 of this machine is excited by the main generator current. At a certain desired value of this current, the counter-e.m.f. of the machine C can be made equal to the bus voltage so that no current will flow through the booster field F_1. When the generator current is below this value, the booster field is excited in such a direction that the battery is charged, and vice versa. *CM* is the motor which drives the counter-e.m.f. machine *C*.

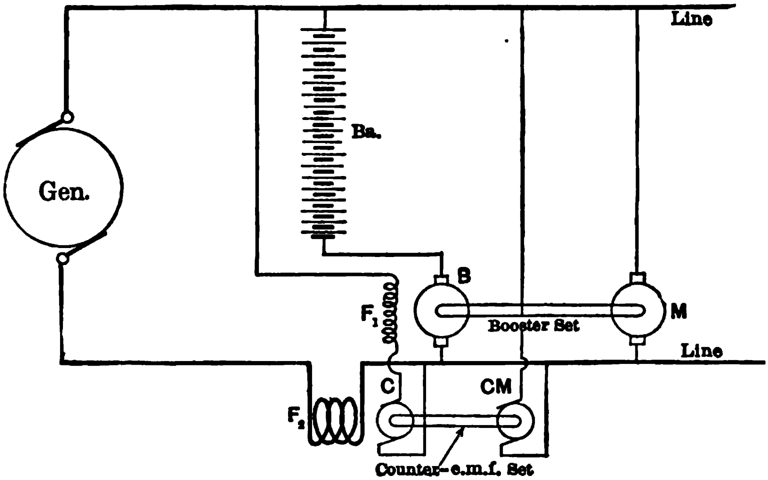


FIG. 345. Automatic booster controlled by a counter-e.m.f. machine directly.

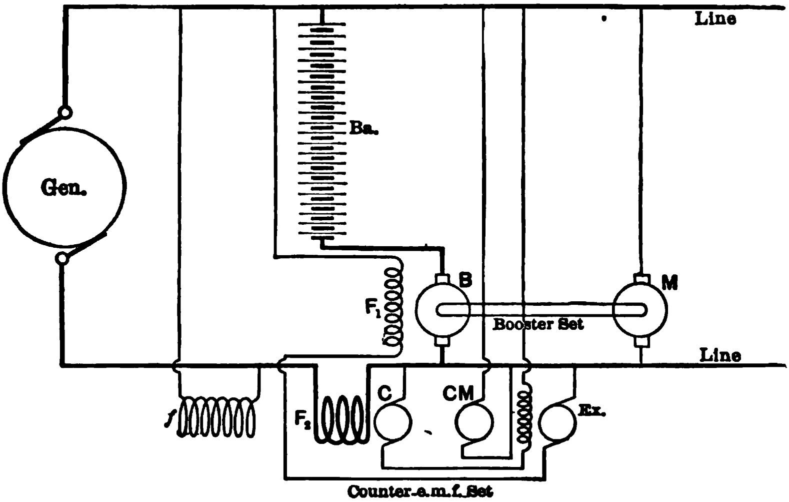


FIG. 346. Automatic booster controlled by a counter-e.m.f. machine through an exciter.

The size of the counter-e.m.f. set can be considerably reduced and the sensitiveness of regulation increased by introducing a relay machine between the machine *C* and the booster field winding (Fig. 346). This additional machine is known as the exciter. The counter-e.m.f.

machine C, instead of acting directly on the booster field, as in Fig. 345, modifies the field of the exciter *Ex.*, which in turn controls the booster field current. The armature of the counter-e.m.f. machine is connected in series with the exciter field, across the main bus-bars. When a normal current flows through the main generator, and consequently through the field F_2, the voltage induced in the exciter armature just balances that across the bus-bars; the booster excitation is then equal to zero. When the generator current is above normal, the voltage in C is higher, and the exciter field is energized in such a direction as to assist the battery to discharge. The opposite takes place when the generator current is below normal. With proper relations, the system keeps the generator load practically constant.

The counter-e.m.f. machine may be provided with an additional field winding, f, which may be connected either directly across the bus-bars, as shown in Fig. 346, or in series with the armature C and the exciter field. The addition of this winding makes the system more flexible and permits of an adjustment for any desired performance of the battery. Further adjustment is made possible by a rheostat in series or in parallel with the field winding F_1, and also by shunting the main series-winding F_2 by adjustable resistances. The machine C is driven by the motor CM. The exciter may be driven by the same motor, or it may be a high-speed machine driven by a separate small motor.

566. EXPERIMENT 25-K. — Performance of a Relay-Operated Battery Booster. — The experiment refers to the systems of booster control such as are described in §§ 564 and 565. The general order of tests is the same as in § 563, except that currents and voltages have to be read in more places, and there are more numerous initial adjustments to be made in order to obtain the best operating results.

CHAPTER XXVI

D. C. MOTOR STARTERS AND CONTROLLERS

567. Classification of Controllers. — A motor controller is a device, or group of devices, which governs, in some predetermined manner, the electric power delivered to the motor governed. The usual basic functions of various controllers are as follows:

(a) To provide safe and proper conditions for starting a motor.
(b) To accelerate or retard the motor, and to vary its speed at will.
(c) To reverse the direction of rotation of the motor armature.
(d) To apply a brake or to make the motor itself act as a brake.

A non-reversing controller, used only for accelerating a motor to full speed, is often called a " starter." When a controller is used to regulate the speed of a motor it is often spoken of as a " regulator."

According to their type of construction, controllers may be divided into the following classes:

Face-plate controllers (Figs. 347 and 352);
Drum controllers of either the cylinder or the cam type (Figs. 354 and 355);
Lever-type controllers (Fig. 349);
Magnetic-contactor controllers (Fig. 351);
Liquid controllers, or liquid rheostats (Vol. II).

Controllers may also be classified according to the type of motor which they govern, such as controllers for shunt motors, for series-wound motors, for squirrel-cage induction motors, for phase-wound induction motors, etc. They may also be more definitely described by naming the principle of regulation, for example, rheostatic control, voltage control, etc.

In some controllers all the basic functions are performed by hand; such devices are called manual controllers. In other controllers all the basic functions are performed by electromagnets; such controllers are called " full magnetic." There is also an intermediate type known as semi-magnetic. When governing functions are performed by electromagnets, an additional device is necessary whereby the operator (or a change in the physical conditions, such as pressure, temperature, etc.) closes a control circuit which in turn operates the electromagnets. Such a device is called a " master switch," and may be in the form of a small drum controller, a float switch, a set of push-buttons, etc. Controllers can be further characterized by the adjectives automatic, semi-automatic, non-automatic, reversing, non-reversing, etc.

Each type of controller has its field of application. The selection of the proper controller depends on the size of the motor and the work to be performed by it. For reasons of safety it is necessary to enclose the live parts of all manually operated controllers. Remote controllers are usually of the magnetic-contactor type actuated by master switches which are entirely enclosed and prevent contact with live parts. The controller itself is located at a distance, and for many applications need not be enclosed.

The purpose of this chapter is to acquaint the student with the simplest and most common types of starters and controllers used with direct-current motors (Chapter XII). Magnetic-contactor controllers, those for polyphase induction motors, and railway controllers are described in Vol. II. The diagrams of connections are subject to innumerable minor variations depending upon the desired purpose in view. It is intended here merely to give a general idea of the typical connections. An exact diagram usually accompanies a controller or may be procured from the maker.

FACE-PLATE D.C. MOTOR STARTERS AND SPEED REGULATORS

568. The Under-Voltage Release. — A typical starter for a small direct-current shunt or compound-wound motor is shown in Fig. 347. It is provided with an under-voltage release consisting of an electro-magnet which holds the starting arm in the running position as long as the coil is energized from the line. Should the main switch be opened, a fuse blow, or the power shut off, the starting arm is released, and returns to its " off " position under the action of the spiral spring at its pivot. The arm is also released should the line voltage drop to a value below which the operation of the motor is not safe, because a sudden return of the voltage to the normal value may cause an excessive rush of current. The coil of the holding electro-magnet is usually in series with the motor field winding; therefore the starting arm is also released should the field circuit become open, thus protecting the motor and the line from possible damage.

Starting resistors are mounted in a box provided with ventilating holes. Taps from these resistances are taken to the contact buttons or segments on the face plate. The release coil is wound on an iron core, and is provided with two pole-pieces. The starting arm is either made of a magnetic material or is provided with an iron armature which is held by the pole-pieces in the running (extreme right) position of the arm.

The motor armature is connected in series with the starting resistance. The field and the release coil form another circuit. When the motor is standing still, the starting arm is in the position shown in the sketch, and the main switch is open. To start the motor, the main switch must be closed first, and then the starting arm moved slowly to the right. On the first notch all of the starting resistance is connected in series with the armature; as the arm is moved farther, the resistance is gradually cut out until on the last notch the armature is connected directly across the line. In this position the starting arm is held by the release electromagnet. In larger sizes of such starters, the starting arm is provided with a laminated copper brush which in the running position completely short-circuits the rheostat between two large contact pieces; these offer a much better contact than a face-plate segment.

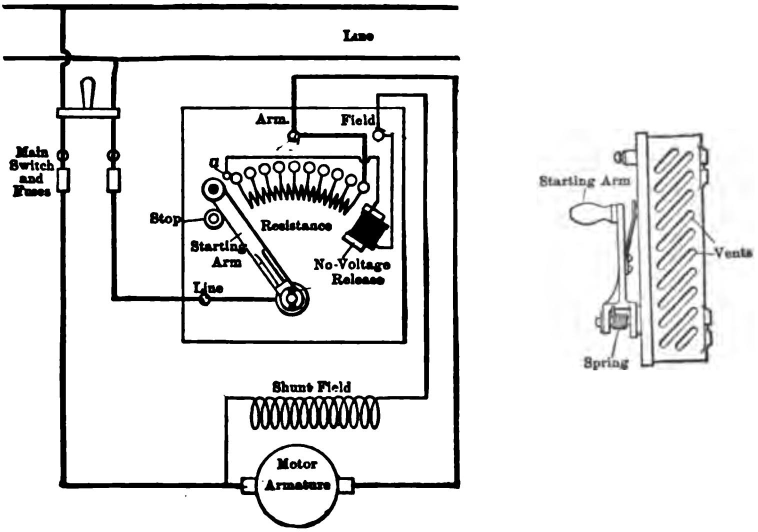


Fig. 347. Motor starter with no-voltage release.

To stop the motor the main switch is opened or the circuit-breaker is tripped. This deënergizes the release coil, and the starting arm flies back to its zero position, under the action of the spiral spring seen in the side view to the right. Whenever the motor is stopped, the whole starting resistance is *automatically* introduced in the armature circuit, protecting the motor for the next starting.

The small auxiliary contact *a*, connected to the first starting button, serves two purposes: (1) It permits the motor field to be energized a moment before the armature circuit is closed, thus allowing for the time lag due to a considerable inductance of the field winding. (2)

It takes up the spark when the starting arm flies back to zero position; after the contact has been considerably burned by this spark, the button *a* can be easily replaced.

By following the electrical connections it will be seen that even with the starting arm in the "off" position the shunt field remains connected across the motor armature. This is done in order to absorb the magnetic energy in the field and to prevent a rise of voltage across the field coils when the circuit is ruptured at *a*. Thus, the spark is not nearly as destructive as it would be if the field circuit were also opened.

It will be seen from the diagram that the part of the starting resistance cut out of the armature circuit is automatically introduced into the field circuit. This is usually of no practical importance, since the starting resistance is but a small fraction of the field resistance. In some cases, the motor field winding and the under-voltage release coil are connected independently across the line, instead of being in series with each other. This is done, for example, when it would be difficult to provide the required number of ampere-turns on the coil in any other way, especially at a high voltage.

569. Overload Protection. — The motor and the starting rheostat are protected from an overload or short-circuit either by fuses or by a circuit-breaker. Formerly, some starting rheostats were equipped with the so-called overload coil connected in series with the line. With such a construction (Fig. 348), when the current exceeds a certain limit, the overload coil *c* attracts the iron armature *i* which strikes and closes the auxiliary contact *p*. This short-circuits the no-voltage release coil, and the starting arm flies back to "zero" position. The armature *i* may be set by means of the screw *r* to open the circuit at a desired value of the current. This form of protection is becoming obsolete, since a small coil with contacts cannot be made either as accurate or as reliable as a separate properly designed circuit-breaker with a well-made time-element attachment. A separate magnetic contactor with an overload relay can also be used. This type of protection is described under magnetic contactors in Vol. II.

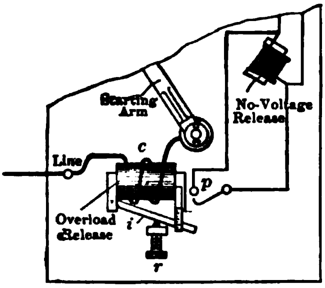


Fig. 348. A starter with an overload release.

570. Multiple-Switch Starters. — With large low-voltage motors, the currents are of such a magnitude that it is difficult to insure a sufficiently good contact between the segments and the arm, using the type of construction shown in Fig. 347. If a manually operated starter is desired, a separate switch may be used for each step (Fig. 349).

The connections are similar to those shown in Fig. 347, including an automatic no-voltage release. The switches are mutually interlocked, so that they can be closed in a certain order only. The first switch to the left, when closed, is held in position by the no-voltage coil, provided the latter is energized. This first switch pushes out of the way a stop which otherwise prevents the second switch from being closed. Now the second switch may be closed; this being done, the stop of the third switch is lifted up, etc. Should the power be cut off, the no-voltage coil releases the first switch, and then all the switches open at once.

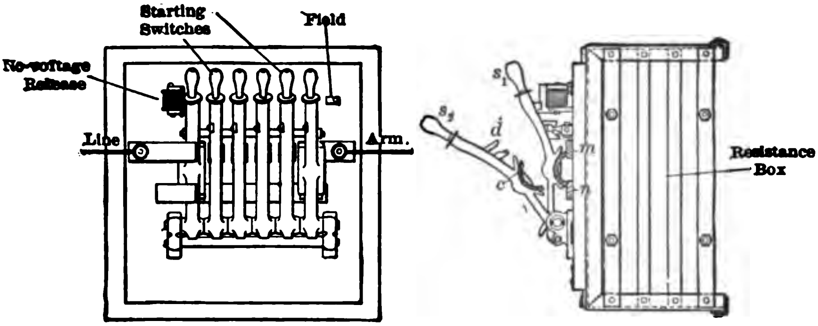


Fig. 349. Unit-switch motor starter with no-voltage release.

Some additional details of construction may be seen in the side view to the right. The switch s_1 is shown closed, the switch s_2 open. The switches are provided with curved copper brushes *c* which close the circuit between a bus-bar *n* and contacts *m*, connected to the sections of the starting resistance. The retaining hooks *d* and their locks are also shown in the sketch.

571. Solenoid Starter. — Instead of operating a face-plate motor starter by hand, the starting arm may be acted upon by an electromagnet energized from an auxiliary circuit (Fig. 350). This is convenient when the motor and the starter are located at an inaccessible place, when it is desired to secure a rate of acceleration independent of the operator's judgment, or when the motor is started and stopped automatically by a float in a water tank, by a pressure gauge in an air reservoir, etc.

With the arrangement shown in Fig. 350, the solenoid is energized when the main switch is closed. The iron plunger then moves the starting arm upward, cutting out the resistance. The rate at which the arm is moved is regulated by the dash-pot. At the end of the travel an auxiliary contact, shown in the sketch, is opened by the arm, and some resistance, usually a lamp, is introduced in the solenoid circuit. The current in the coil is thereby reduced to a value just sufficient to hold the starting arm in the upper running position. When the power is off, or the main switch is opened, the solenoid is deënergized, and the starting arm falls by gravity to its zero position.

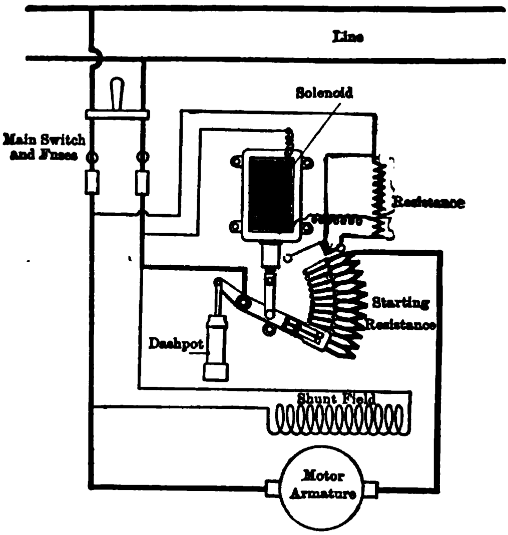


Fig. 350. A solenoid-operated motor starter.

It is not necessary to run the main line to the place where the operator is located. An electrically operated main switch, Fig. 351, may be placed near the solenoid starter, and only small control wires run to the operator's place. By closing a small auxiliary switch the solenoid *C* of the main switch is energized, and in turn it closes the main circuit. This energizes the starter solenoid and the starting arm begins to move.

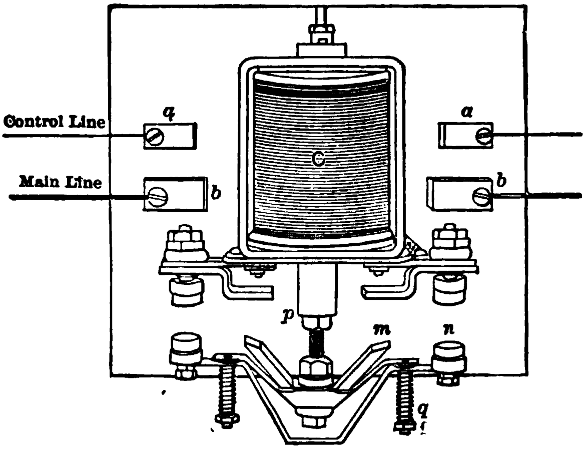


Fig. 351. An electrically operated switch for remote control.

This arrangement is sometimes used with motor-driven pumps and air compressors. The circuit *aa* to the solenoid of the main switch (Fig. 351) is then closed or opened automatically by a float switch, or by a pressure indicator, when the water level in the tank, or the air pressure, is beyond certain limits. This automatically starts and stops the motor, as needed.

The rate at which the rheostat steps are cut out is determined by the frictional resistance of the dash-pot. The rate which is satisfactory at a light load may be too high when the motor starts on a heavy overload. To remedy this, a current-limiting relay may be inserted in the main circuit, with its contacts in the solenoid circuit. When the starting current exceeds a certain limit this relay shunts part of the operating current around the solenoid and thus weakens the magnetic pull. The motion of the lever is retarded or stopped until the main current drops sufficiently, and the relay again allows the solenoid to be energized to its full strength.

With large motors, or when the connections are more complicated, several unit switches, such as the one in Fig. 351, are used, instead of a starting arm. Such magnetic controllers are described in Vol. II.

572. EXPERIMENT 26-A. — Study of a Direct-Current Motor Starter with Under-Voltage Release. — A starting box should be provided of the type shown in Fig. 347, and a suitable shunt-wound motor to be operated in connection with it; also an ammeter, a voltmeter, and means for loading the motor. The theory of the motor will be found in §§ 266 to 269.

(1) Wire up the motor and the starter, and practice starting and stopping. Make clear to yourself the order in which the main switch (or an overload circuit-breaker) and the rheostat handle should be operated, and to what extent the arrangement is "fool-proof;" also what would happen if the operations were performed in a wrong order.

(2) Explain why the handle does not fly back immediately after the main switch is opened; prove the explanation by an experiment. Determine the minimum line voltage at which the coil can hold the arm. Interpose pieces of thin paper between the coil and its armature, and observe the effect on the magnetic attraction.

(3) Apply a certain brake load and start the motor by moving the rheostat arm at a certain definite speed, using a metronome. Every few seconds read instantaneous values of line amperes and volts across the armature. With three observers, after some practice, it is possible to take readings on an instrument every two seconds. A much better way is to use a high-speed recording ammeter and voltmeter (§ 62). Repeat the same experiment with different rates of starting, and with loads of different magnitude.

(4) Measure the total starting resistance and the resistances of sepa-

rate steps. This may be done by applying a steady current and measuring the voltage drop between adjacent segments.

(5) If an electrically operated starter is available, such as shown in Fig. 350, connect it up and operate, in order to observe its action. Take a few readings as before, to characterize the performance of the device quantitatively. Study in particular the dash-pot adjustment, the influence of the kind of dash-pot oil and its temperature, the effect of an abnormal line voltage, of the motor load, etc.

Report. Draw diagrams of the actual connections used during the experiment. Explain your findings concerning the operation of the under-voltage release coil. Give curves showing the variation of motor volts and amperes during the period of starting; show the influence of the load and of the rate at which resistance steps were cut out. Plot the resistance of the steps and the total resistance of the rheostat, to steps as abscissæ. Explain the theoretical reason for which the resistances of various steps are made different. Indicate a possible method of proportioning the resistances of the steps in a new starting rheostat to be designed for a given motor. Give your findings on the operation of the solenoid starter.

573. Speed Reduction by Armature Resistance. — With small motors, when economy of operation is not essential, the speed may be reduced by inserting some resistance in series with the armature (§ 273). This is also the usual method of control in intermittent service, for example, on cranes, hoists, electric cars, etc.

With very small motors, a face-plate controller is used similar to the one shown in Fig. 347, except that the starting arm can remain indefinitely in any position. It is provided with a sector having slots or notches on its periphery. The release coil has a pivoted armature carrying a hook on one end. This hook engages in any of the slots of the sector, and holds the arm in a desired position as long as the coil is energized. When the power is off the armature is released and the spiral spring returns the arm to the " off " position. If no such interlock is provided, a separate under-voltage circuit-breaker should be used to protect the motor should the power be off.

A reversing controller for a series motor is shown in Fig. 352; it is of a type used with small hoists. No interlocking or automatic features are required in this case beyond a separate overload circuit-breaker, since the operator watches the performance constantly.

The regulating lever is made in three pieces separated by strips of insulation. The two outside parts form electrical connections; the middle part is insulated. The function of the controller consists in cutting the resistances in and out. Reversing is done by moving the handle in the opposite direction: this reverses the armature leads.

Magnetic blow-out coils (§ 581) are placed either on the contact arms or on the controller shaft. In the latter case, the iron of the framework is so designed as to complete the magnetic circuit, and to give a blow-out action at the four points at which the circuit is broken.

Only one half of the regulating resistances are used at any one time; therefore one half of them may be omitted and the other half connected to both "forward" and "reverse" contacts. On the other hand, it is of some advantage to use separate resistances, because then they are in the circuit only half of the time and have more time to cool off.

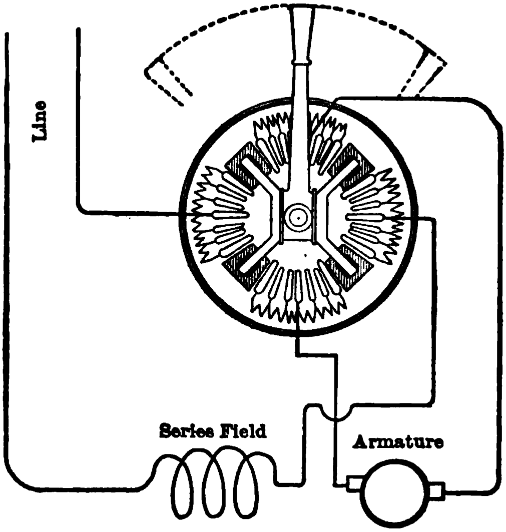


Fig. 352. A face-plate controller for a series-wound motor.

In a rheostat intended for speed control by resistance in the armature circuit, the absolute and the relative values of the resistances of the individual steps depend essentially upon the character of the service which the motor has to perform. Two typical kinds of service are distinguished in industrial practice, viz., the *fan service* and the *machine service*, better called the increasing torque-speed service and the constant torque service. In a fan, the required torque decreases rapidly as the speed is reduced, and vice versa, on account of the nature of air resistance. In a lathe, on the other hand, the torque with a given cut remains essentially constant within a wide range of speeds. The ampere-speed curves in the two cases are entirely different, and so are the values of voltage drop in the series rheostat necessary to produce the same variation in speed.

574. EXPERIMENT 26-B. — Rheostatic Speed Control of a D. C. Motor. — The underlying principles are explained in the preceding section. Particular attention should be paid to the proportioning of the individual resistance steps for different types of motors, shunt, series, and compound-wound, and for different classes of service. The student should see clearly that with any particular setting of the rheostat the motor speed does not remain constant as the load varies, because the voltage drop in the rheostat is a function of the armature current. When experimenting with a series motor it is advisable to have an underload circuit-breaker in the line so as to protect the motor against running away on a light load.

575. Field Control. — In the above-described starting rheostats, no provision is made for regulating the field current of the motor, to vary its speed. If speed control is required, an additional rheostat must be connected in the field circuit (Figs. 202 and 353). The motor should always be started with as strong a field as possible, in order to get a good starting torque, without an excessive rush of armature current. Therefore, the starting and the field rheostats must be suitably interlocked, either mechanically or electrically.

In the controller shown in Fig. 353 the lower row of contacts is connected to the starting resistance and the upper row to the field rheostat. A double lever is provided, the outside arm being for the field contacts and the inside one for the starting contacts (see side view to the right). Only the outside arm is provided with an operating handle. In starting, the two arms are moved together, but the field arm is electrically inoperative, because the field current flows directly through the starting lever, the bar *b*, the under-voltage release coil, and the motor field

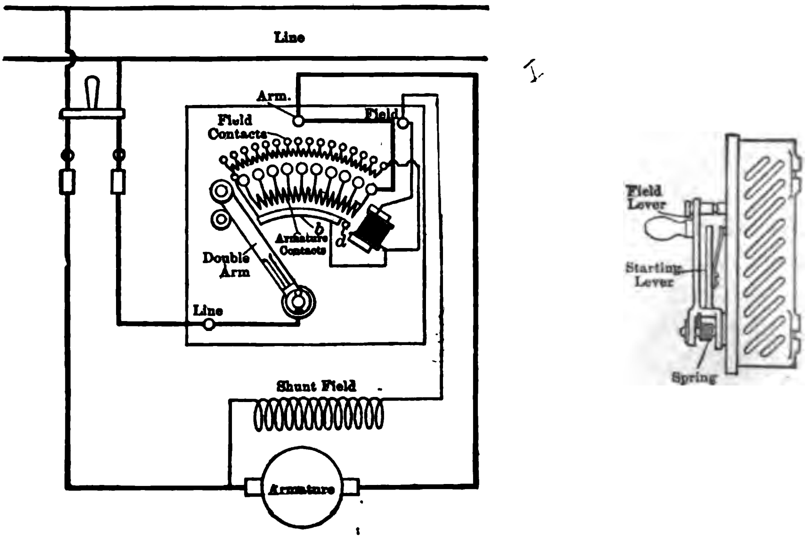


Fig. 353. A combination motor starter and speed regulator.

winding. At the end of the starting period, the starting lever is attracted and held by the under-voltage release magnet, while the field lever may be moved back to increase the speed of the motor. The upper row of contacts is now operative, since the starting lever no longer touches the short-circuiting bar *b*, but rests on the blind segment *d*.

When the main switch is opened, the starting lever is released, and on its way back it strikes the field lever, so that both arms are returned simultaneously to the " zero " position. It will be clearly seen from the above description that it is impossible to start the motor with a weakened field.

In some cases it is preferred to have the field rheostat separate from the starter. In one device of this kind the field rheostat is kept normally short-circuited by the armature of an electromagnet; this insures the starting of the motor with full field. The short-circuit is not removed by energizing this electromagnet, since the armature is too far from it to be attracted. But when the field-regulating arm is brought to its " zero " position, corresponding to the strongest field, it pushes the armature against the electromagnet, and removes the short-circuit. After this, any desired amount of resistance may be added to the field circuit, and the motor speed increased. As soon as the main switch is opened, the armature of the field rheostat is again released and the field rheostat short-circuited, independent of the position of its operating arm.

576. Field-Accelerating and Decelerating Relays. — Besides the two methods for insuring the starting of the motor with a full field, described in the preceding section, an independent relay may be used in the main circuit with its contacts across the field rheostat. As long as the current through the motor armature exceeds the limit for which the relay is set, the field rheostat is automatically short-circuited, no matter in what position its regulating lever may be. When the current drops below that limit the relay again becomes inoperative. An advantage of this so-called *field-accelerating relay* is that it protects the motor not only during the starting period, but also in regular operation, should the motor become overloaded. The relay causes the motor to operate with the full field, that is, at the lowest possible speed and highest torque, until the overload has been removed.

The field accelerating relay, if used alone, without the so-called *field-decelerating relay*, may under certain circumstances cause the following difficulty: Let the motor be running at its maximum speed and let the rheostat be turned rapidly to an intermediate value. The motor field will increase, reversing the current in the armature and causing it to act as a generator, until the speed of the motor has decreased to a balanced value. A relatively small increase in the field strength will usually cause a heavy current to flow, as there is very little resistance in circuit. This excessive current causes the armature of the field-accelerating relay to lift and short-circuit the balance of the field rheostat, thus materially increasing the regenerative effect. The action of the relay under these circumstances is undesirable. In such cases a decelerating relay should also be used for reversing the action of the accelerating relay on regeneration.

The decelerating relay limits the rate of retardation of the motor by preventing too sudden a strengthening of the field due to manipulation of the field rheostat or by other means. It has a compound winding; the series winding is connected in the armature circuit, and the shunt winding across the line or armature terminals. During accelerating periods of the motor the decelerating relay will not operate, as the directions of current through the series and shunt windings oppose each other. However, when the field strength is increased to a sufficient extent to produce regenerative action and the armature current is reversed, the series and shunt coils aid each other, producing a resultant flux large enough to separate the relay contacts, and re-insert the field resistance momentarily. This results in a fluttering action which may occur several times until the motor is operating at normal speed with the rheostat again short-circuited.

577. EXPERIMENT 26-C. — Study of a Field Regulator of D. C. Motor. — The purpose of the experiment is to investigate the performance of a face-plate controller, such as described in the preceding two sections. The characteristics of the motor itself are covered in § 267.

(1) Connect the speed regulator to a motor and practice operating it; make clear to yourself the automatic features of the device.

(2) Measure the speed and the field current of the motor with several positions of the regulating handle.

(3) Measure the total resistance of the field rheostat, and the resistances of the separate steps; also the resistance of the motor field.

(4) Connect an ordinary motor starter (Fig. 347) and a separate field rheostat in place of the combination starter and regulator. Devise an electrical or mechanical interlocking arrangement, which would prevent starting the motor with weakened field.

(5) Provide an overload relay to short-circuit the field rheostat, and experiment with its operation, and with the adjustment of current setting and the time element. Investigate the reversal of the armature

current on regeneration and try the corrective action of a field-decelerating relay.

Report the connections used, and the automatic features of the devices tested. Plot curves of motor speed, field current, and total resistance in the field circuit, to rheostat steps as abscissæ. Describe a method of proportioning the individual resistances of a proposed field rheostat for a given motor, to give desired speed steps at different loads.

578. EXPERIMENT 26-D. — Overload Protection of a D. C. Shunt or Compound-Wound Motor. — The purpose of the experiment is to investigate the relative advantages and disadvantages of the various means for protecting a motor against sudden or sustained overloads. The principal methods in use are enumerated in § 569. Test the principal types of controllers described in §§ 568 to 576. For each device tested ascertain the following features: (a) accuracy of setting in amperes; (b) the time element as a function of current with sudden and with sustained overloads; (c) constancy and reliability in operation under severe industrial conditions of dirt, heat, vibration, rust, etc.; (d) delay in restoring the service; (e) relative expense; (f) immunity against ignorance or malicious tampering. In some of these tests the actual motor may be dispensed with and replaced by an ordinary load rheostat, after the overload characteristics of the motor have been ascertained.

Report. Describe the devices tested, give the numerical data obtained, and the characteristic features (a) to (f). Indicate the advantages and the disadvantages of each method, and its particular field of application.

THE DRUM-TYPE CONTROLLER

579. One of the most common types of manual controllers is the drum-type controller; it is quite widely used with individually driven machine-tools, on small cranes and hoists, on electric cars, and in many other industrial applications. It is used either for governing the main currents, or as a master switch for control currents, when the main controller is of the magnetic-contactor type. There are two types of drum controllers, the cam type (Fig. 354) and the cylinder type (Fig. 355).

In a cam controller the individual contactors are mounted on stationary vertical posts, and the corresponding cams are fastened to a vertical shaft operated by a handle. When the shaft is turned, individual cams force the contactors to close in the desired sequence. The shape of the cams is such that the contacts are made and broken quickly, to prevent burning of the contactor tips. Blow-out coils and arc shields (§ 581) are provided at the contactors at which the circuit is finally broken.

In a cylinder-type drum controller (Figs. 355 and 53), the connections are made between stationary fingers and movable copper strips or segments mounted on the shaft operated by a handle. A controller of this kind is described in detail in the next sections.

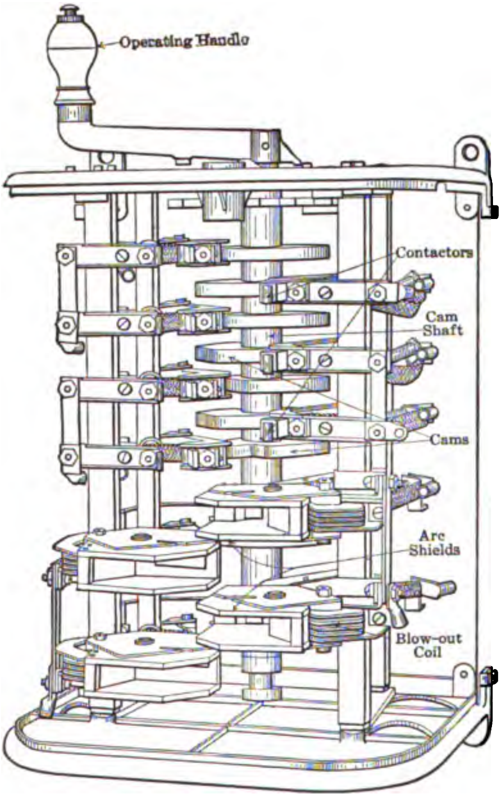


FIG. 354. A cam-type drum controller.

580. Mechanical Features of a Drum Controller. — The drum-type controller shown in Fig. 355 is one that has been quite widely used on street cars, and while intended for two series-motors, is representative of the general construction of drum controllers for other types of motors.

The wires coming from the line, from the motors, and from the starting and regulating resistances, are all connected to stationary "fingers," and these are interconnected in various combinations by the copper segments mounted on the revolving drum. The drum is operated by a handle, and in each position of the handle various fingers are connected in a different way, so as to vary the speed of the motor, the direction of rotation, etc. However complicated the connections inside the controller may be, the operator does not need to know about them; he simply turns the handle in one or the other direction, guided by the indications marked on the cover. A notched wheel is shown on top of the drum, under the cover. A roller is forced into one of these notches by a spring when proper contacts are made inside, thus indicating to the operator the correct running positions.

The particular controller shown in Fig. 355 has two drums and two operating handles. The long drum serves for starting and speed control, while the short one has contact points for reversing the direction of rotation. Many industrial controllers have one drum only, the reversal of rotation of the motor being obtained by turning the handle from the " off " point in the opposite direction.

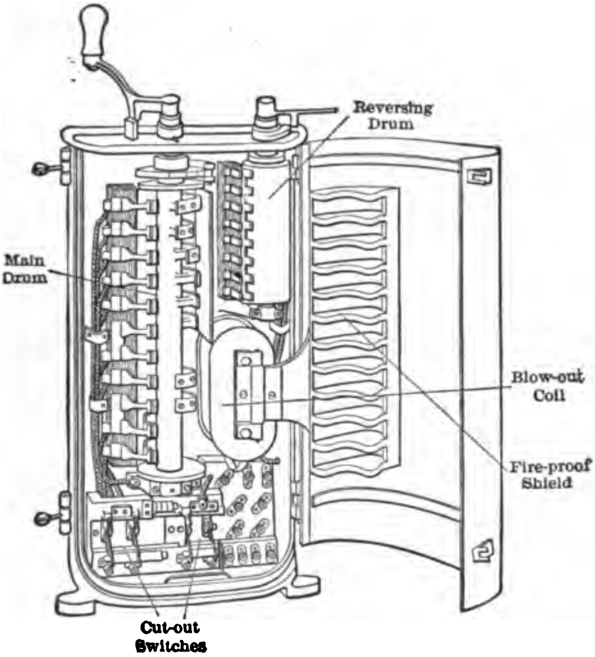


FIG. 355. A cylinder-type drum controller for railway service.

The two handles are mechanically interlocked in such a way that the reversing handle can be operated only when the regulating handle is in the " off " position. Moreover, the regulating handle can be operated only when the reverse handle is either in the " forward " or in the " reverse " position, but not in the " off " position. The purpose of this interlocking is to insure that the last contact is opened and the arc drawn in a proper place on the regulating drum, where a magnetic blow-out field is provided for extinguishing it.

581. The Magnetic Blow-Out. — In a controller which is operated many times a day, some fingers and the corresponding contact strips would soon become burned and pitted by the action of the electric arc when the circuit is ruptured, if means were not provided to reduce sparking.

The usual remedy is a blow-out coil, such as is shown in Figs. 354 and 355. Its action is based upon a fundamental fact of electromagnetism (Fig. 356) that in a magnetic field an electric current tends to move across the lines of force, or, as it is sometimes expressed, to cut them. The pole-pieces, *MM*, are so placed that the general direction of the lines of force is at right angles to the movement of ions in the arc between the electrodes *A* and *B*. The electromagnetic action tends to move the arc outside the field; the length of the arc is thereby increased and it is promptly ruptured. The coil of a blow-out magnet is usually energized in series with the line, so that the arc-expelling action is stronger the larger the current to be ruptured. For small shunted arcs, permanent magnets are sometimes used.

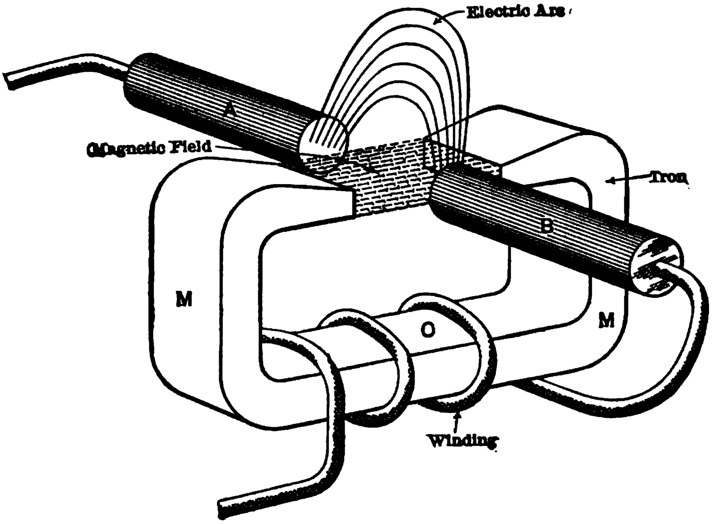


FIG. 356. The action of a magnetic blow-out.

The proper design of the blow-out is very important. The magnetic field must have the correct strength and distribution to reduce the wear on the contacts, and to prevent the arc from flashing over to other parts of the device. The arcing parts are separated from the rest by refractory fireproof partitions, shields, boxes, etc., whose shape depends upon the general design of the controller.

In some types of controllers so-called *arc-splitters* are used in connection with the blow-out magnet. The function of these splitters is to increase the length of the arc, without having it projected much beyond the edge of the arc box. These splitters also cool the arc, which materially assists in rupturing it. These arc boxes are used with magnetic-contactor control and also with drum controllers.

582. Elementary Controller Connections. — Before attempting to read actual diagrams of connections of drum controllers, such as those shown in Figs. 361 and 362, it is advisable for the beginner to become acquainted with the general principles underlying such diagrams. For this purpose, four elementary diagrams are shown below, each illustrating one kind of control only, viz.:

Cutting out the starting resistance in steps (Fig. 357);
Speed control by the field rheostat (Fig. 358);
Reversing the armature terminals for a change in the direction of rotation (Fig. 359);
Change from one terminal voltage to another (Fig. 360).

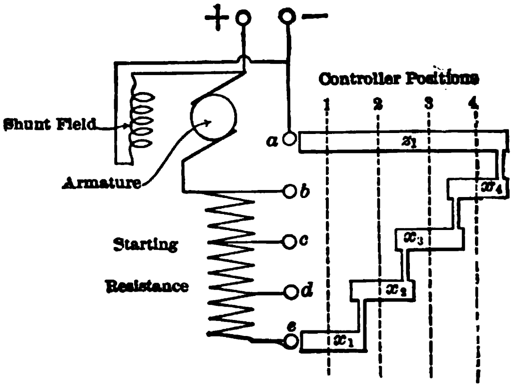


FIG. 357. Connections to the starting resistance.

In all these diagrams the surface of the controller drum is assumed to be developed in a plane. Different fingers are marked *a*, *b*, *c*, etc., while the drum segments are denoted by the letters *x*, *y*, *z*, *u*, and *v*. It will be seen that in most cases the segments are electrically interconnected and their combination acts in the circuit as one piece of metal. Different positions of the controller fingers on the segments are indicated by the dotted vertical lines numbered 1, 2, 3, etc.

In Fig. 357 the starting connections only are shown. On the first notch the fingers *a*, *b*, *c*, *d*, *e* must be imagined as placed on the dotted line 1. It will be seen that the line current from the positive terminal passes through the armature and the whole starting resistance to finger *e*. From there it passes through all the drum segments to the finger *a* and out. On the second notch the finger *d* touches the strip x_2, and the section *de* of the starting resistance is cut out. On the third notch still more resistance is cut out, and finally on the fourth notch the current flows through the armature without any starting resistance in series; this is the running position of the drum.

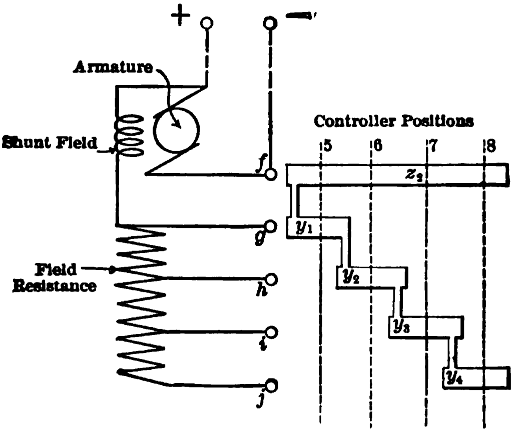


FIG. 358. Connections to the field resistance.

Fig. 358 represents controller connections for speed control by means of a variable resistance in the field circuit; for the sake of clearness the starting connections are omitted. On the fifth notch the field is excited directly across the line, without any resistance in series with it. This gives the strongest field, and therefore the lowest speed. On the sixth notch the resistance between the fingers *g* and *h* is inserted into the circuit, on the next notch that between *g* and *i*, etc., until on the last notch the whole field resistance is in the circuit, and the motor runs at its highest speed.

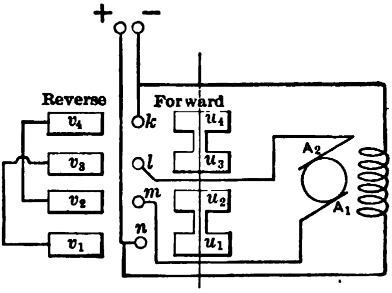


FIG. 359. Connections for reversing the armature.

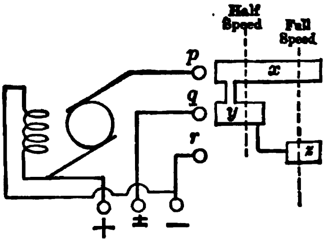


FIG. 360. Three-wire connections.

Connections for reversing the motor are shown in Fig. 359. When the drum is in the " Forward " position, the current from the positive terminal flows through *n*, u_1, u_2, and *m* to the armature terminal A_1, and thence returns to the line through the terminal A_2. When the controller handle is in the "Reverse" position, the current passes through n, v_1, v_3, and l to the armature terminal A_2, thus flowing through the armature in the opposite direction. Therefore, the motor will run in the opposite direction, since the field connections are not reversed.

Controller connections for operating a motor on a three-wire system (§§ 255 and 275) are shown in Fig. 360. In the position marked "Half speed" the armature is connected between the positive and the neutral (±) wires; at full speed it is connected between the positive and the negative terminals.

583. Experimental Controller. — In studying the connections and experimenting with an actual drum controller, the student is handicapped by the fact that the controller is all wired up, and some of the wiring is not accessible. Moreover, the controller is usually intended for a specific duty only, and cannot very well be used for various purposes.

It is therefore advisable to have in the laboratory an *experimental* controller, especially adapted for exercises in wiring. No permanent connections should be made between the strips on the drum, but each strip should be provided with one or more binding posts so that the student himself may establish any desired connections. Some of the segments must be long, others short, and they must be arranged stepwise, for gradually cutting in or out of resistances. Such a controller, if properly designed, is very useful for a study of the operations explained in § 582.

The controller should be mounted horizontally in order to be more accessible, and should have no cover, but only a board on which the fingers are mounted. It is not advisable to have a blow-out coil in connection with it, as the device should be kept as simple as possible. The action of a magnetic blow-out should be studied on a separate electromagnet.

With such a controller the student can realize separately the connections shown in Figs. 357 to 360, and check them by actually operating a suitable motor. Then he can wire the same controller for a combination of starting and field control, starting and reversing, dynamic braking (§ 284), or for any desired combination of standard and special operations.

584. EXPERIMENT 26-E. — Wiring of an Experimental Drum-Type Controller. — The purpose of the experiment is explained in the preceding section. The fundamental connections are described in § 582. At the end of the experiment all the connections should be removed in order that the next group of students may have the benefit of designing their own connections.

Observe separately the action of a blow-out coil in a circuit containing considerable inductance, by interrupting the circuit at a suitable place. At first use a separately excited electromagnet, or a permanent magnet, and observe its action on the arc. Reverse the direction of the main current, also the polarity of the magnet, and in each case notice the effect. Then take a regular blow-out coil, connected in series with the main circuit, and investigate its action. When experimenting with a blow-out coil the student must be careful to have the wires well insulated. The e.m.f. of self-induction, when the circuit is opened, may give quite an unpleasant shock.

Report. Give diagrams of the actual connections used. Describe the tests with the blow-out coil, and show on a sketch where and how a magnetic blow-out could be applied to the controller used in the experiment.

585. Examples of Industrial Controllers. — With a clear understanding of the above simple diagrams, the connections in Figs. 361 and 362 can be followed through with little or no difficulty. Figure 361 represents the connections in a reversing controller. The motor shown is compound-wound, but the series field has nothing to do with the operation of the controller, which can be also used with a shunt-wound motor. The controller has two starting positions, 1 and 2, in

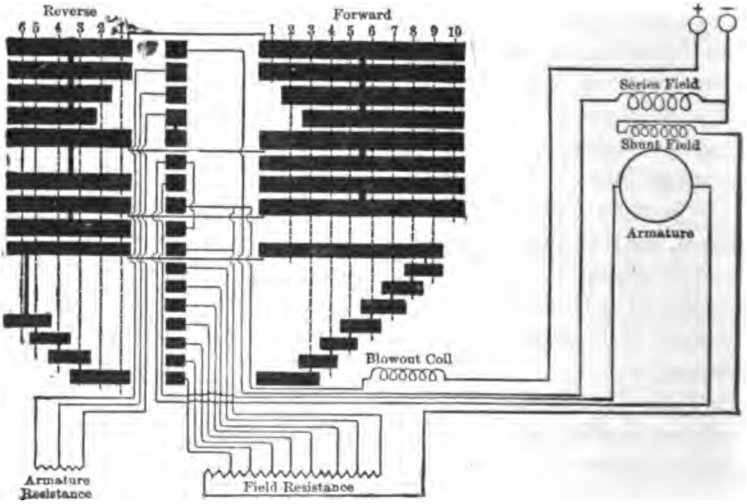


FIG. 361. Connections of a reversing industrial controller.

which the armature resistance is cut out in steps; the other positions are for field control.

The line wires, the armature leads, and one of the field terminals are connected to the fingers corresponding to the long strips, and are in the circuit in all positions. The "Forward" strips and the "Reverse" strips are so connected that the current flows through the armature in the opposite directions. The shorter upper strips are for cutting out the starting resistance in steps. The lower diagonal strips are for gradually introducing resistance into the field. Fewer steps are provided on the "Reverse" to illustrate the possibility of so doing, although in a standard controller identical steps would usually be provided for both directions of rotation.

The controller shown in Fig. 362 performs essentially the same functions as the preceding one, but has a somewhat different arrangement of contacts. It can also be used for dynamic braking (§ 284). In

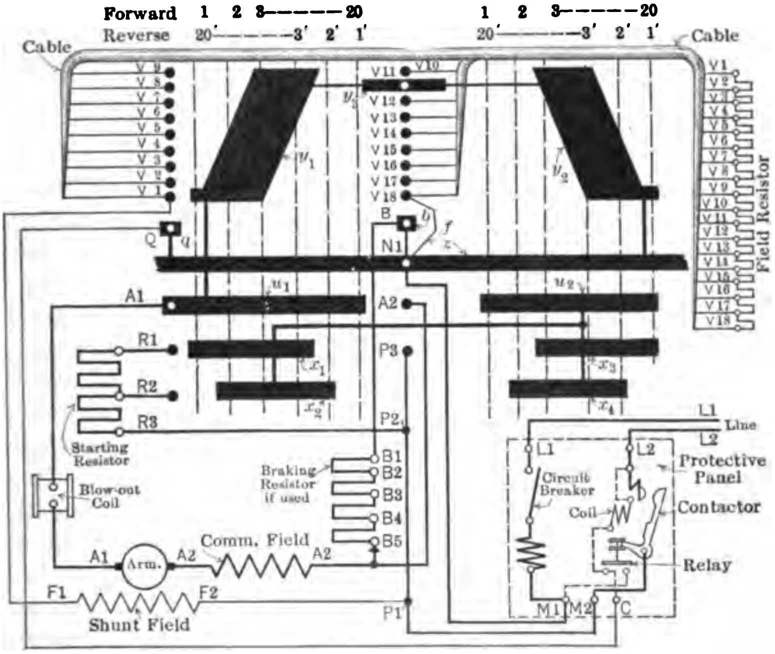


Fig. 362. Connections of a reversing industrial controller.

the lower right-hand corner a protective panel is shown consisting of a single-pole overload circuit-breaker and a clapper-type contactor, automatically operated by an electromagnet.

Assume both the circuit-breaker and the contactor to be open and the controller in the "off" position shown in the diagram. If the operator closes the circuit-breaker by hand, a current will flow through the path $L_1M_1N_1zqQC$, and through the contactor coil to L_2. The contactor armature is attracted and the main circuit is closed between L_2 and M_2. Simultaneously with this motion, the relay contacts are closed, so that the contactor coil is energized directly between M_1 and L_2. This is necessary, because the finger Q does not touch the segment q in some of the operating positions of the controller.

Should the circuit-breaker be tripped, either by hand or because of an overload, the contactor coil is deënergized and the main line is disconnected between L_2 and M_2, as well as between L_1 and M_1. If, instead of the protective panel, an ordinary line switch and fuses are used, the line conductor L_1 is connected directly to the controller finger N_1, and the conductor L_2 is continued as $P_1P_2P_3$. The lead CQ is omitted altogether.

Let us assume the line L_1 to be the positive conductor. Then the finger N_1 and the segment z represent the positive terminal of the line, in so far as the motor is concerned, and the lead $P_1P_2P_3$ is the negative terminal. It is only necessary to follow the armature and the field connections between these terminals.

On the first notch forward the two rows of controller fingers occupy the positions indicated by the two vertical lines marked 1, 1. The armature current follows the path $zA_1A_2u_2x_1R_1R_2R_3P_2$, with the whole starting resistor in the circuit. On the second notch the finger R_2 touches the segment x_2 and the part R_1R_2 of the starting resistance is cut out. On the third notch the terminal A_2 is directly connected to P_3 through u_2 and x_3, and all of the starting resistance is eliminated. After this, the armature connections are not changed, and the field resistance is introduced in steps.

Should the controller drum be turned in the opposite direction, the first step would correspond to the vertical lines marked 1′,1′. In this case A_1 is in contact with u_2, and A_2 touches u_1, causing the armature current to flow in the opposite direction. The rest of the connections is the same.

On the first notch forward the field current follows the path $zy_1V_1F_1F_2P_1$, with no resistance "in." The same field strength is maintained on the next few notches while the armature resistance is being cut out. On the last operating notch (the twentieth), the field is energized from N_1 through the jumper f, contact finger V_{18}, the whole field resistor, and the finger V_1. The motor is then running at its highest speed. In the intermediate positions part of the field resistance is short-circuited by the slanted segments y_2 and y_1, and the controller handle may be left in any of these positions indefinitely.

When the controller drum is quickly returned to the " off " position, the motor is converted into a generator. By following the connections in the " off " position, and assuming the motor to be still running, it will be found that the armature circuit is closed upon itself through the braking resistor B_1 to B_5, while the field winding is excited from the line. The value of the braking resistance can be so adjusted as to reduce the initial generated current to a safe magnitude and to stop the motor within a reasonable interval of time. If dynamic braking is not desired, the braking resistor and the connection to B in the controller are omitted.

586. EXPERIMENT 26-F. — Operation of a Drum-Type Controller with a D.C. Motor. — The purpose of the experiment is to make a study of the connections in a given drum controller. The experiment supplements that described in § 584. The most instructive method of performing the experiment is for the student to trace the actual connections without the use of a diagram. Having made his own diagram, he can then determine what the controller can do, and check his conclusions by actually operating a motor with it. The connections can be traced with a lamp, a voltmeter, or a buzzer, and the resistances measured by the drop-of-potential method or with a simple Wheatstone bridge (Chapter I). If such a problem seems too advanced, or would take too much time, the student may be given a diagram of connections and only asked to check it on the controller and to decide what operations the controller is designed to perform. Then he can check his conclusions by actually operating a motor.

Sketches should be made of the important mechanical features of the controller, of the blow-out coil, interlocking if any, adjustment of fingers, removable parts, arc-resisting partitions, etc. Tests should be made to determine to what extent the controller is fool-proof, and what special instructions should be given the operator. The student should make clear to himself what protective apparatus, such as fuses, a switch, circuit-breaker, an overload relay, etc. should be installed with the controller in order to safeguard, as much as possible, the controller itself, the motor, and the line.

CHAPTER XXVII

ELEMENTS OF TELEPHONY

587. THE two most essential pieces of telephone apparatus are the transmitter and the receiver. Their construction and connections may be understood from the diagram shown in Fig. 363, which represents two subscriber stations connected by a line.

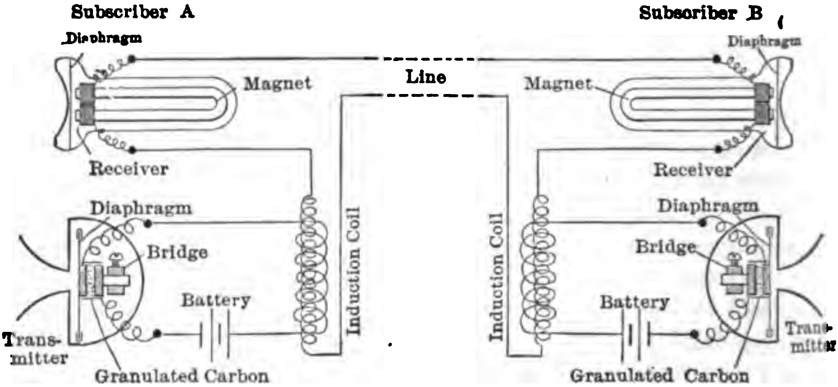


FIG. 363. Diagram illustrating transmission of speech.

The transmitter (Fig. 364) consists essentially of a "microphone" chamber filled with granular carbon, between two carbon electrodes. One of the electrodes is fastened to a thin elastic aluminum diaphragm; the other electrode is rigidly fastened to a metal bridge, or "solid back." The transmitter circuit is closed through a battery, as shown in Fig. 363.

When sounds are produced in front of the transmitter mouthpiece, sound waves strike the diaphragm and set it in corresponding vibrations. The diaphragm periodically compresses and releases the carbon granules in the "microphone" chamber. The resistance of the circuit is thus varied, and correspondingly rapid variations occur in the battery current, giving the effect of small alternating currents superimposed upon the direct current. These alternating currents, acting through the induction coil, or telephone transformer, cause secondary currents to flow through the line and through the receiver of the other subscriber, where a small fraction of their energy is converted back into sound waves.

The receiver (Fig. 363) has a thin iron diaphragm placed near the poles of a steel magnet. The poles are surrounded by coils of wire which are energized by the incoming "talking" currents. These small variable currents change the magnetism in the pole-pieces and hence the attraction between them and the diaphragm. The latter is thus caused to vibrate accordingly, reproducing the sounds spoken at the sending station. The magnet is made of hardened steel so as to retain its initial magnetism permanently. The polar projections are made of very soft iron, in order to respond more easily to small changes in magnetism, without hysteresis effect (§§ 179 and 209).

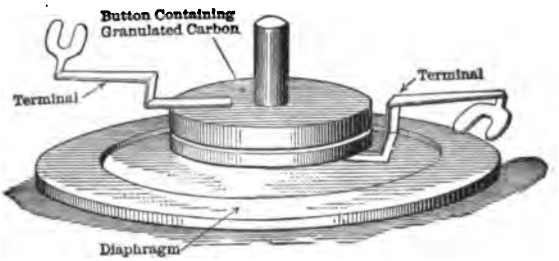


FIG. 364. Telephone transmitter.

The transmitter, the receiver, and the other pieces of telephone apparatus are described more in detail below.

TRANSMITTER AND RECEIVER

588. Description of a Transmitter. — A typical telephone transmitter is shown in Fig. 365. Each carbon electrode is highly polished and soldered to a supporting brass mounting. The two electrodes are placed in a brass cup which, with an auxiliary mica diaphragm, forms the microphone chamber. This chamber is filled almost to the top with carbon granules very carefully prepared and sorted as to size and quality. The cup is lined with paper to insulate it from the granular carbon. The microphone "button" so formed is sealed tight against moisture from the air by means of the brass clamping ring which holds the mica diaphragm against the cup.

The assembled button is fastened in front to the center of the sound-receiving diaphragm. This diaphragm is usually made of hard-rolled aluminum and is held in place against the transmitter front by one or more damping springs. The diaphragm may be flat, or it may have a crimped edge to stiffen it and to make it respond more readily to voice vibrations. It is insulated around the edge with a rubber gasket or with a ring of impregnated linen. The space between the diaphragm and the front plate forms a resonating chamber, and must be such as to give clear sharp articulation, free from the so-called " barrel " tone or unnatural echo effects.

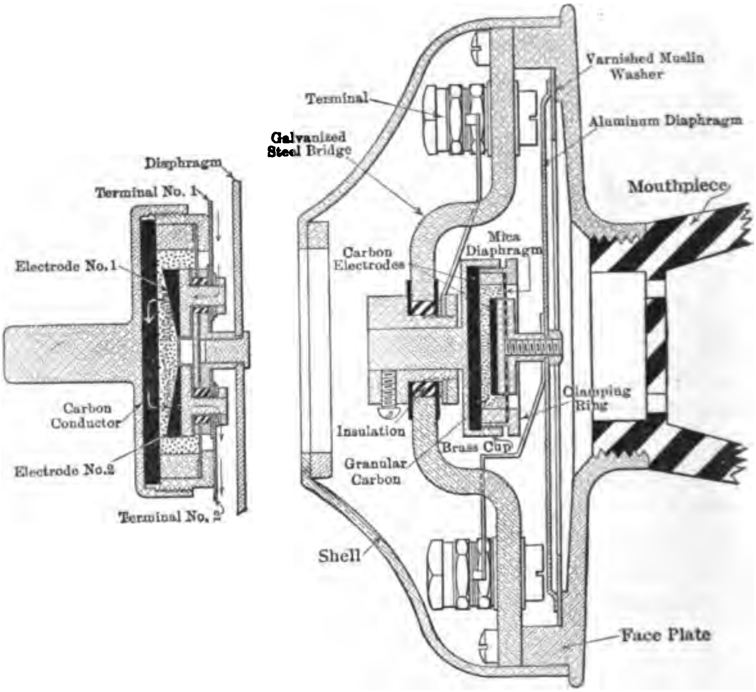


FIG. 365. Cross-section of a standard transmitter to the right; a special microphone button with split front electrode to the left.

The back of the electrode cup has a stud or shank which is held with an adjusting set-screw in the solid steel bridge. When sound waves impinge upon the main diaphragm and make it vibrate, the front electrode compresses or releases carbon granules, the back electrode offering a solid reaction.

One damping spring is applied near the outer edge of the diaphragm and the other half-way between the middle and the edge. Instead of these two springs, a bifurcated spring is sometimes applied directly to the front electrode mounting (centrally damped diaphragm). The purposes of the damping springs are: (a) to limit the amplitude of diaphragm vibration; (b) to cut down side-tones or room noises; (c) to prevent the diaphragm from vibrating in parts that produce over-tones and tend to spoil the quality of transmission; (d) to hold the diaphragm mechanically in place within the transmitter casing.

A transmitter intended for general use must be comparatively insensitive to general noises (room noises) or " side-tones " which interfere with the message itself in the receiver. The diaphragm is therefore purposely damped, and the acoustic design is so arranged that the speaker must hold his lips close to the transmitter mouthpiece and use a firm tone of voice in order to actuate the diaphragm positively.

Each diaphragm has certain natural frequencies to which it responds most readily. Changing the position and the tension of the damping springs results in an increase or decrease of the amplitude of maximum sensitiveness, and, to some extent, alters the pitch of maximum sensitiveness.

The influence of side-tones can also be minimized by opening both sides of the transmitter diaphragm to the disturbing noises. Thus, when these noise waves strike the diaphragm from both sides simultaneously, a balance is produced and the diaphragm is not vibrated. However, when the speaker talks directly against one side of the diaphragm, it is vibrated in the usual manner.

The principal caution to be observed when assembling a solid-back transmitter is the locating of the diaphragm so that the front electrode stud or screw is free from side strains, prior to the clamping of the electrode cup to the diaphragm and to the bridge. This can be accomplished by assembling the diaphragm, electrode cup, and bridge in the ordinary manner, after which the damping springs can be swung into place on the diaphragm. It only remains to set the adjusting screw in the bridge after the transmitter has been tested for freedom of action.

A solid-back transmitter is designed to be self-adjusting when all parts go together correctly. The actual adjusting operations consist in first tapping the edge of the transmitter front lightly with the wooden handle of a screw-driver and then talking loudly into the mouthpiece, thereby causing the rear electrode stud to assume a normal position without strain or tension on the auxiliary mica diaphragm. Now it is only necessary to set the side clamping screw in the bridge so as to hold the rear electrode stud securely in place. Before leaving the factory, transmitters and receivers of the best makes are subjected to an actual talking test, and those that show an appreciable difference from the standard, in " volume efficiency " or quality, are rejected. The volume efficiency of transmitters and receivers is expressed as so many miles of standard cable, above or below a standard instrument, and is determined by the amount of adjustable artificial cable which it is necessary to introduce into one section of a standard test circuit in order to reduce the transmission of the two instruments to the same volume (§ 597).

Permanent-magnet transmitter. Early telephone transmitters were of the permanent-magnet type, similar in their action to present-day receivers (Fig. 363). Any regular telephone receiver acts as a magneto transmitter if one speaks directly and loudly against the diaphragm. The talking currents generated are too small to give the required grade of transmission without amplification. In order to produce more powerful action, the diaphragms of the early magneto transmitters were made large, and the magnets powerful. However, the principle of this transmitter does not seem to permit the production of the same powerful transmission as is obtained with the " microphone " type. At present the only practical application of the magneto transmitter is in cases where a battery is troublesome, and where loud transmission is not essential. The magneto transmitter is also used when it is desired to reduce the space and weight of the transmitting equipment, as for example in some compact designs of linemen's test sets where the receiver is used for a transmitter.

589. Transmitter Electrodes and Granular Carbon. — In order to provide for the greatest possible variation in the resistance of the aggregate carbon granules when the diaphragm is vibrating, it is of importance to know the exact reason for the lowering of the resistance due to a slight pressure. A plausible theory seems to be that the area of contact increases with the pressure. Other theories are based on heating effect, air bubbles, actual change in the resistance of the granules, etc. By selecting a material, shape, size, and quantity of granules such that the foregoing factors are subject to the greatest variations, the best results are obtained.

The transmitters shown in Fig. 363 are supplied with current from local batteries of not more than four dry cells. In city telephone systems the direct current for the transmitters is usually supplied from a common storage battery located in a central office (§ 621). The voltage of such a battery may be 24, 40, or even 50 volts, and while the actual voltage at the transmitter terminals is considerably less, nevertheless it may be higher than that of a few dry cells. For this reason, central-energy transmitters are sometimes made of considerably higher resistance than those intended for service with local batteries. This result is obtained by using granular carbon of somewhat finer mesh.

Sometimes carbon granules do not respond to vibrations of the diaphragm because they have settled into a compact mass; the transmitter is then said to be " packed." Its resistance is lowered so that an excessive current can pass through it, but the resistance does not vary sufficiently to cause a transmission of speech. Usually a packed transmitter can be restored, at least temporarily, by talking loudly into the mouthpiece or by lightly tapping the casing. Some of the causes of packing are: (a) Lack of uniformity in size of carbon granules, small granules filling the interstices between the large ones. (b) Moisture entering the carbon chamber. (c) Wedging of carbon granules between the electrodes so as to spread and lock the latter. (d) Electrode cup too full of carbon granules.

During the first fifteen or twenty seconds after the current is cut on to a quiet transmitter (one in which the diaphragm is not disturbed) there is a rapid increase in resistance, after which the resistance becomes fairly constant. This means that the steady portion of the transmitter current has not the same value during use that it has when at rest. Tests made with recording instruments show instability of resistance of a quiet transmitter between intermittent conversations. For example, the steady current of a quiet transmitter may be greater or less after it has once been talked into and then comes to rest. After the next disturbance of the transmitter diaphragm, this steady current of the quiet transmitter may be lower or higher than before.

With a transmitter having a given resistance variation, the variation in the current is proportional to the steady current in the circuit. Thus, for powerful transmission the current should be increased up to the limit of heating of the electrode cup. Incidentally the increase in volume of the transmission is also limited by the action on the receiver diaphragm, which must not rattle against the receiver pole-pieces.

This principle of boosting the current to obtain maximum transmission is used for long-distance conversations. In case the normal voltage of the central office for local connections is 24 volts, then a 48-volt battery is used when a local telephone is connected to a long-distance line, the impedance of the battery feeds remaining practically the same (§ 623). With the central-office battery of 40 volts for local connections, it is customary to obtain an increased current through the transmitter for long-distance conversations by the use of battery-feed impedances of lower resistance than those used on the local connections.

The polishing of the electrode surfaces increases the current-carrying capacity two or three times, without causing "frying" noises in the transmission. Also the highly finished surface gives improved articulation to the transmission. Extreme care should be used in handling carbon electrodes and granular carbon, so as not to get finger marks on the polished surfaces. These marks cause high transmitter resistance and a noisy instrument.

With the usual construction of the electrodes, the electric current flows directly across the carbon chamber. In one type of transmitter, shown in Fig. 365 to the left, a split front electrode is used for central-energy service; the surfaces of this electrode are inclined toward the center. The two halves of the front electrode are connected to the circuit, while the back electrode serves only as a bridge for more efficient distribution of current. The current flows from one front electrode to the rear electrode and then back to the other front electrode. It is claimed that by having both moving electrodes connected to the diaphragm, fuller advantage is taken of variations in the resistance of carbon granules, since the greatest variation takes place in the near proximity of the vibrating electrode.

590. EXPERIMENT 27-A. — Assembling and Testing of a Telephone Transmitter. — A carbon transmitter, such as is described in §§ 587 to 589, should be carefully assembled, with due regard to the precautions mentioned above. Then it should be tested for the volume and articulation of speech, as affected by varying different factors, such as the position and tension of the damping springs, battery voltage, the initial compression of carbon, the distance of the speaker's mouth from the diaphragm, room noises, etc. Further investigations can be made of the effect of the size and quantity of carbon granules, conditions of packing, care in assembly, etc. Sketches should be made of each part and of the assembly, approximately to scale, and the materials marked. By reading the foregoing description of the transmitter the student will readily find many points of interest to investigate experimentally.

For testing the transmitter, a short telephone circuit can be arranged, as shown in Fig. 363, or even with the induction coils omitted if suitable receivers are available. At one of the stations a standard transmitter and the transmitter under test are placed so as to be connected in the circuit at will by means of a double-throw switch. The relative quality of transmission can then be judged at the other end, by having the same person speak alternately into the two transmitters. Disconnected words or even syllables should be used for such a test, and the percentage of those recorded correctly is a measure of the quality of transmission. A buzzer, an organ pipe, a phonograph record, etc. may also be used as sources of sound. An ammeter should be used in the transmitter circuit and the current kept within reasonable limits.

For more accurate tests a standard circuit should be used (§ 597), and the results expressed in miles of standard cable.

591. The Receiver. — A standard telephone receiver is shown in Fig. 366. It consists of a U-shaped permanent magnet with two soft-iron pole-pieces. The magnet may be bent out of one piece of steel (Fig. 363) or electrically welded out of three pieces. A coil of wire is placed over each pole-piece and an iron diaphragm is mounted in front of the pole-pieces. The whole is enclosed in a hard-rubber or Bakelite shell enclosed in front by the ear cap. The steady magnetic flux of the permanent magnet draws the diaphragm to the pole-pieces, with a very small gap in between, so that the diaphragm is held under a certain tension. Alternating "talking" currents passing through the receiver coils change the magnetic pull and hence cause the diaphragm to vibrate. These vibrations, concentrated by the receiver cap, are transmitted through the air to the listener's ear.

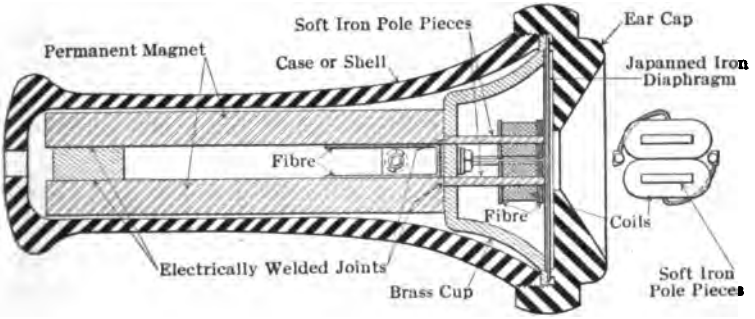


FIG. 366. Cross-section of a telephone receiver.

The magnetic circuit of the receiver has to be *polarized*, in order to cause the receiver diaphragm to vibrate at the frequency of the transmitter diaphragm. With a magnetic circuit consisting of magnetically neutral soft iron only, the diaphragm would be attracted with alternating currents of either direction, thus performing two complete vibrations per cycle. This would reproduce the voice an octave higher than the tone actually used by the distant speaker; that is, it would give his voice a weak, high-pitched, unnatural tone. In a polarized receiver the incoming alternating current gives only one complete vibration per cycle, thus causing the receiver diaphragm to operate in unison with the transmitter diaphragm of the distant telephone.

With the connections shown in Fig. 363, the receivers have to be of the permanent-magnet type, because the battery current does not pass through their windings. By omitting the induction coils, both the battery current and the superimposed alternating currents are made to flow through the receiver coils. In this case the permanent magnets can be replaced by silicon-steel laminations which are magnetized by the battery current. Such a receiver is known as the "direct-current type," and can be made as efficient as a permanent-magnet receiver, provided that the exciting current is kept within certain limits.

Sometimes a permanent-magnet receiver is used with connections that permit the battery current to pass through it. The direction of the battery current must then be such as to strengthen the permanent magnetism and not to weaken it. The process of connecting a permanent-magnet receiver to a line carrying direct current, so as to strengthen the action of the permanent magnet, is called "poling a receiver." The most practical way to "pole" a receiver is by experiment, connecting the receiver-cord terminals to the circuit, first one way and then the other way, testing the magnetic pull of the core ends on the edge of the diaphragm. The connection giving the strongest pull is correct. The sensitiveness of a receiver depends upon the frequency of the current; when in unison with one of the natural periods of vibration of the diaphragm, the sensitiveness is at a maximum. The sensitiveness of the human ear also varies with the pitch of the sound, so that for different observers the same sounds may have different volumes.

Receiver diaphragms are made of soft annealed iron, and are enameled, tinned, or japanned to prevent rusting. If the diaphragm is too thin it gives a high-pitched, shrill tone and is easily buckled by the magnet so as to rattle on the pole-pieces. If the diaphragm is too thick, it gives a "booming volume" which is lacking in clearness.

To reduce hysteresis (§ 209) the iron in the diaphragm must be as permeable as possible, so that through the entire range of current variations, a given amount of increase in current would produce a magnetic effect exactly equal and opposite to that produced by the same amount of decrease in current. To reduce eddy currents (§ 210), the material of the diaphragm should have as high an electrical resistance as possible. The same remarks apply to the material of the pole-pieces or cores.

After the receiver is assembled, the ends of the pole-pieces are ground off to get the correct air-gap. A long air-gap between the pole-pieces and the diaphragm tends to give more faithful reproduction of sound, though it also makes it weaker (reduces the volume). In order to keep the air-gap length constant with temperature changes, the magnet is fastened close to the diaphragm through a metal frame (brass cup), so as to be independent of a considerable expansion of the hard-rubber shell.

Watch-case type receivers, used with a headband by telephone operators, are of the same general construction as the one shown in Fig. 366, except that the magnets resemble a flat horse-shoe or a flat ring.

The pole-pieces are so shaped as to allow the coil windings to occupy the open space within the magnet; the pole ends are located in the same position as in the standard hand receiver.

592. EXPERIMENT 27-B. — Assembling and Testing a Telephone Receiver. — The receiver is described in the preceding section. For test instructions see § 590, keeping in mind the difference in the construction of the two devices. Try the effectiveness of an electromagnetic receiver as a transmitter (see the end of § 588).

593. The Induction Coil and the Repeating Coil. — Telephone induction coils, or transformers, shown in Fig. 363, are employed for the following reasons:

(a) The two windings compose parts of two electric circuits, allowing these circuits to be disconnected metallically, yet inductively connected so as to transfer energy from one circuit to the other.

(b) A local circuit is provided for the telephone transmitter so as to give the best conditions for its efficient operation.

(c) The electric current is changed from uni-directional to alternating, so as to operate the regular polarized telephone receiver more strongly.

(d) By properly selecting the ratio of the numbers of turns, the e.m.f. of the local circuit is stepped up to suit the resistance of the line, and then stepped down to obtain the best efficiency of the receiver.

A typical induction coil is shown in Fig. 367. It has a primary winding consisting of comparatively few turns, and a secondary winding of many turns of fine wire. The windings are placed in an inductive relation to each other, on an iron core made of fine iron wires or of strips of high-permeability transformer iron. The purpose of laminating the core is to prevent eddy-current losses. Such an induction coil differs from ordinary transformers used in connection with light and power transmission in that its magnetic circuit is open instead of being closed. An open magnetic circuit is preferable in telephone substations because

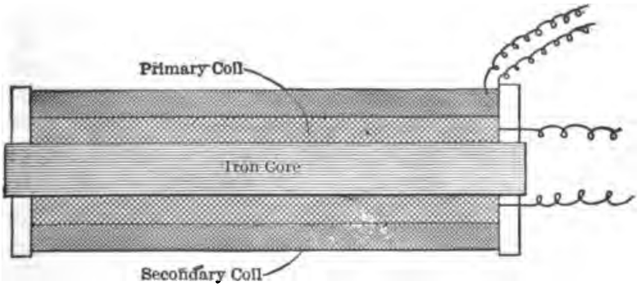


FIG. 367. Induction coil for telephone work.

it does not as readily become saturated under the influence of the direct current flowing in the transmitter circuit. The straight-core type of coil is used because the difference between its efficiency and that of the more costly ring type of core is practically negligible. In simple house telephones where the current is transmitted over a short distance, induction coils are sometimes omitted, and the transmitters and the receivers are connected directly in series.

Pulsating currents produced by the transmitter flow through the primary winding of the induction coil, thereby generating in the other winding alternating voltages which are impressed upon the line. The induction coil is a static transformer, the purpose of which is to connect the transmitter and the receiver to the line in such a way as to produce the maximum amount of talking energy in the line for transmission, and the maximum amount of energy in the receiver for transformation into sound waves. The primary winding provides a low-resistance path for the flow of direct current in the local transmitter circuit.

Because of the high frequencies and slight energy of the voice currents, the hysteresis and eddy-current losses (§ 218) in the core of an induction coil must be kept very low. This is accomplished by designing such cores for low flux densities, using iron of high permeability, and subdividing it thoroughly. Sometimes the iron wires are enclosed in a soft-iron tube with a longitudinal split; this split prevents eddy currents in the tube.

The above-described induction coil is said to have an open magnetic circuit, the lines of force being partly in the air. Induction coils are also made with a closed magnetic circuit, in the form of a torus ring, with two or more windings placed on the ring (Fig. 386). Such coils are more expensive and are used mostly in telephone exchanges. They usually have a one-to-one ratio of turns and are called *repeating coils.* Their functions are explained in the following chapters, in the description of magneto and central-energy switchboards. Where two or more induction coils or repeating coils are placed in near proximity to each other, it is often necessary to encase them in iron covers or surround them with copper sheathing to prevent mutual induction or "cross-talk."

The general theory of repeating and induction coils is the same as that of static transformers used in the transmission and distribution of electric power (Chapter XVI).

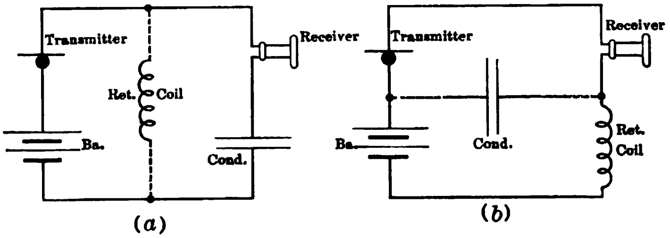
594. EXPERIMENT 27-C. — Effect of an Induction Coil on a High-Resistance Telephone Line. — The purpose of the experiment is to show that with a high-resistance telephone line the quality of speech transmission can be improved by raising the voltage of talking currents by means of an induction coil.

Connect a transmitter, a receiver, and an adjustable non-inductive resistance in series with a few dry cells. Place a buzzer before the mouthpiece of the transmitter, and increase the resistance from zero until the sound in the receiver is barely audible. Determine approximately the value of this resistance. Connect the transmitter and the primary (low-tension winding) of an induction coil in series with the dry cells. Connect the secondary winding to the receiver, in series with an adjustable non-inductive resistance. Place the buzzer in the same position with respect to the transmitter as in the preceding experiment, and determine approximately the resistance required to render the sound barely audible. This resistance will be found to be much greater than that determined in the first test.

In order to be conclusive, this experiment should be performed with transmitters of different resistance and with receivers of different impedance. Induction coils of different ratios of turns should be used for comparison. It must be shown that under the best possible conditions, speech can be better transmitted over a high-resistance line with an induction coil which raises the transmission voltage.

Report. Give the results of the tests. Explain theoretically the function of the induction coil. Consider separately the D.C. and the A.C. components of the transmitter current and of the line current.

595. Separation of Voice Currents by Means of a Condenser and a Retardation Coil. — In many telephone circuits it is necessary to separate high-frequency voice current from the D.C. battery current, without the use of a repeating coil. This can be done on the basis of the following two theoretical propositions:



(a) (b)

FIG. 368. The use of a condenser and retardation coil for separating voice currents.

I. High-frequency currents readily pass through a static condenser, but to a direct current a condenser offers an infinite resistance (Vol. II).

II. An impedance coil of high inductance (§ 138) and low resistance offers an effective barrier to high-frequency currents, while a direct current can freely pass through it. Such an impedance coil is known in telephone practice as a retardation coil.

These principles are illustrated in Fig. 368. In the arrangement (a) the steady battery current circulates through the transmitter and the impedance or retardation coil, while high-frequency voice currents follow the path of the battery, transmitter, receiver, and condenser. In the arrangement (b) the main battery current circulates through the transmitter and the receiver, while its rapid fluctuations, due to sound waves, follow the shunted path through the condenser.

596. EXPERIMENT 27-D. — The Use of Condenser and Retardation Coil in a Simple Telephone Circuit. — The purpose of the experiment is to investigate the action of a condenser and of an impedance coil, as explained in the preceding section. Connect the apparatus as in Fig. 368*a*, with an ammeter in series with the battery, but without the retardation coil. Place a buzzer or some other source of sound in front of the transmitter, and satisfy yourself of the fact that there is no sound in the receiver. Bridge a retardation coil as shown by the dotted lines and observe the effect on the sound. Vary the resistance and the inductance of the coil within wide limits and find the best values for sound transmission. Try the effect of an iron core in the retardation coil. Vary the capacitance of the condenser and find its maximum value beyond which the improvement is hardly noticeable.

If sources of sound of different pitch are available, for example, several organ pipes, find out if the best values of capacitance and inductance are the same for sounds of high and low pitch. Perform a similar experiment with the arrangement (b), first without and then with the condenser.

Report. Give the limits of magnitude of the variables investigated and the range of the values which gave the best results. Explain how a high-frequency current can flow in a circuit containing capacitance in series. Explain in detail the action of the retardation coil upon high-frequency currents.

597. Standard Circuit for Measuring Transmission Losses. — In order to express numerically the quality of various pieces of telephone apparatus with respect to the transmission of speech, a certain standard testing circuit has been agreed upon and is universally used in this country and in England. This circuit consists of a standard transmitter, receiver, induction coil, and condenser at each end of the line, all of definitely specified "Bell" types, and connected in a standard agreed manner to each other and to a "standard" artificial cable through repeating coils.

The standard cable is represented by a sectional arrangement of resistors and condensers equal to the resistance and the capacitance of 32 miles of a certain agreed telephone cable. At the present writing the standard cable for purposes of measurement is defined as one having a linear resistance of 88 ohms per loop mile; linear capacitance between wires, 0.054 m.f. per loop mile; linear inductance and linear leakage, zero. Artificial-cable boxes for testing are made by a number of manufacturers. The cable units are conveniently arranged with switches so that desired lengths can be readily cut in or out of circuit. Such artificial cables are often designed with linear constants different from those of standard cable, in which case conversion factors are used to change tested results into miles of standard cable.

Measurements are made with this circuit by compensating losses in speech transmission with an equivalent length of standard cable (Fig. 369). If it is desired to know the transmission equivalent (efficiency) of a certain telephone circuit, as compared to that of the standard circuit, the latter is handicapped by adding lengths of the standard cable until the transmission through both circuits is the same. If 6 miles of standard cable is required in the standard circuit to get a balance, then the circuit under test is said to have a loss of 6 miles of standard cable. In cases where the circuit under test has a higher transmission efficiency than the standard circuit, the artificial standard cable is added to the circuit under test. If 6 miles of this standard cable is required for such a balance, then the circuit under test has a gain of 6 miles of standard cable.

The same test circuit is used for comparing the efficiencies of particular pieces of telephone apparatus with the corresponding pieces of apparatus used in the standard circuit. For example, if a transmitter is to be tested, it is substituted in the test circuit in place of the standard transmitter, and the quality of transmission is compared to that of the latter. If it is necessary to add 2 miles of artificial cable to the test circuit when using the standard transmitter, in order to bring the speech quality down to that of the transmitter under test, the latter is said to be below standard by 2 miles of standard cable.

Certain precautions are necessary in the handling of the standard circuit in connection with various transmission tests:

1. In some cases it is advisable to have inserted in the test circuit, at all times, about 16 miles of artificial cable, so as to avoid electrical errors due to circuit conditions (reflection losses, etc.).

2. When transmitters are being tested, each transmitter should have its own independent current supply which should not be broken during the period of the test. This is to overcome the period of uncertain

action which exists for the first fifteen or twenty seconds after a circuit is closed through a granular carbon transmitter.

3. When only one particular portion of the circuit or piece of apparatus is undergoing test, as for example a telephone receiver, then only that portion of the standard test circuit should be changed, except for cutting in or out definite amounts of the artificial cable. The purpose of this precaution is to avoid errors due to slight differences which might be introduced if complete sections of one standard circuit were switched to similar sections of another standard circuit.

An adaptation of the standard circuit for testing an induction coil is shown in Fig. 369. It permits a quick comparison of local-battery induction coils. All apparatus in the test circuit, excepting the coils under test and the balancing cable, remain the same throughout a series of tests, so as to avoid errors. The 4 miles of standard cable inserted between the induction coil under test and the first repeating coil represents the conditions of an average subscriber's loop (line circuit). The fixed amount of 28 miles of standard cable, between the two repeating coils, represents the condition of a long-distance line and is great enough to avoid errors in testing which might occur if the line were made short in length. The balancing cable is arranged to be added to or subtracted from the circuit, so as to make the transmission from the "assumed standard" induction coil equal to the coil being compared. When this balance is obtained, the miles of cable added or subtracted represent the loss or gain (comparative efficiency).

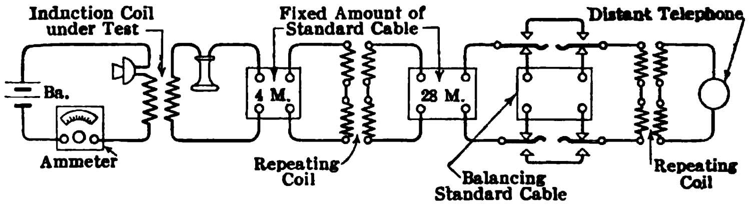


FIG. 369. An adaptation of the standard circuit for testing an induction coil.

The best source of sound is a pure tone of agreed and adjustable intensity and pitch. By measuring the intensity of sound received with the standard apparatus and with the apparatus under test, the amount of artificial cable can be adjusted to make the two intensities equal. Since the measurement of sound intensity is difficult, the current in the standard receiver may be measured by some other means, for example, with a delicate thermo-galvanometer (§ 51). A vacuum-tube amplifier may be used, if necessary. It is also possible to measure the amplitude of vibrations of the receiver diaphragm by attaching a tiny mirror to it and reflecting a beam of light from it on a screen. The surface of the diaphragm itself may be polished to act as a concave mirror. By varying the pitch of the sound in front of the transmitter the efficiency of transmission may be determined within the range of the human voice.

Such methods are more suitable for special investigations; in ordinary tests a trained person speaks into the transmitter and another trained person listens at the receiver and judges the articulation, volume, and quality of speech (squeaky, drummy, barreled, etc.). Disconnected words, or still better, meaningless syllables, should be used, to prevent the listening person from guessing or using his imagination. Sets of syllables have been specially prepared for testing telephone equipment. The percentage of those received and written down correctly is a fair measure of the quality of speech transmission.

While the use of artificial cable provides a means for comparing the transmission loss of an actual circuit with a definite standard, transmission measurements over actual telephone lines are now made with a high degree of accuracy by means of precision types of apparatus. Not only are impedance measurements made on lines, using all of the frequencies contained in the talking range, but in addition, it is possible to apply alternating currents of suitable frequency and strength, and actually measure the received current at the distant end. While this type of measurement involves considerably more in the way of apparatus than is available in an ordinary telephone office, such tests are being made in an increasing number of cases on important telephone lines. In other words the tendency in modern testing practice is to substitute for the human voice generating devices which produce frequencies of predetermined periodicity and strength, and to measure these currents after they have been weakened by passage over the telephone line by means of precision measuring instruments.

598. EXPERIMENT 27-E. — Measurements with Standard Cable. — A description of the standard circuit and its uses will be found in the preceding section. It is desired that the student obtain experience in judging the quality of transmission and in comparing two similar devices of different make or type, such as two transmitters, two receivers, or two induction coils. The results should be expressed in miles of standard cable, with additional remarks to characterize possible faults in the received speech.

THE RINGER AND THE MAGNETO

599. The Ringer. — Telephone bells, or ringers, are operated by alternating current, or in some cases by a pulsating current. The only exception is that of house telephones and intercommunicating systems (§ 607) where battery current is usually employed for ringing. The reasons for using alternating current for ringing are as follows: (a) it is more easily generated at a voltage necessary for transmission over a line, and no commutator is necessary in the generator; (b) the bell itself is more rugged and reliable and has no sparking contact; (c) the bell is not affected by direct current used for talking and can be separated by a condenser; (d) selective party ringing is made possible by using different frequencies.

An alternating-current ringer, or polarized bell, as it is sometimes called, is shown in Fig. 370 to the left. It consists of a permanent magnet *NS* and of an electromagnet through whose winding an alternating current can be sent from the line. A soft-iron armature carries a striker and is so pivoted that the two gongs on top can be struck alternately.

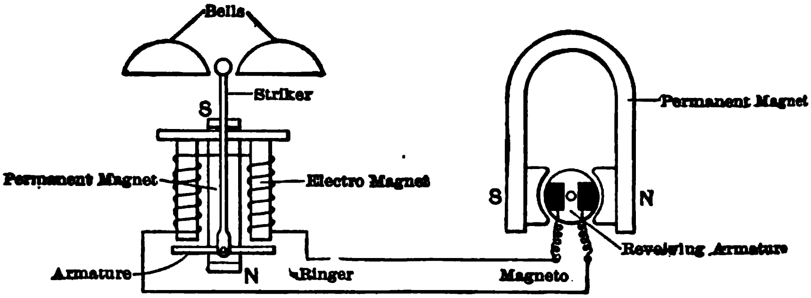


FIG. 370. Magneto-generator and polarized bell.

Without the current, the permanent magnet produces two equal fluxes in parallel through the cores of the electromagnet, and the pull on both ends of the armature is the same. If a direct current be passed through the winding, the flux in one leg of the core is increased and that in the other core reduced. The armature is then attracted to the stronger pole, and the hammer strikes one of the gongs. When the current is reversed the other gong is struck. With an alternating current the gongs are struck alternately at synchronous speed, and produce the effect of continuous ringing.

The present practice is to connect the subscriber's ringer across the line, or between one of the line conductors and the ground. An electrostatic condenser is sometimes used in series with the ringer, to keep out the direct current, while allowing alternating currents to pass. This is necessary when direct current for the transmitter is supplied from the office (central-energy system, Chapter XXIX).

600. Selective Ringing. — With several telephone sets in parallel across the same line wires (party line), and with the above described ringers bridged across the line, all the bells respond whenever a proper signaling current is sent from the central or by one of the subscribers (in the magneto system). It is therefore necessary to use a code of short and long rings to call each individual station. Such a method is annoying to subscribers and leads to confusion. Therefore, several methods of *selective ringing* are in use, with which the code is either simplified or altogether dispensed with. The principal methods are as follows:

(a) *Ringing to the ground.* One-half of the phones on the line have their ringers connected between one of the line wires and the ground, and the remainder between the other wire and the ground. The source of signaling current in the office can be connected at will between either wire and the ground. In this way only one-half of the ringers on the line respond at a time, and the code is considerably simplified. The subscribers' magnetos, if any, and the signals in the office are bridged across the two line wires, so that each subscriber can call the operator.

(b) *Biased bell.* The ringer can be provided with a light spring which will hold the bell clapper as far as it can go to one side. Thus, the armature can be moved from its normal position by a current of one direction only. If an intermittent current of the proper strength and in the proper direction is passed through such a biased bell, the current causes the clapper to strike one gong, and the spring causes the clapper to strike the other gong. An intermittent current has been generally superseded by a combination of an alternating and a direct current. This current has a succession of strong pulses in one direction with weak pulses in the opposite direction between the strong pulses. The strong pulses move the clapper against the spring tension, and the weak pulses assist the spring on the return strokes. By these means an even ring is obtained.

One-half of the ringers on a party line are biased to respond to strong pulses in one direction, the other half to strong pulses in the other direction. Furthermore, one-half of the ringers biased in each direction may be connected between one line wire and the ground and the other half between the other wire and the ground. In this way only one out of every four bells will respond to a signaling current from the office. If there are eight subscribers on the line, the only code needed is one and two rings.

(c) *Harmonic ringing.* The moving part of the ringer can be made to vibrate at a desired natural frequency by supporting it by springs and by adding a proper weight to the clapper. Such a spring, *k*, is shown in Fig. 371*c*. By varying the thickness of the springs and the mass and position of the weights, each bell of a set is given a natural period of vibration which materially differs from that of the others. Such a bell is known as a harmonic or tuned bell. It gives a maximum response to an alternating current of a particular frequency and practically no response if the frequency differs materially from that to which the bell is tuned. The frequencies commonly used are $16\frac{2}{3}$, 25, $33\frac{1}{3}$, 50, and $66\frac{2}{3}$ cycles per second.

With a central office equipped with a source of these frequencies, full selective ringing on either four or five-party lines is obtained by the use of harmonic bells, by merely connecting all bells between the two sides of the lines and ringing each bell with current of the frequency to which it only responds. Full selective ringing may be obtained for an 8 or 10 party line by connecting half of the bells between ground and one side of the line and the other half between ground and the other side of the line.

Various means for generating ringing currents in an exchange are described in § 616 in the next chapter.

601. EXPERIMENT 27-F. — Study of a Polarized Telephone Ringer. — The purpose of the experiment is to become familiar with the ordinary alternating-current ringer used in subscriber sets (§ 599) and with its modifications for selective ringing described in § 600. An alternator should be provided whose frequency can be varied within wide limits (from 15 to 70 cycles, if possible) and whose voltage may be adjusted at will; also a source of direct current, preferably a storage battery, with necessary switches for connecting it in series with the alternator. See also the methods described in § 616.

(1) Examine an ordinary polarized ringer and make a careful sketch of it to scale. Make clear to yourself its magnetic circuit with no current and with a direct current in the winding. Measure ballistically (§ 158) or otherwise the fluxes in all the air-gaps, with different values of direct current in the winding, and decide upon the best magnitude of the current. Adjust different parts of the ringer to get the best results. Measure the resistance of the winding.

(2) Apply an alternating current of the lowest available frequency and at the lowest voltage at which the ringer can operate satisfactorily. Measure the current and the voltage. Since the current wave may be quite distorted, it is better to use a hot-wire ammeter (§ 50), or else a

high resistance in series with the winding. Increase the voltage in steps, keeping the frequency constant, and read the current and the voltage at each step. Note improvements or changes in the sound and in the character of motion. Increase the frequency in steps and take similar measurements at each step, keeping within the range of voltage at which the ringer operates satisfactorily.

(3) Bias the striker with a spiral spring and operate the ringer with a pulsating current obtained by superimposing a direct and an alternating current (battery in series with an alternator). Adjust the component currents so as to keep the resultant current uni-directional, and then so as to have a small reversed current. Obtain by trials the best relationship and adjust the spring tension accordingly. Consider very carefully the kind of ammeter and voltmeter to be used in this experiment.

(4) Load the striker and add flat springs to the armature, to tune the ringer to a particular frequency. Adjust the weights and the springs to make the ringer respond to the desired frequency. Change the frequency of the supply and determine the precision of tuning, that is, find out how much the frequency of current has to be changed before the ringer ceases to respond. At each frequency read the current and the voltage. Try adjusting to the same frequency with a different weight and a different spring tension. Make clear to yourself the advantages and the disadvantages of tuning with a stiff spring and otherwise. Try tuning at some other frequency, far from the first, and make the necessary adjustments until the device operates satisfactorily. Show that the bell will then fail to ring at the first frequency.

(5) Connect an adjustable condenser in series with the tuned ringer. Find the smallest capacitance with which the ringer can be operated at the frequency at which it responds best, and at a reasonable line voltage. Measure the current, the line voltage, the voltage across the ringer, and that across the condenser, the last two preferably with an electrostatic voltmeter (§ 53). Increase the capacitance in steps and also vary the frequency; take readings at each step.

Report. (a) Make a drawing of the ringer and show the paths of the permanent, superimposed, and resultant fluxes. Give the measured values of fluxes and flux densities, and the values for the best operation. Show the effect of adjustments in the ringer. Compute the resistance of the winding. (b) Compute the impedance, the reactance, and the inductance of the winding at different frequencies and voltages (Chapter VI). Explain discrepancies, if any, in the computed values of inductance. (c) Give the best range of voltages and frequencies, draw current and voltage curves, and mark on the curves the approximate relative character and intensity of sound. (d) Describe the test with the biased ringer, explain in detail how the superimposed currents and voltages were measured, and give an approximate curve of current at which the ringer operated best. (e) Describe the experiment on tuning the ringer and give data on the range of its response. (f) Explain the action of the condenser and draw a few vector diagrams showing the currents and the voltages.

602. The Magneto-Generator. — In the magneto telephone system described in the next chapter (§ 610), it is necessary for the subscriber to turn the crank of a small generator in order to call the operator or another party whose telephone is bridged across the same line. Such a generator is shown in Fig. 370, to the right, and in the four sketches in Fig. 371. Its field system consists of a few U-shaped permanent magnets placed side by side and provided with suitable pole-pieces of soft steel. The armature is of the shuttle type, with a single-coil winding. One terminal is usually connected to the shaft and the other to a pin *p* (Fig. 371*a*) mounted on the end of the shaft but insulated therefrom.

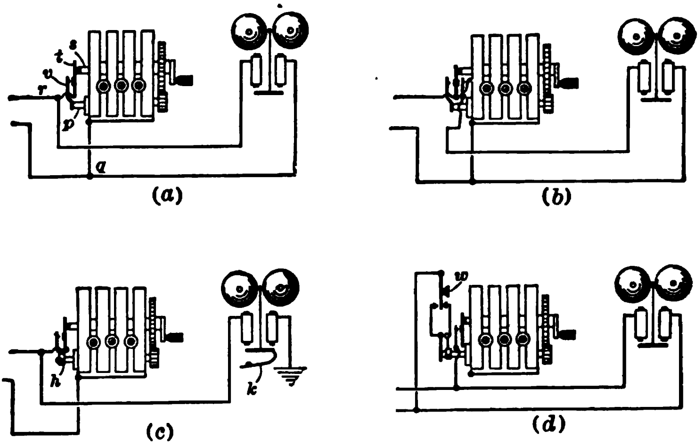


FIG. 371. Alternating and pulsating-current magnetos and their connections to ringers.

In some applications a uni-directional signaling current is required. The above-mentioned pin is then replaced by a two-segment commutator with a stationary brush in contact with it. Such a brush *h* is shown in Fig. 371*c*, and a magneto of this kind is known as a direct-current, or pulsating-current magneto generator.

A magneto, connected across a telephone line, would shunt part of the talking currents, and thus lower the efficiency of speech transmission. Therefore, magnetos are provided with some form of the so-called "generator cut-in," a device which keeps the armature circuit open, except when the crank is turned. The usual method consists in a mechanical arrangement which causes the shaft to move longitudinally and to remain in the new position as long as the crank is being turned. On the grounded side the armature is connected permanently to one of the lines, while the above-mentioned pin *p* makes contact with the line *r* only in the shifted position. A flat spring *t* is permanently connected to *r*, and the spring *v* to *p*. When the shaft *s* is shifted to the left, *t* comes in contact with *v* and the magneto is connected across the line. In the normal position the circuit is open between *t* and *v*.

With the connections (a), the subscriber's own bell rings when he operates his magneto. If this is undesirable, the arrangement shown in (b) may be used. A spring contact *f* is added, which normally connects the bell to the line, and which disconnects the bell when the magneto crank is turned and the shaft is pushed to the left.

The arrangement (c) shows a pulsating-current magneto with a harmonic ringer (§ 600) between one of the lines and the ground. The magneto current in this case will not operate the ringer, and to call a subscriber on the same line it is necessary to call the central, where sources of different frequencies are provided.

The diagram (d) shows a combined D.C. and A.C. magneto generator, and a plain polarized bell. With the push-button *w* in the position shown in the sketch, a direct current is sent to the line, so that other subscribers' bells on the same line do not respond, and the central can be called privately. With the push-button pressed to the left, an alternating current is generated to call the subscribers on the line. It is also possible to call the central and to ring the bells on the line at the same time.

603. EXPERIMENT 27-G. — Test of a Magneto-Generator. — The generator and its functions are described in the preceding section. Examine the available machine thoroughly and make a sketch, approximately to scale. Make a separate sketch of the mechanical details of the endwise movement of the crank shaft; note the materials used. Examine the electrical connections, the armature terminals, the contact springs, etc.

(a) Take a no-load voltage characteristic of the machine, varying the speed within the limits of hand operation. For this test the magneto should preferably be driven by a motor at steady speeds. (b) Take a load characteristic of the generator, preferably using standard ringers in parallel as a load. Keep the speed constant and determine the relationship between terminal volts and line amperes. Perform such runs at several speeds. (c) Connect a resistor between each two ringers to imitate the line resistance, and determine the limit beyond which the remotest bell does not respond satisfactorily. (d) For tests on permanent magnets, see §§ 204 to 207. (e) The wave-form of the induced voltage and the current wave due to the ringers can be investigated with an oscillograph (Vol. II).

Report. Make detailed drawings of the machine and describe the important parts. Draw the no-load voltage curve and the load curves. Explain their shape theoretically. Give the results of the tests with ringers bridged along a high-resistance line, and explain the practical lesson to be derived from this test.

CHAPTER XXVIII

INTERCOMMUNICATING AND MAGNETO TELEPHONE SYSTEMS

604. Classification of Telephone Systems. — Telephone installations differ widely according to the number of subscribers served, the distances covered, and the methods of operation. The following general subdivision will indicate the scope of the field:

(1) House and intercommunicating systems, used in offices, factories, schools, on board ship, etc. Such installations contain comparatively few stations, and the instruments are connected at will by the users themselves, without an operator or automatic central office.

(2) Isolated systems with a single telephone central, manual or automatic, in small towns, in large industrial establishments, hotels, etc. Such systems often have connections with city and long-distance telephone exchanges, and are then called private branch exchanges.

(3) City and suburban systems, with one or more central offices, or exchanges. A call originated in one exchange often has to be completed in another exchange, involving " trunking." Switching operations may be manual, semi-automatic, or full automatic.

(4) Long-distance telephone systems.

With respect to the methods of calling the central and the source of transmitter current, outside the intercommunicating field, telephone installations, are usually divided into *magneto systems* and *central-energy systems.*

In a magneto system each subscriber set is provided with a hand-operated permanent-magnet generator, known as a *magneto* (§ 602). By turning the crank the subscriber sends a current through the line to his " drop " in the exchange. The drop shutter is released, displaying the subscriber's number; this notifies the operator that the calling party desires a connection with another party. A magneto telephone requires a separate local battery for the transmitter (Fig. 363).

In a central-energy system the " central " is called by simply taking the receiver off the hook. This closes the subscriber's circuit and lights a small lamp in the office, calling the operator's attention to the fact that a connection is desired. The direct current for the transmitter, and for operating the various switching relays, is furnished from a battery located in the office, and no batteries are needed in the individual stations. This is a great improvement over the magneto system which involves a considerable expense in battery renewals.

Sometimes magneto telephones are called "local-battery telephones." Such a name is not quite correct; while a local battery is always used with magneto telephones, its use is not confined to magneto telephones. For example, local-battery telephones, without magneto equipment, are sometimes used with central-energy switchboards so as to obtain the lamp signal and at the same time provide a local battery for the operation of the transmitter. Such telephones are described as "central-energy signaling, local-battery talking." They are used on long lines, where the resistance of the circuit makes it impossible to furnish sufficient battery current from the exchange to operate the transmitter at a good efficiency.

Telephone systems may also be divided into manual, auto-manual (semi-automatic), and automatic. The last system is also known as the machine-switching system. In the manual system the operator receives a call and plugs the desired number, if it is not busy. In the full automatic system the subscriber himself performs the necessary distant switching by sending through the line a certain number of impulses which operate proper electromagnetic selectors in the office. In the auto-manual system the operator answers a call as in the manual system, sets the desired number by means of keys, as in an adding machine, and the connections are established automatically.

Telephone lines may be divided into *metallic circuits*, using two wires, and single-wire, or *ground-return*, circuits. Sometimes the talking is done over a metallic circuit, while signaling currents are sent between one of the line wires and the ground. This is one of the methods of obtaining selective ringing on party lines (§ 600).

HOUSE AND INTERCOMMUNICATING TELEPHONES

605. Private Line Connections. — A simple telephone installation consisting of two stations or sets is shown in Fig. 372. Each set consists of the apparatus described in the preceding chapter and of a so-called hookswitch on which the receiver normally hangs. The switch actuates three contacts between springs. In the position shown in the sketch, with the hook "down," the upper contact is closed and the two lower ones are open. When the receiver is lifted and the hook is "up," the upper contact is open and the two lower ones closed.

To call Station *B* from *A*, the receiver is taken off the hook and the push-button, or key, is pressed down. A current flows from the terminal B_2 of the *A* battery through the lower contact on the hookswitch, the terminal L_1, the key of station *B* and its bell, to the terminal B_4 of station *A*. The *B* bell rings, and to answer the call it is only necessary to lift the receiver and to speak into the transmitter. When both receivers are off the hook a single series circuit is established, consisting of both transmitters, both batteries, and the two primary windings of the induction coils. Each receiver, with the secondary winding of its induction coil, forms a separate circuit.

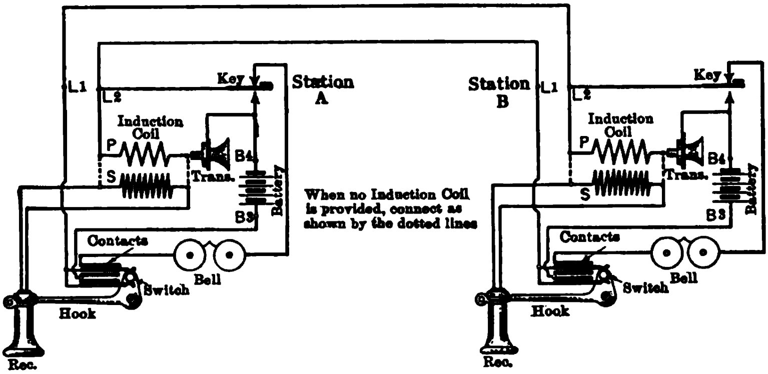


Fig. 372. Connections in a pair of house telephones.

The induction coils can be omitted, provided that low-resistance receivers are used. In this case the series talking circuit also contains the two receivers, as shown by the dotted lines. The receivers, if of the permanent-magnet type, should be properly poled, or else they may be of the electromagnetic type (§ 591).

606. EXPERIMENT 28-A. — Connecting House Telephones with Battery Call. — A typical installation is described in the preceding section, although minor modifications are possible. To get the most benefit out of this experiment, separate parts should be provided, mounted on a board, with suitable clip terminals. The student is to connect them properly so as to make actual conversation possible. After this, a study can be made of the effect of individual factors, for example, the effect of using transmitters and receivers of different resistance, different induction coils, different battery voltage, etc. The effect of the line resistance, inductance, and capacitance may also be investigated, although this problem is covered separately in § 638. Connections can be tried without the induction coils, with properly poled receivers, and with D.C. receivers. In each case the quality of speech should be recorded and, if possible, a theoretical reason found for its improvement or deterioration (§ 597).

607. Intercommunicating Telephones. — A small telephone system (Fig. 373) in which each user himself can make connections with any other telephone on the system, without a central switchboard, is known as an intercommunicating system. A characteristic of such a system is that at least as many wires, or pairs of wires, have to be installed between all the stations as there are stations, and the battery conductors besides. In an individual installation, say a small factory, a large office, a store, or a theater, there is a choice between a private branch exchange, manual or automatic, and an intercommunicating system directly connecting all the subscribers. The selection depends upon the distances to be covered, the amount of traffic, connections to the city and long-distance system, etc.

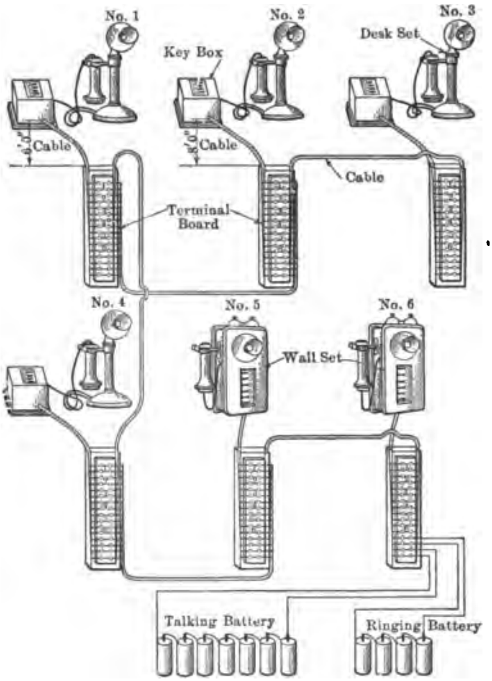


Fig. 373. The principle of an intercommunicating telephone system.

The particular kind of connections for an intercommunication system depends upon the desired kind of service. While a large variety of practical conditions must be met, the following four kinds of service are typical and will give an idea of other possible cases.

(a) *A complete intercommunicating system* is shown schematically in Fig. 373. Each station is provided with a key box, a key being assigned to each of the remaining stations; besides, there is a "home" key. All the required conductors are contained in a cable, and sections of this cable are joined in terminal boxes, one of which must be installed at each station. Two common batteries of dry cells are shown, one for talking, and one for ringing bells. The latter battery may sometimes be dispensed with, the bells being connected across part of the talking battery. If an A.C. lighting circuit is available, the terminals shown connected to the ringing battery can be connected instead to the A.C. circuit through a so-called bell-ringing transformer.

To make a call, the button marked with the name or number of the station wanted is depressed completely, causing certain spring contacts to be made. This permits the ringing current to flow to the station to which this particular key is connected, thereby ringing the bell at that station. When the push-button is released the plunger assumes an intermediate position, breaking the ringing contact and clearing the line for conversation. While conversation is going on, the plunger is automatically held in the talking position by means of a locking plate. The operated key does not resume its normal position until another key is actuated. Talking current, however, is cut off as soon as the receiver is placed back on the switchhook. To answer a call, the connection is made with the calling party by pressing the " home " button and taking the receiver from the hook. It is not necessary to restore the " home " button after a call is answered because this operation is performed automatically when the next call is made.

With the above-described system, as many separate conversations can be carried on simultaneously as there are pairs of telephones in the system. For example, in a system consisting of twelve telephone sets, six separate conversations can be carried on at the same time without interfering with each other. This class of service is generally known as " Selective ringing and selective talking " or " Non-interfering service."

An automatic reset feature may be furnished with intercommunicating telephones, an electromagnet being used to release the key-latching lever or slide when the receiver is replaced on the hookswitch. This applies to the desk stands where the push-buttons are located in a separate box. In the case of the wall telephone, a mechanical release is often furnished, so that the movement of the hookswitch will release the push-button plungers. The majority of manufacturers of intercommunicating equipment prefer to keep the key equipment simple and have the telephone user press the " home " button when answering a call. A few makers furnish the reset feature, but the device requires very careful workmanship, and naturally the maintenance is greater than on the instruments where the subscriber does the resetting by hand.

(b) In some cases the *number of simultaneous conversations* can be *limited to one.* The circuits are then simpler, and three-position switching keys are not required. Each telephone in this system is also equipped with a number of push-buttons for signaling the other stations. By depressing the button marked with the name or number of the station wanted, the bell of that station is rung. No other station in the system is signaled but the one desired; this system is commonly termed "Selective ringing, common talking."

(c) Some intercommunicating telephone installations consist of a *master station* to which a number of other telephones, known as the *outlying stations*, are connected. This system is particularly adapted to residences, country places, hotels, hospitals, schools, etc., where it is desired to communicate from a central point to various rooms, and vice versa.

The master station is equipped with a number of push-buttons, one for each outlying station in the system. By depressing the button marked with the name or number of the outlying station wanted, the bell of that station is rung. The outlying stations are equipped with only one button, which signals the master station when depressed. The service provided by this system is called the "Two-way Service," the name signifying that it is possible to select and ring individually any one of the outlying stations from the master station, and that any one of the outlying stations can call the master station. No provision is made for having the outlying stations call each other. One outlying station can talk to another, however, by first calling the master station and then having the master station signal the other outlying station with which conversation is desired.

(d) It is possible to have a comparatively simple system of intercommunication by connecting *all the telephones in parallel* across the same line. In this case the ringing key, when pressed at any one station, operates the bells of all the other stations, and it becomes necessary to use a code, consisting of long and short rings. There are cases in which such an arrangement is not objectionable and may be installed at a saving in the original cost and upkeep, as compared to the more elaborate systems described before.

608. Connections in an Intercommunicating System. — The electrical connections in an installation, such as is represented in Fig. 373, are shown in Figs. 374 and 375. For the sake of simplicity, only the connections between two stations are shown. In reality there are as many twisted pairs of talking wires as there are stations, and similar connections can be established on any pair.

In Fig. 374 the bell-ringing circuit is shown by the heavy lines. When the button A is depressed to call a station, it first connects the called station's line to the calling station and then connects the ringing battery to the line. The ringing current passes through the bell B at the called station, through the back contacts C of the switchhook at that station, over the side D of the line and through the "way down" contact *E* of the button at the calling station, thence over the other side *F* of the battery line back to the ringing battery, operating the bell *B* at the called station.

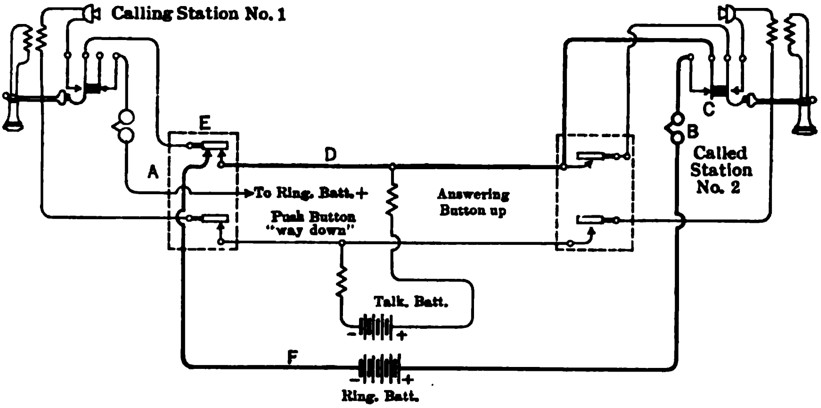


Fig. 374. The ringing circuit in an intercommunicating system.

After the two receivers are lifted off the hooks, the talking connections are those shown by the heavy lines in Fig. 375. The ringing circuit is broken at *A*, this action taking place as the push-button automatically resumes its second position. The two transmitters, *T* and *T*, are now

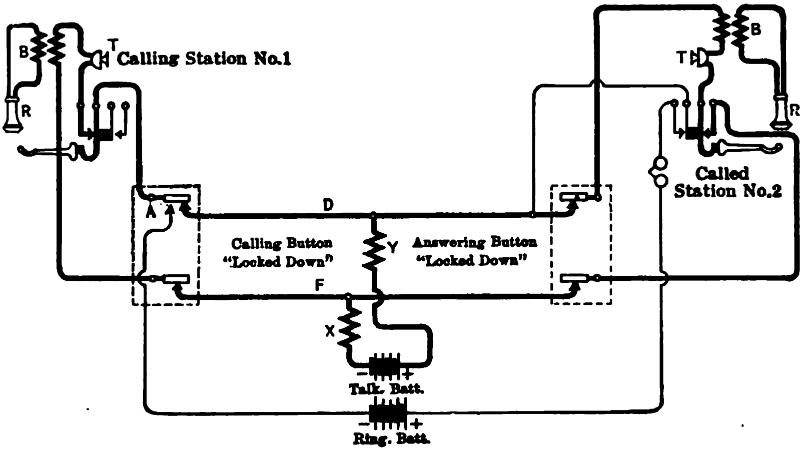


Fig. 375. The talking circuit in an intercommunicating system.

connected directly across the line *D F*, each in series with the primary of an induction coil *B*. The talking battery is also bridged across the line through the two windings, *X* and *Y*, of a retardation coil (§ 595). The function of the retardation coil is to prevent interference of cross talk from other stations which might also be connected for conversation at the same time, as the same talking battery is used for all the telephones in the system. The receivers, *R* and *R*, are each connected in a local circuit which includes the secondary of the induction coil *B* at each station.

An intercommunicating system, such as shown in Figs. 374 and 375, can be used satisfactorily for connections with city exchanges. The circuit is balanced, twisted pairs of conductors being used, and the talking battery is fed through impedance coils so as to give the same character of transmission as is furnished in central-energy exchanges (§ 623).

With the above-described connections as a guide, it is left to the student to devise connections suitable for services (b), (c), and (d) specified in the preceding section.

609. EXPERIMENT 28-B. — Connecting an Intercommunicating Telephone System. — Such systems are described in §§ 607 and 608. For an exercise it is sufficient to have only three stations, with the receivers, transmitters, hookswitches, key boxes, etc. mounted separately, to enable the student to make different connections. (1) Begin with a simple problem, for example, a system of the type (c) or (d) in § 607; lay out the necessary connections on paper, connect the apparatus accordingly and try its performance. (2) Connect up a system described under (b), and investigate its operation. Find out what happens if a third party tries to call someone while a conversation is going on. (3) Connect the available apparatus to make the system described under (a) and shown in Figs. 374 and 375. Try its operation under different conditions that may arise in practice. Devise means for connecting it with a city exchange of the central-energy type. Try a bell-ringing transformer in place of the ringing battery.

Report. Give the actual diagrams of connections used. Describe the special conditions investigated, and the operating characteristics of each system tried.

MAGNETO OR LOCAL-BATTERY TELEPHONE SYSTEM

610. Subscriber's Connections. — The electrical connections of a subscriber's set of the magneto type are shown in Fig. 376, and the assembled set in Fig. 377. The talking circuit consists of a few dry cells in series with a transmitter and the primary of an induction coil. This circuit is closed only when the receiver is off the hook. An alternating-current bell, or ringer (§ 599), is permanently connected, or "bridged," across the line. An alternating-current magneto gener-

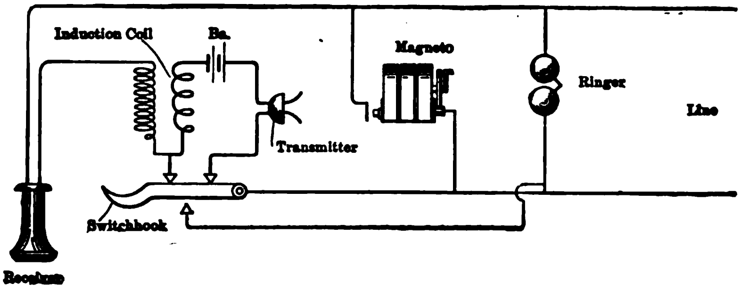


FIG. 376. Connections in a magneto telephone set.

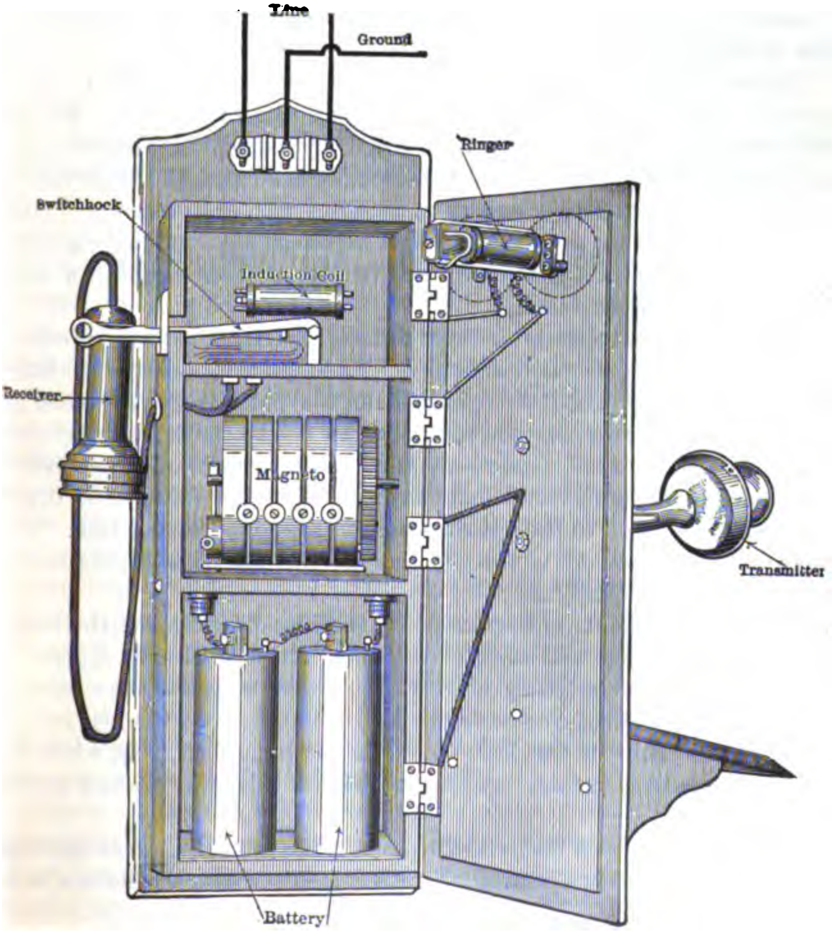


FIG. 377. Magneto telephone set.

ator, used for calling the central and other subscribers on the same line, is also bridged across the line, but the circuit is normally broken, as shown in the sketch, and is closed only as long as the crank is being turned (§ 602).

To call the central, the subscriber turns the magneto handle and then takes the receiver off the hook. The magneto current causes the subscriber's number to be displayed in front of the operator, and she utters the familiar "Number, please." After the connection with the desired party has been made, conversation can take place; each battery current is limited to its local circuit, and only small induced talking currents flow through the line. At the end of the conversation, it is necessary to turn the magneto crank again, in order to display the "clearing-out drop" in front of the operator. She then disconnects the subscribers' lines.

There are cases, especially on rural lines, when it is desired to call a party or the central while one or more receivers on the same line are off the hook. It will be seen from Fig. 376 that all such receivers act as shunts to the ringers, and the latter may not respond. This can be remedied by connecting a condenser in series with each receiver. Such a condenser does not appreciably affect high-frequency talking currents, but greatly increases the impedance of the receiver circuit for low-frequency ringing currents.

Every time the receiver is off the hook, the local battery circuit is closed, and the battery is discharging. Sometimes it is desired to listen without talking. The two contacts on the hookswitch are then so arranged that the receiver contact is closed before the transmitter contact. A special latch, known as the "battery saver," is provided; this catches the hookswitch half-way up when the receiver is lifted. One can then listen to the conversation on the line, but not talk. The latch can be pushed out of the way and the switchhook allowed to come all the way up for talking.

611. Various Kinds of Service. — Several modifications of the ringer and magneto types, and their connections, are described in §§ 599 to 602. Some of the resulting kinds of party-line service are as follows:

1. *Regular bridging-line service.* The connections are shown in Fig. 376. All the bells on the line, including the home bell, ring when the generator crank is turned. A code is used to call other parties on the same line.

2. *Regular bridging with silent ringer.* The home bell does not ring when the magneto is operated (Fig. 371*b*); otherwise the service is as above.

3. *Regular bridging with "non-interfering" push-button.* The office

can be called without ringing the bells of other subscribers on the line, by pressing a key while turning the magneto crank, somewhat as in Fig. 371*d*. This key connects the magneto between one of the line wires and the ground, while all the ringers are connected across the line. To call other subscribers on the line the key is left in the normal position.

4. *Divided-circuit ringing service.* One-half of the ringers are bridged between one line wire and the ground, the other half between the other wire and the ground. The telephone operator must do the ringing, the magnetos being connected across the line.

5. *Biased-ringer, two-* and *four-party signaling.* This system can be used only where the central office is provided with a source of " positive " and " negative " pulsating ringing currents.

6. *Harmonic four-party selective signaling.* Harmonic ringers are used across the line, and a pulsating-current magneto is used to call the central.

7. *Harmonic eight-party selective signaling.* Same as (6), except that the ringers are connected to the ground, four on each side of the line.

612. EXPERIMENT 28-C. — Connecting Bridged Magneto Telephones. — The connections are described in §§ 610 and 611. Information on the component parts of a telephone set will be found in the preceding chapter. No switchboard apparatus is to be used in this experiment, except an answering drop (Fig. 379) across the line and a magneto for ringing subscribers from the central. It is sufficient to have two complete telephone sets across the line; the remainder can be represented by ringers and receivers. Note the make, style number, and other characteristics, such as the impedance or the capacitance, of all the pieces of apparatus used.

The experiment consists in wiring the line apparatus and in trying the arrangements and kinds of service described above. In each case the operation should be tried under normal conditions, as well as under the most severe or abnormal conditions of service, such as high resistance of the line, poor ground connections, metallic line partly grounded, several receivers off the hook, etc. Careful notes should be taken of the quality of service under such conditions.

Report. Give the actual connections and the make and other characteristics of the apparatus used. Describe the performance with each kind of connections, under normal and abnormal conditions. For each kind of service give a practical case in which it is the most appropriate, for example, rural lines, a small town, train dispatching, load dispatching in an extensive power system, line patrol, a road-surveying party, military operations, etc. For each kind of service, state which system would barely meet the requirements, and which one would be the best, irrespective of the cost.

613. Magneto Telephone Switchboard. — The function of a telephone office, or "central," as it is commonly called, is to connect the subscribers whose lines terminate on the switchboard (Fig. 378), or to connect them to other exchanges. The purpose of the various devices shown on the switchboard will be described first, and the required electrical connections will then be explained in detail.

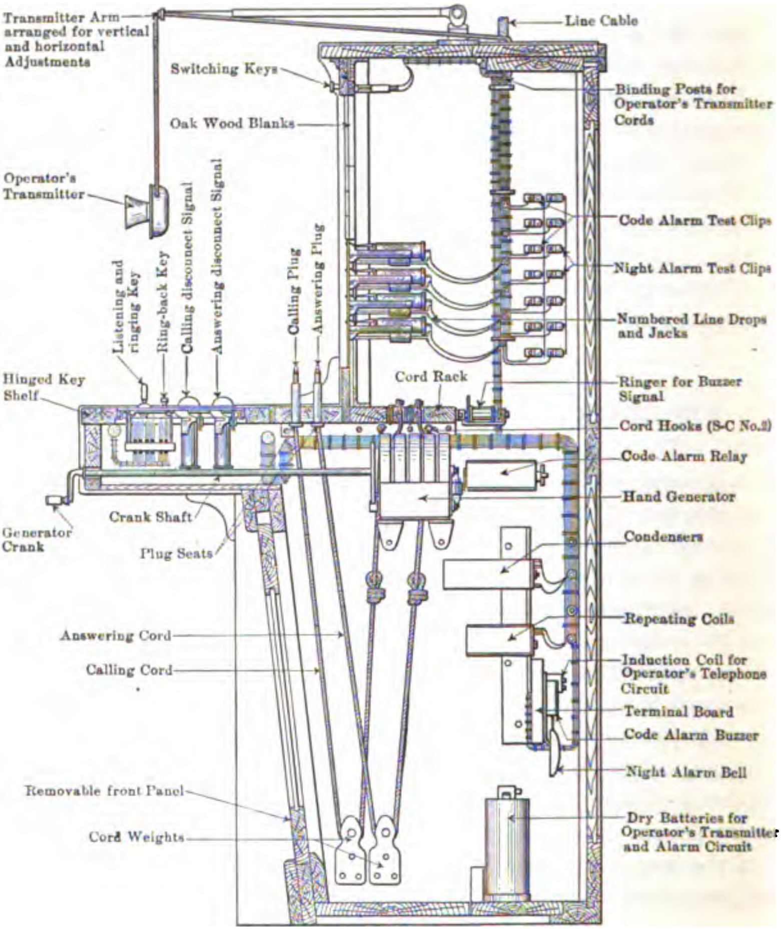


Fig. 378. A cross-section of a magneto telephone switchboard.

A switchboard section accommodates from 50 to 150 lines. Each line terminates in a so-called "jack," four of which are indicated on the vertical panel in front of the operator. When a subscriber turns the crank of his magneto and sends a current over the line, this current passes through the electromagnet of a so-called "line drop" placed above each jack, and causes the release of a shutter, which notifies the operator that a connection is desired. She takes one of the "answering" plugs in front of her and inserts it into the jack below the shutter; this restores the shutter to its normal position. She also turns the corresponding three-way key to the "listening" position and asks for the number of the party to be called. Having received this information, she inserts the "calling" plug of the same cord circuit into the jack of the subscriber to be called, and turns the key to the "ringing" position.

If the exchange is equipped with a power-driven signaling generator, the very act of pressing the key sends ringing current to the line to be called. Otherwise the operator has to turn the crank of the hand generator in front of the shelf. When the party answers, she replaces the key to the middle position and the two subscribers are connected for conversation. A switchboard section is usually equipped with from 10 to 15 cord circuits, enabling a corresponding number of pairs of subscribers to be connected simultaneously.

At the end of the conversation, each subscriber is supposed to "ring off," that is, to give one or two short rings as a signal that he has finished speaking. The corresponding currents operate the so-called "clearing-out drops," or "disconnect signals," in front of the operator; she restores the disconnect signals and pulls the plugs out of the jacks, thereby disconnecting the subscribers. By switching in her listening key while the subscribers are still connected, she can find out at any time whether the connection is still needed or should be broken. In the particular switchboard shown in Fig. 378, the clearing-out shutters are mechanically linked with the listening-key lever, so that they are automatically restored to their normal position by throwing the key lever to the listening position. This eliminates two extra motions that the operator would otherwise have to perform for each call.

614. Switchboard Connections. — A combined jack and line drop is shown in Fig. 379. The upper sketch shows the drop shutter in the normal position, while the lower sketch shows it in the operated position. The line current from a subscriber's magneto passes through the jack springs and the winding of the drop electromagnet, causing the armature to release the drop shutter. When the answering plug is inserted into the jack, it spreads the outer springs and opens the drop circuit. The plug also causes the restoring arm to press on the back of the shutter and to restore it to its normal position. The line is now connected to the switchboard through the tip and the sleeve of the answering plug.

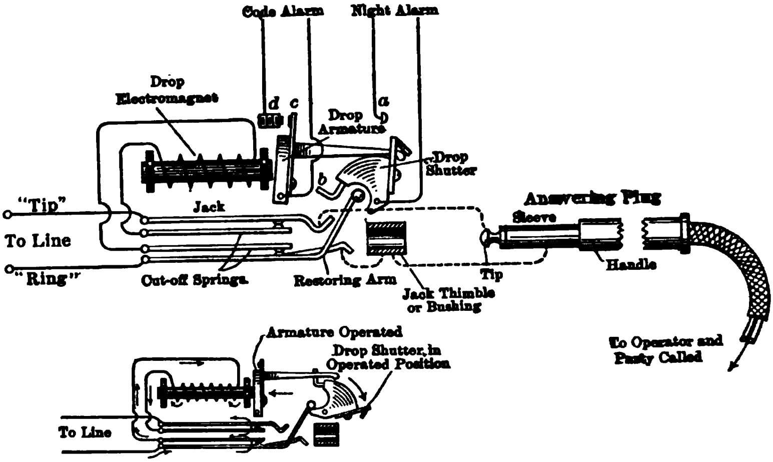


FIG. 379. A combined jack and line drop.

After the connection with the called party has been established, the *cord circuit* between the two corresponds to one of the diagrams in Fig. 380. The incoming voice currents may be made to pass to the other line either directly, as in the lower sketch, or inductively through two *repeating coils*, or transformers, as shown in the upper sketch (§ 593).

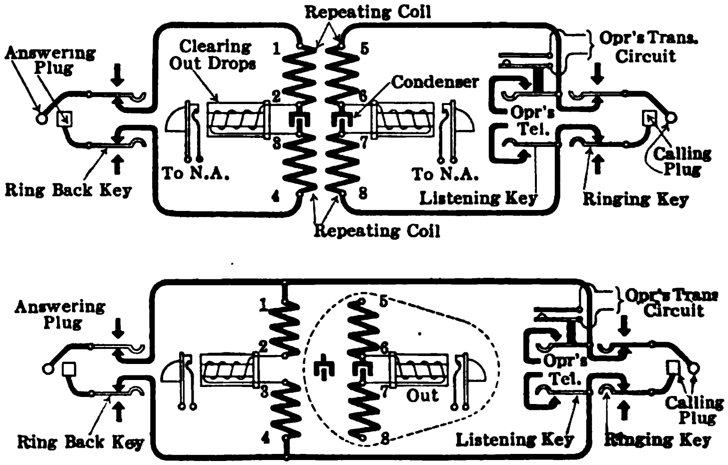


FIG. 380. Two types of cord circuit, direct metallic below and with a repeating coil above.

The lower arrangement is an ideal one from the point of view of the efficiency of transmission, and is sometimes referred to as a "zero loss" connection. It is satisfactory for quiet lines maintained at a high standard, and is used for making long-distance connections. However, under ordinary conditions of rural and town lines, an inductive connection between lines has been found preferable for the following reasons:

(a) If a line is noisy, because of inductive interference or for some other reason, the noise is not fully transmitted through the repeating coils. (b) A grounded line can be connected to a full metallic line without subjecting the latter to the ground. (c) With the ordinary bridged clearing-out drop in the cord circuit of a magneto switchboard, it is possible to tie up a connection between two grounded lines, provided one of the lines is reversed in the jack or cord. The condition then existing is a short-circuit around the clearing-out drop winding, and prevents other subscribers from ringing off. With the repeating-coil type of cord circuit, a reversal of this kind does not prevent the clearing-out drops from being operated. (d) A circuit with repeating coils provides a better arrangement of clearing-out drops, as is explained below. For all these reasons it is considered better practice to connect lines "inductively" rather than "conductively."

Repeating coils used in this connection must be so designed as to be good transformers for high-frequency voice currents and poor transformers for low-frequency ringing currents. The latter condition is essential when a subscriber rings off. His ringing must not be transmitted to the connected line, as it might be interpreted by one of the subscribers on that line as a call. At the same time, the impedance of the coils must be such that a sufficient voltage is established between the condenser terminals to operate the clearing-out drops. Such repeating coils are known as non-ring-through, positive ring-off type.

It will be seen that the use of repeating coils makes double supervision possible, with two clearing-out signals. If only one subscriber has given his ring-off signal, the operator connects her listening key and ascertains whether the other subscriber is waiting for another connection. With the metallic connection shown in the lower sketch, there is but one supervisory signal per cord circuit, and the above advantage is lost.

The connections of the listening and ringing key in its normal position are shown in Fig. 380, to the right. Throwing the lever to the right connects the calling cord to the signaling circuit. Throwing it to the left connects the operator's telephone for listening and talking. The latter connections are shown in more detail in Fig. 381. The terminals marked *5* and *8* are the same as in Fig. 380.

The ring-back key shown in Figs. 378 and 380 is used to signal the calling subscriber, should he hang up the receiver before the connection has been completed, or should the other party request his being called up again before the plugs have been pulled out. Without this ring-back key, it would be necessary to reverse the plug, since the regular ringing key is on the other side of the cord circuit.

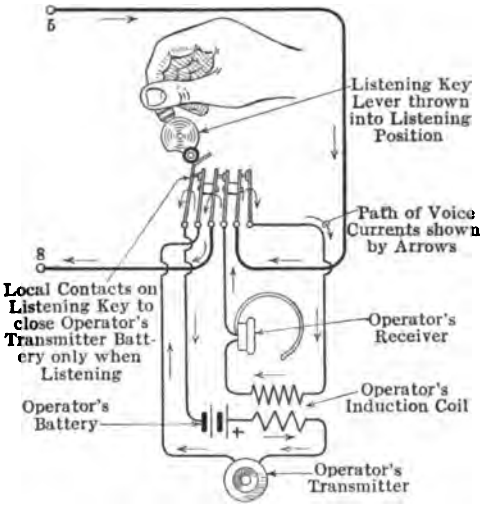

5
Listening Key Lever thrown into Listening Position
Path of Voice Currents shown by Arrows
8
Local Contacts on Listening Key to close Operator's Transmitter Battery only when Listening
Operator's Receiver
Operator's Induction Coil
Operator's Battery
+
Operator's Transmitter

FIG. 381. Connections between the cord circuit and the operator's telephone set.

615. Additional Features of a Magneto Switchboard. — Operating experience has indicated the desirability of some additional features which either improve the service or contribute to the operator's comfort and efficiency. The most important of these features are as follows:

(1) *Night alarm.* In many small installations it is not necessary for the operator to be constantly at the switchboard during the night or during certain hours of the day, when the traffic is very light. There is usually a bell which can be made to ring after a line drop has been operated by a subscriber. In Fig. 379 the local bell circuit is closed between points *a* and *b*. The bell stops ringing when the operator inserts the answering plug into the jack, restoring the drop. The bell circuit is controlled by one of the keys shown on top in Fig. 378, and is not used when the operator is regularly at the board.

(2) *Code alarm.* On party lines on which code ringing by the subscribers themselves is used (§ 600), the operator is not expected to answer every time the line drop operates, because the call may be for another party on the same line. Ordinarily the operator recognizes the code by the rattling of the drop armature, but at night or on Sundays, when she is away from the switchboard, a louder indication is necessary. For such cases there is a buzzer whose circuit is closed through a slow-acting relay, which in turn is operated by a contact on the drop armature. The operator can hear the buzzer repeating the

code, and she comes to the board only when her signal is given. In Fig. 379 the relay contact is closed between points *c* and *d*.

(3) *Hand and power ringing.* In offices operating a considerable number of lines, power generators are used for ringing, and the hand magneto is provided for emergencies only. A special key on the switchboard enables the operator to connect the cord circuits at will to either source of ringing current.

(4) *Buzzer for outgoing signals.* An operator can handle calls most efficiently when she is enabled to recognize the presence of trouble. As an aid in this direction a low-resistance buzzer is included in the generator circuit. The operator soon becomes familiar with the intensity of sound from the buzzer on various types of lines under normal conditions, and is often enabled to detect trouble, even before a subscriber has discovered its presence. Among the uses of this buzzer may be mentioned the following: (a) indication of open line or burned-out fuse; (b) absence of generator current; (c) indication of a short-circuit; (d) aid to operator in ringing code signals on party lines.

(5) *Position switching keys.* A switchboard consisting of two or more positions requires facilities which enable an operator on any position to handle the cords in adjacent positions, as well as her own, during periods of light load. Operators' circuits on adjacent positions may be interconnected or rendered independent by means of position switching keys.

(6) *Transfer circuit.* On switchboards consisting of three or more positions it is impossible for each individual operator to reach all line jacks with a single pair of cords. Under such conditions it becomes necessary to install transfer circuits which serve as extension links from each position to other points on the switchboard where the connections can be completed. The standard transfer circuit terminates in two jacks installed in widely separated positions. In handling a transferred connection, an operator receives and answers a signal in the usual manner. Learning that the line requested is located beyond her reach, she inserts the associated calling plug into a transfer jack giving access to the operator before whom the desired line appears. She then requests that operator to connect the specified transfer circuit with the desired party. A regular cord circuit is used by the second operator to complete the connection. Instructions for completing transferred connections are most satisfactorily passed between operators in a verbal manner on boards not exceeding three or four sections.

(7) *Party-line ringing key.* When selective party ringing is used (§ 600) a special master key is provided for making the proper connections between the cord circuit and the source of signaling current.

616. Ringing Equipment in Telephone Offices. — In addition to hand magnetos and magneto power generators for ringing subscribers from the central office, the regular rotary types of 20-cycle A.C. generators (Chapter XIX) are used in the larger exchanges. Practically all of the medium-sized and smaller exchanges now use the vibrating pole-changer type of A.C. ringing-current generators. When the harmonic or frequency method of selective signaling is used, the central office employs harmonic converters which are vibrating pole-changers adjusted to give the several desired frequencies of ringing current. It is customary to use 20-cycle alternating current for ringing the bells on individual lines, as well as on bridged magneto lines. When the regular harmonic frequencies are available for ringing, the 33-cycle current is used for single-party city lines, which have the ordinary polarized ringer in the telephone. For bridged rural lines, the 33-cycle current is too fast, and a good clear ring on all the bridged bells can be obtained only with the 16-cycle frequency.

The following table gives the various ringing equipments in use; the choice in an individual case depends upon the size of the exchange, the magnitude and the importance of the traffic, and other local conditions.

I. For Straight Ringing Systems

(*a*) Pole-changer type converter deriving its power from a battery of either dry or storage cells.

(*b*) Direct-ringing vibrator converter for use where a commercial 60-cycle alternating-current circuit is available. This apparatus reduces the frequency of the current from 60 to 16 or 20 cycles and the voltage from 110 to 90 or 100 volts.

(*c*) Power generator consisting of a magneto-type alternating-current generator equipped with a pulley for belt connection to a water-motor, gas engine, line shaft, or other rotary machinery.

(*d*) Direct-ringing static transformer for use with a 25-cycle alternating-current power circuit. It steps down the line voltage to 90 or 100 volts, the frequency remaining unchanged.

(*e*) Ringing motor-generator set for use where either an alternating or a direct-current power circuit is available. It consists of a motor of proper type direct-connected to an alternating ringing-current generator mounted on a common base.

II. For Harmonic Ringing Systems

(*f*) Harmonic converter operating from either dry or storage battery, preferably the latter.

(*g*) Composite multi-cycle motor-generator set operating from either a direct or an alternating-current power circuit and delivering ringing currents at the four standard voltages and frequencies.

Pulsating positive and negative currents for selective party-line systems using " biased " ringers can be generated by means of any of the equipments (*a*), (*b*), (*c*), and (*e*), when special attachments are used.

A pole-changer converter consists of an electrically driven vibrator which carries contact springs arranged to reverse a 24-volt direct current from 16 to 20 times per second, thus giving a 24-volt alternating current at a frequency of 16 to 20 cycles, depending upon the adjustments. The alternating current so obtained passes through a transformer and is stepped up to 90 volts or thereabouts.

In the converter mentioned under (*b*) above, the 60-cycle alternating current is first changed to direct current by means of two vibrating rectifiers. This rectified current is, in turn, changed to a 20-cycle alternating current by a pole-changing element and sent through a transformer which delivers the alternating current at 90 volts and 20 cycles. Terminals are also furnished for the delivery of 100-volt 20-cycle current. The pole-changer element is provided with extra weights for the vibrator, so that the frequency can be decreased to 16 cycles per second, if desired.

A harmonic converter is a further development of the pole changer mentioned above. The reversing of the current the required number of times per second is accomplished by means of an electromagnetically driven vibrator carrying the necessary contact points. Each vibrator is tuned and adjusted to a single rate of vibration by means of reed springs and pendulum weights. Condensers are connected across the contact points to reduce sparking. The alternating currents which are obtained are stepped up by suitable transformers.

The composite multi-cycle alternator, used in large exchanges, has four armature windings. A well-known machine of this kind runs at 1000 r.p.m. and delivers 50 watts from each winding at 16⅔, 33⅓, 50 and 66⅔ cycles per second, and at 75, 100, 135, and 175 volts respectively. It can be driven either by a direct-current motor or by a single-phase alternating-current motor, as required. Direct-current motor sets are direct-connected and equipped with a speed governor; alternating-current motor sets require no speed governor but are equipped with a direct-current exciter geared to the generator. Machines of this type are more economical to operate and to maintain than the older four-generator type, as one revolving field is used instead of four. The no-load watt input is reduced quite materially because of the reduction in the number of bearings, in power losses, and in brush friction.

617. EXPERIMENT 28-D. — Operation of a Magneto Telephone Switchboard. — The purpose of the experiment is to become familiar with the construction of a magneto switchboard and with the operator's duties (§ 613). The details of the electric circuits are taken up in the next experiment.

(1) Inspect the available switchboard in detail and note differences, if any, between it and the one described in the text. Count the number of jacks and cord circuits actually installed, and the number for which space is provided.

(2) Operate the switchboard with a few connected lines, under normal and abnormal conditions, varying line resistance, its insulation, the number of bridged stations, etc.

(3) Measure the transmission loss in the cord circuit (§ 597).

(4) Make clear to yourself the functioning of the features described in § 615.

618. EXPERIMENT 28-E. — Wiring a Magneto Telephone Switchboard. — The purpose of the experiment is to study the connections described in §§ 614 and 615. The operation of a finished switchboard is the subject of the preceding experiment. It is sufficient to have two line drops and jacks, and one cord circuit, since all the circuits are identical. The separate devices should be mounted in an accessible manner and provided with clip terminals, necessitating no set-screws or soldering. The student is expected to wire up all the connections, shown in Figs. 379 to 381, and to check them by actually performing the regular operations, beginning with an incoming call and ending with the pulling-out of the plugs after the disconnect signals.

619. The Relative Transmission Efficiency of Various Cord Circuits. — A magneto switchboard circuit should possess the following characteristics in order to assure the greatest satisfaction alike to the subscribers and to the telephone company:

(1) The circuits should introduce the smallest possible loss in voice currents when two lines are connected together.

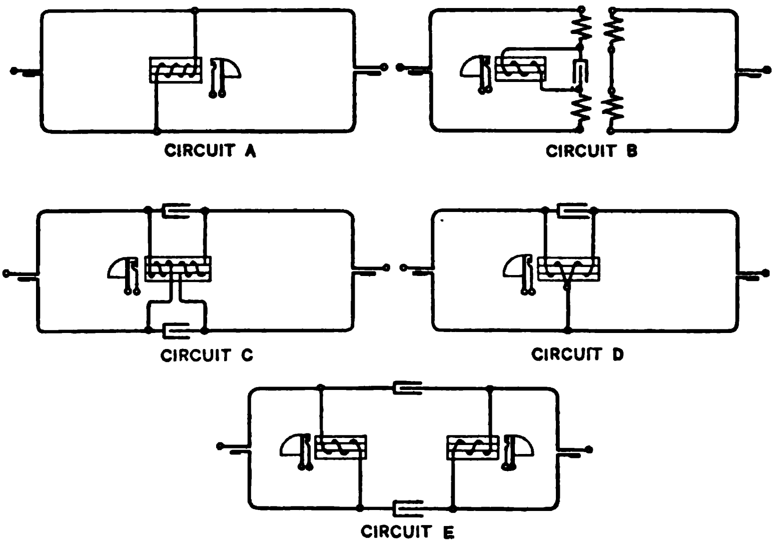
(2) The circuits should be non-ring-through; in other words, when either of the two connected parties signals for a " disconnect " by " ringing-off," the cord circuit should not permit the passage of signaling current to the connected line in such volume as to cause a false ring on that line.

(3) The clearing-out signals in the cord circuits should respond positively under all operating conditions in order that subscribers' lines may be quickly released after finishing conversations.

(4) Each cord circuit should be preferably equipped with two sepa-

rate clearing-out signals, so that the operator may know at a glance whether the parties are still talking, or have finished talking, or one has hung up and the other is awaiting further service.

Besides the standard circuit shown in the upper sketch in Fig. 380, a number of other cord circuits are used or have been used. Some of these are shown in Fig. 382, and since they are somewhat less expensive than the one indicated in Fig. 380, it is important to know their relative efficiency for voice transmission, and the certainty of response of the clearing-out signals.



CIRCUIT A

CIRCUIT B

CIRCUIT C

CIRCUIT D

CIRCUIT E

Fig. 382. Various cord circuits with clearing-out drops.

For the determination of the first of the four above-mentioned points, the standard circuit (§ 597) can be used. The second and third points can be tested directly, by means of lines and bridged instruments of various characteristics. The importance of the fourth point can be ascertained by operating two circuits, one with a single signal, the other with a double signal. It will then be found that, in certain circumstances, the carelessness of a subscriber or the peculiar conditions existing on the line will lead to confusion or delay if a single supervisory signal is used.

620. EXPERIMENT 28-F. — Comparative Study of Magneto Cord Circuits. — The purpose of the experiment and the methods of testing are described in the preceding section.

CHAPTER XXIX

CENTRAL-ENERGY TELEPHONE SYSTEM

621. THE above-described magneto telephone system has the following drawbacks:

(1) Dry cells in subscribers' instruments require renewals.

(2) Turning the crank of a magneto in order to call the operator is too much trouble for a busy person.

(3) The necessity of "ringing off" at the end of a conversation places an additional burden on the user and makes the efficiency of the service depend somewhat upon his act or omission.

These inconveniences have gradually led to the introduction of common-battery or central-energy telephones, and at the present time practically all city and suburban exchanges of any importance are so equipped. A centrally located storage battery has the following advantages:

(1) There are no batteries in subscribers' instruments to be looked after.

(2) The very act of taking the receiver off the hook lights a lamp in front of the operator, notifying her that a connection is desired.

(3) The act of hanging up the receiver causes a supervisory lamp to light in front of the operator, notifying her that the line is to be disconnected.

622. Connections in Subscriber's Set. — The standard connections of subscriber's apparatus are shown in Fig. 383. When the receiver is hanging on the hook, only the ringer circuit is closed, being bridged across the line in series with a condenser (§§ 599 to 601). Thus the bell can be rung from the central to call the subscriber.

When the receiver is lifted, the hookswitch closes the transmitter circuit, allowing the battery current from the office to pass through it. At the same time the receiver is bridged across the transmitter, in series with the condenser. Incoming voice currents, passing through the induction coil, produce secondary currents in the local circuit consisting of the receiver, condenser, and transmitter.

Rapid fluctuations in the battery current, due to talking into the transmitter, cause corresponding alternating currents to flow in the local receiver circuit and are transmitted to the line through the induction coil. Thus, variations in the resistance of the transmittter cause talking currents directly, and also indirectly through the induction coil. For this reason, this particular circuit is known as the *booster circuit*, the induction coil intensifying outgoing voice currents in the line. This results in an improved speech transmission.

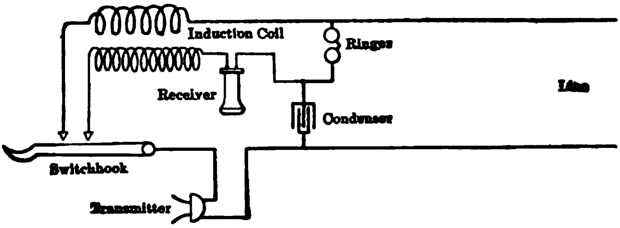


FIG. 383. Connections in a central-battery telephone set (booster circuit).

With the connections shown in Fig. 384, the receiver circuit has no metallic connection with the transmitter, and therefore exerts no booster action. This circuit is being discontinued in favor of the one shown in Fig. 383.

A booster-type instrument may at times be disagreeable to use, because of side-tone in the receiver, especially on long-distance connections when a higher battery voltage is used in the central. This is

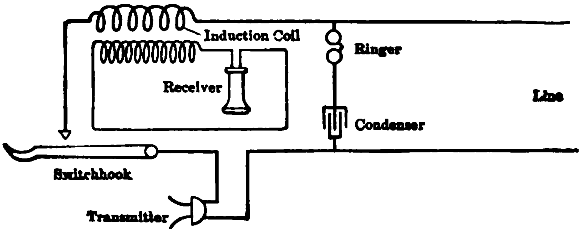


FIG. 384. Connections in a central-battery telephone set (separate receiver circuit).

due to the nature of the connections, the receiver being shunted around the transmitter. The speaker hears his own voice in the receiver and also any external noises that affect his own transmitter, as well as noises caused by the transmitter itself. There is but little side-tone with the circuit shown in Fig. 384.

There are two ways of overcoming this objectionable side-tone: one is to use a push-button for short-circuiting the receiver when talking to a distant subscriber, and the other is to hold the receiver a slight distance away from the ear when talking. The latter scheme is the most practical, as only one hand need be used and there is less danger of losing some of the incoming transmission.

623. Battery Connections. — The common battery in the exchange must be so connected as to supply the necessary direct current for the subscribers' transmitters; at the same time it must not act as a bridged path of low impedance for high-frequency "talking" currents, which would be lost to the receiving station.

The arrangement shown in Fig. 385 for separating direct current from "talking" currents is based on the action of inductance and capacitance. An inductance offers an easy passage to direct current, but effectively prevents the passage of high-frequency currents. Conversely, a condenser offers an infinite resistance to the passage of direct current, but is a comparatively easy path for high-frequency alternating currents (§ 595). Condensers are interposed in the cord circuit, and retardation coils (in the form of supervisory relays) are used in series with the battery. The impedance offered by the windings of these relays prevents the high-frequency talking currents from following the shunt paths across the lines through the battery. With this arrangement, direct current flows from the battery to the substations through the corresponding relays, while talking currents find a direct path from subscriber *A* to subscriber *B* through the condensers. The relays provide a supervisory signal to indicate when the subscriber has placed his receiver on the hook, as is explained later.

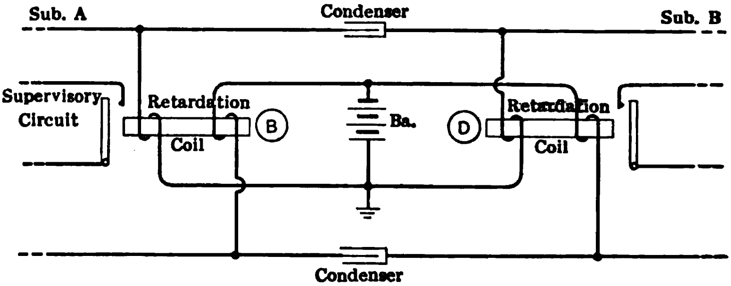


FIG. 385. Battery connections with retardation coils.

It is necessary to subdivide the relay windings in order to maintain the balance of the two line wires to ground. Otherwise, induced currents from sources external to the line itself, flowing through the two line wires, would have unequal paths to ground and would cause noise disturbances in the telephone circuit.

With the circuit shown in Fig. 386, the battery current is separated from the voice currents by means of a transformer, or repeating coil. The battery current flows to subscriber *A* through section *a* of the repeating coil and returns through section *b*. Similarly, sections *c* and *d* are in the battery circuit of subscriber *B*. By following the directions of the currents it will be found that the coils *a* and *b* magnetize the core in the opposite direction to that of *c* and *d*, so that any permanent flux in the core is due only to the difference of the two battery currents.

High-frequency currents from station *A* flow through windings *a* and *b*, and through the battery. The latter offers practically no impedance to voice currents and acts simply as a short-circuit between points *f* and *g*. A secondary voice current is induced by the primary current, through the iron core of the repeating coil, and circulates in the circuit of subscriber *B*, through coils *c* and *d*, and the battery. Thus, the two talking circuits have only a small part in common, namely that represented by the whole storage battery of negligible impedance. The supervisory relays are shunted by non-inductive resistances, r_1 and r_2, which offer but little impedance to high-frequency currents. Each winding of the repeating coil is divided into two sections for the same reason as that in Fig. 385.

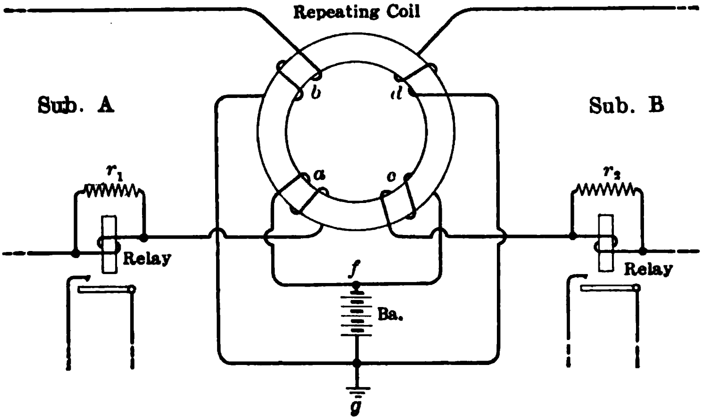


FIG. 386. Battery connections with a repeating coil.

From a transmission standpoint, the advantages and disadvantages of the two arrangements are comparatively unimportant. In cases where it is necessary to connect a metallic circuit to a grounded-line subscriber, a repeating-coil arrangement is generally preferable, because it permits a better line balance to be maintained in the metallic circuit.

The retardation-coil relay generally has a straight magnetic core construction, both to prevent a decrease in impedance to the high-frequency currents due to the action of the direct current, and also to permit the use of an armature. The repeating coil, while requiring an open magnetic structure to prevent lowering of the inductance due to the direct current, does not have the ordinary straight core, since a more efficient coil can be constructed for this particular purpose by using a ring-shaped core and providing it with the equivalent of a relatively short air-gap.

624. EXPERIMENT 29-A. — Subscribers' and Battery Connections in a Central-Energy Switchboard. — The purpose of the experiment is to obtain experience in connecting and testing the circuits described in §§ 622 and 623.

(1) Wire up two telephone sets, as in Fig. 384, and connect them to a storage battery of proper voltage, as in Fig. 385. Having obtained the best possible voice transmission, vary the individual factors, viz., the ratio of turns in the repeating coil, the line resistance, the battery voltage, the capacitance of the condensers, the resistance and reactance of the retardation coil, etc., *one factor at a time.* Determine the effect of each factor upon the efficiency of voice transmission, either qualitatively or by comparison with the standard circuit (§ 597).

(2) Leaving the battery circuit as before, change the receiver connections to those shown in Fig. 383. Find the effect, if any, on voice transmission. Compare the amount of "side-tone" in the two circuits by producing external noises and by using a noisy transmitter.

(3) Connect the battery as in Fig. 386, and determine the effect of different factors, as before.

Report. Describe in detail the effect of varying the different factors. In each case give a theoretical explanation of the observed facts.

625. A Simple Central-Energy Switchboard. — The general appearance of a telephone switchboard for a common-battery system is similar to that shown in Fig. 378, except that small incandescent lamps are used for line and disconnect signals. No dry cells are needed, and the magneto or other source of ringing current (§ 616) is usually power-driven.

In a large modern telephone exchange, laid out for local calls, for trunking to other city exchanges, and for long-distance calls, the electrical connections are quite complicated, and only a few simple diagrams are given below to illustrate the most essential functions. It is more important that the reader understand the functions which such a system is called upon to perform, than the actual details of electrical connections of lamps, relays, etc. The connections are subject to many variations, while the functions to be performed are practically the same in all exchanges.

As an introduction to the subject, a simplified diagram of connections is shown in Fig. 387 and may be studied first. Such connections may be used in a small private-branch exchange, or in a single office where each line terminates in one jack only, without "multiple jacks."

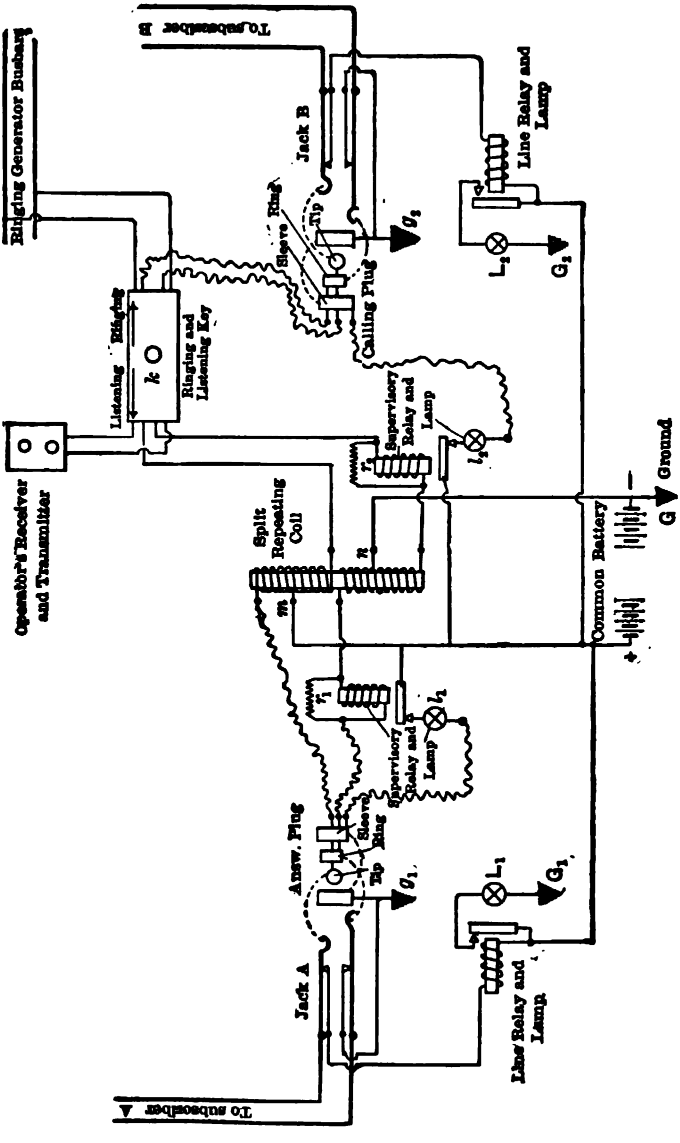


FIG. 387. Connections between two subscribers in a small central-energy exchange.

When subscriber *A* takes his receiver off the hook, the line lamp L_1, placed over his jack, lights up, calling the operator's attention to the fact that a connection is desired. She inserts one of her answering plugs into the jack *A* presses the listening key *k* to the left and asks the subscriber what number he desires. The number being given, she inserts the calling plug of the same cord circuit into the jack of subscriber *B*, who is to be called, and presses the key to the right, thereby ringing the bell of subscriber *B*.

As soon as the calling plug has been inserted into the jack *B*, the supervisory lamp l_2 lights up, and remains lighted until subscriber *B* takes his receiver off the hook. Now the two subscribers may converse and all the switchboard lamps corresponding to these subscribers are extinguished; the connection is indicated by the inserted plugs. As soon as one of the subscribers hangs up his receiver the corresponding supervisory lamp lights up; when both supervisory lamps, l_1 and l_2, are lighted, the operator knows that both receivers have been hung up. She pulls out the plugs and thereby disconnects the stations. If for any reason it is necessary for her to listen to the conversation, she may do so by pressing her listening key *k* to the left.

The battery connections in Fig. 387 are similar to those in Fig. 386, except that the repeating coil is shown with a straight core. The four sections of the winding are shown in a purely conventional way; the correct relative directions in which they are wound are the same as in Fig. 386. The ringing and listening key is similar to the one shown in Fig. 381. With the receiver off the hook, the current from the positive pole of the common battery flows through the line relay under the jack *A*, through the line, through the subscriber's transmitter, back to the station, and to the ground g_1. The circuit is thus completed, since the negative pole of the battery is permanently grounded. The relay attracts its armature and closes the circuit of the line lamp L_1, whereupon the lamp lights through the ground connection G_1. Inserting the answering plug into the jack *A* opens the relay circuit through the auxiliary springs, shown inside the main contacts, and the lamp is extinguished.

The plugs used with this system have three contacts, referred to as the tip, the ring, and the sleeve. The talking circuit is completed through the tip and the ring, while the sleeve contact connects the supervisory lamp l_1 to the ground g_1. Its circuit is opened by the armature of the supervisory relay shown above the lamp; it is also open when the answering plug is not in the jack *A*. The supervisory relay is inserted in the main line circuit and holds the lamp circuit open as long as direct current is flowing through the line *A*; in other words, as long as the receiver is off the hook. When the receiver is replaced, the supervisory lamp lights up. The operator then pulls the answering plug out of the jack, thereby breaking the connection to the ground through the sleeve contact, and the lamp goes out

The coils of supervisory relays are usually encased in a copper sheathing, so that they can be placed close together without "cross-talk" due to mutual induction between two talking circuits.

Selective ringing. Party-line ringing is accomplished either by ringing to the ground or by using alternating currents of different frequencies and tuned bells (§ 600). With this "harmonic" system of ringing, bells are bridged across the line without ground connections. There are some objections to the use of the ground in selective ringing, in addition to the cost of installation and maintenance. All such systems require that the telephone bells be connected to definite sides of the line, and any reversal between the bell and the jack contacts in the exchange interferes with the selective arrangement. Furthermore, if the party who is being called takes his receiver off the hook, ringing current passes through this low-resistance bridge to the opposite side of the line, and under certain conditions of line resistance a bell on the opposite side of the line may be unintentionally rung. In order to prevent this trouble, the opposite side of the line is grounded by the ringing key; but if the line is fairly long, or the ground at the telephone is not of low resistance, a sufficient amount of current may nevertheless be shunted through the ringer to operate it.

Condensers are used in series with harmonic ringers, so that this system is particularly suitable for central-energy telephones. It is not possible to use a condenser in series with a positive and negative pulsating system where the so-called "biased ringers" are used. In some cases very high non-inductive resistances are placed in series with these biased bells so as to prevent leakage of the battery current and still allow the positive and negative pulsations to act directly on the ringer winding. This is not a satisfactory system, for if the resistances are made sufficiently high to prevent leakage and not to interfere with the operation of the central office relays, the ringers will not receive a sufficient amount of ringing current for good clear operation. This leakage difficulty is overcome by installing a special alternating-current relay in each telephone, connecting its winding in series with a condenser and bridging the two across the line. When positive or negative pulsating current is projected over either side of the line, it passes through the relay windings and back on the other side of the line, actuating all of the relays and, through contacts on the latter, connecting all the biased ringers to ground. Then one of the ringers is operated to correspond to the particular ringing current used. As soon as the ringing current is removed from the line, all the relays return to normal so as to disconnect the ringer windings from ground. This system works well in practice, but requires an extra piece of apparatus in the telephone and retains the ground connections.

626. EXPERIMENT 29-B. — Wiring a Small Common-Battery Switchboard. — The student is given the necessary jacks, lamps, relays, cords, etc.; also a few subscriber sets, and an operator's telephone. The experiment consists in connecting up and operating the switchboard, as described in § 625 above. Having obtained a satisfactory working of the system, the student should determine the amount of time necessary for making a connection, from the instant when the first subscriber takes his receiver off the hook to the instant when the second subscriber's bell begins to ring.

Report. Give rough sketches of the devices used, and the exact diagram of connections employed, at least where it differed from that shown in Fig. 387. Give a diagram of connections of the operator's set. Mention the adjustment of relays, etc., if any.

627. Multiple Jacks and the Duties of A and B Operators. — In a large office each outgoing line terminates in more than one jack (Fig. 388). One of these jacks is associated with a line lamp and the necessary relays, and is called the answering jack. The other jacks usually have no such equipment and are known as multiple jacks (see, however, § 630, under " operator's bar "). An operator has within her reach a limited number of answering jacks and a large number of multiple jacks for calling subscribers. When a call comes, she inserts an answering plug in the corresponding jack and, having ascertained the number desired, she touches the thimble (sleeve) of the multiple jack of that number with the tip of the calling plug. If the line is busy she hears a click in her receiver and notifies the calling party. Otherwise she inserts the plug and rings the desired party. Thus, with multiple jacks an operator is able to connect a great many more subscribers than those for whom she has line lamps and relays.

In very large offices it would be too expensive, or even impossible, to duplicate all the jacks within the reach of each operator. In such cases the operators are divided into two classes, class *A* and class *B*. The *A* operators have before them answering jacks and as many multiple jacks as space will permit. The positions of the *B* operators are provided only with multiple jacks and are connected with those of *A* operators by means of *trunking lines* and call or *order wires*.

When an *A* operator receives a call for a subscriber for whom she has no multiple jack within her reach, she connects her telephone to a *B* operator's telephone over one of the order-wire circuits, and gives her the number desired. The *B* operator connects an idle trunk with the desired line and at the same time reports back to the *A* operator the number of the trunk used, whereupon the *A* operator connects the calling plug to that trunk. The busy test is made by one or the other operator according to the type of connections used. At the end of the conversation, when the *A* operator releases the trunk by removing her plug, a supervisory lamp lights in front of the *B* operator and she disconnects the called line.

The traffic between offices in the same local-service area is usually carried on by similar means. The *A* operator in the calling subscriber's office informs a *B* operator in the called subscriber's office of the desired connection, and gets a trunk number assigned to her, over which the call is completed.

The cord circuit equipment used by *A* operators consists of two cords equipped with plugs which are used for connecting together two subscribers, or in connecting a subscriber with an inter-office trunk. The inter-office trunks usually terminate at the *B* operator's position in single cords equipped with plugs.

In large metropolitan areas it is quite common to omit all subscribers' multiple jacks at the *A* positions and to handle calls to subscribers in the same office through *B* operators. This method is more economical, especially in offices where a large per cent of the calls are for subscribers in other offices, in which case the multiple jacks at the *A* positions would be used infrequently.

In large multi-office local areas, where each office must have trunk connection with each other office, it is frequently found more economical to operate *A*—*B* trunks in series or in tandem, as it is called, than to provide each office with direct trunks to each other office. Calls to groups of offices are then routed through one of the offices which is centrally located with respect to the others in the group. That office is provided with *B* operator's position equipment, which is arranged for connecting incoming call-circuit trunks, known as tandem trunks, with outgoing call-circuit trunks, known as completing trunks. Such a *B* operator's position is known as a tandem *B* operator's position, or simply as a tandem position.

The answering jack in Fig. 388 differs from that in Fig. 387 in that it has no auxiliary springs for extinguishing the line lamp. When the receiver is taken off the hook, the circuit is completed through the line relay, so that the line lamp is lighted. When the operator inserts an answering plug she thereby connects the sleeve of the jack to the live (ungrounded) pole of the battery. A current then flows through the winding of the cut-off relay; this relay becomes energized and opens the circuit of the line relay, thereby extinguishing the lamp. Thus the cut-off relay takes the place of the inner contacts in Fig. 387.

The following two additional features are indicated schematically at the bottom of Fig. 388, and similar features are shown more in detail at the bottom of Fig. 389. (a) A *night-alarm relay* can be used in connection with the line lamp, to call the operator by an audible signal, if she is not required to be on duty regularly during certain hours. (b) For each horizontal row of line lamps in front of the operator, there is a larger *pilot lamp* which lights when any of the lamps in that row is lighted. This makes the supervision of the incoming calls easier.

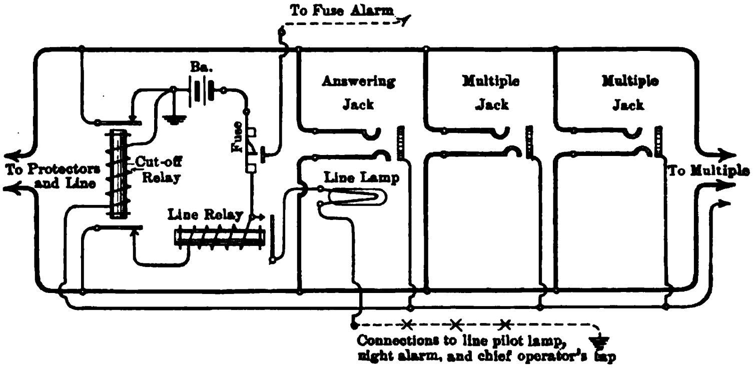


FIG. 388. Multiple jacks, and a line lamp with its relays.

628. Cord Circuit with Automatic Features. — A cord circuit embodying a number of automatic features (absent in Fig. 387) is shown in Fig. 389. These features are made possible by the use of several relays marked alphabetically with the letters A to H, and by the other relays shown on the top and bottom of the diagram. The windings of these relays are permanently connected on one side either to the live terminal of the battery or to the ground, so that a relay is energized when its circuit is closed on one side only.

One and the same storage battery is used with all the relays, and separate batteries are shown in the diagram only to simplify the connections on paper. In the description of the connections below, the live terminal of the battery is referred to as Ba(A), Ba(B), etc., the letter in the parentheses indicating the particular relay with which the battery is shown associated. The ground connections are similarly denoted Gr(A), Gr(B), etc. The contact points of each relay are marked with the corresponding small letters. Thus, the contacts of relay A are denoted by a_1, a_2, a_3, etc. The numbers marked at the relays give the resistance of their windings in ohms.

The battery circuit is identical with that shown in Fig. 385. The calling subscriber's transmitter is supplied with battery current through the retardation coil *B*, which also serves as a relay. The called subscriber's telephone is connected through the retardation coil *D* and contact point g_5 on the ring side, and through the same coil and contacts, g_2 and c_5, on the tip side.

Automatic listening. When a subscriber removes his receiver from the hookswitch to call the central, the line lamp lights, as explained in § 627. The operator plugs the answering cord (Fig. 389) into the jack, and is thereby automatically brought in talking connection with the calling subscriber, no listening key being necessary, except for recall (§ 630). This saves time and effort in making connections. As soon as the answering plug has been inserted, a current flows from Ba(A) through the high-resistance winding HR of relay A, through the sleeve and the cut-off relay (Fig. 388) to the ground. The " make " contacts of A are thereby closed. The relay B is also energized by the battery current flowing to the subscriber's line and the " make " contacts of B are closed. A current path is therefore provided through Ba(E), winding E, and contacts c_2, b_3, and a_2, to the ground Gr(A). The relay E is energized and the contacts e_2 and e_3 are closed, connecting the operator's telephone across the answering cord.

Secret service. When the operator plugs the calling cord into the line desired, the relay C is energized through the sleeve and the line cut-off relay (Fig. 388) to the ground. The contact c_2 is broken, relay E is deënergized, and the operator's set is disconnected from the cord circuit. The operator is thus prevented from listening while a conversation is going on. An emergency listening key is described in § 630.

Double lamp supervision. As long as current is flowing into the calling subscriber's line, the contact b_1 is broken and the supervisory lamp L_1 is dark. When the receiver is replaced on the hook, a circuit is established from the live pole of the battery through L_1, b_1, a_2, to the ground, and the lamp is lighted. When the operator removes the plug, the contact at a_2 is broken and the lamp is extinguished. The circuit of the supervisory lamp L_2 of the called subscriber is through the contacts g_4, d_1, and c_3 to the ground. When his receiver is off the hook, the contact d_1 is open and the lamp is dark. It lights when the receiver is hung up, and is extinguished when the calling cord is disconnected, because then the contact at c_3 is opened.

629. Automatic Ringing. — The cord circuit shown in Fig. 389 is arranged for four-party harmonic ringing (§§ 600 and 616). Four sources of different frequencies are shown schematically on top, at *k*. Each cord circuit is equipped with four ringing buttons, and the operator

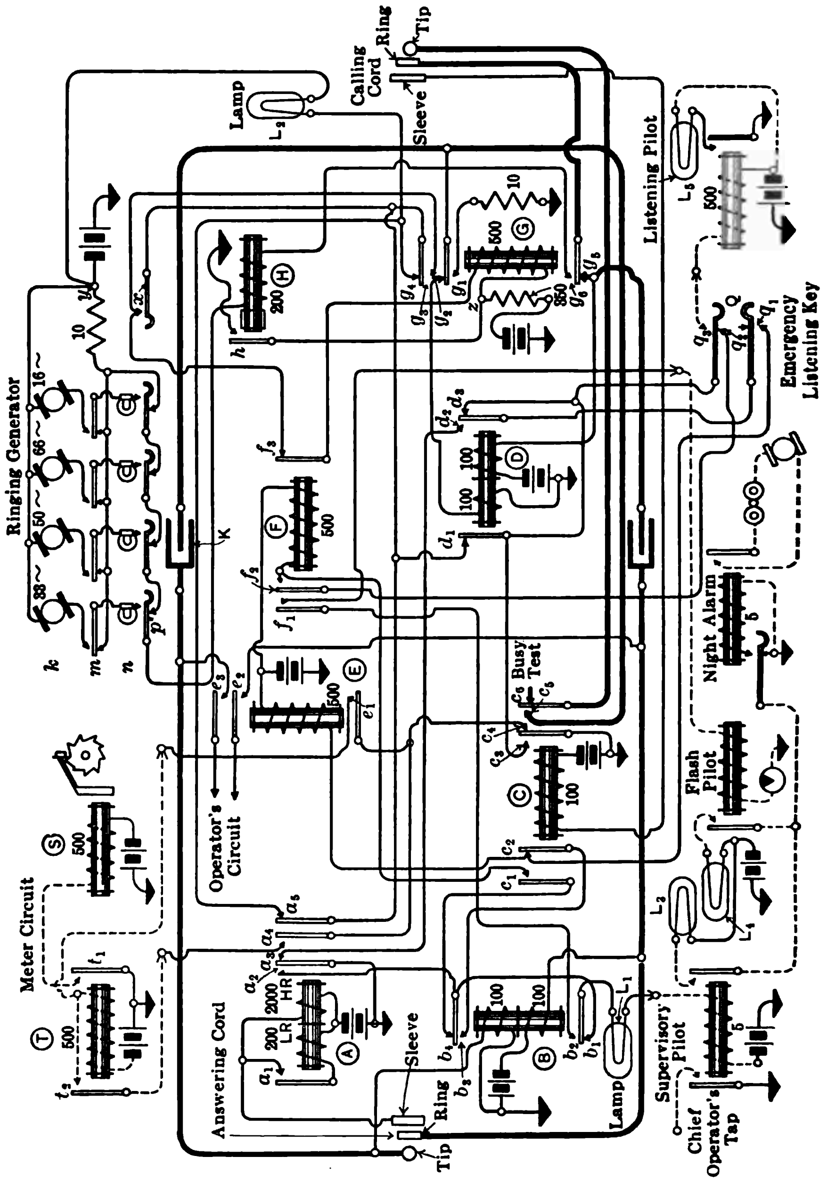


FIG. 389. A common-battery cord circuit with automatic features.

only has to press the proper button for an instant, after having plugged the called subscriber's jack. Intermittent ringing then goes on automatically until the subscriber answers, or until the calling subscriber abandons the call.

When the ringing key is depressed, its end-springs close the contact x, in the upper right-hand corner of the diagram. A circuit is established from Ba(G), through the winding of relay G, following the path f_3, x, d_1, c_3, and Gr(C). The relay G now closes its contact g_3, and the foregoing circuit between Ba(G) and Gr(C) is maintained after the operator releases the contact at x.

Ringing current is now applied to the called line through the ringing generator, one of the interrupter springs m, lamp protection n, key contacts p, relay H, and contact g_6, to the ring side of the line. The tip side of the line is connected to the ground at G through the contacts c_5 and g_1. It will be noted that the upper terminals of the ringing generator are grounded through the battery. The relay H is inoperative on alternating current and responds to direct current only. The interrupter m is designed to apply the ringing current for one or two seconds and to leave the circuit open for four or five seconds.

Ringing tone. During the period of ringing on the calling cord, the subscriber on the answering cord is receiving a ringing tone which indicates to him that the operator is ringing his party. This tone is due to a small part of the ringing current shunted through his telephone. That is to say, from c_5 most of the ringing current goes to Gr(G) through g_1, but a small part is shunted through the condenser K into the calling subscriber's line and returns through Gr(B). This tone varies in intensity and pitch in accordance with the frequency of the generator current.

Instantaneous generator cut-off. As long as the transmitter circuit of the called subscriber is open, no direct current can flow into his line, because the harmonic ringer is in series with a condenser (Figs. 383 and 384). But as soon as he removes the receiver from the hook a metallic circuit is established from the point y in Fig. 389 through p and winding H, following the same path as the ringing current described above, except that in the subscriber's set the direct current follows the transmitter circuit and not that of the ringer. As mentioned before, the relay H responds to direct current only and the contact h is closed. This allows the current from Ba(G) to flow from point z directly to Gr(H), around the winding of the relay G. The latter is deënergized and the ringing circuit is broken. Thus, the ringing generator is automatically cut off the instant the subscriber answers the call.

The calling subscriber abandons the call. Frequently a subscriber who has called for another leaves the telephone before his party has answered. When he returns his receiver to the hookswitch, the relay B is deënergized, and a circuit is closed through the winding of relay F, following the path Ba(E), F, c_1, b_4, a_2, and Gr(A). The contact

f_8 in the ringing circuit is opened, stopping the ringing on the cord already connected to the subscriber called for.

Reverting call. In party-line service, a subscriber on a party line may wish to call another subscriber on the same line. There are also occasions when the operator has to call subscribers for routine service. Such service requires the use of the calling half of the cord circuit only. When a subscriber makes such a " reverting " call, the operator requests him to hang up his receiver, to enable her to ring the party to be called. This being done, the operator's automatic cut-in relay E will become deënergized, thereby causing the operator to resort to the emergency listening key for monitoring (supervising) the connection. The same key is used when the operator uses only the calling cord to call a subscriber.

The emergency listening key is shown in the lower right-hand corner of Fig. 389, and when operated it closes the contact q_1 which causes the relay E to be energized, so that the operator's circuit is placed across the cord circuit. After the calling party has replaced his receiver for the reverting-call operation, relay A will be normal. It will also be normal when only one cord is used. At the same time the relays G and C will be operated, while relays D and F will not be energized. The calling supervisory lamp will be lighted and will remain so until the party on the calling cord has answered, when the light will be extinguished.

The circuit is as follows: Beginning with point y of the battery at the calling supervisory lamp L_2, through a_5, d_1, c_3, and to the ground Gr(C). As soon as the party answers, the relay H will trip the ringing. Relay G thus becoming normal, provides a path for the operation of relay D, which in turn breaks the contact supplying ground to the calling supervisory lamp. In this manner the calling-cord lamp is extinguished.

630. Some Additional Features of Multiple Switchboard. — The cord circuit shown in Fig. 389 possesses some other features of interest, intended to improve and to expedite the service. Some of these features are described below.

Dark keyboard. Supervisory lamps are dark after intermittent ringing begins. The circuit of lamp L_1 is broken at b_1, that of L_2 at g_4. One of the chief advantages of a dark keyboard is the supervision obtained when a calling party abandons his call. In this case both supervisory lamps will light, indicating that a full disconnect is demanded.

The supervisory pilot relay, connected to the answering-cord lamp L_1, operates when a regular disconnect is indicated, and lights the pilot lamp L_3. Such a pilot lamp is common to several cord circuits and is so mounted as to attract the immediate attention of the operator.

The same pilot relay also actuates a night bell, when its key is in the operated position.

Automatic flash recall on the answering supervisory lamp may be used at any time after a connection has been completed, when the calling party again desires to attract the operator's attention. To do this, the calling party depresses the hookswitch lever *once*. This act causes the answering lamp, together with the flash pilot, to light intermittently, until the operator closes the emergency listening key Q or pulls down the calling cord. When the hookswitch is depressed, current flows from Ba(E) through F, c_1, b_4, and a_2, to Gr(A). The relay F thus being energized, another circuit is established from Ba(E) through F, f_2, q_2, c_3, and Gr(C). Thus, the relay F is now kept energized by its own contact, and current continues to flow through it when the subscriber lets the hookswitch go, thereby breaking the contact at b_4.

The flashing of the answering pilot lamp will now commence, the circuit being closed from the battery through the supervisory pilot relay, lamp L_1, contacts b_2, f_1, and the flash pilot relay, to the intermittent ground. The flash pilot lamp L_4 will also respond, as well as the night alarm, if its switch is closed. When the operator switches in her emergency listening key, the flashing circuit is opened at q_2.

Emergency listening key and pilot. Having received a recall signal, the operator switches in her emergency listening key Q, opening the contact q_2 and closing the contact q_1. The flashing stops and a current flows from Ba(E) through E, c_2, q_1, d_3 and c_3, to Gr(C). The relay E being energized, the operator's circuit is cut in and she can talk to the subscriber. The relay D must be deënergized in order that the contact d_3 may be closed. This means that the called subscriber must hang up his receiver before the emergency listening key becomes operative. This is to prevent direct listening-in on a conversation. With the listening key Q in the operated position, the contact q_3 is closed and the pilot lamp L_5 is lighted. This enables the supervising operator (monitor) to watch the use of the emergency listening key.

Operator's bar. In some cases two or more answering jacks in multiple (Fig. 388) are provided, each with a line lamp. In this case the first operator who happens to be at liberty answers the call, and thus the load is more uniformly distributed among the operators. With such an arrangement it is necessary to prevent two operators from securing a talking connection with the same line at the same time. This is done by means of relay A (Fig. 389) which is provided with a high-resistance winding HR and a low-resistance coil LR. As soon as the first answering plug has been inserted, the HR winding of the relay becomes energized through the sleeve and the line cut-off relay.

This causes the contact a_1 to close, connecting the *LR* winding in parallel with the *HR* coil. Should a second operator insert her answering plug in a multiple jack, the battery current through the *HR* coil of her relay *A* will be shunted by the low-resistance winding of the first relay, and not enough current will flow through it to cause its operation. Therefore the telephone set of the second operator will not be connected to her cord circuit, and she will be unable to communicate with the calling subscriber.

Meter circuit. In order to determine the load being handled on a given position, the cord circuits of each position are often provided with a common meter. The meter operates when the answering cord is inserted in a line across which a subscriber is waiting. It is released immediately when the operator inserts the calling plug. The meter and its two relays, *S* and *T*, are shown in the upper left-hand corner of Fig. 389. The relay *T* is energized from Ba(*T*), through e_1, c_4, and Gr(*C*). This causes the contact t_1 to be closed, energizing the relay *S*, which in turn operates the meter register by one notch. At the same time the contact t_2 is also closed, establishing a circuit for the coil *T* through a_4, instead of e_1. Both the relay *S* and the register remain in the operated position until the calling plug is inserted. Then the relay *C* opens the ground contact c_4 which releases the relay *S* and the meter. This manner of control prevents the further operation of the meter should the calling party shake his hook before a connection is completed.

631. Operator's Set and Busy Test. — The operator's telephone set used with the cord circuit in Fig. 389 is shown in Fig. 390, to the left.

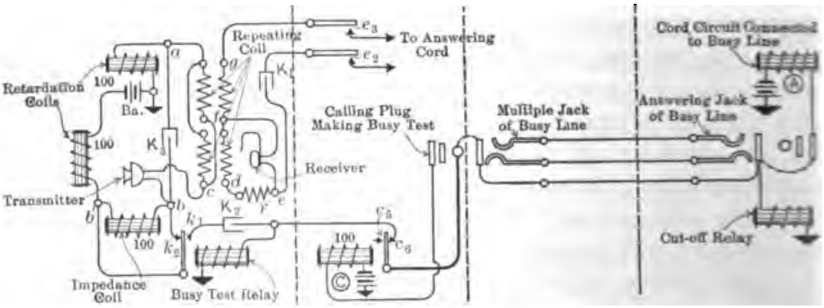


FIG. 390. Operator's set and busy test.

The transmitter circuit includes the primary of a repeating or induction coil, and is connected to the battery through two retardation coils (§§ 595 and 596). These coils prevent talking currents from passing through the common battery and thereby causing " cross-talk " with other circuits. An easy path for voice currents is provided through the shunted condenser K_3.

The secondary circuit is arranged in a form resembling a Wheatstone bridge (§ 16), the four branches being formed by the two windings of the repeating coil, a non-inductive resistance r, and the line between the contacts e_2 and e_3. The condenser K_1 and the resistance r are adjusted to give a balance for voice currents for the average line conditions. The receiver is connected in the position in which a galvanometer would be used in an ordinary bridge.

This arrangement of the secondary parts is known as the *anti-side-tone circuit,* and its purpose is to prevent the outgoing transmission, including the operator's own voice and local noises, from reaching her receiver and interfering with good listening. With a fairly well balanced Wheatstone bridge, the galvanometer current is nearly zero, so that any currents induced by the two equal branches of the primary winding of the repeating coil in the secondary winding go out to the line without passing through the operator's receiver. On the other hand, the incoming voice currents follow the easier path, and pass through the receiver rather than through the much higher impedance of the winding fd and resistance r. Thus, while the outgoing transmission is nearly lost for the receiver, the volume of the incoming transmission is practically the same as with a straight series connection.

Busy test. Before the operator inserts a calling plug, she touches the thimble (sleeve) of the corresponding jack with the tip of the plug (Fig. 390). If she hears a click in her head telephone she informs the calling subscriber that the line is busy. The reason for this click is as follows: When an answering jack has a plug in it, the battery potential is impressed on its sleeve and on the sleeves of all the multiple jacks through the relay A. This is the same relay A as is shown in Fig. 389. Therefore, when the tip of an operator's plug touches one of the jack sleeves, a current flows from Ba(A) to the ground through the contact c_6 and through the high-impedance winding of the busy-test relay in the operator's circuit. The relay opens the contact k_2, causing the transmitter current to flow through the impedance coil $b'b$. The sudden change in the transmitter current causes a secondary current to be induced in the receiver circuit, and the operator hears the above-mentioned click. When she removes the tip of the plug, the contact k_1 is closed again and the spark is taken up by the condenser K_2.

When a busy test is made by a B operator (§ 627) she reports the fact to the A operator by inserting the plug of the assigned trunk in a special "busy-back" jack to which a tone circuit is connected through an interrupter. The A operator's calling supervisory lamp is thus flashed at regular intervals, and the calling subscriber hears the interrupted tone. He usually hangs up before the *A* operator identifies the flashing signal and reports to him.

632. EXPERIMENT 29-C. — Operating a Common-Battery Multiple Switchboard. — The purpose of the experiment is to become familiar with the automatic features of a switchboard, such as those described in §§ 627 to 631. The switchboard, or at least part of it, should be previously wired, in order that the student may concentrate on its functions rather than on details of connections. The latter study is the subject of the next experiment. The order in which the operator's functions are performed, and the special cases which arise, are described in the text above, and these functions should be performed by the student, one by one. Abnormal conditions should also be studied, such as a short-circuit, an open circuit, or a ground on the line; the subscriber leaving his receiver off the hook; low battery voltage, abnormally high line-resistance, a relay failing to act, etc.

633. EXPERIMENT 29-D. — Wiring a Central-Energy Switchboard with Automatic Features. — This experiment supplements the preceding one; attention should now be paid primarily to the details of construction of the separate elements and to their electrical connections, rather than to the operation of the switchboard as a whole. Sufficient apparatus should be provided for one cord circuit, and all the relays should be mounted in an accessible manner and provided with clip terminals. First wire up the answering jack and enough of the cord circuit to satisfy yourself that the line lamp lights when the receiver is taken off the hook, and is extinguished when the answering plug is inserted. Then add the relay which connects the operator's circuit to the subscriber, then the busy test, the ringing circuit, etc., until all the operations have been checked. Make sketches of the parts and connections which are different from those described in the text.

CHAPTER XXX

TELEPHONE LINES

634. A LONG telephone line is an important factor in the efficiency of voice transmission, and the following experiments are intended as an introduction to the study of problems connected therewith. Comparatively little work on telephone lines can be done with the apparatus available in an elementary laboratory, and without going into advanced theory of electric and magnetic circuits. For location of faults, insulation measurements, and the properties of lines with distributed constants, see Vol. II.

The three features of telephone lines considered below are:

(1) Phantom circuits;
(2) Loading;
(3) Inductive interference.

635. Phantom Circuits. — In long-distance telephony it is possible to save a considerable number of parallel lines by combining them, as shown in Fig. 391. AA' and CC' represent two metallic lines, connected at each end to a switchboard in the usual manner, and known as *side circuits* or *physical circuits.* BB' is a *phantom circuit* also connected to a jack at each end, but not carried through between the switchboards. Instead, it is connected to the middle points of transformers or autotransformers bridged across the physical lines. Either transformers with two windings (repeating coils), or single-coil transformers (impedance coils) are used. The latter are less expensive, but repeating coils make it possible to raise the voltage for transmission and also insulate the switchboard electrically from the line. For purposes of illustration, coils of one type are shown at one end and of the other type at the other end.

With this arrangement, three independent conversations can be carried on, with only four wires instead of six. When the lines are properly balanced, there is no appreciable "cross-talk." Voice currents in line AA' do not tend to flow into BB' or CC', if points a and a', and also c and c', are kept at the same potential. Moreover, the A' currents do not flow through the impedance coil because of its high inductance. Wires *1′* and *2′ in parallel* constitute a continuation of the conductors *3* and *3′*, the direction of an instantaneous current in the phantom circuit being shown by the arrowheads. The currents in the two halves of a repeating or impedance coil flow in opposite directions, and the inductive drop is reduced to a small amount.

In order to prevent cross-talk, identical side-circuits must be used, and symmetrical conditions must be maintained with respect to freedom from the ground, mutual induction, etc. Special transposition (§ 639) is also necessary. The repeating or induction coils must be balanced, and the windings subdivided and grouped in such a way that the currents of the phantom circuit will induce no e.m.f.'s in the side circuits, and vice versa. The preferred type of coil is one with a toroidal core (Fig. 386) and with two inner and two outer windings.

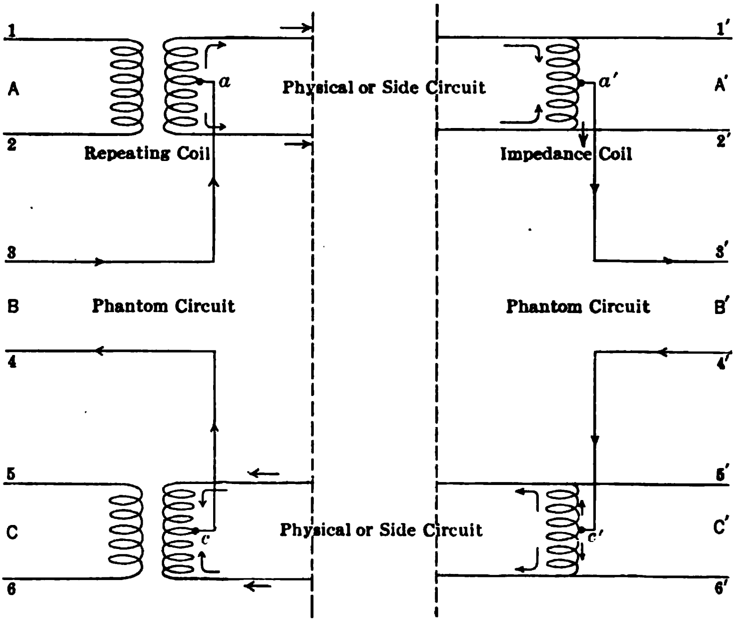


FIG. 391. A phantom circuit between two physical circuits.

A grounded phantom circuit, or half-phantom, can be obtained by connecting one side of the phantom, say *3*, *3'*, as in Fig. 391, and connecting the conductors *4*, *4'* to the ground. Such a circuit is liable to be noisy and subject to cross-talk, because of earth potentials and lack of transposition.

With unusually good insulation and with careful transposition of line sections, it is possible to superimpose a phantom upon two phantoms, obtaining a so-called hyper-phantom. Thus, with four full metallic circuits, seven conversations may be carried on simultaneously, four over the physical circuits, two over the phantom circuits, and one over the hyper-phantom.

636. EXPERIMENT 30-A. — Study of a Phantom Circuit. — Phantom circuits are explained in the preceding section; the purpose of the experiment is to study the electrical relations and the effect of unbalancing upon cross-talk.

(1) Connect three lines as in Fig. 391; use transformers or auto-transformers adapted for 60 cycles, 500 cycles, or some such frequency with which ordinary measuring instruments and considerable amounts of power may be used. Connect a source of A.C. power at A and an adjustable load at A'. Also connect another source at C and a load at C'. Connect a third source of power at B, and an adjustable load at B'. Provide the necessary measuring instruments at both ends and in the middle portion of the line (use a polyphase board, § 58). Show that power from each source is transmitted independently, and is not perceptibly affected by variations in current, voltage, or frequency in the other two circuits. Make clear to yourself the value of the superimposed current in the middle portion of the lines, especially when the two currents are of different frequency, or when one of them is a direct current.

(2) Unbalance the lines and show that the currents in the three circuits mutually affect each other, both by conduction and by induction. Disconnect the sources of power in two circuits and measure the amount of power received in each from the third.

(3) Replace the transformers by corresponding telephone coils and repeat the foregoing tests with telephonic frequencies, using transmitters and receivers. Use a different source of sound in each circuit, for example, a buzzer, a metronome, an organ pipe, a phonograph, a human voice, etc. Adjust for the best conditions of minimum interference. Then unbalance individual factors and determine those which have to be balanced with special care in order to prevent audible interference.

Report. (1) Give the data on superimposed alternating and direct currents, and check a few observed values by computation; draw a few vector diagrams, especially with unbalanced conditions. (2) Explain the factors upon which cross-talk chiefly depends. (3) Describe the experiment with telephonic currents and give the best conditions for a satisfactory phantom circuit.

637. Loaded Telephone Lines. — With a long telephone line, without amplifiers, only a small part of the energy of outgoing voice currents reaches the receiver. A large amount of energy is lost in the unavoidable leakage to the ground and in the ohmic resistance increased by the skin effect (§ 149). This gradual falling off of the transmitted energy is known as the attenuation of the line. The numerical value of attenuation is a function of the line constants, viz., its resistance, inductance, capacitance, and leakage, all per unit length of the line. The attenuation is also a function of the frequency of the line current and increases with the frequency. The human voice consists of simple tones of various pitch, and the electric current corresponding to each frequency is attenuated by a different amount, so that the received currents have a larger proportion of the deeper tones than those at the sending station. This loss of higher harmonics results in indistinct articulation, confusion of vowels, changed consonants, etc.

By using copper conductors of as large a cross-section as economic conditions will permit, and by providing for the best possible maintainance of the line, attenuation is reduced to a minimum. The remaining line constants (inductance and capacitance) must be of such a magnitude as to equalize the attenuation of currents of different frequencies within the audible range. Mathematical investigation shows that the latter condition is fulfilled when the ratio of the resistance r of the conductor to the insulation conductance, g, is equal to that between the inductance L and capacitance C, that is

$$r/g = L/C \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In practical units, r is in ohms, g in mhos, L in henries, and C in farads, all four per unit length of the line. Such a line is known as a *distortionless circuit.*

In actual aerial lines and in cables, the capacitance is higher than that according to the above rule, and with long lines it becomes necessary to increase the inductance in proportion. This artificial increasing of the line inductance is called *loading the line.* A theoretically ideal solution consists in introducing a distributed inductance, or "continuous loading," for example, by wrapping iron wire around each copper conductor (Krarup cable). Another alternative is to insert concentrated inductances at such short intervals that their effect is nearly the same as that of distributed inductance. Such inductance coils are known as loading coils or Pupin coils, and the method itself is called discontinuous, or "lumpy," loading. There should be more than three coils per wave length of the highest important frequency, in order to approximate the effect of continuous loading. Otherwise, wave reflections take place at each coil, with a corresponding loss in transmitted energy.

The usual form of loading coil is one with toroidal core (Fig. 386);

both the outgoing and the incoming conductors being wound on the same core. The energy loss in the core, and the effective resistance of the winding, must be kept down to a minimum. Loaded lines must be maintained at a uniform and high degree of insulation in order to preserve the theoretical ratio of the constants as nearly as possible.

Propagation of currents in telephone lines can be conveniently studied in the laboratory by using a so-called *artificial line.* Such a line consists of numerous sections in series, and each section has a series inductance coil, a shunted condenser, and a proper amount of resistance and leakage. By using a sufficiently large number of sections, the effect of concentrated capacitance becomes nearly the same as if it were uniformly distributed, and the influence of " lumpiness " is reduced to the desired minimum. Loading coils can be introduced into such an artificial line and their effect studied directly by measuring the transmitted and received currents, or by talking and listening (§ 597). See also the chapter on transmission lines in Vol. II.

638. EXPERIMENT 30-B. — Influence of Telephone Line Constants Upon Quality of Speech Transmission. — The theory is explained in the preceding section. The quality of transmission can be measured by one of the methods described in § 597. For a complete study a suitable artificial line, with adjustable constants, is necessary, but a general idea may be gotten from a few simple experiments. Such experiments may be made with low-frequency currents, say 25 or 60 cycles (or better 500 cycles) using ordinary A.C. ammeters, voltmeters, and wattmeters for measuring the input and the output. The results may be sufficient to illustrate the general trend of the influence of various factors. Some of the following suggestions presuppose that a fairly well subdivided artificial line is available.

(1) Investigate the influence of line resistance alone, on the volume and articulation of transmitted speech. Increase the resistance in steps until the speech becomes unintelligible.

(2) Connect non-inductive shunts across the line in one or more places, to imitate leakage, and vary both the amount of leakage and the line resistance. Find out whether the effect of the same leakage conductance is more noticeable when it is nearer the transmitter or the receiver end of the line.

(3) Take a line of low resistance and high inductance, with and without iron, and determine the loss in transmission with high-pitch and low-pitch sounds, i.e., with high- and low-frequency currents.

(4) Connect several small condensers between the line wires at differ-

ent places, and determine the combined effect of resistance and capacitance.

(5) Connect inductance coils in series with the line, between the condensers, and note the improvement in transmission. Adjust for the best results, with different amounts of leakage.

(6) Investigate the effect of "lumpiness" in the line, that is, of concentrated capacitance as against more uniformly distributed capacitance.

Report. Give separately the observed effect of each factor. Explain the obtained facts theoretically, at least qualitatively. Show how nearly the measured relationships check with the above-given rule for distortionless transmission.

639. Inductive Interference and Transpositions.—When a current is flowing through a transmission or telephone line, a magnetic flux is produced in the surrounding medium, the magnitude of the flux density at any point being proportional to the instantaneous current. The flux distribution depends upon the spacing of the conductors. If there is another line circuit in the vicinity, and the current in the first circuit varies with the time, then the changing magnetic flux induces in the second circuit an e.m.f. which may be a source of disturbing or dangerous currents. This phenomenon is known as *inductive interference.* Telephone lines are subject to troublesome interference on the part of paralleling power circuits, and there is also some mutual interference among adjacent individual telephone circuits, such as those supported by the same poles (cross-talk). See under "Mutual Inductance" in Chap. VI.

The usual remedy consists in *transposing* both the power and the telephone conductors at regular intervals. In this manner, equal lengths of a given telephone circuit can be subjected to equal and opposite fluxes, and the total induced e.m.f. made equal to zero. With several metallic and phantom circuits (§ 635) on the same pole, the actual scheme of transposition becomes quite complicated. For diagrams of the usual cases the reader is referred to electrical handbooks and to special books and articles on the subject.

An experimental study of transposition can be made in the laboratory by using miniature lines and comparatively strong currents, in order to obtain measurable induced e.m.f.'s. Elongated coils of several turns may be employed in place of single line loops, so as to magnify the effect. At the ends of each transposition section, terminal boards should be arranged for connecting the adjacent sections in any desired order.

Before making such an experimental investigation the student should become familiar with the general shape of the magnetic field around a single loop of parallel wires. A theory and a sketch showing lines of force will be found in many works on electricity and in treatises on transmission lines. Keeping this flux distribution in mind, the best relative position of the loops for a minimum of interference can be foreseen theoretically.

A 25-cycle, or even a 60-cycle power current is of too low a frequency to produce a disturbing tone in a telephone line, unless it is quite loud. However, a power current is usually accompanied by higher harmonics, and these harmonics produce in telephone receivers disturbing noises which make it difficult or impossible to understand transmitted speech.

Besides the above-described magnetic interference, a high-voltage power line may cause dangerous electrostatic potentials in adjacent telephone lines. Such potentials cannot be removed by transposition, but can be reduced by connecting so-called *drainage coils* at intervals across each telephone line. A drainage coil is an impedance coil with the middle point of its winding grounded. No appreciable voice currents can flow through it, but the average potential of the line is reduced to a safe value, not very different from that of the grounded connection. The high potential due to the power line is nearly the same for both telephone wires, and the discharge currents flow to the ground through a drainage coil in the opposite directions, somewhat as shown at a' in Fig. 391.

640. EXPERIMENT 30-C. — Transposition of Telephone and Power Lines. — The purpose of the experiment is to measure induced e.m.f.'s due to mutual induction of parallel lines, and to study various transpositions by means of which interference is reduced to a minimum. A possible experimental arrangement is suggested in the preceding section. Frequencies of a few hundred cycles per second will be found most convenient, and primary currents should be large enough to induce voltages which can be measured without any special refined means.

(1) Excite a loop to represent a single-phase power line and measure the e.m.f. induced in a parallel loop, with different distances between the loops and with a different angle between their planes. Determine the positions of maximum and minimum interference.

(2) Transpose the conductors in the two halves of the primary loop and observe the result. Do the same with the secondary loop, without transposing the primary.

(3) Take a three-phase power line with a symmetrical spacing, and measure the e.m.f.'s induced by it in a single loop. Find the positions of maximum and minimum interference. Divide the total length of the power line into six equal sections, and by transposing the conductors

in adjacent sections "screw fashion" reduce the induced e.m.f. to a minimum. Try other methods of transposition and find their effect upon the secondary induced e.m.f.

(4) Represent experimentally the conditions corresponding to a single-phase railway with ground return, and paralleling a telephone line, (a) grounded and (b) full metallic. Measure the relative amount of mutual induction in the two cases and show the advantage of the full metallic circuit.

(5) Arrange a few full metallic circuits corresponding to telephone lines on the same cross-arm or on two adjacent cross-arms of a pole. Try different transpositions and find the most favorable arrangements for minimum cross-talk.

(6) Extend the preceding case to include one or more phantom circuits.

Report. Give the results obtained, and explain them theoretically, at least qualitatively. State a few definite rules for transposition, covering the standard cases met with in practice.

641. Advanced and Special Work in Telephony. — Chapters XXVII to XXX cover some of the elements of telephone engineering. A chapter on high-frequency measurements, with some reference to radio work, will be found in Vol. II. A student interested in more advanced phases of communication engineering will find information in numerous books and articles on the subject. Among the large and important telephone topics not touched upon in the text are the following:

Semi-automatic or auto-manual system of making connections between subscribers.

Automatic or machine switching.

The use of vacuum tubes or audion repeaters and amplifiers.

Simultaneous telegraphy and telephony.

The use of carrier frequencies and tuned circuits for simultaneous transmission of several messages over the same long-distance line.

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